USE OF LINEAR PROGRAMMING TO EVALUATE ALTERNATIVE PARKING SITES

E. M. Whitlock, Wilbur Smith and Associates

The author has related a detailed study of the development of a parking plan at Carnegie-Mellon University. Because it was possible to establish competitive plans and values between development and operating costs and user costs, a simplex network obtained from linear programming was used to determine that combination of facilities to serve 1980 parking needs with maximum utility and minimum cost. Application of the program assisted in determining the relative importance of each site with regard to its proximity to the many campus generators of parking demands. The time-distance relations of sites and parkers were measured on the assumption that each competitive site was developed with a given number of spaces in terms of cost of trip.

•THE SUCCESS of off-street parking facilities depends largely on their location relative to generators of parking demand. Other factors, such as charges, ease of access-egress, and functional design, also affect usage. When special-purpose parking facilities are planned, additional values can be given to the types of users who are expected to patronize the system. Those facilities would normally serve large office buildings, hospitals, institutions, universities, and other land uses having a number of different categories of users.

A CASE STUDY

In 1968-1969, Carnegie-Mellon University started a planning program to expand its off-street parking. There were 1,625 curb and off-street spaces at various locations on and adjacent to the campus, and those spaces served daily terminal needs of more than 5,000 staff, faculty, students, and visitors. By 1980, based on normal growth, the daily campus population is estimated to be 7,500. Estimates of parking demand are 3,400 spaces, based on study of travel, parking, and socioeconomic facts.

Final planning resulted in a study of selected alternative sites (Fig. 1) varying in size from about 2,000 spaces at sites A (Panther Hollow) and C (Skibo Hall) to 100 spaces in a surface lot near site F (near WQED radio station). Physical characteristics, given in Table 1, included integrated air-rights development, conventional free-standing parking structures, a subterranean garage, and 3 surface lots. Because land was not a cost factor, development costs versus utility of the sites to serve various categories of parkers were final determinants in the evaluation.

THE LINEAR PROGRAM

Primary reasons for selecting linear programming as the analysis tool were relative ease of adoption (off the shelf), economy of computer runs, and direct applicability to measure relative scale of capital, operating, maintenance, and user costs competitively. The objectives of using the linear program were

1. To distribute a maximum number of parkers from each building of campus activity to each potential parking location with minimum cost,
2. To test alternate combinations of off-street parking facilities, and
3. To identify parking facilities with the least cost to all parkers.

The inputs of the program included

1. A network description of campus buildings, facilities, and generators of activity and parking demand;
2. The number of potential parkers in each building in the categories of faculty, staff, and student;
3. Estimates of absolute costs of development, operation, maintenance, and parker assignment; and
4. Capacity of each parking facility.

The outputs of the program included

1. An input network description to authenticate possible errors,
2. An output network description to identify the flow and cost over each link and parker accumulation at each facility, and
3. A tabulation of the total cost for each facility and the daily estimate of cost per parker.

The network logic is shown in Figures 2 and 3; account is taken of parking supply-demand relations and the relative cost values of the transportation network connecting origins and destinations of parkers. The testing phase is shown in Figure 3 where different values of time and parking cost can be applied.

Mathematically, the equations describing the program are directed to seek a set of values that, moving from origin to destination, are equal to or greater than zero. With this case, the sum of the costs of traversing the network by the parkers will be minimized, subject to certain cost constraints imposed at each end of the parker's trip. The essential equations to be solved are as follows:

$$\Sigma_j f(i, j) = a(i)$$

where $a(i) > 0$ and $i = 1, \ldots, m$.

$$\Sigma_i f(i, j) = b(j)$$

where $b(j) > 0$ and $j = 1, \ldots, n$. A set of $f(i, j) \geq 0$ must be found such that $z = \Sigma_i \Sigma_j c(i, j) f(i, j)$ is minimized.

$$\Sigma_i a(i) = \Sigma_j b(j)$$

By definition,

$f(i, j)$ = quantity of parkers destined from place of parking to generator;
$c(i, j)$ = value of person's time consumed in traveling to parking facility (by vehicle) and from parking space to destination (on foot), plus parking cost;
$a(i)$ = supply and cost of parking spaces at origin i;
$b(j)$ = demand for spaces at destination j; and
z = total trip and parking cost.

