Appendix B – Design Examples

B.1 MULTI-SPAN PRECAST CONCRETE GIRDER MADE CONTINUOUS WITH COMPOSITE DECK

This is one of the most common types of structures used for freeway bridges and overpasses. This three-span precast/prestressed girder example features a single long span in the middle along with two short side spans, as shown in Figure B.1. A uniform depth is used to reduce set-up costs and improve aesthetics. It is intended that the side spans are short enough so the minimum flexural provisions control the design in the positive bending regions.

Seventy-two inch bulb-tee girders are featured in this example since the bottom flange tends to be relatively narrow, thus limiting the amount of rotational ductility that can be sustained in the negative bending region.

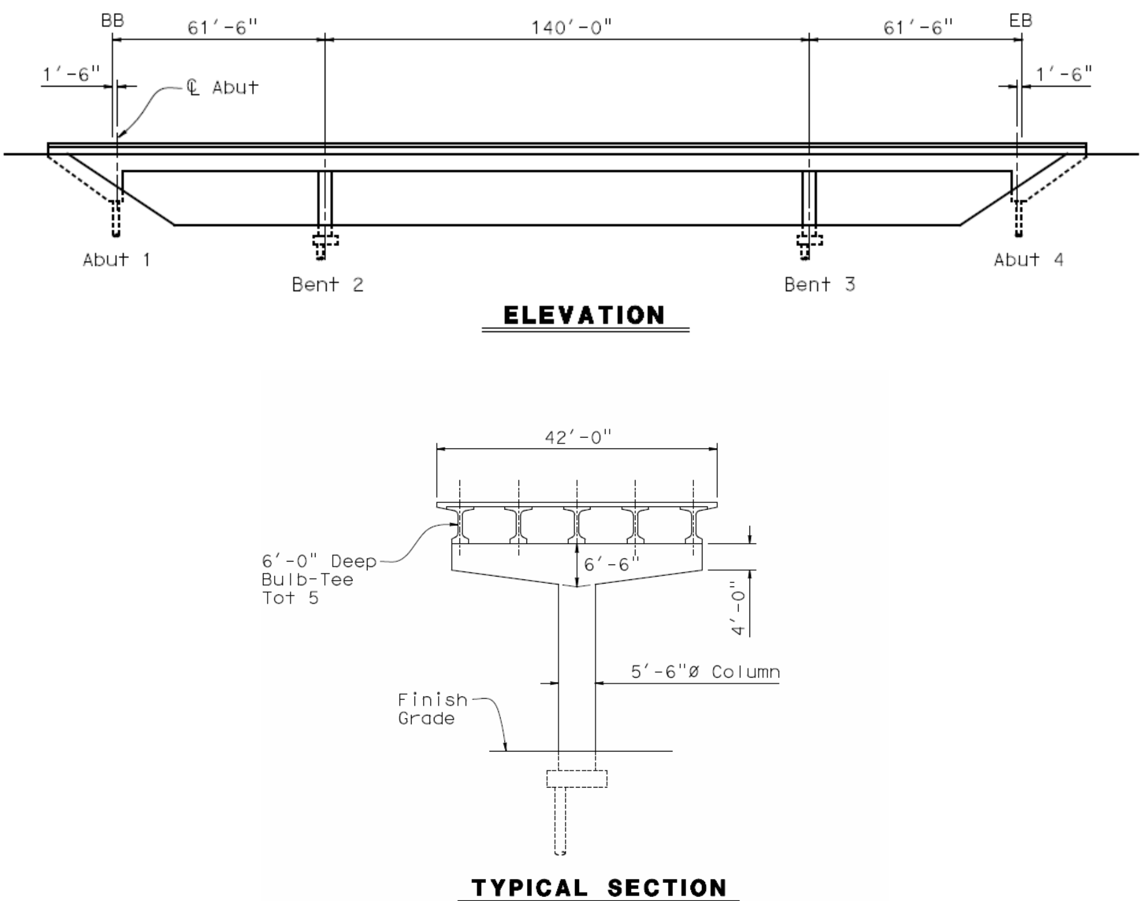

Figure B.1 – Precast Concrete Girder Made Continuous with a Composite Deck

BRIDGE DESCRIPTION

Bridge dimensions

The bridge is 42.0 ft wide and 6.83 ft deep at the supports. The 6.0 ft deep bulb-tee girders are spaced at 9.0 ft on center. The deck is 8.0 in. thick. The columns are circular with a diameter of 5.5 ft.

