INSTABILITY ANALYSIS OF A VEHICLE NEGOTIATING A CURVE WITH DOWNGRADE SUPERELEVATION

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This study was initiated as a result of numerous skidding accidents occurring at locations where highway geometrics include a combination of downgrade, curve, and superelevation. Mathematical equations are developed for obtaining all the wheel forces (both normal and lateral) of a vehicle negotiating such a curve at the instant of incipient skidding for a variety of parameters. Factors that appear to be most important in regard to critical skidding velocities are the lateral coefficient of friction between the tire and the road surface and driver maneuvering. Factors that appear to have little influence are superelevation (if relatively small), crosswind velocity, and type of vehicle (excluding tractor-trailers).

•ON INTERSTATE 95, at the interchange with US-1, numerous skidding accidents have occurred in recent years. As the tested coefficient of friction between treaded tires and the road surface at this location is well above 0.4 (even when wet), other reasons for the many skids were investigated. At this site there is a 1-deg horizontal curve, a downhill grade of 2.6 percent, and a transverse superelevation of 0.0156 ft/ft. The combination of these geometric conditions has never been investigated for its effects on skidding; therefore, this mathematical study was initiated to obtain a quantitative means of predicting critical skid velocities for a variety of parameters. (As a result of observations at this site, other sites with downgrade superelevation have been inspected and also found to be associated with numerous skidding accidents. This finding suggests that skidding on a downgrade superelevation is not an isolated problem but perhaps a general one requiring special attention.)

The various parameters studied include highway grade, superelevation, vehicle weight, vehicle geometry, crosswind velocity, road surface condition (coefficient of friction), and driver correction maneuvers (nontracking of the intended roadway path).

ANALYSIS

Figures 1 and 2 show the free body diagrams of the vehicle projected in 2 vertical planes. Included are the centrifugal D'Alembert forces caused by the vehicle negotiating a downhill curve to the right. It is assumed that the center of mass will not shift its relative position appreciably because of the dynamic forces acting on the elastically sprung body mass.

Also, for simplicity, the lateral wind force L is assumed to act through the center of mass, which is justified on the basis that, at the high vehicle speeds at which skidding takes place, the wind force is but a very small fraction of the total lateral force.

Other notations are defined as follows:

- W = gross weight of vehicle;
- g = acceleration due to gravity (32.2 ft/sec^2);
- R = radius of curvature of vehicle path (this value may or may not coincide with radius of road, for a driver maneuver within the width of the roadway could control the value of R used);

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Figure 1. Forces and dimensions in vertical plane (rear view).

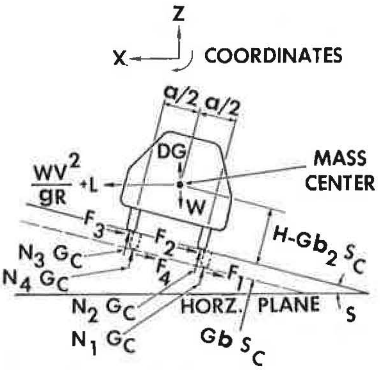

Figure 2. Forces and dimensions in vertical plane (side view).

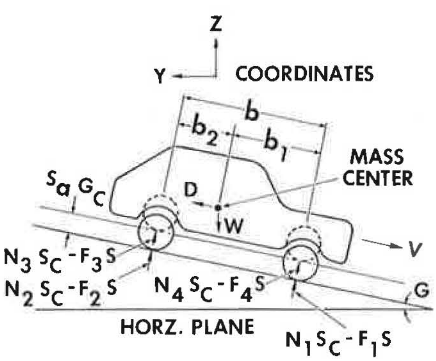

Table 1. Critical speeds for sample cases.

Case	Maneuver	R (ft)	Cross-wind (mph)	Condition Change	Critical V (mph)
1	Tracking curve		0		164.6
2	Tracking curve		40[a]		151.0
3	Tracking curve		40	Icy pavement, m = 0.1	74.9
4	Corrective maneuver	1,000	0		68.7
5	Corrective maneuver	1,000	40		63.0
6	Corrective maneuver	1,000	40	Superelevation of 0	61.2
7	Corrective maneuver	1,000	40	Superelevation of 0.0312 on 1	64.9
8	Corrective maneuver	1,000	40	m = 0.4	74.0
9	Corrective maneuver	1,000	40	m = 0.5	83.5
10	Corrective maneuver	1,000	40		61.4
11	Corrective maneuver	1,000	40		63.7

[a]Blowing from inside of curve to outside.

