RATIONAL DESIGN OF CONTINUOUS AND INTERMITTENT AGGREGATE GRADINGS FOR CONCRETE

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The author has earlier presented an improved method for the design of aggregate gradings and demonstrated its applicability to the design of dense asphaltic compositions (1). Subsequently, further work has been done to confirm that both continuously graded and gap-graded bituminous mixes can be satisfactorily designed according to the method and to show that, for highway and airfield pavement surfacing purposes, certain principles governing skid resistance over a practical range of speeds can be incorporated into the design method. The current study describes the application of the method to the design of concrete mixes. For gap or intermittent gradings, the relevance of studies of interparticle voids is demonstrated with respect to critical ratios of entrance and occupation (measured for typical aggregates) and a newly defined term, the critical ratio of dilation. The influence of gradings, designed according to this rational method, on concrete workability and strength is discussed. Reference is also made to the design of pavement concrete mixes to meet the different requirements of skid resistance for low-speed and high-speed conditions on roads and airfields.

•THE properties of concrete are highly dependent on the structural arrangement of included aggregate particles, and the literature contains many references to the study of aggregate gradings. However, comparatively few of the methods that have been proposed for selection or design of aggregate gradings have considered adequately the packing properties of the aggregate particles. Rather, these have assumed that the optimum grading follows some simple mathematical law or law of past experience, irrespective of variations of shape from source to source, or with variation in crushing plant or method, or from size to size within the same source and irrespective of the several other factors that affect the structural packing of particles. Other methods of grading design that have considered some of these factors have neglected others or have treated them as factors to be dealt with in isolation, ignoring their interaction.

In the present method, a simple experimental procedure has been developed for assessing the packing properties of intended component aggregates in the laboratory, from the results of which may be determined the optimum percentage of fine material to obtain maximum density in a two-component mix. It has been subsequently verified that this approach can be extended into the field of multicomponent mixes, enabling maximum-density gradings of both the intermittent and continuous types to be derived. These gradings, in contrast to those of a more empirical nature (2-6), take into account any change in aggregate packing properties that may occur at any size level.

In addition to the important effects on packing of particle shape, the method recognizes the importance of any lubricating or adhesive coatings, surface static effects on small particles, degree and type of compactive effort, and the effect of external boundaries including external form and minimum thickness of any section and of internal boundaries such as reinforcement of various degrees of congestion.

The complexity of the interactions of these factors makes it impossible to derive reliable porosity values (upon which the design calculations will be based) by way of correction factors for the variables concerned. Accordingly, it is considered that all assessments of porosity should be made in containers that reproduce as nearly as practicable the critical minimum dimension(s), external form and attitude, and internal boundary effects of the proposed concrete member. Representative samples of the actual aggregates to be used are employed in the laboratory tests and under a compactive effort varied in accordance with the anticipated effort on site.

Four factors determine the porosity of a system of packed aggregate particles: the mode of packing, the size distribution, the shape (in which is included surface texture), and the shape distribution. Other factors have been mentioned in the literature, but all exert their influence ultimately via one of these four main factors.

AGGREGATE GRADING DESIGN

The factors that have been mentioned previously as being of major importance in influencing the packing and porosity of aggregate particles have in the proposed method been recognized as those that should be considered in the design of aggregate gradings.

It has further been recognized that, for this design purpose, it is unnecessary to undertake detailed and time-consuming measurements of influencing parameters such as particle shape, surface texture, specific surface, area-volume ratio of container, surface area of reinforcement, and compaction energy and that the total and mutually interacting effects of these parameters can be most simply taken into account by simulative laboratory tests in which measurements are taken of the mass porosity for each considered size component. The mass porosity is the measured porosity in a system of packed aggregate particles, excluding any internal porosity of the particles.

Justification for this approach has been given in an earlier paper (1) in which it has been shown by reference to theoretical and practical studies of packing of two-component systems that the maximum density is achieved at an optimum percentage of the fine component, this optimum being a function of the porosities of the individual separate components and the size ratio between the components; i.e.,

$$\begin{array}{l}\text{Optimum}\\ \text{percentage of}\\ \text{fines (by}\\ \text{volume)}\end{array} = f\left\{\begin{array}{llll}\text{particle shape and} & \text{container wall} & \text{compactive} & \text{size ratio}\\ \text{surface texture,} & \text{and internal} & \text{effort} & \\ \text{surface charge,} & \text{boundary} & & \\ \text{lubricating and} & \text{effects} & & \\ \text{adhesive coatings} & & & \end{array}\right.$$

$$= f\left\{\begin{array}{llll}\text{particle friction} & \underbrace{\text{boundary effects} \quad \text{compactive effort}}_{\text{measured porosities of fine and coarse components expressed as}} & & \text{size ratio}\end{array}\right.$$

$$= f\left\{\begin{array}{lll}P_{av} & P_{diff} & \text{size ratio}\end{array}\right.$$

where P_{av} = average porosity of separate coarse and fine components, P_{diff} = difference between porosity values of coarse and fine components, and where differences are positive if the coarse component has the higher porosity and negative if the fine component has the higher porosity.

Figure 1 shows how, for two-component systems, the optimum percentage of fine material for minimum porosity in the mix varies significantly with average particle shape, differences in particle shape, compactive effort, and boundary wall effects.

The upper points (F and C) on the vertical axes of the graphs of Figure 1 represent the specific void content (void ratio) and porosity values for hypothetical cases of packing of aggregates of certain generalized shape categories under different compactive efforts and in different size containers.

Lines are drawn from the specific void content value of any considered component to the partial specific void content of the other component plotted on the opposite axis [following the methods of Furnas (7, 8) and Powers (9)]. The partial specific void content of a component may be defined as the contribution made by it to the total void content of the mix, assuming that no dilation of the other component is caused by its presence. This assumption is only valid for the theoretical case of size ratio equals zero, where, for example, the fine particles can be considered to be infinitely small. Thus, the partial specific void content of any coarse aggregate (C_p) equals zero because coarse aggregate particles can, under this assumption of no dilation of the other component, be added to a system of fine aggregate without any addition of voids to the mix; i.e., such particles add their own volume, but no more than their own volume, to the system.