Prestress force

Analysis of the girders was performed using Conspan to include evaluation of live loads, permanent loads and prestress. In the analysis, each girder supports its own weight, and the fluid weight of the deck. Loads placed on the bridge after the deck has been placed are assumed to be resisted by the composite girders that span continuously across the interior supports. Based on the analysis, a total of 10 and 32 – 0.6 inch diameter strands for interior and girders of Spans 1 and 2, respectively, meet service and strength limit state requirements. For Span 2, a total of 6 strand are draped within the web to control stresses at the ends of the girders.

All assumed prestress losses were assumed to occur in developing the factored cracking moment, as described herein.

Material Properties

f'_c = 7.5 ksi (girders)

f'_{ci} = 5.5 ksi (girders)

f'_c = 4.5 ksi (deck)

f_y = 60 ksi

E_s = 29,000 ksi

f_{pu} = 270 ksi

E_{ps} = 28,500 ksi

MOMENT PROFILES

Moment demand profiles along with minimum reinforcement requirements are plotted in Figure B.2, which demonstrate that minimum flexural reinforcement requirements control for nearly all of Span 1 and most of Span 2. Under negative bending, the section is designed as a reinforced concrete inverted T-girder, and the minimum reinforcement requirement impacts curtailment of rebar designed to resist negative bending forces at Bent 2.

Analysis of the girders was performed using Conspan to include evaluation of live loads, permanent loads and prestress. Based on the analysis, a total of 8 and 32 – 0.6 inch diameter strands for an interior girder of Spans 1 and 2, respectively.

The factored cracking moment (M_{cr}) is constant under negative bending when the deck is in tension because there is no prestress assumed in the analysis. M_{cr} under positive bending varies with the amount of prestress. For Span 1 the prestress is constant along the bottom flange with exception at the ends of the beams. As stated previously, strands in Span 2 are draped at the ends to control stress, and the variation in prestress varies accordingly.