PLANNING VALUES

Parking demands were determined by relating existing densities of people in the various classroom, administration, and activity buildings for each hour in the normal school day. In the calculations, assumptions were also made with regard to locations of new buildings anticipated (with their estimated parking demands).

Figure 4 shows the location of major parking generators, and Table 2 gives the parking spaces required for the generators excluding dormitories. The data given in Table 2 are based on distribution of computed parking demands by duration of time per building. The main academic thrust is in the enclosed loop comprising Doherty, Hamerschlag, Scaife, and Baker Halls; the Graduate School of Industrial Administration; and the College of Fine Arts. Student activities are generally centered in Skibo Hall near the track oval, and the university's business offices are in Warner Hall. Walking distances among those buildings are not excessive (Fig. 5) and range from about 150 ft in the loop to more than 1,000 ft to the fraternity-dormitory area across Forbes Avenue.

Figure 1. Possible off-street parking facilities.

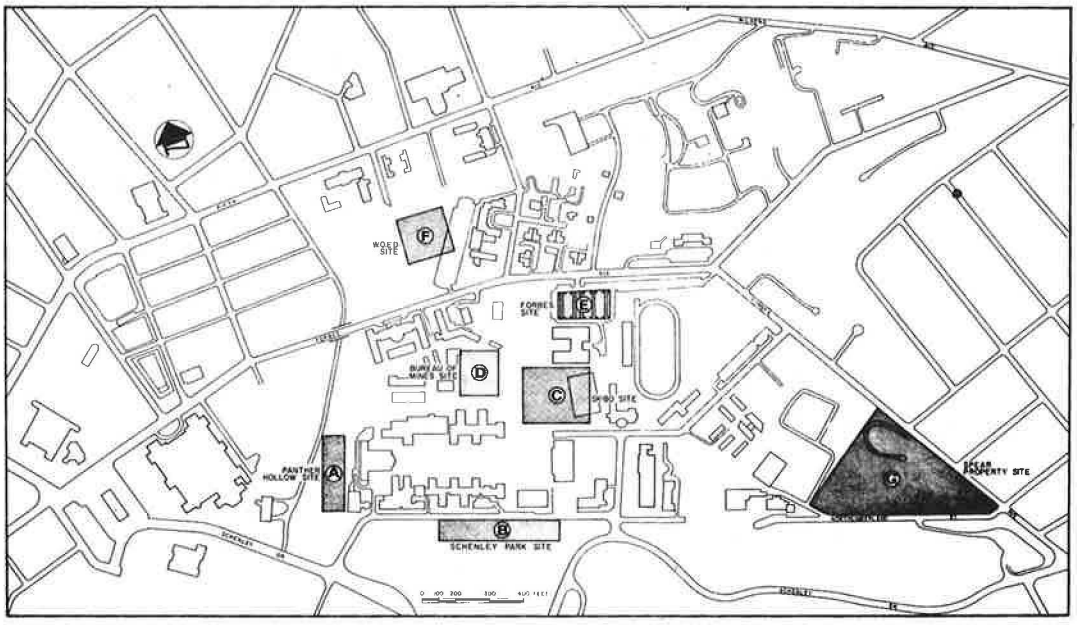


Table 1. Characteristics of possible parking facilities.

Site	Location[a]	Type	Maximum Capacity (spaces)
A	Panther Hollow	Air-rights structure	2,000
B	Schenley Park	Above-grade structure	1,800
C	Skibo Hall	Below-grade structure	2,000
D	Bureau of Mines	Above-grade structure	1,500
E	Forbes Avenue	Surface lot	300
		Above-grade structure	1,200
F	WQED	Surface lot	100
		Above-grade structure	1,000
		Below-grade structure	1,000
G	Spear property	Surface lot	960

[a]Figure 1 shows site location.

Figure 2. Network logic.

