METHODS FOR THE DETERMINATION OF REQUIRED BLENDING PROPORTIONS FOR AGGREGATES

Sandor Popovics, Northern Arizona University

The general numerical method and several of its special forms, along with three graphical forms, are presented for finding the required blending proportions for mineral aggregates. These methods are analyzed, their limits of applicability are given, and the special features, including the advantages and disadvantages of each method, are discussed. Eleven examples illustrate the application of the presented methods. These are also used to make the comparisons among the various methods more meaningful.

•IT often occurs in practice that two or more mineral aggregates of differing grading should be blended to comply with a given grading specification.

A set of blending proportions with given aggregates for approximation of a required grading can usually be obtained by trial and error, provided that such a set does exist. This method, however, is quite time-consuming and cumbersome. There are numerous other methods for estimating the needed proportions more directly.

The linear theory of blending, as well as several of its particular forms, is discussed in this paper. Although the methods discussed here pertain primarily to grading, most of them are suitable for other blending estimations as well.

GENERAL THEORY OF BLENDING

The common mathematical basis of the various graphical, semigraphical, and numerical methods for finding the needed blending proportions can be stated in the following way.

The fulfillment of (n - 1) grading conditions requires, in general, the combination of at least n aggregates of differing grading. These conditions can be written in the form of a system of (n - 1) simultaneous linear equations and inequalities containing the needed blending proportions as unknowns. To these an n th equation should be added to take care of the obvious condition, namely, that the sum of the blended aggregates is equal to the total of the combined aggregate. Thus, the general mathematical form of this system is something like

$$\left.\begin{array}{l}
c_{11} x_1 + c_{12} x_2 + \dots + c_{1n} x_n = b_1 \\
c_{21} x_1 + c_{22} x_2 + \dots + c_{2n} x_n = b_2 \pm \Delta b_2 \\
\vdots \\
c_{i1} x_1 + c_{i2} x_2 + \dots + c_{in} x_n \geq b_i \\
\vdots \\
c_{(p-1)1} x_1 + c_{(p-1)2} x_2 + \dots + c_{(p-1)n} x_n < b_{(p-1)} \\
x_1 + x_2 + \dots + x_n = 1
\end{array}\right\} \qquad (1)$$

where

c_{ij} = the i th grading characteristic of the j th aggregate to be blended,
x_j = the sought blending proportion for the j th aggregate,
b_i = the given i th condition for the combined grading,
p = number of grading conditions plus one, and
n = number of the aggregates to be blended.

n can be smaller, equal to, or larger than p. So long as $n \geq p$, there are, in general, an infinite number of sets of x_i-values (but at least one set) that satisfy this system. Out of these, however, only those sets represent blending proportions where none of the included x_i-values is less than zero because a negative blending proportion is physically meaningless. The lack of such a nonnegative set of solutions indicates that it is impossible to fulfill all the given (p - 1) grading conditions with the given n aggregates.

In other words, system 1 is applicable for every kind of blending problem of aggregates. In most cases it provides the exact solution (if there is any) to a grading problem. The disadvantage of the numerical method is that it becomes increasingly lengthy and cumbersome when n and/or p is larger than three, unless a computer is used.

It follows from the space-filling role of the aggregate particles that the correct way to express their blending proportions is by absolute volume. In this case, the grading of the combined aggregate should also be expressed in terms of absolute volume. Only when the specific gravities of all the given aggregates to be blended can be considered identical may the proportions be expressed by weight.

SPECIFIC FORM OF SYSTEM 1

The mathematically simplest form of system 1 is when p = n, and each condition is represented by an equation with a single number on the right-hand side. Here the number of the aggregates available for blending is one greater than the number of the given grading conditions. Mathematically this problem is represented by a system of n linear equations with n unknowns. Because the determinant of the coefficients cannot be zero in blending problems, there is one and only one set of solutions of this system. If none of the solutions is less than zero in this set, the solutions represent the only combination of the n given aggregates that can fulfill all the (n - 1) grading conditions.

When there are more equations than unknowns in the system, it is mathematically impossible, in general, to find an exact set of solutions for the system of equations. In other words, when the number of aggregates to be blended is less than the number of grading conditions plus one, all the grading conditions cannot be fulfilled in an exact fashion. It still may be possible, however, to find a set of blending proportions by the least-squares method that provides the best approximation for the combined grading to all the given conditions (1).