For the fine aggregate (again when size ratio equals zero), it is possible to add small particles, up to a certain proportion, to a system of coarse aggregate particles without increasing the bulk volume of the system. In other words, these particles reduce the void content of the system. The partial specific void content of the fine aggregate (F_p) is therefore negative (-1), each added particle deducting its own volume from the total void content.

The lines drawn as indicated from C to F_p and from F to C_p (Fig. 1) indicate the drop in voids that would occur by adding increasing quantities of one component to the other. Beyond the point of intersection O, from whichever direction it is approached, the opposite trend takes over. The point of intersection therefore represents the lowest possible void content for the combined fine and coarse aggregates blended at their optimum proportions.

For mixes of finite size ratio, i.e., between 0 and 1, the presence of one component will always dilate the other. Thus, no void contents lower than the construction lines drawn can exist, and the total range of void contents for all possible blends lies within the shaded triangle bounded by the straight line (size ratio equals one) and the upper portions of the size-ratio-equals-zero lines (i.e., FO and OC).

For the theoretical case of size ratio equals zero, Figures 1a, 1b, 1c, and 1d show that the optimum proportions for minimum voids are dependent on the shape characteristics of the aggregate. For example, if both fine and coarse aggregates are of high sphericity, high roundness particles packing individually to low porosities of, say, 30 percent, then a low percentage of fines (23.1) is indicated (Fig. 1a).

However, if both components are angular, packing to high voids values of, say, 60 percent, then the optimum percentage of fines is higher, i.e., 37.5 (Fig. 1b). Still more extreme differences in the optimum proportions are noticed if aggregates of contrasting shape properties are combined, e.g., 47 percent fines for an angular coarse aggregate combined with a rounded fine aggregate (Fig. 1c) contrasting with 21 percent fines for a rounded coarse aggregate combined with an angular fine aggregate (Fig. 1d).

Figures 1e and 1f show further that, even if the aggregate type is not changed, a difference in compactive effort itself produces a difference in the optimum proportion, from 28.6 to 33.3 percent fines in the case illustrated.

The further effect of boundaries on packing is shown in Figures 1g and 1h, which show that, even if aggregate type and compactive effort are held constant, a change in the dimensions of the section to be filled also leads to a change in the optimum proportions, from 28.6 to 40.0 percent fines in the case shown. It has been hypothesized in this case that, although both fine and coarse aggregates would be affected by boundary walls so as to increase their porosity, the coarse aggregate would be more affected by a reduction in the size of the container than the fine aggregate (10, 11).

Although all effects in Figure 1 have been shown by reference to the theoretical case of size ratio equals zero, it has been established that, for real systems with finite size ratio, the same trends are apparent but that the value of the optimum is affected by the size ratio.

In the light of this analysis, the approach in the present study has been to consider that a common law might exist for the proportioning of particles in a two-component system, based on the size ratio and on the measured porosities of each of the components but irrespective of the means by which the porosities were produced.

In application of the method to the design of two-component and multicomponent systems, the following procedure is adopted:

1. Representative samples of the aggregates to be used are obtained by accepted sampling procedures. The sizes to be tested are chosen as those that are to be used

in the mix in question. Thus, for a continuous grading, all sizes from maximum to minimum are tested in the same arbitrarily divided subgroups as those from which the final mix will be made up. If the mix is to be intermittently graded, only the selected sizes will be subject to test.

2. Tests are performed to determine the mass porosity (i.e., excluding internal porosity) of the aggregate components. These tests are performed under conditions that simulate as closely as possible the field case with respect to container dimensions and compactive effort.

A container is made up in a laboratory, which simulates the field section, i.e., vertical slab, horizontal slab, column, etc., as closely as practical, with or without reinforcement as designed. The dimensions of the container will normally include the minimum dimension(s) of the field section. Thus, in the case of design for a slab, the minimum dimension of the container will be the slab thickness. The remaining dimensions are not critical in this case. In practice, it is considered satisfactory if the remaining dimensions are at least two to three times the minimum thickness.

The compactive effort applied to the aggregate is determined on the basis of the anticipated effort on site. Heavy compaction with a vibrating plate is, for example, simulated in the laboratory by vibration of the aggregate components to a maximum density condition on a vibrating table. The maximum density condition is assessed as that at which no further decrease in volume can be observed with time or with change in amplitude of vibration. Hand placing with no vibration would be simulated by dropping the aggregate at random into the container from a scoop held approximately 2 in. above the aggregate level and applying no further effort.

3. For each of the components, the mean equivalent spherical diameter (ESD) is calculated [ESD = $\sqrt[3]{(6V)/\pi}$, where V = average volume per particle, calculated from the average weight per particle and the known bulk specific gravity, taking not less than 500 particles]. The ESD for particles too small to be counted out is obtained by extrapolation from a series of determinations on larger aggregate from the same source.

4. The calculated values of porosity and equivalent spherical diameter (columns 2 and 3, Tables 1 and 2) are input to a computer program that has been prepared for the design method (12), from which the output is the data in columns 4 through 12 of these tables. The output gives the design grading as proportionate volumes of the component sizes (column 8). The completion of the design by conversion to proportionate weight (column 13, Table 1; column 14, Table 2) percentage of weight (column 14, Table 1; column 15, Table 2), and summation percentage of weight (column 16, Table 2) are separately performed by reference to specific gravity differences among component aggregates.