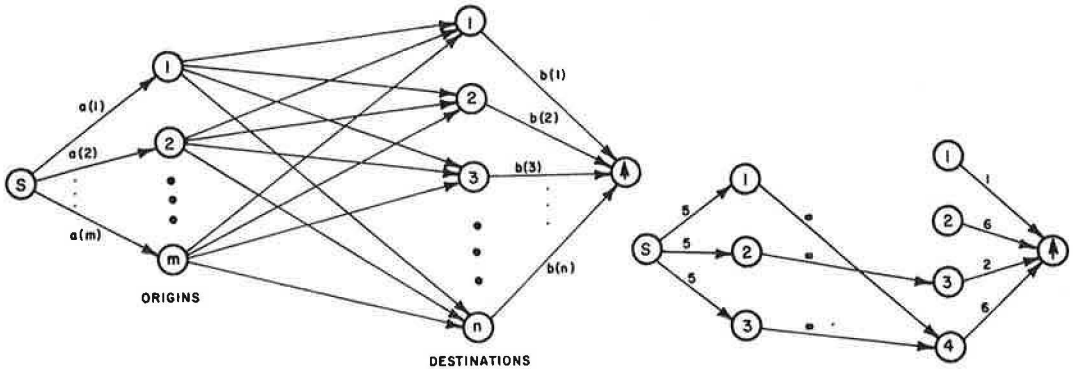


Figure 3. Maximum flow assumptions.

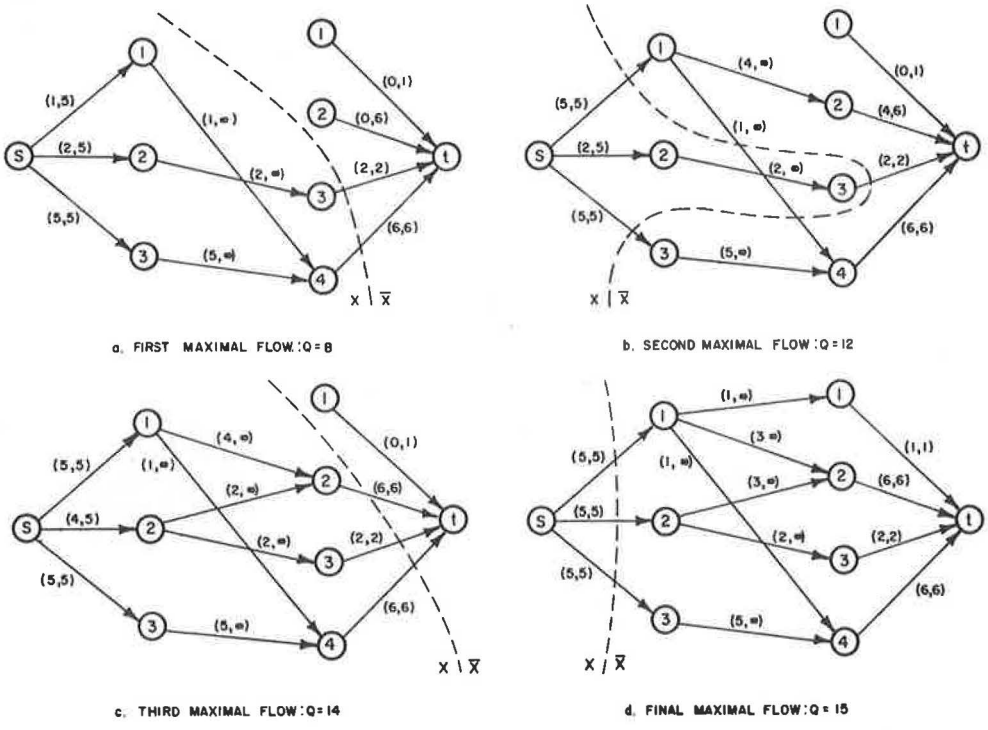


Table 2. Parking spaces required for major generators.

		Parking Spaces Required					
		For Faculty		For Staff		For Students	
Generator	Location	1975	1980	1975	1980	1975	1980
81	Baker-Porter Hall	100	105	182	195	764	803
82	Scaife Engineering	25	26	48	52	190	200
83	Hamerschlag Hall	16	18	43	45	154	161
84	Doherty Hall	44	47	71	76	410	430
85	College of Fine Arts	42	46	13	14	181	190
86	Margaret Morrison Carnegie College	22	23	38	40	119	126
87	Graduate School of Industrial Administration	14	15	23	25	96	102
88	Warner Hall	6	7	165	176	—	—
89	WQED, Dramatic Arts	30	32	77	83	70	74
90	Computer Science Building	37	39	84	90	149	157
Total		336	358	744[a]	796[b]	2,133	2,243

[a]Includes 131 visitors. [b]Includes 138 visitors.