A typical case for the application of the system of n linear equations with p = n unknowns is when the conditions specify (n - 1) points within the 0 and 100 percent points through which the cumulative sieve curve of the combined grading must pass. A numerical illustration, for three aggregates, is given in example 1.

EXAMPLE 1

Three mineral aggregates, A, B, and C, are given. Their gradings are shown in Figure 1. Determine their x_A, x_B, and x_C blending proportions required to yield a combined grading that passes through the 20 percent point at the No. 30 particle size and the 40 percent point at the No. 4 particle size.

The needed three blending proportions can be calculated from the following system of linear equations where the coefficients were taken from Figure 1:

$$\left.\begin{aligned} 0.52x_A + 0.10x_B + 0.05x_C &= 0.20 \\ 0.92x_A + 0.60x_B + 0.10x_C &= 0.40 \\ x_A + x_B + x_C &= 1.00 \end{aligned}\right\} \qquad (2)$$

From this, $x_A = 0.303$, $x_B = 0.102$, and $x_C = 0.595$; that is, approximately 30 percent should be blended from aggregate A, 10 percent from aggregate B, and 60 percent from aggregate C.

When only two aggregates are to be blended, so that the combined sieve curve must pass through the y_o-point, the blending proportions can be calculated directly from the following two formulas, provided that y_o is between y_1 and y_2:

$$x_1 = \frac{y_2 - y_o}{y_2 - y_1} \tag{3}$$

$$x_2 = \frac{y_1 - y_o}{y_1 - y_2} \tag{4}$$

with the availability of the check

$$x_1 + x_2 = 1 \tag{5}$$

where y_1 and y_2 are the pertinent ordinates of the sieve curves of aggregates 1 and 2 respectively.

Equations 3 and 4 are obviously the solutions of the corresponding system of two linear equations but otherwise similar to Eq. 2. Thus, they take into account only one point, through which the combined grading curve is required to pass.

EXAMPLE 2

Use Eqs. 3 and 4 for the determination of the blending proportions for aggregates B and C shown in Figure 1 such that the amount of particles in the combined grading passing through a No. 4 sieve is 40 percent.

By taking the appropriate y-values from Figure 1,

$$x_B = \frac{10 - 40}{10 - 60} = 0.60 \text{ and } x_C = \frac{60 - 40}{60 - 10} = 0.40$$

The graphical interpretation of Eqs. 3 and 4 reveals that (a) the sieve curve of any combination of two aggregates must fall in its total length within the sieve curves of the two aggregates and (b) the distance between these two sieve curves at any particle size is divided by the combined sieve curve in the ratio of the x_1 and x_2 blending proportions employed.

This consideration may help one select a suitable value of y_o in a blending problem. It can also be seen in Figure 1 that there is no combination of aggregate A and B that could fulfill the 40 percent condition given in example 2.

Note also that there is a range of solutions for the linear system, instead of a unique solution, when (a) the number of aggregates to be blended is greater than the number of grading conditions plus one, and/or (b) some or all of the grading conditions are given in the form of inequalities or in the form of ranges rather than as single numbers.

The range of solutions provides a certain flexibility for the engineer in selecting the most suitable set of blending proportions based on economical or other considerations.

EXAMPLE 3

Calculate again the blending proportions for the case discussed in example 2, but with the condition that $y_o = 40 \pm 5$.

The upper and lower limits of the sought x_B blending proportion and the corresponding x_C-value can be calculated again by substituting successively $y_o = 45$ and $y_o = 35$ into Eqs. 3 and 4. The results are $(x_B)_{up} = 0.70$ and $(x_C)_{up} = 0.30$ and $(x_B)_{lo} = 0.50$ and $(x_C)_{lo} = 0.50$.

That is, any quantity of aggregate B between 50 and 70 percent combined with aggregate C in the corresponding quantity of 100 $(1 - x_B)$ percent yields a combined grading in which the amount of particles passing the No. 4 sieve is the required 40 ± 5

percent. From this range, one pair of blending proportions can be selected on the basis of the local conditions, such as the price and availability of the two aggregates.

