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BALANCED DESIGN AND FINITE-ELEMENT ANALYSIS OF CULVERTS

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This paper outlines a method for treating culverts in embankments. Design is accomplished with approximate relations based on empirical determination of arching and maximum induced moment. Relations are provided that permit determining the factor of safety against failure and collapse in the various possible modes. This permits the adjustment of designs to achieve a desired balance in these modes against failure and collapse. Discussion is provided regarding finite-element analysis of resulting designs. A three-dimensional linear finite-element solution is provided for the design example. It is shown that linear solutions predict deflections reasonably well but that they overestimate thrusts and moments. The area of utility of the approach as compared to other methods is discussed.

• A BODY of knowledge germane to the culvert problem has resulted from efforts to develop design and analysis methods for buried shelters in order to resist the effects of nuclear weapons. This paper adapts that information to the design of culverts. The principal goal is to provide a method of solution for culvert problems excluding the class of small culverts where handling and durability govern the design.

A classification of conduits, including a distinction of those governed by handling and durability, may be found elsewhere (1). NCHRP Report 116 (1) reviews the older design methods and delineates their limitations and deficiencies. Also, it proposes two new methods: one based on foundation settlement considerations and the other based on elastic theory relations (2, 3).

An approximate approach that circumvents some of the limitations of the elastic theory has been formulated elsewhere (4). The use of this method is limited to fully buried cylinders under high loads. It utilizes the elastic theory and an empirical arching relation.

Among the contributions from weapons effects work is the development of relations for modeling earth materials (5, 6). Some of these relations have been incorporated in nonlinear finite-element programs that permit far better analysis of soil-structure systems than has been possible in the past (7, 8, 9).

The approach recommended here is to use the approximate method to obtain approximate designs and then to use the finite-element method to analyze the resulting designs. The importance of the soil and of the relative stiffness in controlling system behavior is emphasized. It is control of these independent parameters that permits efficient designs.

The following notation is used in this paper:

A_o = maximum active arching,
A_s = area of section at springing,
b = width of base of embankment,
c = distance from midplane to extreme fiber of plate,
$\bar{C}$ = constant that depends on the mode of buckling,
$\tilde{C}$ = constant that depends on the Poisson's ratio of steel,

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$C_{xx}, C_{xy}, \ldots C_{ys}$ = material property constants,
C_1, C_2, C_3, C_4, C_5 = constants to account for different bedding conditions and flow characteristics of the soil,
D = cylinder mean diameter,
d_e = depth to the plane of equal settlement,
E = Young's modulus of elasticity,
E' = modulus of soil reaction,
$f(\ldots)$ = function of several variables,
G_{xy} = shear modulus,
h = height of embankment above the invert,
I = moment of inertia,
N = thrust at the spring line,
P = surface pressure,
$p_{1_{cr}}$ = transitional buckling stress,
t_e = equivalent thickness of culvert,
Δx = horizontal diametral extension,
Δy = vertical diametral extension,
ϵ_c = unit vertical strain of culvert across diameter,
ϵ_s = unit vertical strain in the soil,
ϕ = angle of friction,
γ = unit weight of soil,
σ_{allow} = allowable stress,
σ_y = yield stress, and
Ω = arching coefficient.

BASIC CONSIDERATIONS

At the outset, it is worthwhile to establish a few basic definitions and to review the dominant knowns and unknowns of the culvert problem. For present purposes, failure will connote the occurrence of visible distress as indicated by wall crushing or excessive cracking; buckling; plastic deformation (other than local); separation or rupture of seams and joints; or excessive deflection that impairs the functional performance or psychological acceptance of the installation. This definition requires the establishment of suitable design criteria (1, 4). Designs should be evaluated in terms of their failure and collapse loads.

Dimensional analysis and experiments on buried cylinders in a uniform granular soil field (10) show that to a first approximation

$$\frac{y}{D} = f\left(\frac{p_i}{M_s}, \frac{L}{D}, \frac{M_s}{EI/D^3}, \frac{d_o}{D}\right) \tag{1}$$

in which y = radial deflection of the crown, D = cylinder mean diameter, p_i = interface load at the crown, M_s = effective secant confined compression modulus at a stress equal to the applied load, L = cylinder length, EI = cylinder wall stiffness, and d_o = depth of cover over the crown.

For culverts, the effect of L/D is usually negligible, and the behavior can be expressed by the remaining four nondimensional pi-terms. Equation 1 may also be deduced from the elasticity theory on neglecting the influence of Poisson's ratio.

