ACCEPTANCE SPECIFICATION OF COMPACTED SOILS

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An investigation of the failure probability of linear elastic heterogeneous embankments is reported, and illustrations are given for incorporating those results into acceptance specifications. Failure is defined in terms of deformations. Uncertainties in the soil stiffness include the spatial uncertainty and the uncertainty resulting from limited sampling during the inspection process. The results give a quantitative measure of the failure probability as influenced by the embankment height and slope angle, the slope geometry, the mean and standard deviation of the soil stiffness, Poisson's ratio, the number of samples taken during the inspection, and the mean and standard deviation of the measurements taken during the inspection. A hypothetical problem illustrates the use of the results, which can be tabulated for easy field use. The influence of neglecting the spatial uncertainty of the soil properties in acceptance requirements is also illustrated.

• ONE PURPOSE of compaction specifications is to provide assurance that the properties of the compacted soil comply with the values of the design. If the strength or stiffness of the compacted soil is less than the design value, a failure is more likely to result. On the other hand, if the strength or stiffness of the compacted soil is greater than the design value, additional compaction effort and, thus, additional cost will be expended.

Even with very rigid product control, soil properties such as moisture and density vary from point to point within the compacted mass. The variation results from changes in the borrow material and changes in the placement conditions and is reflected by the compactive effort and environmental conditions. Extensive statistical data on moisture and density in compacted soil are readily available (2, 3, 12). An uncertainty in the soil properties also results from testing errors and incomplete sampling for product control. Decisions for accepting or rejecting a unit of compaction work must be made, however, on the basis of the imperfect information furnished by the sampling and testing data. In view of the uncertainties associated with the information available for making a decision, a rational criterion for accepting or rejecting a unit of work should be based on statistics and probable risks.

A contractor's risk arises from the possibility that an acceptable unit of work may be rejected. An owner's risk arises from the possibility that an unacceptable unit of work may be accepted. Both risks are dependent on the statistical parameters of the compacted soil, testing method, and associated costs of the risks. To evaluate the risks requires that a method for determining the performance of statistically heterogeneous earth structures be available. The purposes of this paper are to present, in terms of a failure probability, the deformation behavior of statistically heterogeneous earth embankments as represented by an elastic material and to show how that information can be incorporated into earthwork acceptance specifications. This paper is concerned with acceptance specifications and not with product control requirements. Product control is a separate subject. It is assumed in this paper that sufficient consideration has been given to product control requirements.

NOTATION

The notation used in later sections of this paper is defined as follows:

a = soil constant;
b = soil constant;

c = soil constant;
C_{VE} = coefficient of variation of E;
$\overline{E}^*$ = measured mean stiffness;
$\overline{E}_d$ = design mean stiffness;
$f(\cdot)$ = probability density function;
F_s = safety factor, in terms of deformations;
H = embankment height;
n = number of samples;
$\overline{E}$ = mean soil stiffness parameter;
$\overline{k}$ = dimensionless deformation parameter;
$P[\cdot]$ = probability;
P_f = failure probability;
P_{fm} = maximum permissible P_f;
s_u = undrained shear strength;
S_E = measured variance;
w = moisture content, percent;
$\overline{w}_d$ = design mean w;
$\overline{w}^*$ = measured mean w;
$\overline{x}$ = normalized variable;
z = normalized variable, student t distribution;
α = $S_E/\overline{E}_d$;
β = $\overline{E}_d/\overline{E}^*$;
β_c = critical;
δS = deformation;
$\delta\overline{S}$ = mean deformation;
γ = soil unit weight;
γ_d = soil dry density;
$\overline{\gamma}_d$ = design mean γ_d;
$\overline{\gamma}_d^*$ = measured mean γ_d;
σ_k = standard deviation of k; and
η = $\overline{E}/\overline{E}^*$.

PERFORMANCE MODEL

An understanding and a quantitative evaluation of the performance of an earth structure with statistically heterogeneous properties are required for the development of a meaningful and workable statistical acceptance specification. Consider the performance of the earth embankment shown in Figure 1. Although conventional practice is to evaluate the performance of an embankment by using safety factors based on limiting equilibrium analysis, it is likely that in the future more emphasis will be placed on estimates of deformations (5). That change in emphasis is made possible by the recent developments of numerical methods to analyze nonlinear stress-strain behavior. In view of the recent interest in the estimate of deformations, this paper will use deformations rather than stresses to define failure. The performance may be evaluated on the basis of the deformation of the embankment surface. Large displacements, which may constitute failure, at interior points of an embankment should be reflected by the displacements of the surface. Thus, only surface displacements are considered. The embankment is divided into a finite number of elements as shown. The magnitude of soil properties, such as strength and compressibility, may differ from element to element. In view of this unknown variation in the soil properties, the deformation at any point is unknown. However, the uncertainty in the deformation can be described in probability terms, as will be mentioned later.

Most statistical data on compacted soils are in terms of moisture content and dry density. For a given soil and compaction method, strength and compressibility parameters of the compacted soil can be related to the molded moisture content and dry density (8, 11). If this functional relation is known, the statistical characteristics of strength and compressibility can be derived from the statistical data on moisture and density. The coefficient of variation of the compressive shear strength of a compacted plastic

clay was found to be 0.4, even though the coefficients of variation of moisture and density for compacted soils are typically on the order of 0.10 and 0.05 respectively (17). When the soil stiffness modulus is proportional to the soil strength, the coefficients of variation of the stiffness and strength are equal.