Example 2 is a special case of the more general blending problem previously discussed.

A system similar to Eq. 2, and consequently formulas similar to Eqs. 3 and 4, can be used for the calculation of blending proportions when the fineness modulus, or the specific surface, is used for the grading characterization rather than the sieve curves. This is shown for two aggregates in the following example.

EXAMPLE 4

In what proportions should a fine aggregate with a fineness modulus of $m_f = 2.2$ be blended with a coarse aggregate of $m_c = 8.0$ to obtain a combined grading with a fineness modulus of $m_o = 5.4$?

By using formulas similar to Eqs. 3 and 4,

$$x_f = \frac{8.0 - 5.4}{8.0 - 2.2} = 0.45 \text{ and } x_c = \frac{2.2 - 5.4}{2.2 - 8.0} = 0.55$$

Graphical Methods in General

Certain forms of the linear system presented as system 1 may lend themselves conveniently to graphical or semigraphical solutions. Such methods can be advantageous beyond their visuality because they may be faster than numerical methods, they may provide the totality of the solutions, and they may be applicable for computer graphics. Their common disadvantage is that they are valid only for more or less special cases.

The limited accuracy or, in certain cases, the approximate nature of graphical methods is not particularly harmful because fluctuations in the gradings of the aggregates to be blended make a blending proportion precision better than 1 percent meaningless anyway.

First Graphical Method

An approximate graphical method for the determination of blending proportions, very much the same as the one to be described, was probably first offered by Rothfuchs (2). There are two restrictions concerning the applicability of the method: It is suitable only for grading problems when all the gradings are characterized by sieve curves, and no more than two of the n given individual sieve curves may overlap significantly at any point, as viewed from the horizontal axis. Even with these restrictions, however, the method cannot provide exact solutions for two reasons: The actual sieve curves of the aggregates to be combined are not used, but straight lines are substituted for their curves; and the larger the overlapping of two such grading straight lines, the poorer is the approximation of the solution obtained.

The main advantage of this method is its simplicity, and this simplicity is hardly affected by the number of aggregates to be blended. Thus, it is particularly useful for large n-values.

The mathematical justification of this method is that, when there are no double or triple overlappings in the n sieve curves, the system of the corresponding n linear equations can be arranged in the form of a Gaussian elimination algorithm. This permits the successive determinations of the unknowns (blending proportions) one by one. The graphical method discussed here is an approximate form of this procedure, as can be seen from a comparison with the presented graphical interpretation of Eqs. 3 and 4.

The use of the method is illustrated in example 5 for four aggregates.

EXAMPLE 5

Determine the blending proportions for the four aggregates shown by the continuous lines in Figure 2 such that the combined gradings approximate the specified sieve curve shown by dotted line. The procedure for solution is as follows.

The sieve curves of the four aggregates to be blended are approximated by fitting straight lines. These are represented by the dashed lines shown in Figure 2. The opposite ends of these straight lines are joined together as shown by the dot-and-dash lines. The blending proportions sought then can be determined as the differences of the ordinates of the successive points, marked by circles, where the joining dot-and-dash lines intersect the dotted line representing the required grading. In our example, as shown on the right-hand side of Figure 2, 4 percent of aggregate 1, 15 percent of aggregate 2, 24 percent of aggregate 3, and 57 percent of aggregate 4 should be blended for the approximation of the specified grading.

The difference between the specified sieve curve and the actual sieve curve of the determined combination of the four aggregates is also shown in Figure 2 by the shaded areas.

If the combined grading is specified by a pair of limit curves rather than a single sieve curve, there are, as a rule, an infinite number of solutions to the blending problem. Here the acceptable ranges of the blending proportions can be obtained by applying the foregoing method for the upper limit and then repeating the procedure for the lower limit. On the other hand, if any of the dot-and-dash lines do not intersect the sieve curve specified for the combined grading, the grading problem has no solution.

Another simple graphical method is recommended by the British Road Research Laboratory (3) for the determination of n blending proportions for passing through (n - 1) specified points of a combined sieve curve. The mathematical form of this method is again the Gaussian elimination algorithm; therefore, no double or triple overlapping of the sieve curves is permitted here, either. In other words, the grading problem in example 5 can be solved with this method, but the one in example 1 cannot. Under these conditions, this method provides the exact solution to the given system of linear equations within the accuracy of a graphical procedure. On the other hand, the method becomes more and more complicated with the increase of the value of n.