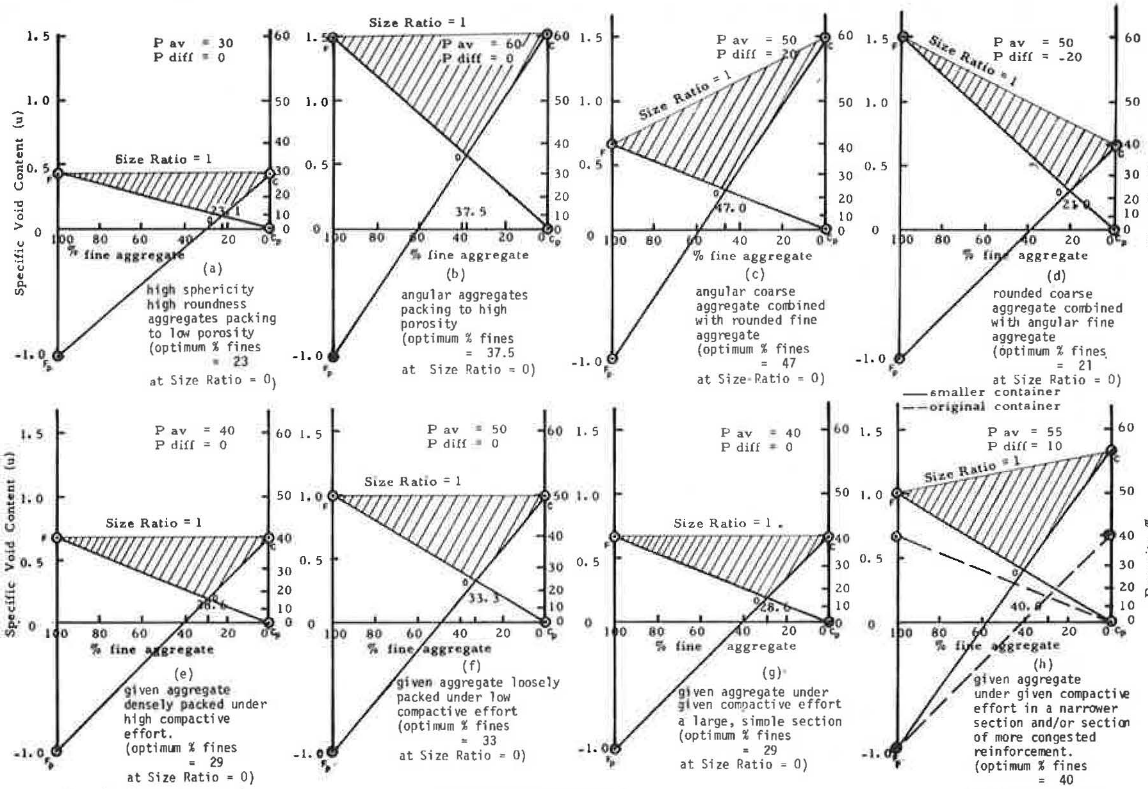
It should further be noted that, if the aggregate has been tested according to quarry sizes (i.e., with its attendant oversize and undersize material), the design percentage of weights will be relevant to the quarry sizes. A grading curve for the mix is plotted by correcting for oversize and undersize and calculating the summation percentage on this basis.

Table 3 gives the application of the method to two- and three-component mixes of a wide variety of particle shape combinations. Predicted porosities for the proposed method at the two-component stage and the final three-component stage are generally close to the measured value with a similar order of accuracy. Mixes with up to 15 components have similarly shown good agreement between predicted and measured porosities (1).

Design of Continuous Gradings

In this type of grading system, there is by definition some representative portion of each size between the selected maximum and minimum sizes (and normally continuous with the size of cement particles in the case of concrete). It has sometimes been taken that a continuous system will be of the form of $p = (d/D)^n \times 100$, where p = percentage finer than size d, d = the particular sieve size considered, D = the maximum size, and n = an exponent (generally in the range of 0.2 to 0.5). Also, much of the literature has been devoted to discussion of the merits of the various values of n.

Figure 1. Effect of particle shape, compactive effort, and boundary on optimum percentage of fine aggregate for maximum density in two-component mixes.



Note: Specific void content = e/(i - e) where e = unit voids in a system of packed aggregate particles.

Table 1. Computation for continuous grading for concrete (mix C, Table 5).

Sieve Size (1)	Porosity (percent) (2)	Equivalent Spherical Diameter (mm) (3)	Porosity Avg[a] (percent) (4)	Porosity Difference[b] (percent) (5)	Size Ratio[c] (6)	Optimum Percentage of Fine[d] (7)	Proportional Volume[e] (8)
½ in. to ⅜ in. (crushed rock)	46.5	11.24	—	—	—	—	50.0
⅜ in. to ¼ in.	45.9	7.51	46.2	+0.6	0.66	50.0	50.0
¼ in. to No. 7	45.0	4.73	44.8	-0.3	0.5253	40.9	69.2
3/16 in. to No. 200 (concrete-sand)	28.1	0.414	35.3	+14.4	0.073	41.6	120.0

Sieve Size (1)	Cumulative Volume[f] (9)	Mean Equivalent Spherical Diameter[g] (mm) (10)	Relative Contraction[h] (11)	Mix Porosity (percent)[i] (12)	Proportional Weight (13)	Percentage of Weight (quarry sizes) (14)
½ in. to ⅜ in. (crushed rock)	50.0	11.24	—	46.5	50.0	17.7
⅜ in. to ¼ in.	100.0	9.00	0.048	44.7	50.0	17.7
¼ in. to No. 7	169.2	5.729	0.088	42.5	69.2	24.5
3/16 in. to No. 200 (concrete-sand)	289.2	0.91	0.450	20.8	113.0	40.0

[a] Average of b(line n)/l(line n - 1).
[b] l(line n - 1) - b(line n).
[c] c(line n)/j(line n - 1).
[d] Derived from reference 1 or computer program.
[e] Proportional volume = [i(line n - 1) x g(line n)]/[100 - g(line n)].
[f] Sum of column 8.
[g] Mean ESD = $1 \Big/ \left[\sum_{i=1}^{n} (p_i/d_i) \right]$ where $p_1, p_2, \ldots p_n$ are the proportions by volume of particles of diameter $d_1, d_2, \ldots d_n$ respectively for any number of components.
[h] Derived from reference 1 or computer program.
[i] Mix porosity = a - relative contraction [a - (p_{coarse} x p_{fine}/100)] where a = p_{coarse} or p_{fine}, whichever is the lower; relative contraction = column 11; p_{coarse} = column 1 (line n - 1); and p_{fine} = cloumn b (line n).

Table 2. Design calculations for four-component gap-graded concrete.