Statistical data on natural (as compared to compacted) soils indicate that the coefficient of variation of soil strength ranges between 0.2 and 0.6 (13, 17). Those values are restricted to what are referred to as "grossly uniform deposits" (9). Such deposits contain only one soil type. It is reasonable to assume that the coefficient of variation of compacted soil strength is of the same order as those of natural soil deposits. Hence, in this study, values for the coefficient of variation are taken to range from 0.3 to 0.6. That variation in soil properties includes both spatial variations and variations resulting from the testing method. At the present time sufficient knowledge is not available to separate those variations. If the testing variation and bias tendencies are known, they could be incorporated into the analysis with little additional effort.

The finite-element analysis and a simulation technique to account for the statistical variation of soil properties within the embankment can be used to estimate the variation of the deformation of any point within the compacted embankment. Further details on the method for generating these data are given by Kraft and Yeung (7). As an illustration, the variations of the mean deformation for different slope angles at selected locations on an embankment are shown in Figure 2. Although the coefficient of variation of the soil stiffness was 0.6, the maximum coefficient of variation of the embankment deformation was only 0.25. The indeterminancy of the earth mass tends to reduce the uncertainty in the deformation at locations where larger displacements occur as compared to the uncertainty of the soil stiffness (7).

A state of failure can be defined if the maximum deformation of the embankment surface exceeds some critical value. The magnitude of the critical deformation depends on factors such as the function of the earth structure and aesthetics. For the sake of argument and for the purpose of illustration, the critical deformation is taken as the mean design deformation times an appropriate safety factor F_s for a preselected location on the embankment, such as points 1 or 2 shown in Figure 2. Other locations for the critical deformation can be taken at the discretion of the engineer.

According to Gould and Dunnicliff (4), crest settlements of modern embankment dams on dense foundations are typically 0.2 to 0.6 percent of their height. Limiting shear strains within the embankment are in the range of 0.03 to 0.1 radian. Inclinometer deflection readings in dams that have performed satisfactorily are typically 0.01 radian. These values for limiting and observed deformations and strains may serve as first estimates of a design criterion based on deformations rather than limiting equilibrium.

It is recognized that soils possess neither linear nor elastic stress-strain characteristics. However, for the safety factors commonly used against shear failures, the shear stresses within the earth structure may be of magnitude where the assumption of linearity serves as a good first approximation. D'Appolonia, Poulos, and Ladd (1) have demonstrated that the initial settlement of footings supported by a bilinear elastic material depends on the elastic modulus, ultimate strength, and initial shear stress ratio. For safety factors against shear equal to approximately 2.0 and low initial shear stress ratios, which are typical of overconsolidated soils, the elastic theory can be used to calculate the initial settlement, even though the soil stress-strain curve is nonlinear. In view of the limited state of quantitative knowledge on the subject of this paper, it also seems logical that a linear model be examined before a more complex stress-strain relation is pursued that requires a much larger amount of computer time. The safety factor in terms of deformation would be smaller than the safety factor against a shear failure, as shown in Figure 3.

FAILURE PROBABILITY

An analytical evaluation of the failure probability of a compacted earth embankment will assist in making a decision to accept, reject, or perform additional sampling on a unit of work. If the mean stiffness parameter E and variance are known, the failure probability is the probability that the selected deformation will exceed the mean design deformation times the safety factor. If the deformation is normally distributed, the

failure probability is

$$P_f = P(\delta < \delta\overline{F}_s \mid \overline{E}) = 1 - \int_{-\infty}^{\overline{x}} \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}}\, dx = \int_{\infty}^{-\overline{x}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\, dx \qquad (1)$$

where $\overline{x} = \overline{k}\,[(F_s - 1)/k]$, and σ_k is the standard deviation of the deformation parameter $\overline{k}$, which is a dimensionless parameter $(\delta\overline{E}/H^2\gamma)$. The parameters $\overline{k}$ and σ_k depend on the slope geometry, slope angle, Poisson's ratio, and C_{VE}.

The use of the normal probability density function, at least as a first approximation, may be justified on the argument that the deformation at any point is the result of the sum and multiplicative mechanisms of the interactions of the elements. For coefficients of variation less than approximately 0.3, both the log normal and normal distributions are very similar. If the well-known "central limit theorem" is used, the multiplicative mechanisms would tend to be log normally distributed and the sum mechanisms would tend to be normally distributed. If the coefficient of variation is small, the normal probability density function could then be used to approximate the total effect of the sum and multiplicative mechanisms. Although exceptions may be found to this argument, it is the opinion of the authors that the normal distribution serves as a very good first approximation for the larger mean deformations, which are the ones of predominant interest. Also, an examination of the coefficient of skewness and the coefficient of peakedness of the deformations obtained from the simulation results suggests that a normal distribution is a reasonable distribution for this problem.

For purposes of illustration, the vertical deformation at the center of an earth embankment will be used to define a state of failure. For the trapezoidal and triangular embankments shown in Figure 4, the relation between the failure probability and the safety factor is shown in Figure 5 for different slope angles.