Various applications of the method are demonstrated in examples 6 through 9.

EXAMPLE 6

Determine the blending proportions for aggregates 1 and 2 (Fig. 2) by using the British graphical method such that the amount of particles in the combined grading passing through a No. 50 sieve is 14 ($\approx 100 \times 4/27.5$) percent. The steps of the solution are as follows (Fig. 3).

A square diagram is prepared with percentage scales along three sides as shown in Figure 3. The amount of material in aggregate 1 that passes through a No. 50 sieve (50 percent) is marked off along the left-hand vertical axis. The amount of material in aggregate 2 that passes through a No. 50 sieve (0 percent) is marked off along the right-hand axis and is joined by a straight line to the point on the left-hand axis (thick sloping line called sieve-size line). This sieve-size line, representing the No. 50 sieve size, intersects the horizontal line representing the required percentage of material passing the No. 50 sieve in the combined grading (14 percent). This intersection is marked again by a circle. The sought percentage of aggregate 1 is indicated on the top scale by the vertical line (line of combination) drawn through this point of intersection. In this example, it is approximately 28 percent.

When aggregates 3 and 4 are to be combined such that the amount of particles in the combined grading passing through a ½-in. sieve is 22 ($\approx 100 \times 15.5/72.5$) percent, the same graphical method provides approximately 20 and 80 percent blending proportions for aggregates 3 and 4 respectively (Fig. 4). Note that this combination does not contain particles passing through the ⅜-in. sieve.

This graphical method also has the advantage that it can provide the complete combined grading with little additional work. This is shown in example 7.

EXAMPLE 7

Determine several points of the sieve curve representing the combination of aggregates 1 and 2 that was obtained in example 6. The steps of this determination are as follows (Fig. 3).

The sieve sizes for the complete grading of aggregate 1 are marked along the left-hand axis of Figure 3 according to the corresponding percentage passing values. Aggregate 2 is represented similarly on the right-hand axis. Each point on the left-hand axis is joined by a sloping straight line (sieve-size line) to the point with the same sieve size on the right-hand axis. Any point of the combined sieve curve (that is, the percentage of the combined aggregate passing through any sieve) is the ordinate of the point of intersection that the vertical line of combination (as defined in example 6) makes with the corresponding sloping sieve-size line. The process is shown in Figure 3 by the thin continuous lines. It can be seen that, in our example, the points of the combined sieve curve related to the Nos. 100, 50, 30, 16, 8, and 4 sieve sizes are approximately 1, 14, 42, 58, 72, and 100 percent respectively.

The British graphical method is also applicable to the case when the aggregates should be blended such that the sieve curve of the combined grading falls within a pair of specified limit curves.

EXAMPLE 8

Determine the blending proportions for aggregates 3 and 4 (Fig. 2) by using the British method such that the combined gradings fall within the following limits:

Sieve Size	Specified Limits, Total Percentage Passing
$1\frac{1}{2}$ in.	100
1 in.	50 to 73
$\frac{3}{4}$ in.	30 to 60
$\frac{1}{2}$ in.	18 to 45
$\frac{3}{8}$ in.	15 to 40
No. 4	7 to 20
No. 8	0

The steps of the procedure are as follows (Fig. 4).

The sieve-size lines are constructed for aggregates 3 and 4 in the same way as was discussed in example 7. The specified upper and lower limits are marked off on each sieve-size line as shown in Figure 4. The points representing the lower limit are connected, and so are the points representing the upper limit. Any vertical line of combination that can be placed between these limits without intersecting either of the limits represents a blending proportion for aggregates 3 and 4, which produces a combined grading falling between the specified limits. Figure 4 shows that any proportion for aggregate 3 between 18 and 35 percent combined with the corresponding amount (between 82 and 65 percent) of aggregate 4 will yield gradings that comply with the specification.

If the upper limit and lower limit curves intersect each other, the blending problem, as stated, has no solution.