Figure 4. Major parking generators.

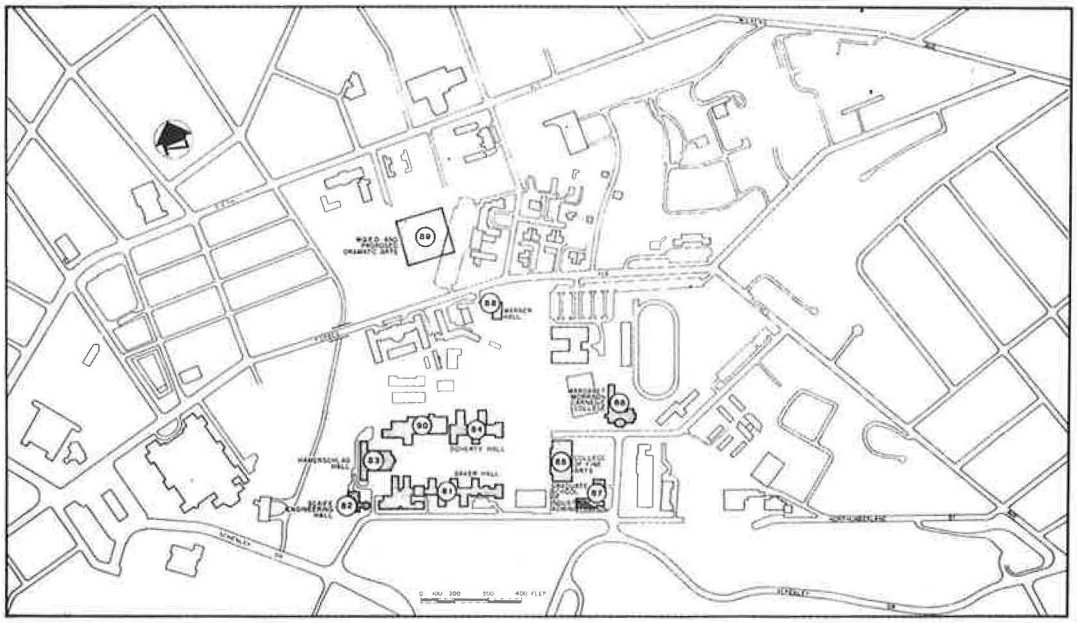
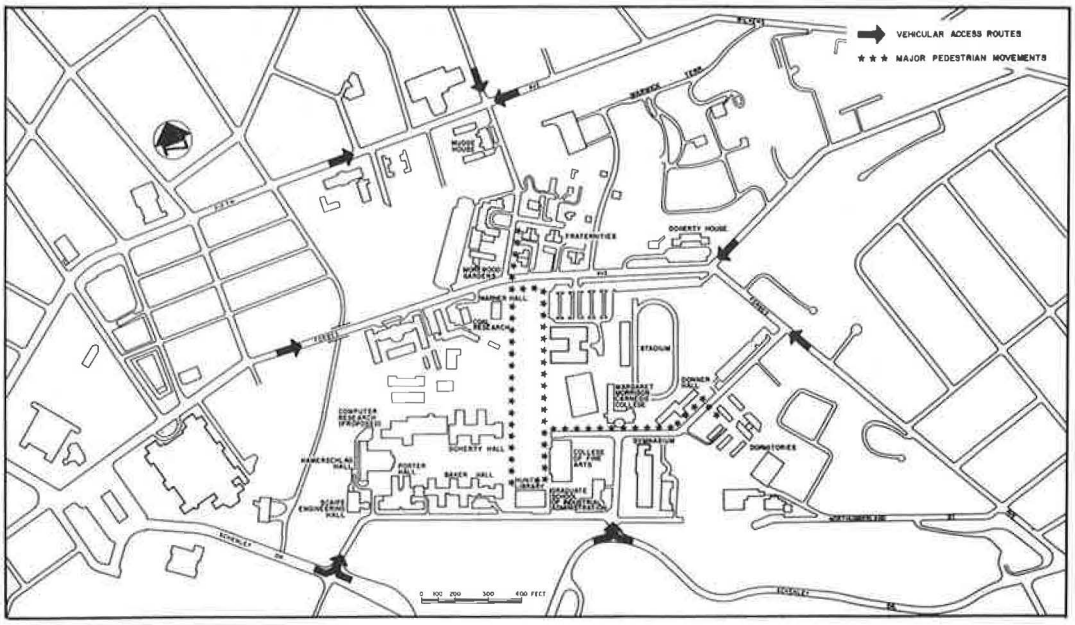


Figure 5. Vehicular access and major pedestrian movements.