Sieve Size (1)	Porosity (percent) (2)	Equivalent Spherical Diameter (mm) (3)	Porosity Avg (percent) (4)	Porosity Difference (percent) (5)	Size Ratio (6)	Optimum Percentage of Fine (7)	Proportional Volume (8)	Cumulative Volume (9)
½ in. to ⅜ in. (basalt)	45.7	10.06	–	–	–	–	48.35	48.35
No. 7 to No. 14 (sand)	32.6	1.73	39.1	+13.1	0.172	51.65	51.65	100.0
No. 36 to No. 52	37.0	0.3651	33.52	-6.96	0.127	15.96	18.97	118.97
No. 150 to No. 200	37.5	0.09164	29.43	-16.06	0.067	10.7	14.4	133.37

Sieve Size (1)	Mean Equivalent Spherical Diameter (mm) (10)	Relative Contraction (11)	Calculated Mix Porosity (percent) (12)	Actual Mix Porosity (percent) (13)	Proportional Weight (14)	Percentage of Weight (15)	Summation (percent) (16)	Size Ratio[a] (adjacent) (17)
½ in. to ⅜ in. (basalt)	10.06	–	45.7	–	48.35	37.5	100.0	–
No. 7 to No. 14 (sand)	2.88	0.145	30.0	28.5	47.9	37.2	62.5	0.172
No. 36 to No. 52	1.37	0.455	21.4	21.1	18.55	14.4	25.3	0.211
No. 150 to No. 200	0.544	0.47	15.9	16.7	14.05	10.9	10.9	0.251

Note: Further explanation of columns 4 through 12 is given in Table 1, footnotes a through i.
[a]c(line n - 1)/c(line n).

Table 3. Composition and predicted porosities.

Mix	Material	British Standard Sieve Size	Recommended Proportions (percent)	Predicted Porosity (percent)	Measured Porosity (percent)
G1	Rounded gravel	¾ in. to ½ in.	60.3		
	Crushed gravel	¼ in. to 3/16 in.	28.4	31.0 (2 components)	30.5
	Leighton buzzard sand	No. 14 to No. 25	11.3	19.8 (3 components)	21.9
B1	Basalt (equidimensional)	1½ in. to 1 in.	34.0		
	Crushed gravel	5/16 in. to ¼ in.	46.9	30.0 (2 components)	32.1
	Leighton buzzard sand	No. 7 to No. 14	19.2	23.0 (3 components)	24.5
B2	Basalt (disks)	1½ in. to 1 in.	35.9		
	Crushed gravel	¼ in. to 3/16 in.	46.7	30.3 (2 components)	32.2
	Leighton buzzard sand	No. 14 to No. 25	17.5	20.7 (3 components)	23.5
B3	Basalt (blades)	1½ in. to 1 in.	17.7		
	Crushed gravel	5/16 in. to ¼ in.	49.9	32.0 (2 components)	34.8
	Leighton buzzard sand	No. 7 to No. 14	32.3	24.5 (3 components)	24.4
B4	Basalt (rods)	1½ in. to 1 in.	31.3		
	Crushed gravel	5/16 in. to ¼ in.	51.8	30.0 (2 components)	31.8
	Leighton buzzard sand	No. 7 to No. 14	17.0	24.0 (3 components)	24.6
B5	Basalt (equidimensional)	½ in. to ⅜ in.	45.3		
	Crushed basalt	3/16 in. to No. 5	30.3	34.9 (2 components)	33.8
	Leighton buzzard sand	No. 14 to No. 25	24.4	22.8 (3 components)	23.9
B6	Basalt (rods)	½ in. to ⅜ in.	42.5		
	Crushed gravel	3/16 in. to No. 5	36.2	34.5 (2 components)	33.1
	Leighton buzzard sand	No. 14 to No. 25	21.4	21.9 (3 components)	24.7
B7	Basalt (equidimensional)	1 in. to ¾ in.	57.5		
	Crushed gravel	3/16 in. to No. 5	30.9	29.2 (2 components)	30.6
	Leighton buzzard sand	No. 14 to No. 25	11.6	21.7 (3 components)	23.9
B8	Basalt (disks)	1 in. to ¾ in.	37.8		
	Crushed gravel	3/16 in. to No. 5	40.0	32.1 (2 components)	33.0
	Leighton buzzard sand	No. 14 to No. 25	21.3	22.8 (3 components)	23.5
B9	Basalt (blades)	1 in. to ¾ in.	31.4		
	Crushed gravel	3/16 in. to No. 5	44.4	32.1 (2 components)	33.7
	Leighton buzzard sand	No. 14 to No. 25	24.3	22.8 (3 components)	24.6
B10	Basalt (rods)	1 in. to ¼ in.	35.6		
	Crushed gravel	¼ in. to 3/16 in.	44.3	31.0 (2 components)	32.8
	Leighton buzzard sand	No. 14 to No. 25	20.1	20.8 (3 components)	22.7

However, the author proposes that a more rational approach is to adjust the proportion of any constituent in accordance with its own packing properties. This is the effect of the proposed design method in that the calculations result in steeper portions of the grading curve (hence allowing greater quantities) for components that have good packing properties, i.e., tend to pack to low porosities, and less steep portions (hence limiting the content) for components with poor packing properties.

Thus, for example, the fallacy is revealed of introducing a quantity of crushed gravel in order to "correct" a grading that deviates from some supposed ideal "type" grading by absence of some middle sizes, without paying attention to the packing properties and effects of the added constituent.

The "type" gradings given in Road Note 4 (5) and similar standards are subject to criticisms similar to those that follow unquestioningly some oversimplified mathematical law because these gradings do not take account of the properties of the aggregates that are to comprise them. Further, as has been stated by Hughes (13), "the object of good mix design is to utilize the available material as economically as possible so as to obtain a hardened concrete of the required minimum quality, and any arbitrary preference for particular aggregate gradings tends to defeat this object." Although "type" gradings have the advantage that they are easy to apply and give generally reliable concretes in practice, they tend to favor richer mixes and to lead to the rejection of aggregates that do not conform to the grading but that would be capable of being used successfully in a mix designed to suit their special characteristics.

An example of a computation for a continuous grading by the proposed method is given in Table 1, and a section of a concrete mix made to this grading is shown in Figure 2.

The method adopted for the completion of the design of a concrete mix from the designed aggregate grading is described following the section on gap gradings.

Design of Gap Gradings

Gap gradings are defined as systems of aggregate in which certain size components are missing. The component(s) may be excluded either by circumstance or by choice.