When more than two aggregates are to be combined, the blending proportions can be obtained by the repeated application of the British method: Two appropriately selected aggregates should be blended first, and then this combination should be treated as one material to be combined with another, and so forth.

An example for four aggregates is presented in the following section.

EXAMPLE 9

Determine again the blending proportions for the case discussed in example 5 but by using the British graphical method. The procedure is as follows.

An appropriate combination of aggregates 1 and 2 is found. Then an appropriate combination of aggregates 3 and 4 is found. These steps have been taken in example 6. Thus, these two combinations should be blended again such that the amount of particles passing through the No. 8 sieve in the new combination is 19 percent. Because it has

Figure 1. Aggregate gradings for examples 1 and 10.

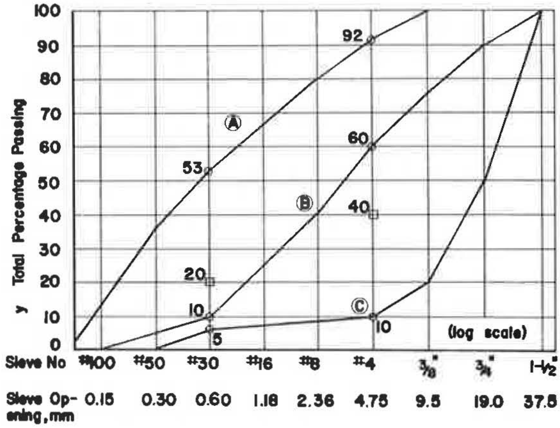


Figure 2. Graphical determination of the blending proportions needed to approximate the specified grading.

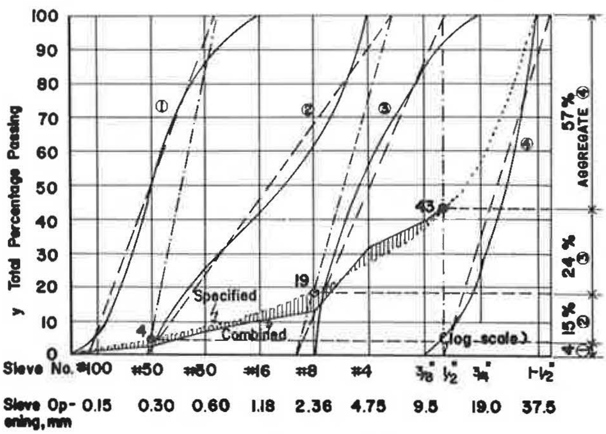


Figure 3. Application of British method.

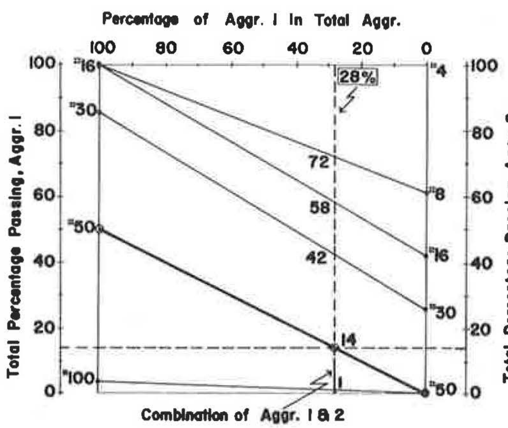
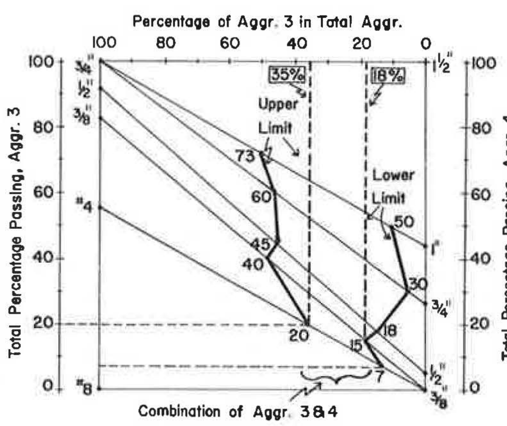


Figure 4. Determining range of blending proportions using the British graphical method.