The results shown in Figure 5 are based on statistical estimates of the means and variance of the deformation, as obtained by Kraft and Yeung (7), who used finite elements and a simulation technique. For the purposes here, it is assumed that the statistical estimates are exact. The uncertainty in the mean $\overline{E}$ is usually larger than the uncertainty in the deformations resulting from the simulation process. The simulation results are based on 100 to 200 samples, whereas the mean soil property in the field for a given section of compaction is based usually on 10 or fewer samples. Therefore, the assumption that the statistical estimates are exact is considered to be reasonable.

For a given embankment height, a change in slope angle implies a change in $\overline{E}$ for a given safety factor. The safety factor can be altered by changing the slope angle and embankment height and by holding $\overline{E}$ constant. The results shown in Figure 5 demonstrate that the failure probability depends on C_{VE} and slope angle rather than solely on the safety factor. Hence, if the safety factor concept is to be used with a consistent reliability, the magnitude of the safety factor should be selected by giving some consideration to C_{VE}, Poisson's ratio, slope angle, and slope geometry for the particular job. The results shown in Figure 5 demonstrate that, for a given set of conditions, the triangular embankment is more reliable than the trapezoidal embankment. These results give some quantitative insight into the factors that affect the failure probability of embankments and may prove useful in the decision-making process during the design. However, this paper is concerned with illustrating how these results can be used during the acceptance stage of the embankment construction.

Because only a limited number of samples are taken during the inspection process, the mean E and the variance are not known with certainty. The uncertainty in E resulting from limited sampling can be approximated by the student t distribution.

$$f(z) = \frac{1}{\sqrt{\pi(n-1)}} \frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)} \left(1 + \frac{z^2}{n-1}\right)^{-n/2} \qquad (2)$$

where $z = \overline{E}^* - \overline{E}/\sqrt{S_E^2/[n(n-1)]}$; $\overline{E}$ is the actual mean of the earth mass; S_E^2 and $\overline{E}^*$ are the calculated variance and mean of E respectively, based on the n samples taken; and

$\Gamma(\cdot)$ is the gamma function. Combining Eqs. 1 and 2 and performing some mathematical manipulation give the failure probability with limited knowledge.

$$P_f = P(\delta > \bar{\delta} F_s) = \int_0^{\infty} P(\delta > \bar{\delta} F_s \mid E)\, f(\bar{E})\, d\bar{E}$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{-\frac{\bar{k}}{\sigma_k}\left(F_s - \frac{\beta}{\eta}\right)} \frac{1}{\pi\alpha\beta} \sqrt{\frac{n}{2}} \frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)} \left(e^{-x^2/2}\right) \left[1 + n\left(\frac{1-\eta}{\alpha\beta}\right)^2\right]^{-n/2} dx\,d\eta \quad (3)$$

in which $\alpha = S_E/\bar{E}_d$ and $\beta = \bar{E}_d/\bar{E}^*$, where $\bar{E}_d$ is the design mean of E, and $\eta = 1 - [\alpha\beta/\sqrt{n(n-1)}]\, z = \bar{E}/\bar{E}^*$.

In the limit $\{(-\bar{k}/\sigma_k)\,[F_s - (\beta/\eta)]\}$ of Eq. 3, values of η are restricted to being positive inasmuch as E is positive. Mathematically the student t distribution accounts for both positive and negative values of η. However, for the range in the α and β values of practical interest, the probability density function describing η is for all practical purposes generally 0 on the negative η axis if n is larger than 5. If n is smaller than 5, Eq. 2 must be replaced by a probability density function appropriate for describing the uncertainty in the mean of a positive random variable, and appropriate changes would be required in subsequent equations using Eq. 2.

The parameter $\bar{k}/\sigma_k$, which is used in Eq. 3, is a function of the C_{VE}. Both the standard deviation of E and C_{VE} are uncertain. Although it is possible, with additional mathematical effort, to include the uncertainty of C_{VE} in the analysis, the additional effort is deemed unwarranted at this stage. For given soil conditions and normal product control measures, a reasonable estimate for the coefficient of variation of the soil property can be made even with a limited number of samples. If a reasonable estimate of C_{VE} can be made, the ratio $\bar{k}/\sigma_k$ can be determined for a given slope angle and embankment geometry as suggested by Kraft and Yeung (7).

A typical result obtained from Eq. 3 is shown in Figure 6. As the reciprocal of β increases, the failure probability decreases. The magnitude of the decrease in P_f is influenced by C_{VE}, the slope angle and geometry, Poisson's ratio, α, F_s, and the number of samples taken to determine β. The larger the number of samples is, the greater the confidence will be that the actual mean E is equal to the measured mean $\bar{E}^*$; hence, for a given β the failure probability decreases as n increases.

Treating $\bar{E}$ as a random variable accounts for the possibility that the actual mean may be greater than the design $\bar{E}_d$ and, thus, accounts for the contractor's risk. At the same time, the possibility that the actual mean is less than the design mean is also accounted for in probabilistic terms; thus, the owner's risk is accounted for. If the maximum permissible failure probability can be established, the results obtained with Eq. 3 can be represented in a graphical or tabular form readily usable for field execution. Before that representation is illustrated, however, a few comments on the maximum permissible P_f are in order.