Total parking demand by 1980 for all users of the 10 selected generators is expected to be about 3,400 spaces. The demand is 3,200 spaces in 1975 and 2,550 spaces in 1967.

Because the analyses for relative importance of parking sites were sensitive to cost variables of user's time, 1980 parking spaces were divided according to the estimated number in each user category as follows:

Category	Number	Parking Spaces
Faculty	575	358
Staff	1,150	796
Students	5,750	2,250
Visitors	—	130
Total	7,375	3,534

For assumed development costs, the program was used because of its capability to account for total trip costs of all parkers. Development costs of estimated new parking facilities then became key to the total economic implication of building a new facility; the assumption was that user income would self-liquidate the investment. Those values were established by taking into account design, construction, financing, operating, and maintenance costs at the parking end of the trip. Each cost factor, however, was determined separately (Table 3). Development costs exclude land acquisition.

The program measured relative attraction of sites based on travel and parking costs and on the value of time for various categories of users. Therefore, the daily annual implication of amortizing development costs from user revenues was further stated in a breakdown of annual per-space development costs on the basis of a 20-year amortization period. Based on self-liquidation from user charges, the higher the annual cost was, the greater the penalty applied to the user's trip. The annual per-space development costs were further extended for faculty, staff, and students based on the following frequencies of use:

Category	Parking Days	Percent of Parkers
Faculty	230	32
Staff	310	43
Student	180	25

A similar set of unit values was developed for operating and maintenance costs for the various users of the university's parking program. Those values are also given in Table 3.

Whereas the absolute values can be expected to vary depending on circumstance in other applications of the program, the important issue here is the ability to place different relative values on the cost of providing parking.

At the destination (generator) end of the parking trip, a measure of relative user importance was used to ascertain "place utility" of the alternative parking facilities. If the facility were convenient to the generators, a lesser penalty was applied to the trip because the pedestrian trip would be shortened. The indexes of user walking time are as follows:

Category	Estimated Avg. Weekly Income ($)	Walking Time Cost ($/min)
Faculty	230	0.110
Staff	145	0.070
Student	40	0.017

Only in special circumstances, such as a university, hospital, or other "place generators," can those varying time values be logically applied. The time values were deter-

mined from average weekly income levels of the faculty, staff, and student parkers. Visitors were not considered in this phase of the study because they constitute a relatively insignificant portion of the annual total of parkers at the university. It was also difficult to arrive at an appropriate income range for the visitor parkers.

Manual adjustments for service levels of street accessibility were made after the linear programming model was applied. They took into account minimum travel paths based on existing capacity and observed congestion measured by actual vehicular travel times.

DEVELOPMENT OF ALTERNATIVE PARKING PLANS

A number of alternate parking plans were hypothesized in the analyses. Six of them had location and capacity variables that supplied the optimum number of spaces for each user type and generator. Optimum-flow and least-cost comparisons were derived by relating parking sites, size, user type, all travel and parking costs, and destinations. Figure 6 shows a conceptual network diagram indicating the myriad of relations that could be tested by altering the previously mentioned assumptions.

RESULTS OF MODEL APPLICATION

A summary of the 6 plans tested is given in Table 4. The plans were modified according to the number of parking spaces expected to be provided at the 7 proposed sites. According to relative demands, proximity of new facility, and cost indexes (facility and user), each received assignments of vehicular flow likely to be related to the parking sites.