The design method proposed may be utilized in either case. If circumstance (i.e., availability of aggregate sizes) dictates the absent size(s), the method involves no change from that employed for continuous gradings. If with the object of minimizing the porosity of the grading a choice of sizes is made, it is suggested that this choice be based on consideration of three properties of the void system within the packed aggregate, namely, the critical ratios of occupation, entrance, and dilation. The first two of these terms were originally defined by Fraser (14) in terms of the diameters of the largest spheres that could respectively (a) occupy the void spaces in the structure and (b) pass along the "throats" between adjacent cells. These diameters were expressed as ratios to the diameters of the host particles. Fraser computed ratios of entrance and occupation for the loosest (cubical), densest (rhombohedral), and intermediate packings of spherical particles. Lees (15) described the application of these concepts to systems of typical aggregate particles, i.e., to loosely and densely packed gravels and crushed rock of various shape categories (Fig. 3), and determined the critical ratios for such systems by means of a void impregnation and dissection technique. Measurements by this technique indicated that, for a wide variety of particle shapes, these values for loose packing are in the region of 0.22 to 0.29 for the critical ratio of entrance and 0.33 to 0.44 for the critical ratio of occupation.

In application of the study of void characteristics to the design of gap gradings, it is the author's view that the optimum size ratio to be employed in selection of sizes should lie between the critical ratios of entrance and occupation for loose packing of typical nonspherical aggregates.

The reasons for choosing these conditions are as follows (it is assumed throughout that maximum and minimum sizes are fixed):

1. Loose packing—The densest packing of one component cannot exist in the presence of other components; i.e., dilation of one component by the other will always occur in cases of finite size ratio. The dense packing case is therefore irrelevant where two or more aggregate size groups are combined.

2. Typical concrete aggregates—The critical ratios for packings of spherical particles are of academic interest only and have no relevance to the packing of typical concrete aggregates. Compare the critical ratios for cubical and rhombohedral packing of spheres with those for loose and dense packing of typical aggregates given in Table 4.

3. Lower limit, critical ratio of entrance—If a very low size ratio is employed, voids within the coarser aggregate of a considered pair will be occupied by groups of very small particles plus their attendant voids. With a higher size ratio, opportunity is provided for comparatively larger particles to occupy a proportion of these sites, replacing solid plus voids with solid matter. A greater number of component sizes are therefore included than when the size ratio is small, and hence the porosity of the aggregate mass, using optimum proportions of the components, is lower than can be achieved with gradings based on smaller size ratios.

4. Upper limit, critical ratio of occupation—Size ratios greater than the critical ratio of occupation imply that the coarser aggregate particles of the considered pair are so far separated as to have largely lost contact with one another even when there is only one particle of the finer size present in each void. With still higher ratios, this dispersal effect intensifies, the number of components becomes large, and in the limit the grading becomes continuous. In this condition, the advantage of increasing the number of components is offset by the tendency to disperse the larger particles.

5. Size ratio between the limits of critical ratios of entrance and occupation—Use of a size ratio between these limits acts to prevent segregation of the fine particles from the coarse during normal handling of the bulk concrete because the fine particles would be too large to filter out from the voids within the framework of coarser particles.

Critical Ratio of Dilation

It was mentioned previously that the particles of the coarser component of any pair would begin to lose contact with each other should the size ratio with the fine component exceed the critical ratio of occupation, even if only one such fine particle existed in each coarse aggregate void. Clearly, even at or below the occupation size, similar dispersal of the coarse component would occur if more than one fine aggregate particle per void was in place.

These considerations led to the concept of the critical ratio of dilation, which is defined as the size ratio at which the fine aggregate dilates the coarse aggregate to the state of its loosest packing when both components are combined at their optimum proportions, at the compactive effort designed to produce the densest state of packing. The use of aggregates to this size ratio aims at ensuring that the coarser particles are not dilated away from each other beyond the point of mutual contact in a fully compacted mix. In doing so, it recognizes that the choice of size ratio should not be made in isolation but only in consideration of the proportions in which the components will be mixed.

The concept of the critical ratio of dilation (CrD) is shown in Figure 4. For size ratios greater than CrD, the dilation of the coarse aggregate by the fine would be to porosities greater than the maximum porosity, and thus the coarse aggregate particles would begin to lose contact within the mix. For low size ratios less than CrD, the coarse aggregate is dilated to a state between its own densest and loosest packings. This in itself is satisfactory for two-component mixes, but in the context of multi-component mixes too low a size ratio may lead to some of the disadvantages mentioned previously, such as an insufficient reduction of porosity in the total mix and segregation of fine aggregate and coarse aggregate.

An analysis of experimental data obtained in the laboratory showed that, for a wide variety of shape combinations, the critical ratio of dilation lay in the region of 0.23. This value, it will be noted, lies within the range of critical ratios of entrance and occupation previously noted (0.22 to 0.44), confirming the conclusion drawn from studies of interparticle voids that, for rational gap-grading design, the size ratios chosen should preferably lie within this range, preferably toward the lower end of the range. Availability of aggregate sizes is such that it is not always possible to satisfy the criterion of size ratio very closely, and it is not suggested that the choice is critical. A number of gap-grading designs have now been made up according to the proposed method utilizing size ratios as near as possible to 0.23. Table 2 gives one example of these gradings, and Figure 5 shows a section of the concrete made to this grading.

Brief comment may be made on other methods of design of gap gradings for concrete that has been developed by other workers. Bate and Stewart (16) and Stewart (17) point to the desirability of avoiding honeycombed zones in a concrete, which might arise out of an inability of the fine particles to filter into the voids between the coarse aggregate. Such an aim leads to their adoption of the "admittance size," a synonym for the entrance size, as the basis of design. These authors refer to the critical ratio of entrance of 0.154 calculated by Fraser for rhombohedral packing of spheres, but Stewart states that in practice the admittance size "will be found to be 0.125D, where D = diameter of coarse aggregate." This represents a smaller size ratio than that recommended previously. In this approach the fine aggregate is generally considered as one component and the coarse aggregate as another, i.e., a two-component system.

Vallette (18, 19) recommended size ratios in the order of 0.20 to 0.33 for multi-component gap-graded mixes with the stated aim that each size of particle shall be able to fit into the voids of the next larger size. A close similarity is noted between these values and those proposed in this paper. Bahrner (20) commented on Vallette concrete as having the appearance of "a heap of stones when it leaves the mixer. If, however, it is properly proportioned and placed in the forms, it will be found that after vibration (all Vallette concrete has be be vibrated) it will yield a very smooth and uniform surface." This has also been the author's experience in gap gradings designed to the method proposed in this paper.