DESIGN CRITERION

Because the acceptance specifications should reflect the design philosophies and design considerations in some way, these comments are made on the design criterion. The optimum selection of the failure probability may be based on minimizing the total expected cost of the embankment. The total expected cost consists of the construction cost (usual maintenance costs are assumed to be included) and the expected risk cost. For a given embankment geometry and soil, the construction cost usually increases with a decrease in the failure probability. The risk cost (or failure cost) increases with an increase in the failure probability. Adding the construction cost and risk cost

Figure 1. Simulation of statistically heterogeneous embankment by finite elements.

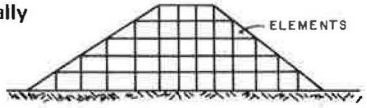


Figure 2. Mean slope deformations of a statistically heterogeneous embankment.

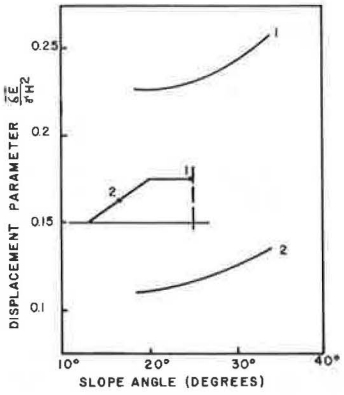


Figure 3. Concept of safety factor.

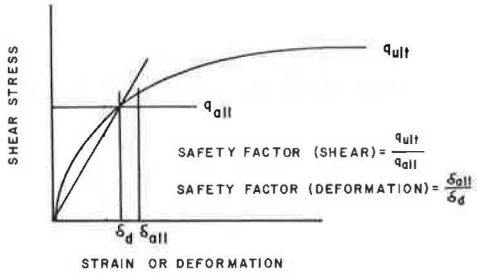


Figure 4. Embankment geometry.

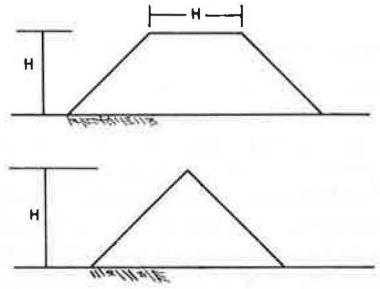
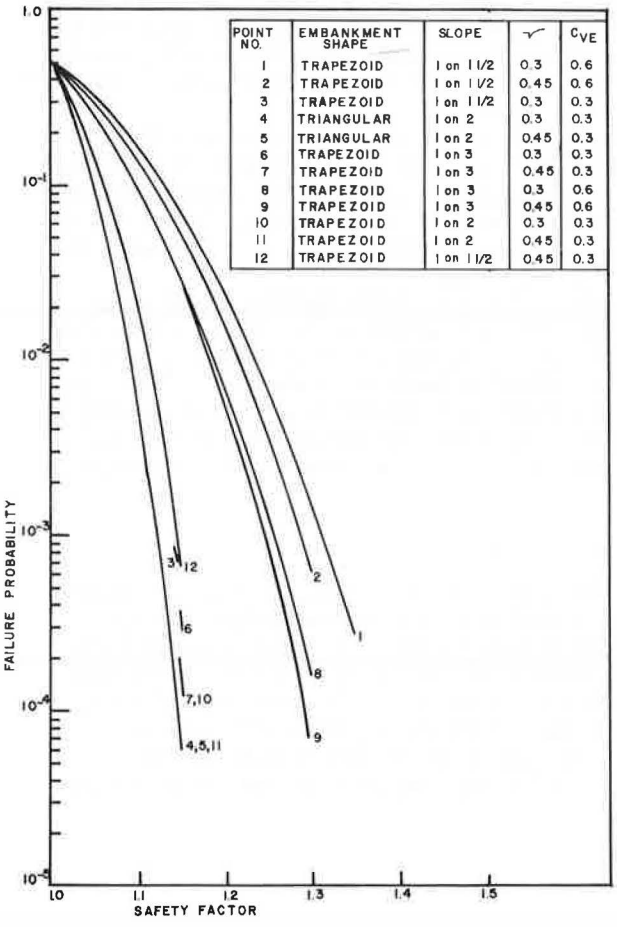
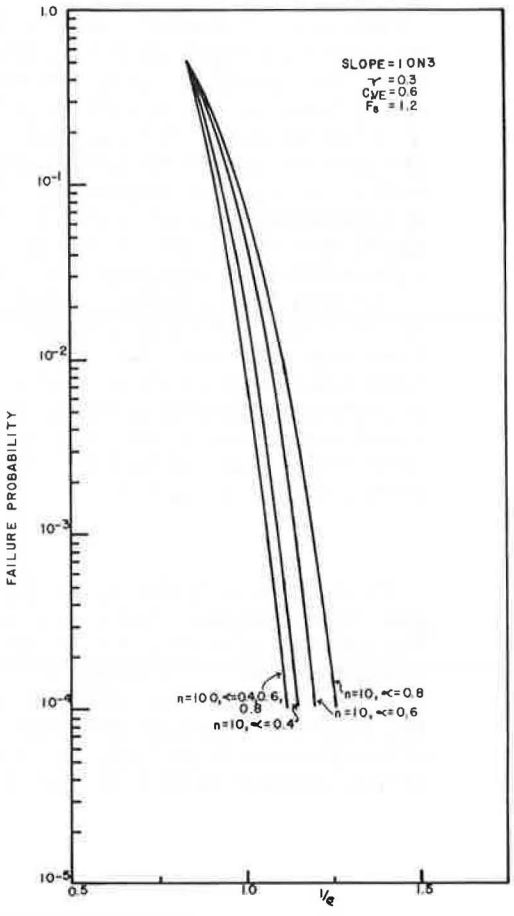


Figure 5. Influence of safety factor on failure probability knowing the mean soil stiffness.