The parking concepts tested had the following space capacities based on realistic functional plans previously prepared:

Site	Spaces
A	1,000
B	900
C	1,200
D	960
E	300
F	100
G	960

The main variables evaluated in the testing procedure involved whether the parking facility was a surface lot or a structure. In plan 1, for instance, there were parking garages at sites A, B, C, and D and surface lots at sites E, F, and G. With these, appropriate unit costs were used to achieve a relative measure of utility. Table 5 gives the assigned "utility" of the first alternate plan tested and supports the expected favorable acceptance of the implied cheaper, more convenient parking plan. It also supports, by favoring sites A, B, D, E, and F, the relative attraction of strategic locations for the new parking facilities. Table 5 also gives the assigned utility of the other plans that were tested.

The plan finally recommended is that parking concept expected to achieve maximum flow and least cost to the university (Fig. 7 and Table 5). Though extensive and perhaps energetic from a monetary standpoint, it will best serve 1980 terminal needs of the campus. Expected needs are about 1,200 spaces in Panther Hollow, 1,800 spaces in Schenley Park, and 350 spaces in the surface lot along Forbes Avenue.

As expected in an analysis of this type, the facilities that are costly to develop and those sites that are more distant from generators of campus activity did not prove to be the best choices.

CONCLUSIONS

On occasions when time-cost values can be placed on development costs and categories of users, a relative measure of the optimum utility of a given parking site can be

Table 3. Estimated parking facility development and operating costs.

Parking Facility	Development Costs (dollars)					Operating Costs (dollars)			
	Per Space	Per Space per Year[a]	Per Day per Capita			Per Space per Year	Per Day per Capita		
			Faculty	Staff	Student		Faculty	Staff	Student
Surface lot	800	64	0.030	0.040	0.020	15	0.007	0.009	0.005
Above-grade structure	3,000	240	0.110	0.140	0.080	55	0.024	0.032	0.015
Below-grade structure	5,000	400	0.180	0.240	0.140	90	0.039	0.053	0.031
Surface lot plus shuttle	900[b]	70[b]	0.032[c]	0.043[c]	0.022[c]	—	0.015[d]	0.020[d]	0.012[d]

[a]Based on 5 percent interest rate for 20 years.
[b]Based on 20-year period and capital investment of approximately $100,000 for 4 buses, 2 for each 10-year operating period.
[c]Based on addition of bus operation.
[d]Includes salaries and maintenance for bus operation.

Figure 6. Simplex network.

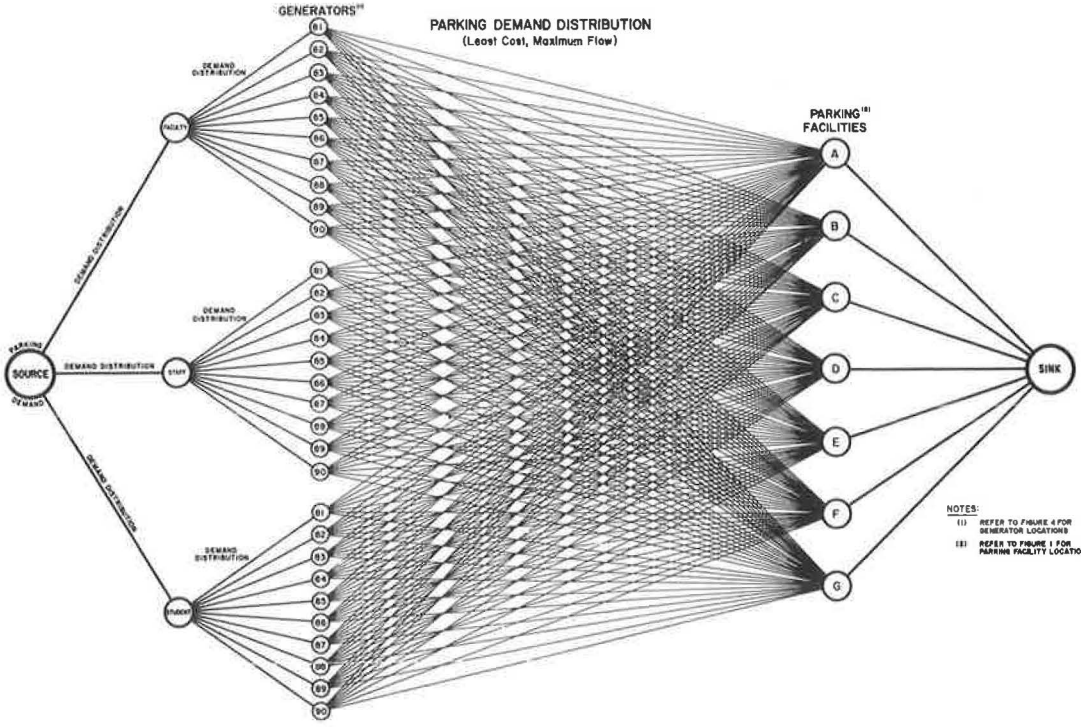


Table 4. Summary of parking plans.