From past analyses, tests, and experience, we know the following:

1. EI/D^3 strongly influences the magnitude and distribution of the load on the culvert;
2. Soil stiffness, M_s, and variations of the soil field in the vicinity of the culvert (bedding and backfill) strongly influence behavior;
3. Time effects are usually small except for clayey soils (with clayey soils, time effects result in a shift in load to the pipe);
4. The possibility of shear failure in the soil in most circumstances is slight until the onset of a culvert failure;
5. Live loads due to traffic are significant only for depths of cover of less than about 5 ft;

6. Extensional flexibility does not affect the behavior or design of steel, aluminum, or reinforced concrete culverts; and

7. Use of the constrained (one-dimensional compression) modulus is advantageous because it is relatable to dry density, vane shear strength, and certain other useful indexes of field soil conditions and because it is a standard, widely used laboratory test.

Principal uncertainties or deficiencies in knowledge of culverts are attributable to insufficient data on the influence of variations in the adjacent field around the pipe—particularly the manner in which behavior is influenced by bedding, backfilling, and backpacking; deficiency of measurements on actual pressure distribution, especially in regard to interface shear; and inadequate criteria and definitions relating to failure and factor of safety. Also, analytic methods available in the past were inadequate for designing culverts under high fills or conduits with shallow overburden. Within limits, the elastic theory is suitable as a basis for design.

A principal weakness of the elastic theory is that it does not account for arching as it occurs in soils. According to the elastic theory, the minimum thrust possible at the spring line in any culvert is equal to pr; but it is known from numerous experiments that the thrust is usually less than pr (where $r = D/2$) (10, 11, 12, 13). A comparison of thrust calculated from an empirical arching equation (4) and from the elastic theory shows a 20+ percent divergence. From these results, it is clear that thrust cannot be determined directly by the elastic theory. Moments from the elastic theory are also in error, especially at high loads.

Elastic theory is adequate for determining deflection if (a) the boundary conditions correspond to those in the theory, (b) the soil field surrounding the culvert is uniform, (c) M_s is taken as the secant modulus to the stress-strain diagram at a stress corresponding to the applied load, (d) the shear stresses do not exceed the shear strength of the soil, and (e) time effects are negligible. Conditions (a) and (b) often are not met in practice; consequently, adjustments to the applied load and the effective modulus may be necessary. Space does not permit an elaboration of how such adjustments are made; however, if one recognizes that behavior is governed primarily by the relative strain in the soil with respect to the average strain over the height of the inclusion, the method of adjustment is fairly obvious.

Other considerations that aid in converging on a sensible design approach are as follows:

1. Deflection and wall-crushing are the most likely failure modes where existing seams are properly designed.

2. Moments are more variable than deflections and thrusts.

3. Deflections are the most predictable quantity and are the easiest to measure in a completed installation. However, deflection is not the best criterion for the design of a flexible culvert because, for such culverts, deflection has a weak dependence on culvert stiffness. By elimination, then, wall strength and soil stiffness are the appropriate principal bases for culvert designs where handling and durability do not govern.

Ideally, one would like to achieve an equal probability of survival in all possible failure modes. This means that, for a "balanced design," a higher factor of safety is required against buckling than against wall-crushing and that a higher factor of safety is required against wall-crushing than against deflection failure. Also, one might logically have separate factors of safety against failure and against collapse.

A final item of background is desirable regarding the use of the elastic theory versus the Iowa formula (1, 14). Actually, the Iowa formula conforms closely to the elastic theory as may be seen by writing the deflection equations in a similar form (with $\Delta x \cong \Delta y$) as follows:

Elastic theory (full slip):

$$(\Delta y/D)/(p/M_s) = \left(C_1\{[M_s/(EI/D^3)] + 1\}\right)\Big/\left(C_2\{[M_s/(EI/D^3)] + 9\}\right) \tag{2}$$

Iowa formula:

$$(\Delta y/D)/(p/M_s) = \{C_3[M_s/(EI/D^3)]\}/(C_4\{[M_s/(EI/D^3)] + C_5\}) \tag{3}$$

These two equations differ appreciably only for $M_s/(EI/D^3) \gtrsim 10$. For $M_s/(EI/D^3) > 10^3$, the two equations give essentially identical results.