POINT NO.	EMBANKMENT SHAPE	SLOPE	γ	C_{VE}
1	TRAPEZOID	1 on 1 1/2	0.3	0.6
2	TRAPEZOID	1 on 1 1/2	0.45	0.6
3	TRAPEZOID	1 on 1 1/2	0.3	0.3
4	TRIANGULAR	1 on 2	0.3	0.3
5	TRIANGULAR	1 on 2	0.45	0.3
6	TRAPEZOID	1 on 3	0.3	0.3
7	TRAPEZOID	1 on 3	0.45	0.3
8	TRAPEZOID	1 on 3	0.3	0.6
9	TRAPEZOID	1 on 3	0.45	0.6
10	TRAPEZOID	1 on 2	0.3	0.3
11	TRAPEZOID	1 on 2	0.45	0.3
12	TRAPEZOID	1 on 1 1/2	0.45	0.3

Figure 6. Influence of sampling results on failure probability.



results in the total expected cost as shown in Figure 7. The failure probability corresponding to the minimum cost represents the optimum for the stated conditions. Provided such cost functions can be established, the maximum permissible failure probability, denoted as P_{fm}, can be determined with this approach.

An alternative, which is more straightforward but less quantitative, for establishing P_{fm} is to arbitrarily select P_f as some small number. The smallness of the number should be such that for all practical purposes the failure probability is 0. This alternative is used here, and 2 values of the maximum permissible P_f (namely, 10^{-2} and 10^{-4}) are used for purposes of comparison.

PRODUCT CONTROL CHARTS

For a given slope geometry, Poisson's ratio, C_{VE}, and α, the value of $1/\beta$ corresponding to P_{fm} can be determined for different values of n from graphs such as those shown in Figure 6. These limiting or critical values of $1/\beta$ will be denoted as $1/\beta_c$. Only values of $1/\beta$ greater than $1/\beta_c$ constitute an acceptable unit of work. Values of $(1/\beta_c)$ plotted against the number of samples for a variety of conditions are shown in Figure 8. The results show that a much larger value of $1/\beta_c$ is required for values of n on the order of 5 than for values of n on the order of 10 to 20. Values of $1/\beta_c$ larger than 1 correspond to measured values of the mean $\overline{E}^*$ being greater than the design mean $\overline{E}_d$. Because of the advent of more rapid field-measuring devices, the cost and time of taking 10 to 20 samples in a given section are not at all prohibitive. The attainment of more samples allows for a reduction in the required $1/\beta_c$, which the contractor would probably welcome, and at the same time does not increase the risk, as defined by the probabilities, to the owner or engineer.

On the basis of test results from n samples, a decision must be made to accept or reject a unit of work as currently compacted. A unit of work may conveniently be taken as the volume compacted during a specified time period such as 1 day. Methods for defining units and for randomly selecting subunits within the unit for testing are available in the literature (12, 16) and will not be repeated here. Because the results of the model presented in this paper are based on a relatively homogeneous earth mass, the acceptance or rejection of each unit of compacted soil must be made on the basis of test results solely from the respective unit. Thus, a relatively uniform completed project is ensured. If the cross section consists of zones of different materials and if the zoning conditions are known ahead of time, it would be feasible to develop the corresponding control charts.

The results shown in Figure 8 demonstrate that, as the design safety factor F_s increases, the value of $1/\beta_c$ decreases. In other words, the greater the conservatism in the original design is, the more relaxed the acceptance requirements can be. The presented results provide a quantitative measure of those factors.

As the maximum permissible failure probability decreases, the value of $1/\beta_c$ increases. The results shown only include a comparison of $P_{fm} = 10^{-2}$ and $P_{fm} = 10^{-4}$. For practical purposes, a failure probability of 10^{-4} or smaller may be considered as 0.

The coefficient of variation of a compacted soil property is dependent on many factors including the soil itself. For soils with a greater natural heterogeneity, the coefficient of variation is likely to be larger than for more uniform soils even with strict product control (12). A comparison of Figures 8e and 8f demonstrates that a larger $1/\beta_c$ is required for the more heterogeneous soil. As an approximation, $1/\beta_c$ corresponding to a C_{VE2} can be obtained from

$$\left(\frac{1}{\beta_c}\right)_2 \sim \frac{C_{VE2}}{C_{VE1}}\left[\left(\frac{1}{\beta_c}\right)_1 - 1\right] + 1.0 \qquad (4)$$

where $(1/\beta_c)_1$ corresponds to the condition for C_{VE1}.

Although most of the above results are intuitively obvious, they do provide a quantitative measure. It is illustrated in the following section how these results can be applied in terms of moisture and density.

Figure 7. Optimization concept.