Bahrner proposed a modification of Vallette concrete that would allow the fine aggregate to be used as one undivided aggregate, in view of the difficulties that would sometimes be found in obtaining the closely defined single sizes of fine aggregate proposed by Vallette. This he referred to as skeleton concrete (a return to a two-component system). It is to be noted that a 10 percent cement mortar surplus was recommended over that which would be calculated from the void-filling criterion adopted for design purposes. For too low a cement mortar surplus, in this case 0.5 percent, Bahrner observed that the mix lacked cohesion and was subject to segregation.

Concrete Mixes With Designed Aggregate Gradings

An aggregate grading (gap or continuous) designed by the method here proposed may be included in a concrete mix by any of a wide variety of published methods of concrete mix design (including methods of trial mix formulation). However, in a further stage of development of the present design process, the method adopted for the completion of the design of a concrete mix from the designed aggregate grading has been as follows:

1. Select water-cement ratio on the basis of required level of compressive strength (5, Fig. 1; 21). A control ratio appropriate to the anticipated level of supervision on site will also be used to determine the required level of compressive strength, as in normal practice (5, 21).
2. From the calculated specific gravity of the unhydrated cement paste at the selected water-cement ratio, determine the weight percentage of paste to fill the voids in the designed grading as previously computed.
3. Hence, determine the cement content, water content, and aggregate content (as percentage by weight of total mix).

The following is an example of this method as used to compute a four-component gap-graded concrete (Table 2):

1. Selected water-cement ratio = 0.4;
2. Specific gravity of ordinary portland cement = 3.15;
3. Bulk specific gravity of mixed aggregate = 2.725;
4. Voids in mixed aggregate = 15.9 percent (in 2.5-in. deep mold, with maximum compaction by vibration table);

5. Specific gravity of water-cement ratio = 0.4, and specific gravity of unhydrated cement paste = [(28.5 x 1) + (71.5 x 3.15)]/100 = 2.54;

6. Hence, 15.9 percent (volume) = (15.9 x 2.54)/[(15.9 x 2.54) + (84.1 x 2.725)] = 14.98 percent (by weight); and

7. Therefore, cement content = (14.98 x 71.5)/100 = 10.71 percent (by weight), water content = (14.98 x 28.5)/100 = 4.27 percent (by weight), and aggregate content = 85.02 percent (by weight) (aggregate-cement ratio ≃ 8:1).

The characteristics of this mix were as follows: compacting factor 0.84; Vebe time 25 sec; compressive strength (7-day) 5,530 psi.

In the case of air-entrained mixes, adjustments can be made in the calculation of cement content and water content to allow for the volume of air required; e.g., if the total required entrapped plus entrained air = 4 percent, then, in the case of the preceding mix, the following holds:

1. Voids to be filled by cement paste = 15.9 - 4 percent = 11.9 percent (by volume);
2. 11.9 percent volume = 11.18 percent wt (as calculated previously);
3. Therefore cement content = (11.18 x 71.5)/100 = 7.99 percent (by weight); and
4. Water content = (11.18 x 28.5)/100 = 3.19 percent (by weight) and aggregate content = 88.82 percent (by weight) (aggregate-cement ratio ≃ 11:1).

Workability—It will be noted that the mixes proportioned as described have been designed with no apparent reference to workability. It is recalled, however, that the measurements of porosity on which the design is based are carried out with as close a simulation as possible to anticipated compactive effort and to the external and internal geometry of the site section. The aggregate grading and the aggregate, cement, and water contents are thus all determined with reference to factors that are recognized as imposing different requirements for workability. It follows from the normal trend of porosity values measured in these tests that low compactive efforts, narrow or complex sections (including narrow pipes for pumped concrete), sections with congested reinforcement, etc., all of which demand high workability mixes, will, by their influence on the design factors, tend to increase the content of the finer aggregate components and to decrease the aggregate-cement ratio, compared with mixes designed for high compactive effort and simple, massive, and unreinforced sections. Maximum aggregate size is adjusted to the site section geometry in the proposed method, as in all recognized mix design methods.

The proposed method determines the aggregate grading and the mix proportions from measurements that are related to the ability of the aggregate particles to pack together under given conditions of boundary interference and effort. The criterion of workability is thus included as an intrinsic part of the initial design procedure of aggregate grading and selection of mix proportions in contrast to the more usual assumption that the grading would be a prior fixed parameter and that the workability requirement would be sought as a final stage in the mix design process by adjustment of the aggregate-cement ratio.

In the mixes designed according to this method, a check on workability has been made by use of the compacting factor and Vebe tests (Tables 5 and 6).

Strength and Economic Factors—The object of the proposed design method is to take account of the particular packing properties of the component aggregates, therefore allowing a more effective and more economic use of available aggregates including those of nonstandard shape and grading characteristics, and to relate these in terms of density and workability to the anticipated site compactive effort and site geometry. In an early trial it has been confirmed that leaner mixes of equivalent strength can be designed by the proposed method than by following a standard mix formulation with the same aggregates. For example, mix C (proposed design method, Tables 1 and 5) and mix S (standard mix, 5, 6) were both designed to give a 28-day strength of 4,160 psi (to satisfy the strength requirement of 4,060 psi for pavement quality concrete, 22). Assuming good site supervision, a 75 percent control factor was used. Thus the designed strength became 5,550 psi, giving a water-cement ratio of 0.6 (5). A high compactive effort requiring only a very low workability mix was assumed in both cases.

Figure 2. Continuously graded concrete (mix C).

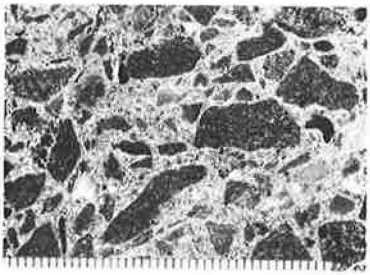
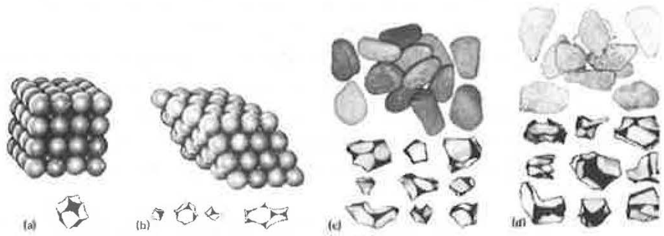


Figure 3. Interparticle voids.