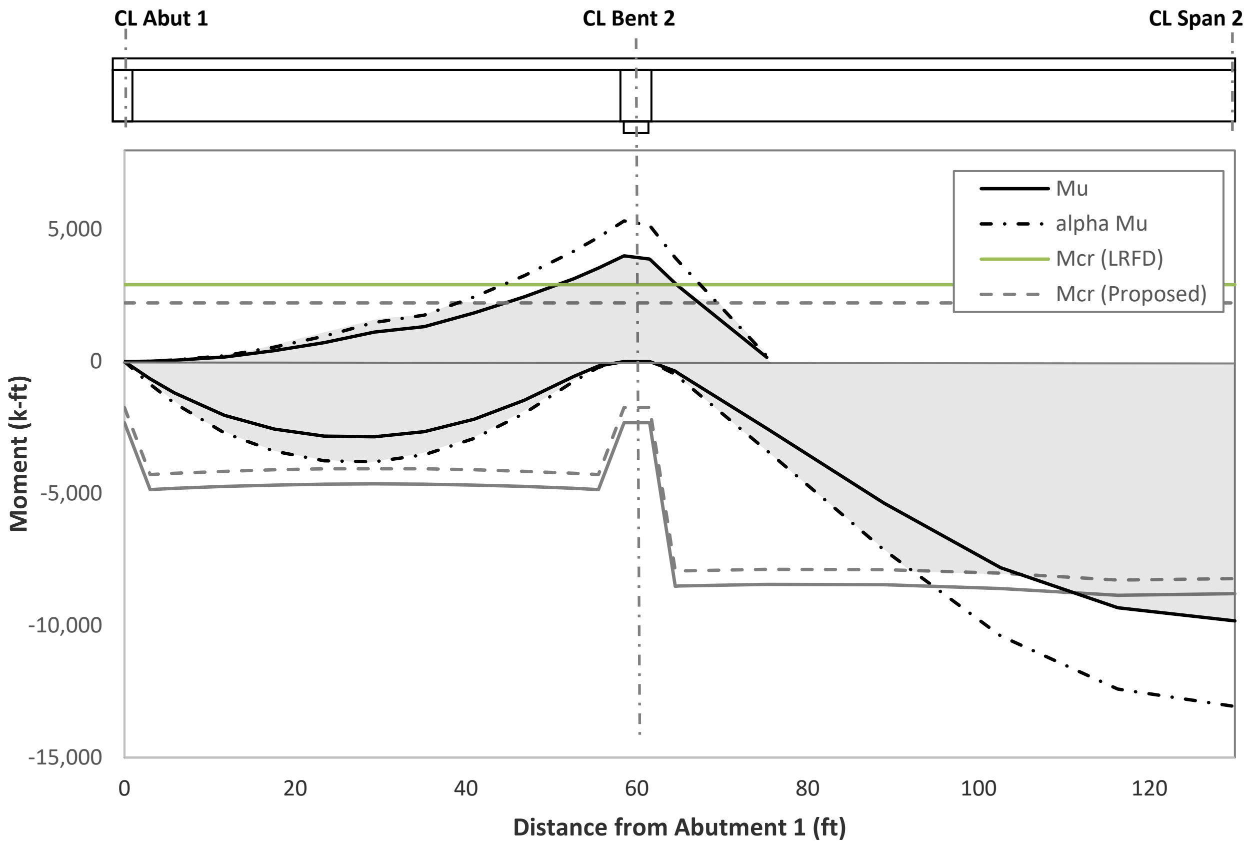

Figure B.2 – Factored Moment and Minimum Reinforcement Moment Profiles for Both AASHTO LRFD and Proposed Requirements

The controlling strength limit state moment profiles are shaded for both positive and negative moments. These profiles are based on the proposed method of developing M_{cr}. As shown, minimum reinforcement provisions are the controlling moment load case for along the entire length of Span 1 under positive bending and roughly 75% of the negative bending region. In Span 2, minimum reinforcement provisions control along roughly half of the span.

MINIMUM REINFORCEMENT CALCULATIONS

Span 1 Midspan Positive Moments

Section Properties

	Girder	Composite	
Area (A) =	1,063	1,733	in^2
Depth (h) =	72.83	80.83	in
Inertia (I) =	755,589	1,404,252	in^4
Centroid (y_b) =	37.2	52.51	in
Section Modulus (S_{top}) =	21,207	49,585	in^3
Section Modulus (S_{bot}) =	20,312	26,743	in^3
Modular ratio (n) = E_d/E_g =	0.775		
Strand centroid (x) =	2.5		in

Design Moments

Factored Moment (M_u) = 2,850 kft (Strength I)
Non-composite Self weight Moment (M_{dnc}) = 859 kft

AASHTO LRFD

Flexural cracking variability factor (γ_1) = 1.6
Prestress variability factor (γ_2) = 1.1
Ratio of yield to ultimate reinforcement (γ_3) = 1.0
Area of prestress (A_{ps}) = 2.17 in.^{2} total 10 - 0.6 inch strands
Working prestress (f_{ps}) = 183 ksi (assume 20 ksi losses)
Effective prestress force (P_f) = A_{ps} f_{ps} = 397 k
Compressive stress (f_{cpe}) = P_f ($1/A$+e/S) = 1.06 ksi
Prestress eccentricity (e) = y_b - x = 34.7 in.
Modulus of rupture (f_r) = 0.24 $f'_c{}^{0.5}$ = 0.657 ksi
$M_{cr} = \gamma_3\left[\left(\gamma_1 f_r + \gamma_2 f_{cpe}\right)S_c - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right)\right]$ = 4,830 kft
1.33 M_u = 3,760 k - ft controls

Proposed Method

Flexural cracking variability factor (γ_1) = 1.6 $h^{-0.15}$ = 1.20
Prestress variability factor (γ_2) = 1.1
Ratio of yield to ultimate reinforcement (γ_3) = 1.0
Area of prestress (A_{ps}) = 2.17 in.^{2} (total 10 - 0.6 inch strands)