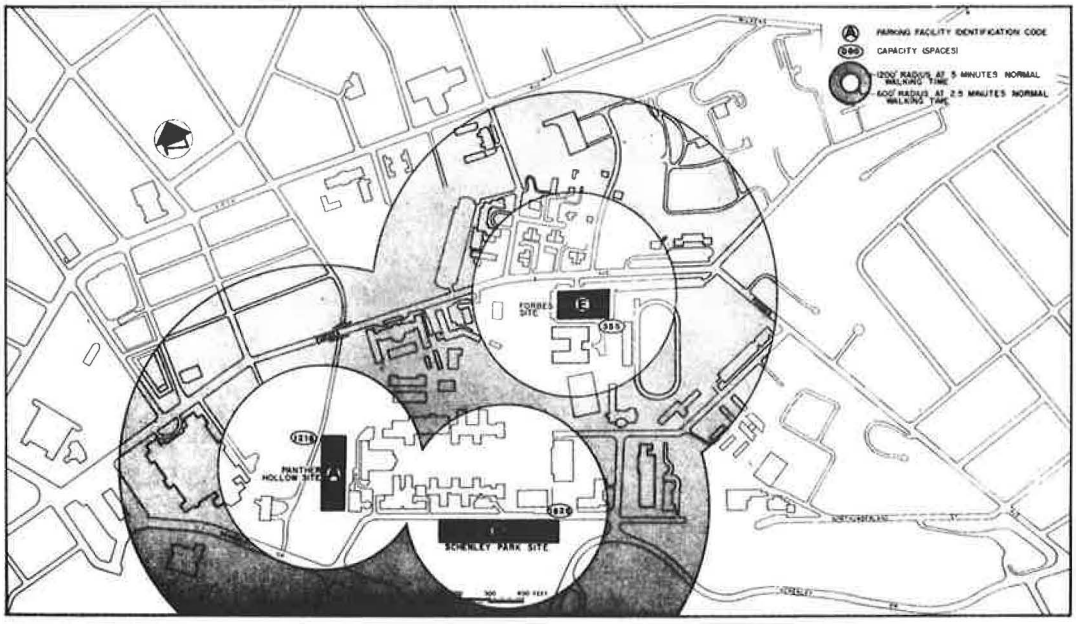
Site	Plan 1	Plan 2	Plan 3	Plan 4	Plan 5	Plan 6
A	Structure	Structure	Structure	Structure	Structure	Structure
B	Structure	Structure	Structure	Structure	Structure	Structure
C	Subsurface	Subsurface	Subsurface	Subsurface	Subsurface	Subsurface
D	Structure	Structure	Structure	Structure	Structure	Structure
E	Surface	Surface	Surface	Structure	Structure	Structure
F	Surface	Structure	Subsurface	Surface	Structure	Subsurface
G	Surface	Surface	Surface	Surface	Surface	Surface

Table 5. Optimum-flow and least-cost comparisons for parking facility alternatives.

		Flow of Vehicles[a]													
	Practical Capacity	Plan 1		Plan 2		Plan 3		Plan 4		Plan 5		Plan 6		Recommended Plan	
Site	(spaces)	1975	1980	1975	1980	1975	1980	1975	1980	1975	1980	1975	1980	1975	1980
A	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,133	1,216
B	900	900	900	900	900	900	900	900	900	900	900	900	900	1,740	1,826
C	1,200	103	137	125	161	171	177	51	62	—	—	—	—	—	—
D	960	710	960	810	960	846	960	890	960	855	960	855	960	—	—
E	300	300	300	300	300	300	300	272	375	384	451	485	537	340	355
F	100	100	100	74	76	54	60	100	100	74	86	—	—	—	—
G	960	—	—	—	—	—	—	—	—	—	—	—	—	—	—

[a]For plans 1 through 6, flow of vehicles indicates relative acceptance of parking facility given in Table 4. Maximum acceptance is achieved when the number of vehicles equals the practical capacity. For the recommended plan, the flow of vehicles accounts for the number of spaces required for each forecast year if the recommended facilities are constructed.