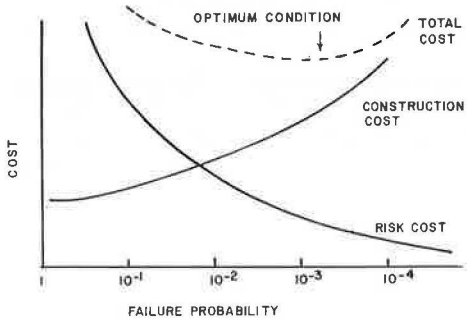
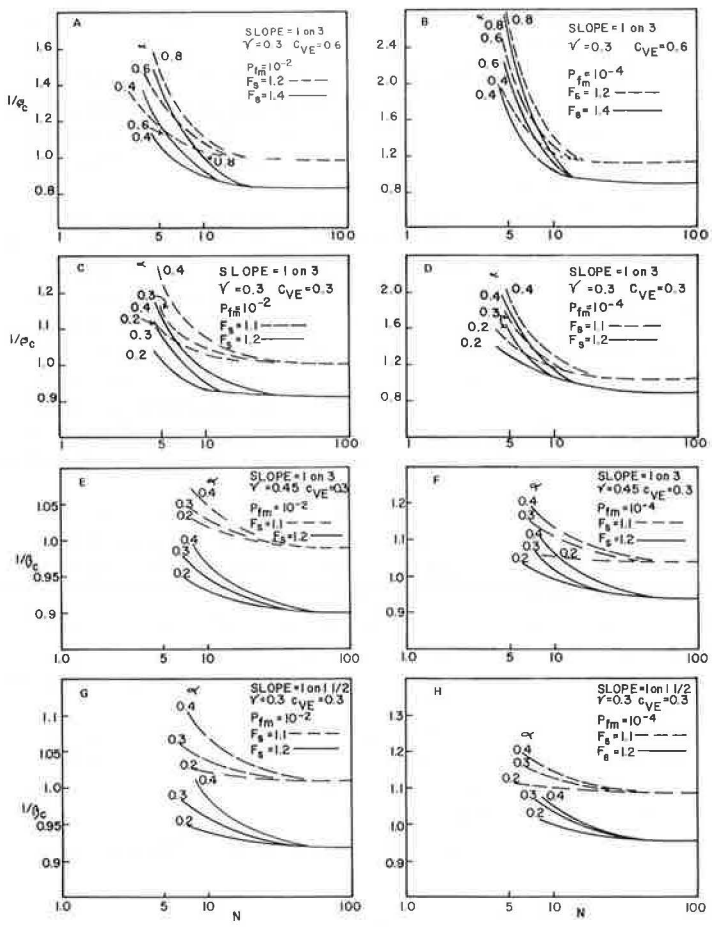


Figure 8. Acceptance requirements.



MOISTURE AND DENSITY CONTROL

Strength, stiffness, swelling potential, and permeability of the soil govern the performance of an earth mass and for a given soil and compaction technique can be related to the moisture and density of the soil. Because field control is based on moisture and density, the following arguments are presented to illustrate the application of the results found in this study.

For a particular soil, relations can be established between dry density and strength and between water content and strength. In this example, the data obtained by Seed and Chan (10) for a plastic clay are used. The strength relation can be approximated by the linear equation

$$s_u = a\,\gamma_d + bw + c \tag{5}$$

where s_u is the undrained shear strength, γ_d is the dry density, and w is the water content expressed as a percent. The constants for the plastic clay are equal to 11,570 cm, -0.488 kg/cm^2, and -6.15 kg/cm^2.

Inasmuch as the above data pertain to laboratory-compacted specimens, it may be questioned whether Eq. 5 can be applied to field-compacted soils. However, Seed and Chan have shown that different laboratory-compaction procedures lead to about the same ultimate strength for a given water content and dry density. Agreement between the strengths of laboratory- and field-compacted soils has been reported by Holtz and Ellis (6). Hence, in the subsequent analysis, it is assumed that there is no difference between the shear strength of laboratory-compacted and field-compacted specimens.

It must be noted that the constants a, b, and c may be different for different soils. Also the linear approximation of the strength may not be applicable for all soils. However, there are insufficient data to establish the nature of such variations. Hence, this analysis is restricted to soils similar to those studied by Seed and Chan.

A further assumption is required to relate the elastic modulus E to the undrained strength. For purpose of demonstration, the elastic modulus is taken to be proportional to the undrained strength. Hence, the β value in terms of moisture and density is

$$\beta = \frac{a\,\overline{\gamma}_d + b\overline{w}_d + c}{a\,\overline{\gamma}_d^* + b\overline{w}^* + c} \tag{6}$$

where $\overline{\gamma}_d$ and $\overline{w}_d$ are the mean values used in the design, and $\overline{\gamma}_d^*$ and $\overline{w}^*$ are the measured means (dry density and water content). Equation 6 is based on γ_d and w being statistically independent. The validity of that assumption is discussed further in later paragraphs. If, for an acceptable unit of work, β must be less than β_c, then

$$\overline{\gamma}_d^* + \frac{b}{a}\overline{w}^* + \frac{c}{a} - \frac{\overline{\gamma}_d + \frac{b}{a}\overline{w}_d + \frac{c}{a}}{\beta_c} \geq 0 \tag{7}$$

For a given job, the parameters a, b, c, $\overline{\gamma}_d$, $\overline{w}_d$, and β_c are known. Using the equality sign, Eq. 7 is shown in Figure 9 in terms of different β_c values, which can be determined from Figure 8. If the measured values $\overline{\gamma}_d^*$ and $\overline{w}^*$ plot above the line for the respective β_c value, the unit of work is considered acceptable. The results shown in Figures 8 and 9 can be combined into a single graph; however, there is no apparent advantage in consolidating them. Figure 8 can be used to establish β_c, and then Figure 9 can be used with the corresponding value of β_c.