In its usual form, the Iowa formula contains a so-called modulus of soil reaction, E′, and coefficients to account for different bedding conditions and flow characteristics of the soil. By letting $M_s/(EI/D^3)$ go to infinity in Eqs. 2 and 3, it may be shown that $M_s \approx 1.1$ to 1.5E′. For a value of the bedding coefficient of 0.1, $M_s = 1.22E'$. Use of the bedding coefficient is an indirect means of accounting for different arching conditions. It is desirable to determine arching directly and to use M_s instead of E′. These features and the foregoing requirements and reasoning are reflected in the design procedure that follows.

Because the main deficiency in our knowledge concerns the load-carrying capability of the soil, a few words of clarification are in order.

SOIL PARAMETERS

The term "constitutive properties" (or laws) is used in continuum mechanics to mean the set of equations that define the stress-strain properties of the media of concern. Rather refined constitutive relations have been developed to represent earth materials (5, 6), and some of these have been incorporated in the computer programs that will be discussed later.

Fortunately, the required constitutive relations for static loading are relatively simple because unloading is not usually involved. For approximate calculations, the following are required: an average value of Poisson's ratio, ν_s; sometimes the cohesion, c; the coefficient of lateral earth pressure, K_o; the angle of friction, ϕ; and the effective secant modulus at a stress equal to the applied load, M_s. One should keep in mind that, for embankments with large height-to-width ratios, the effective value of K_o may be less than in fully buried installations. Of the preceding properties, the secant modulus is of dominant importance.

Soil elements at different locations throughout an embankment are subject to different confining stresses; thus it would be expected that the effective modulus would vary throughout the cross section. For a properly designed and compacted embankment, the effective modulus in the bottom central region should be nearly equal to the confined compression modulus. The effective modulus at a given depth should correspond to the overburden stress at that depth, except near the sides. Near the sides the effective soil modulus decreases, but so does the load; consequently, the conditions at the midbottom region may be presumed to control the design. (Stress conditions near the ends are discussed later in the paper.) These comments are predicated on the assumption that the width of the embankment is large ($\gtrsim$ 10D) compared to the diameter of the pipe. If this is not the case, the effective soil modulus will have to be appropriately reduced.

Designers of culverts should appreciate the wide range of values that M_s may have depending principally on the soil type, the placement methodology, and the applied stress. M_s may vary from essentially zero for saturated clays to several hundred thousand for granular (locking) materials under high stresses. One should also be aware of the large variability in M_s attributable to placement. As an example, in laboratory tests (11) where care was taken to replicate placement of dry sand in a test bin by using the sand-fall method, variation of M_s from the mean value was ±20 percent; certainly a greater variation must be expected in field installations.

Because of the wide variability in soil properties, the use of an unduly refined design procedure does not seem appropriate. The need is for a design method based on the proper criteria that contain the principal parameters in correct relation to one another.

DESIGN PROCEDURE

A proposed design procedure for culverts in embankments is given by means of an example. An example employed by others (1) is used so that comparisons can be made.

We are required to select a section for a 60-in. diameter steel culvert under a 20-ft embankment of soil weighing 120 pcf. The basic steps in the design are as follows:

1. Determine the vertical stress at midheight, p_s, and the vertical stress at the elevation of the crown, p_v.

Dead load: $p_s = [h - (D/2)]\gamma = [20 - (5/2)](120/144) = 14.6$ psi

$p_v = (h - D)\gamma = (20 - 5)(120/144) = 12.5$ psi

Live load: 0

2. Determine M_s corresponding to the stress at midheight of the culvert from confined compression test results. For carefully placed granular fills compacted to at least 85 percent AASHO T-99, one may use $M_s = 1{,}000\ p_s^{0.8}$ if no test results are available (15). In the present example, use $M_s = 1.22\ E' = 1.22 \times 700 = 854$ psi to conform to the comparison design (1).

3. Estimate the arching A and calculate the thrust from

$$N = p_v(1 - A)(D/2) \tag{4}$$

The relations $A = 0.2 - 0.2[1 - (d_o/D)]^2$ for $d_o/D \leq 1.0$ and $A = 0.2$ for $d_o/D > 1.0$ may be used if better bases for the estimate do not exist (16). Bedding angle, projection ratio, and certain other factors will influence A:

$$N = 12.5\ (1 - 0.2)\ (60/2) = 300\ \text{lb/in.}$$

4. Calculate the stiffness required by the handling criterion $D^2/EI \leq 0.0433$:

$$EI = 60^2/0.0433 = 83{,}200\ \text{lb-in.}^2/\text{in.}$$

Thus, $M_s/(EI/D^3) = (854 \times 60^3)/83{,}200 = 2{,}220$.