L = lateral force due to wind on the vehicle (approximately equal to the product of 0.00254, lb, the projected side area of the vehicle, ft^2, and the crosswind velocity, mph^2);
D = aerodynamic drag of vehicle (assumed to act through center of mass);
H = height of center of mass of vehicle above a level road surface;
N_1, N_2, N_3, N_4 = normal forces on wheels;
F_1, F_2, F_3, F_4 = lateral forces on wheels;
G = grade of road, radian (small angles assumed);
S = superelevation of road, radian (small angles assumed);
G_c = cosine G;
S_c = cosine S;
V = velocity of vehicle;
m = coefficient of side slip friction on right front wheel; and
m_2, m_3, m_4 = coefficient of side slip friction on other wheels.

The following dynamic equations can be written (assuming steady-state dynamic conditions). The horizontal equation in the X-direction is

$$(WV^2/gR) + L - SG_c(N_1 + N_2 + N_3 + N_4) - S_c(F_1 + F_2 + F_3 + F_4) = 0 \tag{1}$$

The vertical equation in the Z-direction is

$$- W - S(F_1 + F_2 + F_3 + F_4) + S_cG_c(N_1 + N_2 + N_3 + N_4) + DG = 0 \tag{2}$$

The DG term is equal to WG^2 (from the constant V relation), which is very small and is neglected. The roll equation about the Y-axis is

$$- (F_1 + F_4)(H + Gb_1G_c) - (F_2 + F_3)(H - Gb_2G_c) + (aG_c/2)(N_3 + N_4 - N_1 - N_2) = 0 \tag{3}$$

The pitching equations are (for 0 acceleration)

$$N_2 = (b_1/b_2)N_1 \tag{4}$$

$$N_3 = (b_1/b_2)N_4 \tag{5}$$

The yawing equations are (for 0 acceleration)

$$(b_1/b_2)F_1 = F_2 \tag{6}$$

$$(b_1/b_2)F_4 = F_3 \tag{7}$$

To solve for the 8 unknown wheel forces requires an additional relation provided by the slip equation

$$F_1 = mN_1 \tag{8}$$

As determined by Saibel and Tsao (1) and based on the sprung vehicle system, the first wheel to skid on cornering is the one on the right front.

The 8 equations listed can be solved simultaneously to obtain the tire forces as follows:

$$F_4 = (JWU + EAU - PTW)/(PM - QU) \tag{9}$$

$$N_4 = (TW + F_4M)/U \tag{10}$$

$$N_1 = (N_4SBG_c + F_4BS_c - A)/J \tag{11}$$

$$F_1 = mN_1 \tag{8}$$

$$F_2 = (mb_1/b_2)N_1 \tag{12}$$

$$F_3 = (b_1/b_2)F_4 \tag{7}$$

$$N_2 = (b_1/b_2)N_1 \tag{4}$$

$$N_3 = (b_1/b_2)N_4 \tag{5}$$

where

$B = (b_1 + b_2)/b_2$;
$J = B(-mS_c - SS_c)$;
$E = B(-mS + S_cG_c)$;
$A = (WV^2/gR) + L$;
$P = BG_c(ES + JS_c)$;
$T = -2Hm - (2b_1Hm/b_2) - aBG_c$;
$U = BG_c[TS_c - a(SBm + BS_cG_c)]$;
$Q = SB(BmS_c - SBG_c) - EBS_c$; and
$M = TSB - E[2H + 2Gb_1S_c + (2b_1H/b_2) - 2b_1GS_c]$.

These equations, arranged as they are in sequences, can easily be solved on a digital computer for any set of conditions.

To ascertain the critical skid speed, one merely programs the computer to start with some lower limit velocity (such as 40 mph) and to calculate the value of

$$m_4 = F_4/N_4 \tag{13}$$

for successively higher values of V (at small increments) until the value of m_4 equals or just barely exceeds the assigned value m. m_4 is the coefficient of friction of the left front wheel, the second wheel to initiate skidding, as determined by Saibel and Tsao (1). When $m_4 = m$, the entire vehicle may be assumed to be unstable, for the 2 rolling rear wheels offer no resistance against yawing. This criterion thus establishes the critical skid velocity.