The coefficients of variation for moisture content and dry density are typically on the order of 0.1 and 0.05 respectively, as evidenced by published data (2, 14, 15). Most of the moisture and density conditions will be between ±2 standard deviations from the mean. If the mean is equal to the design condition, the moisture and density condition should fall within the shaded region shown in Figure 9 for the design conditions shown.

It is well established that the control of the moisture content is crucial for obtaining the proper density. Hence, some dependence between moisture and density is to be expected. However, for a moisture content range of 2 to 3 percent, there is evidence

that moisture and density may be assumed to be independent, at least for some soils. Torrey (14) presents results from 5 materials in the construction of the Littleville Dam on Westfield River in Massachusetts. His results illustrate that the assumption of independence may serve as a good first approximation. With that assumption, it can then be stated that the probability that a unit of work will satisfy the specification increases as the area of the shaded region below the respective $1/\beta_c$ line (Fig. 9) decreases. One may also observe that the probability of satisfying the specification is less if the measured mean moisture content is on the wet side than if it is on the dry side for the given soil.

Obviously, consideration must be given to several factors such as swelling potential, strength, stiffness, and permeability in establishing the testing procedure for determining the relation between soil stiffness and moisture-density. For the concepts of this paper to be applied, the attainment of the soil stiffness as related by moisture and density must incorporate those considerations. If the relation is nonlinear, appropriate modifications in the formulation must be made. For example, it has been reported (18) that for soils compacted on the dry side of optimum moisture the strength increases with the square of the dry density.

$$s_u = a + b\gamma_d^2 \tag{8}$$

Again, if the elastic modulus is taken to be proportional to the undrained strength, the β value is

$$\beta = \frac{a + b\overline{\gamma}_d^2}{[a + b(\overline{\gamma}_d^*)^2 + S_\gamma^2]} \tag{9}$$

where S_γ^2 is the measured variance of the dry density. The mean of the square of a random variable is not the same as the mean squared. The quantity in the brackets of the denominator of Eq. 9 is the mean of the square of γ_d. For a silty clay, the constants a and b were found to be -11.25×10^6 kg/cm^2 and 3.95×10^6 cm^4/kg, based on a density ranging between 1.68 and 1.76×10^{-3} kg/cm^3.

For other relations among strength, moisture, and density, the appropriate modifications can be made. The following illustration is limited, however, to conditions representative of Eq. 6.

ILLUSTRATION

The design conditions shown in Figure 9 are applied to a 1:3 embankment with a design safety factor of 1.2 and a maximum permissible failure probability of 10^{-4}. The values of $1/\beta_c$ as influenced by the sample size n are given in Table 1; a $C_{VE} = 0.3$ (Fig. 8d) is used. If only 5 samples are taken and if the shaded area shown in Figure 9 is representative of the field condition, the probability of accepting the work is much smaller than if 10 or more samples are obtained. On the other hand, if the field moisture conditions are on the wet side of the design values, resulting in the shaded area being shifted to the right and possibly downward, the probability of accepting the work will be small regardless of the number of samples taken.

As a means for illustrating the use of the results of the paper, suppose that a mean moisture content and dry density of 20 percent and 1.8×10^{-3} kg/cm^3 and an α value of 0.25 have been obtained from 10 samples. For the previously stated condition, should the unit of work on which the data are based be accepted? From Figure 8d, a value of $1/\beta_c$ equal to 1.08 is estimated. Because the point (20 and 1.8×10^{-3}) plots above the line corresponding to $1/\beta_c = 1.08$, the unit would be accepted.

The pertinent charts or graphs can easily be used by field personnel. The additional effort required is not prohibitive, and the use of charts reduces the risk of computational errors.

Most statistical specifications for compaction control give consideration only to the uncertainties resulting from limited sampling. The model developed here considers the influence of both spatial uncertainties and sampling uncertainties; therefore, it is of interest to compare these results with those obtained when only limited sampling is considered. If the design is based on the mean soil property governing the performance

Figure 9. Compaction requirements in terms of moisture and density.

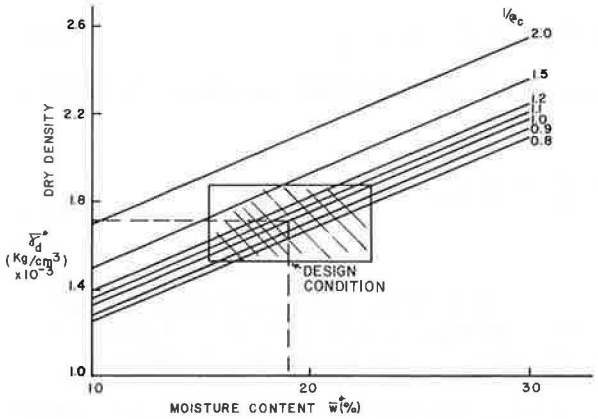


Table 1. Values of $1/\beta_c$.

n	α = 0.2	α = 0.3	α = 0.4
5	1.28	1.56	1.80
10	1.04	1.08	1.16
100	0.92	0.92	0.92

Figure 10. Acceptance requirements neglecting spatial variations.