5. Determine the moment using Figure 1 (experimental curve) with

$$p_1 = p_v\ (1 - A) = 12.5\ (1 - 0.2) = 10\ \text{psi}$$

$$M = 0.005\ p_1\ D^2 = (0.005 \times 10.0)60^2 = 180\ \text{in.-lb/in.}$$

6. Determine the equivalent flat plate thickness required to resist the thrust and moment based on a factor of safety of 2 against yielding of the total section. Use the approximate relation

$$\sigma_{allow} = \sigma_y/FS = (N/A_s) \pm (Mc/I)$$

where $A_s = t_e \times 1$ and $I = t_e^3/12$. Substituting, $33{,}000/2 = 300/t_e + (180 \times 6)/t_e^2$ then

$$t_e = 0.265\ \text{in.}$$

7. The corresponding stiffness is

$$EI = [(29 \times 10^6)\,0.265^3]/12 = 44{,}950\ \text{lb-in.}^2/\text{in.}$$

which is less than 83,200 lb-in.²/in.; therefore, handling governs. Use a 12-gauge plate with $2\frac{2}{3}$- by $\frac{1}{2}$-in. corrugation (17). Spangler chose the same corrugation but a 10-gauge plate (1). Calculations are given as follows for both gauges:

$$\left.\frac{EI}{D^3}\right|_{12} = \frac{(29 \times 10^6)0.00343}{60^3} = 0.461 \quad ; \quad \left.\frac{EI}{D^3}\right|_{10} = 0.604$$

8. Determine $M_s/(EI/D^3)$ and find the corresponding value of $(\Delta y/D)/(p_v/M_s) \cong \epsilon_c/\tilde{C}\,\epsilon_s$ using Figure 2 with $\tilde{C} = (1 + \nu_s)(1 - 2\nu_s)/(1 - \nu_s)$:

$$\left.\frac{M_s}{EI/D^3}\right|_{12} = \frac{854}{0.461} = 1{,}854 \; ; \quad \left.\frac{M_s}{EI/D^3}\right|_{10} = 1{,}413$$

$$\left.\frac{\Delta y/D}{p_v/M_s}\right|_{12} \cong 2.46 \quad ; \quad \left.\frac{\Delta y/D}{p_v/M_s}\right|_{10} = 2.44$$

9. Calculate the depth to the plane of equal settlement (4) or use $d_e = D$ for $h \geq 2D$; then, determine the arching coefficient

$$\Omega = (2d_e/D)\,[(\epsilon_c/\epsilon_s) - 1]$$

and find the arching using Figure 3

$$\Omega_{12} = (2 \times 60)/60\ (1.83 - 1) = 1.66$$

$$A = 0.2 = A_{assumed}$$

therefore OK. When the predicted and calculated arching values do not agree, iterate as necessary to bring the two values into agreement.

10. Determine the conformance with design criteria and the factor of safety for the various possible modes of collapse. For deflection, from step 8,

$$\Delta x_{12} \cong \Delta y_{12} = 2.46(p_a/M_s)D = 2.46(14.6/854)60 = 2.52 \text{ in.}$$

$$\Delta y_{10} = 2.38 \text{ in.}$$

$$\Delta x/D = (2.52/60)100 = 4.2 \text{ percent} < 5 \text{ percent}$$

therefore OK.

$$FS\Big|^{12}_{caving} = 0.20/(\Delta x/D) = 0.20/(1.88/60) = 6.4$$

An alternate would be to calculate the buckling load corresponding to the second mode and use $FS\,|_{caving} \approx p_{1_{cr(2)}}/p_1$. For wall crushing, by limit-equilibrium of the soil block above the culvert,

$$FS\,|_{wall} = [2(1 - A)(2cd_e + \sigma_y t_e)/\sigma_y t_e(D - 2d_e K_o \tan\phi)]$$

For granular soils, $c = 0$, $d_e = D/2K_o$, and $A_o \approx \tan\phi$

$$FS\,|_{wall} = [2(1 - A)/(1 - A_o)] = [2(1 - 0.3)/(1 - 0.87)] = 10.8$$

For seam strength, load = 300 × 12 = 3,600 lb/ft and capacity (17) = 23,400 lb/ft

$$FS\,|_{seam} = 23{,}400/3{,}600 = 6.5$$

For transitional buckling (4),

$$p_{1_{cr}} = \overline{C}\sqrt{M_s(EI/D^3)}$$

where

$\overline{C} = 6\sqrt{\tilde{B}\tilde{C}}$;
$\tilde{B} = 0.75$ for $\nu_s = 0.3$, $D/b < 0.2$; and
$\tilde{C} = (1 + \nu_s)(1 - 2\nu_s)/(1 - \nu_s)$.