SAMPLE SOLUTIONS

Although the developed equations are complete in themselves, 11 sample solutions or cases are presented to provide some physical interpretation for various typical conditions. The geometric conditions existing at I-95 and US-1 will be taken as standard, namely, a downhill grade of 2.6 percent, a superelevation of 0.0156 on 1, and a 1-deg curve to the right (R = 5,730 ft). For the basis of comparison, m will be taken as 0.3, except where noted; g is 32.2 ft/sec^2. Table 1 gives the maneuver being effected by the 3 types of vehicles and the critical speeds in relation to crosswind and vehicle path radius.

Cases 1 through 9 are for a standard American-made passenger car with the following properties: W = 4,000 lb (with passengers); b_1 = 4.5 ft; b_2 = 5.5 ft; H = 2.0 ft; a = 5.25 ft; and a projected side area = 50 ft^2.

Case 10 is for a small foreign car with the following properties: W = 2,200 lb (with passengers); b_1 = 4.5 ft; b_2 = 3.5 ft; H = 1.6 ft; a = 3.7 ft; and a projected side area = 35 ft^2.

Case 11 is for a large bus with the following properties: W = 34,400 lb (with passengers); b_1 = 14 ft; b_2 = 8 ft; H = 5 ft; a = 8 ft; and a projected side area = 390 ft^2.

CONCLUSIONS

Although the 11 cases given in Table 1 are but samples, some conclusions can be drawn concerning those factors that appear to be significant and those that do not.

Among the most significant factors is driver control. In case 1 a vehicle correctly tracking the curve skids at 164.6 mph (far above normal highway driving speeds and perhaps even beyond the possible speed of the vehicle itself), whereas in case 4 a slight corrective movement or swerve in the direction of the curve decreases this skid speed to 68.7 mph (close to the posted speed of 65 mph).

Another important factor, which is under the direct control of highway engineers, is the coefficient of friction. A comparison of cases 5, 8, and 9 shows that by increasing this coefficient critical speeds can be increased from 63.0 mph for $m = 0.3$ to 74.0 mph for $m = 0.4$ to 83.5 mph for $m = 0.5$, the last being well beyond the range of the posted speed limit. (Case 5 is a fairly severe condition but not an improbable one. Skidding in this case occurs below the posted limit of 65 mph.) An icy road, for which the coefficient is of the order of 0.1, is of course a serious condition as can be seen by comparing cases 2 and 3. Ice on this roadway approximately halves the critical speed.

Parameters that appear to have relatively little influence are crosswind, degree of superelevation, and type of vehicle. The effect of crosswind is shown by comparing cases 1 and 2 or cases 4 and 5. In each set of cases, a fairly brisk crosswind of 40 mph decreases the critical speed by only about 7 percent. The influence of superelevation is illustrated by cases 5, 6, and 7. At the existing superelevation, the critical velocity is 63.0 mph; at 0 superelevation, the critical speed is 61.2 mph; and at double the existing superelevation, the critical speed is increased to only 64.9 mph. (All of these superelevations are relatively small.) For the 3 entirely different vehicles, cases 5, 10, and 11 show that the critical speeds are all practically the same (63.0, 61.4, and 63.7 mph respectively). Because of limitations in the formulation of the given theory, these conclusions apply only to 4-wheel, solid-body vehicles and not to tractor-trailer vehicles that possess additional wheels and additional degrees of freedom because of the pin-coupling between the tractor and trailer.

As all theories should be corroborated with tests, it is recommended that these conclusions be further investigated by physical tests with either small-scale models or full-scale vehicles. Needed in particular are data on side slip friction and driver steering wheel reaction in rounding a curve. Additional theoretical studies could also be made of the dynamic behavior of coupled vehicles, such as tractor-trailer or car-camper units, as well as of various transient effects, such as braking, transition curves, road bumps, and wind gusts. Refinements to the "rigid body" theory presented could also be made by including the dynamic effects of the springing of the vehicle itself.

Nonetheless, pending such further studies, it is believed that the results of this current study are sufficiently valid to suggest that all high-speed, curved highways with downgrade superelevation possess high friction capabilities under all weather conditions. The specific amount needed can be determined for any given site condition by the solution of Eqs. 8 through 13 along with Eqs. 4, 5, and 7 as described in this paper.

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