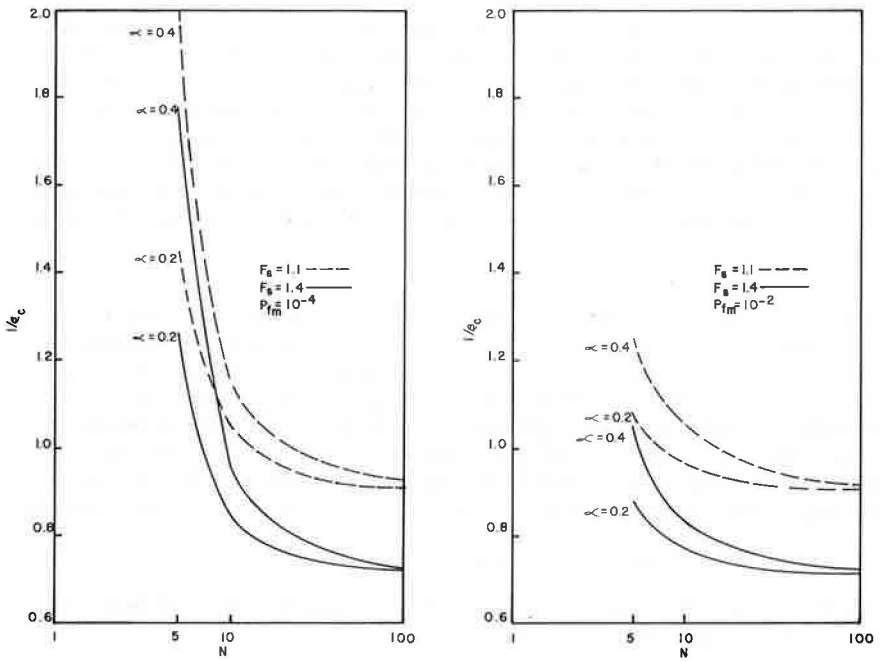
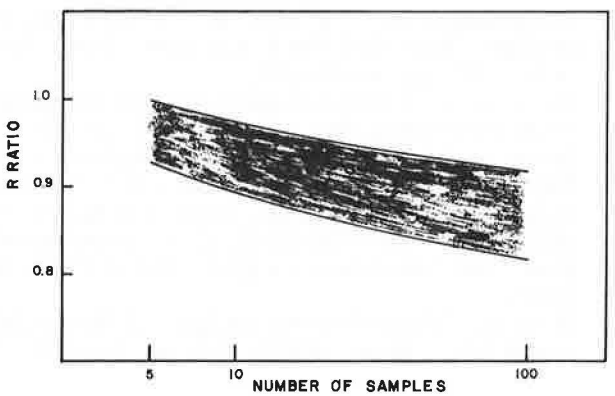


Figure 11. Effect of not including the influence of spatial variations in acceptance requirements.



of the earth mass, the failure probability is

$$P_f = P\left[\delta > F_s\bar{\delta}\right] = P\left[\bar{E} < \bar{E}_d/F_s\right] \tag{10}$$

The student t distribution is used to account for the uncertainties contributed to limited sampling, and the failure probability is

$$P_f = 1 - \int_{-\infty}^{\frac{1 - \beta/F_s}{\beta}\sqrt{n(n-1)}} f(z)\,dz \tag{11}$$

Equation 11 can be used to generate curves similar to those shown in Figure 8. The values of $1/\beta_c$, if the influence of spatial uncertainties is neglected, are not influenced by the angle of the slope, the geometry of the slope, or the magnitude of Poisson's ratio.

The importance of neglecting those factors can be estimated by comparing the results shown in Figure 10 with those shown in Figure 8. The ratio R of the $1/\beta_c$ requirement neglecting the spatial uncertainty (Eq. 11) to the $1/\beta_c$ requirement considering both spatial and sampling uncertainties (Eq. 3) will be used for purposes of comparison. An R value greater than 1 indicates that neglecting the spatial uncertainty is conservative. A range of the R values obtained for slope conditions shown in Figure 8 is shown in Figure 11. Neglecting the spatial variation of the soil properties in the acceptance criterion, as shown in Figure 11, can result in the acceptance requirements being too low. The influence of the spatial variation increases as the number of samples increases, if both uncertainties (spatial and limited sampling) are considered. For small values of n, the uncertainty due to limited sampling has the predominant influence on the acceptance requirements; whereas, for larger values of n, the uncertainty due to spatial variation has the predominant influence.

SUMMARY

The results of the study provide quantitative insight into the factors affecting the acceptance specifications of earth compaction. The results can be used for establishing acceptance specification guidelines and are readily adapted to field use. Several simplifying assumptions were made. However, it is the opinion of the authors that the results clearly demonstrate that, if the acceptance is based on the measured mean as compared to a design value, 10 or more samples should be required for each unit of work. Because random sampling is a necessity for the use of these results, the size of each unit of work should be limited so that it is convenient to obtain random samples.

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