For $\nu_s = 0.33$, $\tilde{C} = 0.742$, and $\overline{C} = 4.5$

$$p_{1_{cr}} = 4.5\sqrt{854 \times 0.461} = 89 \text{ psi}$$

$$FS\Big|_{\text{buckling}} = 89/10 = 8.9$$

11. Determine longitudinal deflection and tension, bending, and durability requirements in the manner suggested elsewhere (1). Other methods yield essentially the same design (1). For large diameters, where handling no longer governs, resulting designs are different.

One could argue that the minimum factor of safety of 6.4 is excessive for many installations; however, reducing the factor of safety for such small culverts is not possible unless special handling provisions are instituted. The actual factor of safety in an installation is probably greater than 6 because the value of M_s used (854 psi) is low for granular fills if reasonably good construction controls are maintained.

Principal advantages of the proposed method are that it permits treatment of embankment-culvert systems of all materials, depths of cover, and sizes; incorporates a rational method for accounting for arching; considers all potential modes of failure and collapse and enables achievement of a "balanced" design where the factors of safety in the different modes are in a desired ratio to each other; incorporates the means of accounting for moments; and enables the logical design of systems with backpacking (4).

The principal deficiency of the proposed method is that resulting designs for large-diameter culverts are strongly influenced by induced moments that are subject to relatively large variations. Also, the method involves the use of empirical relations with constants that are not as yet defined for cohesive soils.

Other deficiencies of the proposed design method are almost too obvious to mention. Clearly, test data on large pipes and conduits are needed for Figure 1. No tests on prototype culverts with the required measurements are known to exist although instrumentation exists to obtain the needed quantities. Empirical exponents are also desired for cohesive soils (tests are being planned to obtain the data). The method can be used with reduced accuracy without the data by assuming a conservative value of arching. In spite of these limitations, the method represents an improvement over prior ones because (a) it accounts for all principal variables, (b) all possible modes of failure are considered, and (c) the factor of safety in all modes can be estimated and, based on these values, adjustments can be made to the design to achieve a desired "balance" among them.

Important designs developed with the preceding methodology can be analyzed by using the finite-element method as is indicated in the following section.

FINITE-ELEMENT ANALYSIS

Once a culvert is selected, two- or three-dimensional finite-element analyses can be performed on the system. One may obtain two-dimensional linear or nonlinear solutions that incorporate either small- or large-deformation theory (8, 18, 19, 20). Also, the linear small-deformation code has been modified to permit accounting for interface slip and boundary separation (21). Some of the referenced codes have been modified to account for the presence of initial stresses and for dynamic loading. Examples of the use of some of these two-dimensional codes can be found elsewhere (19, 21, 22).

If other than stresses and deformations on a transverse section are desired, a three-dimensional solution is required (9). The remainder of this paper is devoted to presentation of a linear three-dimensional solution of the problem employed in the example design (with 10-gauge plate) of the previous section. The geometry and material properties used are given in Figures 4a and 4b respectively. Only one-quarter of the soil-culvert system was analyzed because of symmetry. Note that different moduli are used for the different layers because of differing dead load stresses at different depths. The elastic modulus at the culvert corresponds to the M_s used in the design example.

One thousand, five hundred and sixty-eight 8-node hexahedron solid elements were used to represent the soil, and 84 shell elements were employed to model the culvert. The culvert was represented by a plate of equivalent thickness (0.387 in.) to properly model the stiffness of its transverse section. No correction was made to account for cor-

rugations; thus, the longitudinal stresses in the cylinder would be expected to be greater than those where circumferential corrugated plate is used. Stresses in all elements and deflections at all node points were obtained as output data; however, space limitations permit visual display of only a small portion of these data.

Vertical soil stress contours in the Y-Z plane and in the X-Y plane are shown in Figures 5 and 6 respectively. As may be observed, the interface stress at the crown is about 15 psi as compared to 10 psi predicted with the arching relation in the example design. At the invert, the normal stress is about 16 psi. The horizontal stress at the spring line is about 15 psi. It is interesting to note from Figure 6 that the vertical soil stress is greater at about 1 to 2 ft above the culvert than it is at the crown.