Figure 7. Recommended off-street parking facilities and associated areas of influence.



determined. If given the variables of cost at the origin and destination ends of the trip (parking cost versus value of user's time), the linear program can quite readily and inexpensively correlate the pertinent factors to ascertain the optimum plan for off-street parking. The circumstances where model application is most effectively achieved are where categories of users can be quantified by type and time value. The program will aid site planners to develop a parking plan that is most cost-effective.

DISCUSSION

Edgar Elias Osuna, Instituto de Urbanismo, Universidad Central de Caracas

The subject of the paper appears to be quite interesting in the sense of applying some linear programming techniques, basically of the Hitchcock-Koopmans type, in a field where until now most of the trials of this kind have been unsuccessful.

In effect, for many years these techniques have been used, not quite successfully, in the treatment of problems related to urban flows, urban activity locations, and the like. The reasons for failure may be attributed to the fact that in most cases the optimization of the studied systems has been based on the assumption of an optimum collective behavior. However, what really occurs is an aggregate of individual decisions that are basically independent of each other and that pursue individual benefit or optimality. Obviously this result is not always the optimum for the entire system. The case presented here might not be an exception for this failure unless some provisions are made.

In effect, the formulation of the problem here states that at the optimal solution the expected parking demand from each building should be allocated to a specified set of parking facilities in such a way that total trip and parking supply cost is at a minimum. Once this solution has been found, the cost of parking supply is fixed (defined by the resulting facilities plan), while trip costs of parkers is a variable function of the allocation pattern resulting from parkers' decisions. If the solution is kept optimal, this pattern should be its resulting set of f(i, j), which would be the same set as the one obtained with a Hitchcock-Koopmans problem where the a(i) and b(j) were the supply and demand sources as given by the facilities development plan and the university buildings and the c(i, j) were as defined in the previous equations of the paper, except that parking supply cost would not be included this time.

As stated at the beginning of this discussion, however, every parker will try to maximize his own benefit, and the resulting allocation pattern will not be the required optimum unless some special control policies are used. A very important feature of the paper is that this can certainly be done in this case. In effect, here we can state "parking policies" that will require or encourage parkers to behave so that the desired optimum for the system is achieved. We can, for example, assign restrictive parking permits in the following way.

Let D_i^k be the parking demand of K-category (faculty, staff, or student) generated by building i and $f^k(i, j)$ be the number that should be allocated to facility j for the optimality condition. In a first approximation, we should assign to the D_i^k parkers only $f^k(i, j)$ permits that allow them to park on facility j only.

Obviously this control policy would be highly inconvenient. In the first place, D_i^k is only a fraction of the total population of potential parkers in K-category from building i. It probably represents their expected number at the moment of maximum accumulation.

This implies that the number of parking permits for facility j given to those in K-category from building i should be a number larger than $f^k(i, j)$, and probably proportional to it. Second, under any circumstance, the number of parkers of any type, from any building, at any facility, and during any time of day is a random variable, and the system will surely be designed in such a way that there will be a non-zero probability that a potential parker could find no space to park at his pre-assigned facility, or at any other one, at the time of requiring it.

This points out that permits should not be absolutely restrictive in the sense that they should allow for parking in some other facility (or facilities) in case the assigned

one is full. To discourage any particular parker, however, to go for personal reasons to some of those other facilities instead of his assigned one even if that one is not full, a penalty, or differential fare, could be established. An appropiate penalty or differential fare could be determined, for example, on the basis of the expected increase in cost to the system caused by the misallocation of the parker or on the basis of his personal value system or on the basis of both.

In summary, the point I have tried to make in these comments is that the solution (recomended plan) is optimal only as long as parkers behave in a prescribed way, which is not necessarily the best way for them as individuals. The characteristics of the system studied in the example, however, do allow for simple control policies that could guarantee a behavioral pattern expectedly close to optimal.

The author must be congratulated for such an interesting paper, and I hope these comments will contribute at least a small portion to its better understanding and application.