(a) cubical packing of uniform spheres
(b) rhombohedral packing of uniform spheres
(c) single size rounded gravel aggregate (blades)
(d) single size crushed basalt aggregate (blades)

Table 4. Critical ratio of entrance and critical ratio of occupation values.

	Loose Packing		Dense Packing	
Aggregate	CrE	CrO	CrE	CrO
Spheres	0.414	0.732	0.154	0.225, 0.414
Rounded gravel (average)	0.24	0.37	0.21	0.32
Crushed igneous rock (average)	0.28	0.41	0.23	0.40

Figure 4. Critical ratio of dilation.

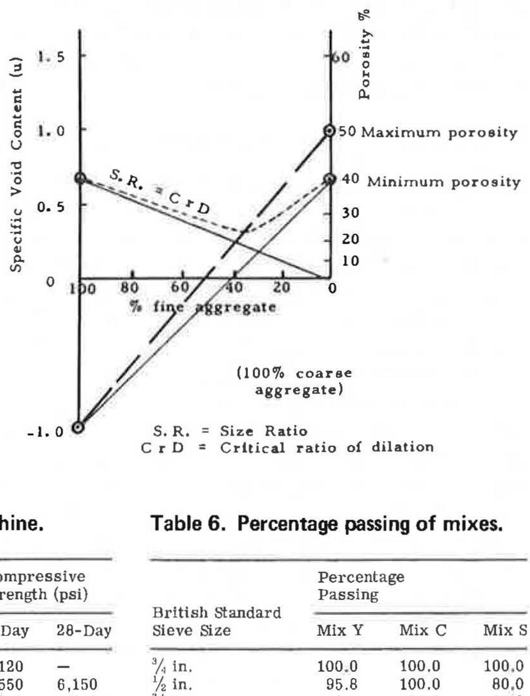


Table 5. Mixes tested on variable-speed internal drum machine.

Mix	Texture	Water-Cement Ratio	Aggregate-Cement Ratio	Compacting Factor	Vebe Time (sec)	Compressive Strength (psi) 7-Day	28-Day
Y[a]	Open	0.6	7.2	0.80	21	3,120	—
C[a]	Closed	0.6	8.5	0.73	30	4,550	6,150
S[b]	Closed	0.6	6.3	0.84	4	3,260	5,460

[a] Lee's design method used. [b] Road Note 4 (5) method used.

Table 6. Percentage passing of mixes.

British Standard Sieve Size	Percentage Passing Mix Y	Mix C	Mix S
3/4 in.	100.0	100.0	100.0
1/2 in.	95.8	100.0	80.0
3/8 in.	89.6	81.5	55.0
1/4 in.	81.2	60.1	43.0
3/16 in.	49.7	34.7	35.0
No. 7	22.9	27.3	28.0
No. 25	9.9	20.4	14.0
No. 100	2.3	3.5	2.3
No. 200	0.7	0.6	0.4

In the proposed design method mix, this was related to the very high compactive effort utilized in obtaining the design factors of aggregate porosity. In the standard mix, the aggregate-cement ratio for the given water-cement ratio was read off the column for "very low workability" (5).

The aggregates (a Precambrian graywacke and a concreting sand of glaciofluvial origin) were assumed to be in an air-dry condition, and the appropriate adjustment in water-cement ratio was made to allow for water absorption (6).

Both mixes were air-entrained for frost resistance, allowing an air content of 4.5 percent, by addition of an air-entraining agent at the rate of 1 cc of agent to 2 kg of cement.

Mix C slightly surpassed the design strength of 5,550 psi, with 6,150 psi at 28 days, with an aggregate-cement ratio of 8.5. The standard mix, mix S, achieved slightly under the design strength, i.e., 5,460 psi with an aggregate-cement ratio of 6.3.

The results indicate that satisfactory results can be achieved with leaner, and hence more economical, mixes by the proposed method.

As in most cases where a rational approach to mix design has been recommended, it is recognized that any complexity in the method may dictate that it is uneconomic to use that method for small jobs. The suggestion is made that the present method is most applicable where comparatively large amounts of concrete are to be placed and where site conditions will remain uniform with respect to aggregate type, section dimensions, reinforcement, etc. for a reasonable period.

Laboratory Studies of Skid-Resistant Concrete

It has elsewhere been shown (23) that the grading design method can be used to design bituminous mixes of controlled void content whose special feature is that they have a dense, impermeable internal structure but at the same time possess a surface with a system of intercommunicating channels of high drainage efficiency. The anomaly is explained by the special characteristics of voids at a boundary. The boundary in this case is the upper compacted surface. It is suggested that the same requirements exist and the same design philosophy can be applied in concrete as in bituminous surfacings.

Attention has also been drawn by Holmes, Lees, and Williams (24) to the different bulk water drainage requirements of low-speed and high-speed sites, and the suggestion is made that, where circumstances are appropriate, surfaces should ideally be designed to suit the special skid resistance requirements of the highway or airfield environment. It is in this context that the present design method offers scope for design of a range of surface macrotextures to suit particular highway and traffic circumstances. The importance of producing and preserving a suitable level of microtexture for all categories of speed has also been stressed by these authors.

Valuable work has been done by members of the British Road Research Laboratory (25-28) and others on the influence on skid resistance of cut, flailed, and formed grooves and of fine and coarse aggregate types. The present section describes an initial study that is in progress on the design of concrete mixes of high skid resistance properties, as an alternative to grooving treatments or for areas in which such treatments are not available.