also been established in example 7 that the amount of particles passing through a No. 8 sieve in the combination of aggregates 1 and 2 is 72 percent, whereas it is 0 percent in the combination of aggregates 3 and 4, the needed blending proportions for the combination of combinations can be determined as shown in Figure 5 by a thick line. Accordingly, approximately 26 percent should be taken from the combination of aggregates 1 and 2 and 74 percent from the combination of aggregates 3 and 4. Therefore, the needed percentages of the four individual aggregates in the total aggregate can be calculated from these values and from the values obtained in example 6 as follows: aggregate 1 (100 x 0.26 x 0.28 = 7.3 percent), aggregate 2 (100 x 0.26 x 0.72 = 18.7 percent), aggregate 3 (100 x 0.74 x 0.20 = 14.8 percent), and aggregate 4 (100 x 0.74 x 0.80 = 59.2 percent).

There are noticeable differences between these blending proportions and those determined in example 5, but these new proportions are practically identical to the exact values that could have been calculated by the numerical method from the same grading conditions. As a result of this identity, the combined grading of example 9 passes through the No. 50, No. 8, and ½-in. points of the specified sieve curve (Fig. 2), whereas the combined sieve curve of example 5 does not. Nevertheless, one can ascertain from a comparison of the line of combination in Figure 5 with the specified and combined sieve curves in Figure 2 that the quality of the overall fit provided by the two graphical methods is more or less the same.

Note also that, if the specified sieve-size line does not intersect the horizontal line representing the required percentage of passing material in the combined grading, the grading problem, as stated, has no solution.

The Triangular Method

This semigraphical method utilizing triangular charts is much more general than the previous two in that it can handle sieve curve, fineness modulus, specific surface, or other grading specifications, or combinations of these, in the form of both equalities and inequalities. It is also suitable for blending on the basis of the probability of grading, that is, with a consideration of the expected grading fluctuations in the aggregates to be blended (4). The triangular method provides the exact solutions within the accuracy limit of graphical methods. Without the application of computer graphics, however, it is restricted essentially to cases where all the gradings included in the blending problem are considered as consisting of not more than three appropriately defined aggregate fractions.

The mathematical basis of the method (5) is that, in the case of transformation of a triangular system of coordinates into another one, the x_i new coordinates, on the one hand, are solutions of a system of three linear equations similar to Eq. 2 but, on the other hand, they can also be determined by measuring certain p distances in the new system (Fig. 6) and substituting them into the following formulas:

$$\left.\begin{aligned} x_A &= \frac{p_2}{p_1 + p_2}\,\frac{p_3}{p_3 + p_4} \\ x_B &= \frac{p_1}{p_1 + p_2}\,\frac{p_3}{p_3 + p_4} \\ x_C &= \frac{p_4}{p_3 + p_4} \end{aligned}\right\} \qquad (6)$$

where it can be verified that $x_A + x_B + x_C = 1$.

Shaefer was probably the first to recommend these formulas, without an exact mathematical justification, for the determination of blending proportions (6).

Before the application of this method is demonstrated, it should be pointed out that the grading in a triangular system is represented by a point rather than a curve. For instance, if an aggregate consists of 53 percent of pan to No. 30 particles (fine sand), 39 percent of No. 30 to No. 4 particles (coarse sand), and 8 percent of No. 4 to 1½-in.

particles (gravel), this grading is represented by point A in the triangular system shown in Figure 7. Incidentally, this is the same grading that is represented by sieve curve A in Figure 1.

The use of triangular diagrams for the determination of blending proportions is illustrated in the following two examples. More details and pertinent information are given elsewhere (5).

EXAMPLE 10

Determine again the blending proportions for the case discussed in example 1 but by using the triangular-diagram method. The procedure is as follows (Fig. 7).

All gradings should be considered as consisting of three fractions, the limits of which are determined by the specified points of the combined grading. In our example the specified points are at the No. 30 and No. 4 sieve sizes. Therefore, the three fractions to deal with are fine sand, coarse sand, and gravel.