Working prestress (f_{ps}) = 183 ksi (assume 20 ksi losses)
Effective prestress force (P_f) = A_{ps} f_{ps} = 397 k
Compressive stress (f_{cpe}) = P_f ($1/A+e/S$) = 1.06 ksi
Prestress eccentricity (e) = y_b - x = 34.7 in.
Modulus of rupture (f_r) = 0.24 $f'_c{}^{0.5}$ = 0.657 ksi
$M_{cr} = \gamma_3\left[(\gamma_1 f_r + \gamma_2 f_{cpe})S_c - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right)\right]$ = 4,250 k-ft
α M_u = 3,760 k-ft controls
Strength Modification Factor (α) = 1.33 ϕ = 1.33
Resistance factor (ϕ) = 1.0 (prestressed, tension controlled)

Moment Resistance

Prestress stress at ultimate (f_{ps}) = f_{pu} (1 - 0.28 c/d_p) = 268 ksi
Neutral axis depth (c) = a/0.85 = 1.67 in.
Flexural depth (d) = h - x = 78.3 in.
Compression flange width (b) = 108 in.
Stress block depth (a) = A_{ps} f_{ps}/(0.85 $f'c$ b) = 1.42 in.
Nominal Resistance (M_n) = A_{ps} f_{ps} (d - a/2) = 3,770 k-ft
Resistance factor (ϕ) = 1.0
Moment Resistance (M_r) = ϕ M_n = 3,770 k-ft
Net tensile strain (ε) = $\frac{\varepsilon_c(d-c)}{c} + \frac{f_{cpe}}{E_g}$ = 0.135 > 0.005 Tension controlled

Negative Bending

AASHTO LRFD

Factored Cracking Moment (M_{cr}) = γ_3 γ_1 f_r S /n = 2,910 k-ft (LRFD 5.6.3.3)
Modular ratio (n) = E_d/E_g = 0.775
Yield-to-ultimate ratio (γ_3) = 0.67 – ASTM A615 Grade 60
Flexural cracking variability factor (γ_1) = 1.6
Modulus of rupture f_r = 0.24 $f'c^{0.5}$ = 0.51 ksi - deck concrete (LRFD 5.4.2.6)
Minimum reinforcement (A_s) = 8.7 in^2 - minimum required to resist M_{cr}
Compression flange width (b) = 29.53 in.
Effective structure depth (d) = h - x - 1.5 d_b = 77.0 in.
Tension reinforcement diameter (d_b) = 1.25 in. - assume No. 9
Compression block depth (a) = A_s f_y/(0.85 f'_c b) = 4.62 in.
M_n = A_s f_y (d - a/2) = 3,250 k-ft

$\phi =$ 0.9

$M_r = \phi M_n =$ 2,920 k-ft - OK

Proposed Method

Factored Cracking Moment (M_{cr}) = $\gamma_3\ \gamma_1\ f_r\ S\ /n$ = 2,190 k-ft - (LRFD 5.6.3.3)

Modular ratio (n) = E_d/E_g = 0.775

Yield-to-ultimate ratio (γ_3) = 0.67 – ASTM A615 Grade 60

Flexural cracking variability factor (γ_1) = 1.6 $h^{-0.15}$ = 1.2

Member depth (h) = 6.74 ft

Modulus of rupture $f_r = 0.24\ f'c^{0.5}$ = 0.51 ksi - deck concrete (LRFD 5.4.2.6)

Minimum reinforcement (A_s) = 6.5 in^2 - minimum required to resist M_{cr}

Compression flange width (b) = 29.53 in.

Effective structure depth (d) = $h - x - 1.5\ d_b$ = 77.0 in.

Tension reinforcement diameter (d_b) = 1.25 in. - assume No. 9

Compression block depth (a) = $A_s\ f_y/(0.85\ f'_c\ b)$ = 3.45 in.

$M_n = A_s\ f_y\ (d - a/2)$ = 2,450 k-ft

$\phi =$ 0.9

$M_r = \phi M_n =$ 2,200 k-ft - OK

Summary

Both the current and proposed provisions for minimum flexural reinforcement control the amount of prestress in Span 1 and the amount of longitudinal deck reinforcement over most of bridge length, as shown in the moment profiles of Figure B1-1. Therefore, the bar cutoff lengths are directly impacted by minimum reinforcement.