Horizontal stress contours in the Y-Z plane are shown in Figure 7. Perhaps the most significant aspect of the horizontal stress is the rapid dispersal of the stress concentration adjacent to the spring line.

Stresses and forces in the culvert are shown in Figures 8 and 9. The contour plots in Figure 8 show the longitudinal and circumferential stresses in the extrados of one-half of the developed longitudinal section. At the center spring line the circumferential stress in the extrados is about 11,100 psi.

The peak longitudinal stress is about 18,000 psi for the modeled plate; however, it would be less for a longitudinally corrugated culvert.

Forces and deflections on the transverse sections at midlength and one-quarter of the total culvert length from one end are shown in Figure 9. It is interesting to note that the thrust at the spring line is about double the thrust at the crown and invert. Moments at the crown and spring line are about equal in amplitude but of opposite sign.

Horizontal diametral expansion at the center section is 1.27 in., and the vertical diametral shortening was 1.35 in. The corresponding vertical deflection determined in the design was 2.38 in. The deflection by the Iowa formula, excluding deflection lag, was 2.14 in. Absolute displacement of the invert at the center section was 4.77 in. This is the amount of camber that should be provided initially to ensure that the longitudinal axis of the culvert is straight when the embankment is completed.

In the example used, design deflections are in approximate agreement with values from the finite-element solution. Peak thrusts and moments are about a factor of 2 larger than in the design. The reason for this is that the elastic theory does not properly account for the arching in granular soils. Two-dimensional elastic theory indicates that, for $M_s/(EI/D^3) = 1{,}850$, the thrust at the spring line is 1.32 p_r. Experimental data and the arching theory give the thrust as 0.8 p_r. The latter value is considered correct for granular soils; however, elastic theory results may be more nearly correct for other than granular soils. Of course, knowledge of stress distribution from elastic finite-element solutions is useful. Codes with constitutive relations that properly model soil behavior must be used to obtain correct amplitudes.

Incidentally, stresses and deflections are expected to be lower than the values from the example for installations in granular soils. The reason is that the modulus of properly compacted granular fill would be an order of magnitude greater than that used. The low value was used to permit comparison of the results with existing designs for the same problem.

Although the example design and analysis were for a steel culvert, the same methodology is applicable to concrete culverts. The principal difficulty in the design and analysis of concrete cylinders is that the effective section modulus changes with load as fine cracks develop around the perimeter. As a consequence, concrete cylinders are not as "rigid" as is often presumed.

SUMMARY

This paper presents the bases for improved design and analysis methods for culverts. The design method illustrated is the only known method that permits accounting for arching and moment as an integral part of the design procedure. This is unimportant for small metal conduits under moderate fill heights because handling and durability usually govern designs under those circumstances. For culverts of cementitious materials and for all culverts under high loads, induced moments become important,

Figure 1. Moments in buried cylinders.

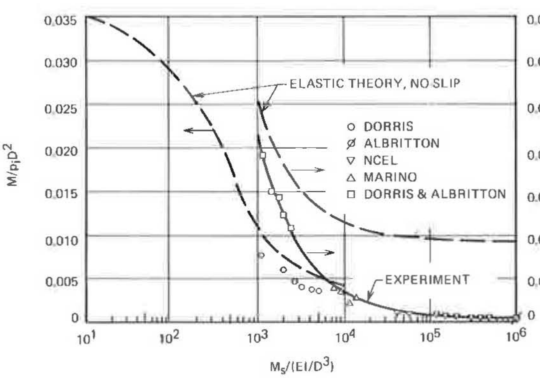


Figure 2. Deflections of buried cylinders (corresponding to average of full- and no-slip cases from the elastic theory and $\upsilon = 0.3$).

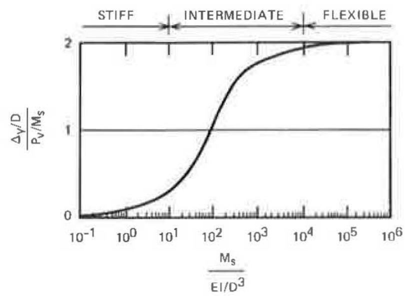


Figure 3. Plot for determining arching over culverts in granular soil.