An equilateral triangular system of coordinates is prepared with percentage scales along the three sides as shown in Figure 7. Each side is an axis for one of the three aggregate fractions previously mentioned. The grading point of aggregate A can be obtained by marking off 53 percent on the fine sand scale and 39 percent on the coarse sand scale and checking whether this point cuts out 8 percent on the gravel scale. This is shown in Figure 7 for point A with dashed lines. The grading points for aggregates B and C, as well as for the specified combined grading (P), are similarly plotted. Points A, B, and C are connected with straight lines to form another triangle. Then another straight line is drawn to pass through point P and one of the A, B, or C vertexes. In our example, point B is selected arbitrarily, and point X is marked off where the BP line intersects the AC side. The p distances are measured from Figure 7 as follows:

$p_1 = AX = 2.65$ in. $\qquad p_3 = BP = 1.50$ in.
$p_2 = CX = 1.35$ in. $\qquad p_4 = PX = 0.15$ in.

By substituting these values into Eq. 6, the sought blending proportions can be calculated as follows:

$$x_A = \frac{1.35}{4.00}\,\frac{1.50}{1.65} = 0.307$$

$$x_B = \frac{0.15}{1.65} = 0.091$$

$$x_C = \frac{2.65}{4.00}\cdot\frac{1.50}{1.65} = 0.602$$

$$\text{Total} \qquad \overline{1.000}$$

A comparison of these results with the solutions of Eq. 2 in example 1 shows good agreement. Note that only such blending problems have solutions where the point P is within the ABC triangle.

EXAMPLE 11

In this example the triangular method is applied for a more complicated blending problem (5).

Assume that materials a, b, and c are given. Their compositions are given in Table 1. Determine the blending proportions required to yield the specified grading given in Table 1.

If the gradings of the available materials are plotted as points a, b, and c in Figure 8, then the points of the shaded area represent the full range of all possible mix proportions that will comply with the grading requirements. It can be seen that in this

Figure 5. Determination of blending proportions for four aggregates using the British graphical method.

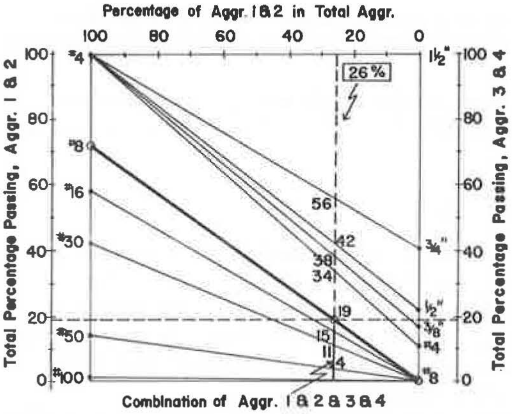


Figure 6. Determination of the trilinear coordinates.

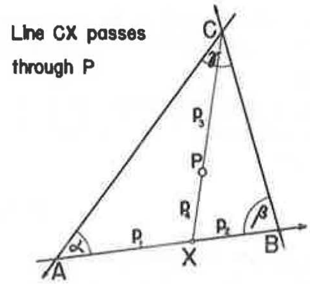


Figure 7. Triangular method for the determination of the blending proportions.

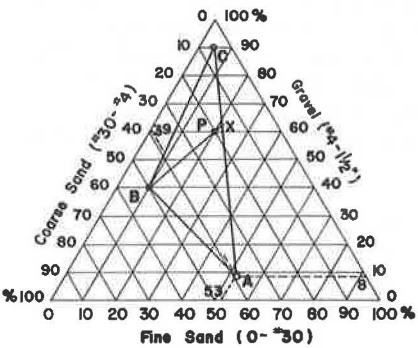


Figure 8. Proportioning mineral aggregates by the triangular method.

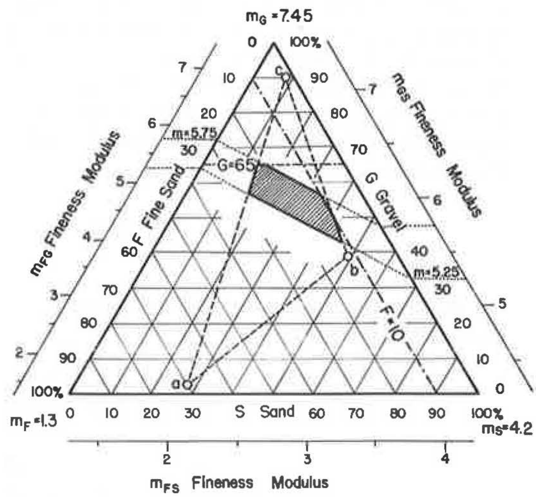


Table 1. Grading data for materials a, b, and c.