Two contrasting types of concrete surfacing have been designed in accordance with the proposed method, namely, an open-surface texture mix (mix Y)(Fig. 6) and a close-surface texture mix (mix C). These two mixes and the mix designed to standard composition (mix S)(6) have been compared, all at a water-cement ratio of 0.6, for cube-crushing strength and also for skid resistance properties on the variable-speed internal drum machine (Fig. 7). This machine, designed by Williams and Lees (29), is, so far as these authors are aware, the only laboratory machine in existence on which it is possible to carry out with a tire wear-polishing cycles and rolling and locked-wheel friction tests on any chosen tire compound-road material combination on the same apparatus. Its main features are as follows:

1. It can be used for studies of complete concrete and bituminous surfacing materials;
2. The wear-polishing procedure can be varied by use of different water flows, types of abrasive, and tire slip-angle conditions;

3. The wear on the surface (e.g., under the action of normal and spiked tires) and on the tire tread can be determined;
4. The tire-road friction can be measured in both the peak rolling and the locked-wheel slide conditions; and
5. These friction coefficients can be obtained over a range of speeds from 0 to 70 mph and under various water-depth, slip-angle, inflation-pressure, and load conditions.

Some properties of the concrete mixes studied in this investigation are given in Tables 5 and 6. The coarse aggregate used for all mixes was a Precambrian gritrock of high polishing resistance. The fine aggregate used in mix Y was the crushed fines of this rock and in mix C and mix S was a concreting sand (Zone 2, British Standard 882)(30). In all tests the sample surface was wetted with a thin film of water, which was maintained at constant thickness by adjustment of the rate of water flow relative to the varying drum speed. Tires used were all plain-treaded cross-ply tires with a standard tread compound. Separate groups of tires were used for the wear-polishing cycles and for the braking tests to ensure that the results of the latter were not affected by tire wear.

A contact pressure of 32.8 psi, realistic in comparison with that of an average private automobile, was achieved with a 30-psi tire inflation pressure and a 140-lb wheel load.

Wear-polish cycles were carried out at a speed of 40 mph at a slip angle of 6 deg in wet conditions.

At an early stage in the test, following an initial calibration stage of some 50,000 revolutions, braking tests were performed at speeds of 20, 30, 40, 50, and 60 mph at 0-deg slip angle. Twelve tests were performed at each speed, and the peak and locked-wheel friction values were averaged.

Figure 8 shows the performance in locked-wheel friction of the 3 mixes over this range of speeds.

Confirmation of the frequently observed trend in road tests for the friction of close-textured surfaces to decrease more rapidly with speed than that of open-textured surfaces is clearly seen; e.g., the open-textured surface mix Y maintains a friction torque value of 4.70 m-kg at 60 mph (≡4.70 coefficient of friction) compared with that of 2.54 m-kg (≡2.54 coefficient of friction) for the standard close-textured surface mix S. These mixes had virtually identical friction values (5.73 m-kg ≡5.73 coefficient of friction) at 20 mph.

Further braking tests have been performed at intervals of 50,000 revolutions of wear-polish cycles. The braking tests in this series have been performed at 30 mph. No change in the order of merit has been observed up to 400,000 revolutions. Further tests and analysis of results are in progress.

CONCLUSIONS

A new method for the design of aggregate gradings has been applied to the design of continuously graded and gap-graded concretes. For normal dense concretes, the grading and cement content are calculated on the basis of simple laboratory tests for aggregate packing relative to the anticipated compactive effort on site and the geometric complexities of the site section, i.e., to factors that control the aggregate structural arrangement and the workability requirement of the mix. Concrete mixes made by using this method in the laboratory have been shown to possess high strength for economical compositions.

Concrete mixes can also be designed that have a controlled void content above the minimum possible. The application of this facility to the design of open-surface texture pavement quality concretes, which have good skid resistance at high speeds, is under investigation. Results to date show that these mixes possess good frictional properties that tend not to fall off as rapidly at high speeds as normal, dense, untextured concrete surfaces.

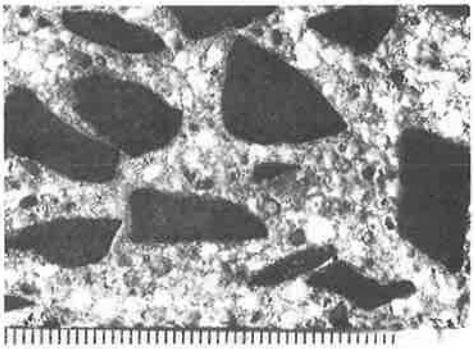


Figure 5. Four-component gap-graded concrete.

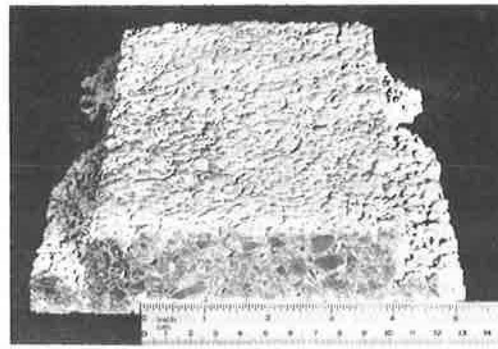


Figure 6. Surface texture and internal structure of mix Y.

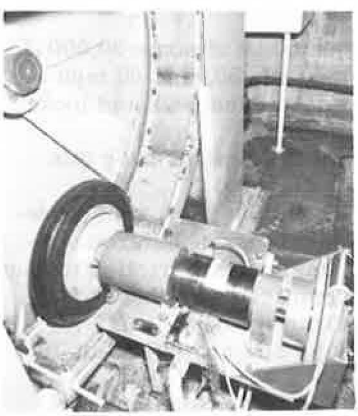


Figure 7. Variable-speed internal drum machine.

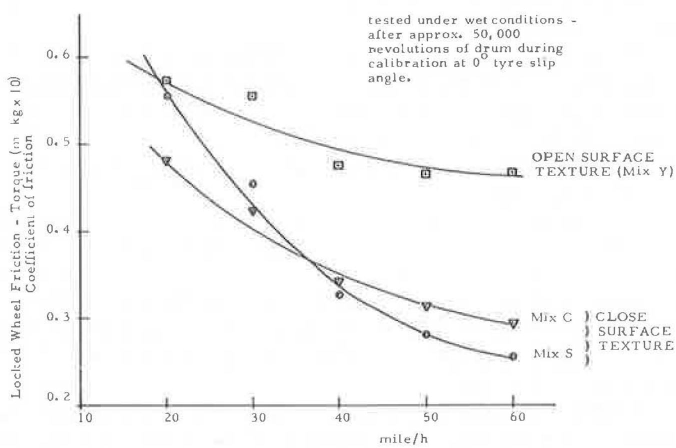


Figure 8. Friction versus speed on concrete surfaces.

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