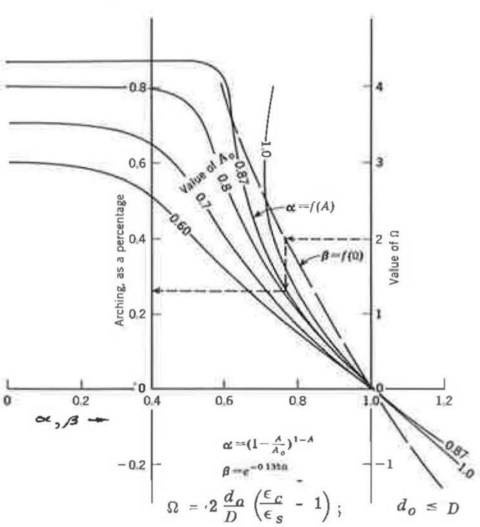


Figure 4. Soil-culvert system model.

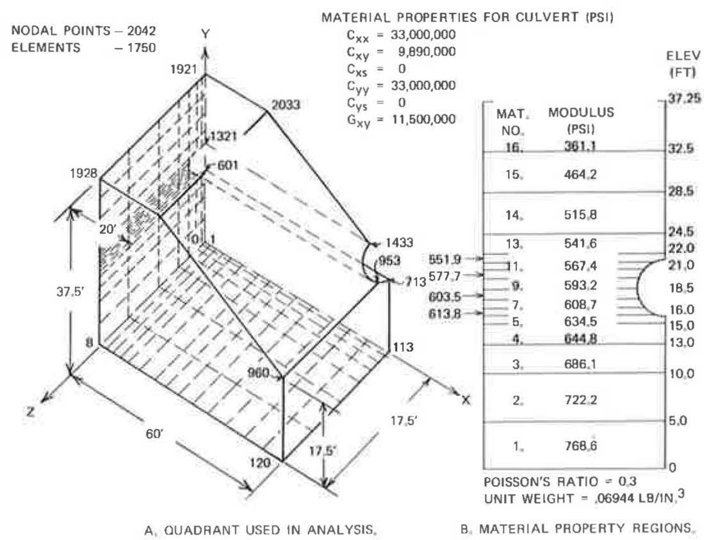


Figure 5. Vertical soil stress contours (Y-Z plane).

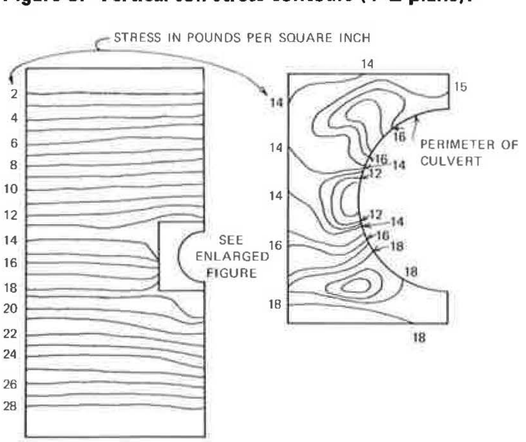


Figure 6. Vertical soil stress contours (X-Y plane).

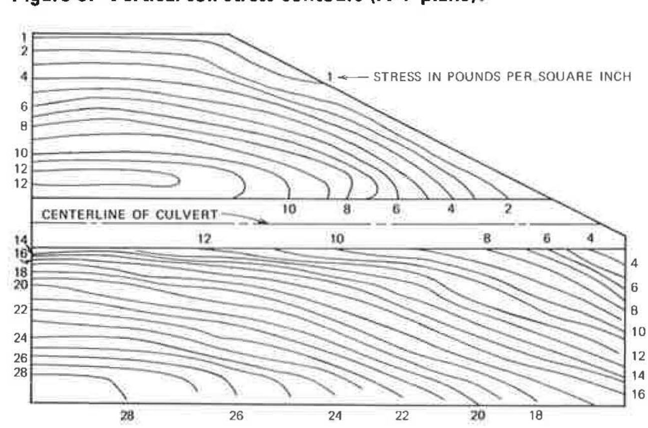


Figure 7. Horizontal soil stress contours (Y-Z plane).