Material	Gravel[a] (percent)	Sand[b] (percent)	Fine Sand[c] (percent)	Fineness Modulus (ASTM Designation C 125-68)
a	3	27	70	2.3
b	38	49	13	5.05
c	90	7	3	7.05
Specified grading	< 65	None	> 10	5.5 ± 0.25

[a]Coarser than No. 4 sieve. [b]No. 4 to No. 16 sieve. [c]Finer than No. 16 sieve.

Table 2. Limits for permissible blending proportions of materials a, b, and c.

		Extreme Proportions Within Specifications (percent)			
Material	Limit	Blend 1	Blend 2	Blend 3	Blend 4
b	Lower	0	4	61	80
	Upper	4	61	80	92
c	Lower	← -0.6b + 63 →			
	Upper	-0.43b + 72	-0.6b + 73	-0.84b + 87	100-b
a	Lower and upper	← 100 - b - c →			0

case there are many such mix proportions. The construction of the limits of the shaded area is as follows: The positions of the two parallel, sloping dotted lines are determined by the specified values (5.5 ± 0.25) of the fineness modulus of the combined grading; the two dot-and-dash lines are determined by the specified limits for the respective gravel and fine sand contents; and the three dashed lines are determined by the actual gradings of aggregates a, b, and c to be blended.

One of the mix proportions can be determined by the semigraphical method from any point of the shaded area and the triangle abc. This has been illustrated in the preceding example. By the successive application of the same method to each corner point of the shaded area, the lower and upper limits of the desired mix proportions can be obtained for all three mineral aggregates. Thus, the full range of the blending proportions that will comply with the specified grading requirements is determined. These lower and upper limits are given in Table 2. It can be seen that the amount of material b may vary between 0 and 92 percent, and the amount of material c may vary between 7.8 and 72.0 percent. If it has been decided to take, for instance, 50 percent of material b, then, according to Table 2, the amount of material c may vary between 33 and 43 percent, and the amount of material a may vary between 7 and 17 percent to comply with the specified grading requirements.

CONCLUSIONS

The general mathematical form of the blending problem is system 1, which contains the sought blending proportions as unknowns. Depending on the number and nature of the grading conditions, there can be an infinite number of sets of blending proportions, a single set, or no set at all that satisfies all the given conditions. The numerical method is applicable for every kind of blending problem of aggregates. In most cases, it provides the exact solutions; but it becomes increasingly cumbersome when the number of aggregates to be blended is more than three, unless a computer is used.

The graphical method discussed first is simple even for a large number of aggregates to be blended, but it is applicable only in special, although practically important, cases. Also, the solutions it provides are usually not exact.

The restrictions concerning the applicability of the British graphical method are essentially the same as those for the first graphical method. The British method becomes more complicated with the increase of the number of aggregates, but the solutions obtained are the exact solutions. It also provides the grading of the combined aggregate with little additional work.

The triangular method is simple and still more general than the other two graphical methods. It provides also the exact solutions. It is an ideal method for blending three aggregates. In cases more complex than this, however, it may become overly complicated, unless computer graphics is used.

REFERENCES

1. Neumann, D. L. Mathematical Method for Blending Aggregates. Jour. Construction Div., Proc. ASCE, No. CO2, Sept. 1964, pp. 1-13.
2. Rothfuchs, G. Betonfibel, Band 1 (Concrete Primer, Vol. 1). Bauverlag GmBH, Wiesbaden and Berlin, 1962.
3. Design of Concrete Mixes. Department of Scientific and Industrial Research, Road Research Laboratory, London, Road Note 4, 1950.
4. Sargent, C. Economic Combinations of Aggregates for Various Types of Concrete. HRB Bull. 275, 1960, pp. 1-17.
5. Popovics, S. Theory and Application of Triangular Diagrams. RILEM Bull., Paris, New Series 22, March 1964, pp. 37-43.
6. Shaefer, T. E. Application of Trilinear Charts to Aggregate Grading Control. Concrete, London, Vol. 36, April 1930.