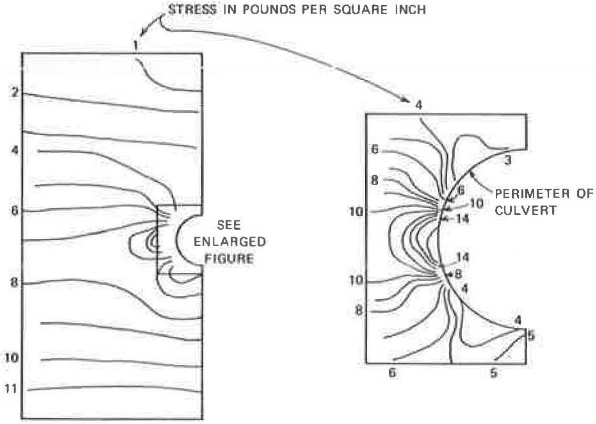


Figure 8. Stress contours on developed longitudinal half-section.

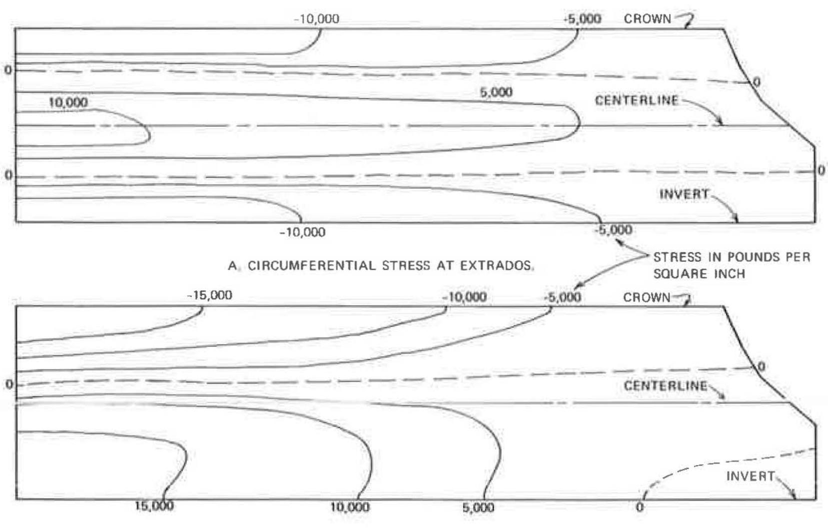
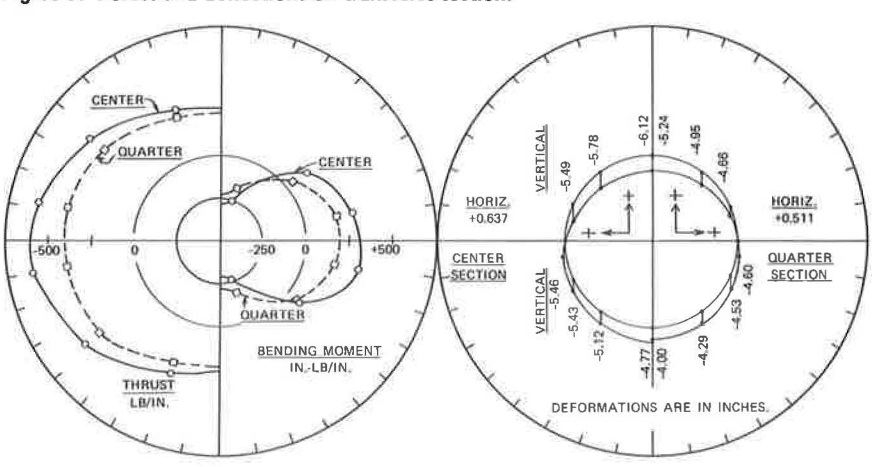


Figure 9. Forces and deflections on transverse section.



and economics require the use of arching. It is in these circumstances that the method given is useful.

Important installations warrant detailed analysis, and the finite-element method is useful for this purpose. A variety of two-dimensional codes may be used, including one that accounts for interface slip and boundary separation. Linear three-dimensional solutions, as illustrated in the text, may also be obtained; however, the magnitudes of the resulting thrusts and moments will be markedly larger than those in actual installations in granular soils.

Key points in the design of culverts are as follows:

1. If adequate construction procedures are followed, handling and durability considerations govern the required section properties of metal culverts less than about 5 ft in diameter under fills or equivalent loads less than about 30 ft in height (1).
2. For larger diameters or equivalent fill heights, good design and economics require consideration of moment and arching. With the larger diameters, achieving soil control via quantitative measurements becomes essential.

The methods outlined here, together with the use of backpacking in accordance with the relations found elsewhere (4), should permit more accurate and efficient designs of culvert systems than has been possible heretofore.

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