The proposed method significantly reduces the minimum reinforcement from the current provisions in the LRFD. For the negative bending regions, the proposed method reduces the factored cracking moment by 25%. The reduction is less pronounced for the positive bending regions because of the effects of prestress.

B.2 CAST-IN-PLACE CONCRETE BOX GIRDER

A three-span cast-in-place concrete box girder bridge shown in Figure B.3 that is commonly built in California and Nevada is the subject of this design example. As with the first example, the side spans are far shorter than the end spans while the depth of the bridge is constant for along the entire length. Because the bridge is monolithic, the bridge resists all loading continuously including any prestress forces. All prestress consists of continuous post-tensioning that runs full length of the bridge. To control camber and reduce friction losses, the post-tensioning tendon midspan eccentricity is reduced in the shorter spans were flexural demands are reduced.

For this type of structure, it is more economical to design the post-tensioning cables for service loads, and add mild reinforcement in localized areas as needed to resist strength limit state loads including minimum reinforcement provisions. It is anticipated that minimum flexural reinforcement will control the design of this mild reinforcement in these side spans.

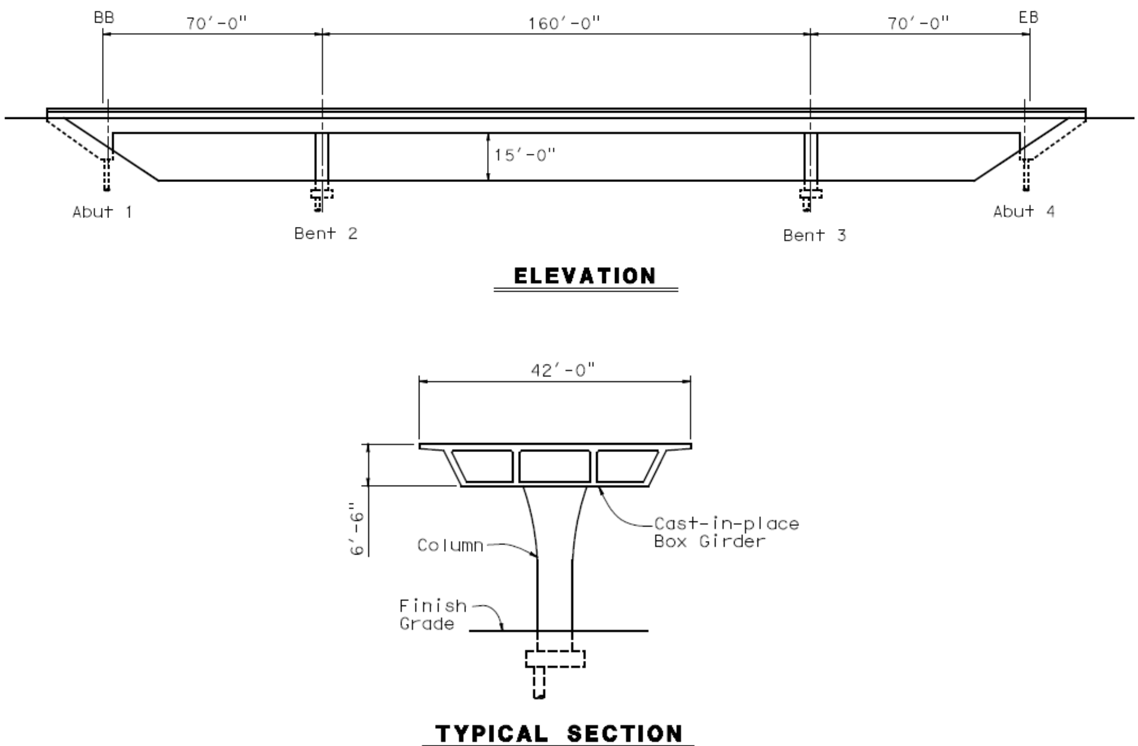

Figure B.3 – Cast-in-Place Box Girder

B 2.1 Bridge Layout

The bridge is 42.0 ft wide and 6.5 ft deep. The girders are spaced at 11.0 ft on center and are flared from 12 in. to 18 in. at the abutments and the bents. The soffit is flared to 12 in. at the bents. The columns are circular with a diameter of 6.0 ft.

Material Properties

$f'_c = 4ksi$

$E_c = 3{,}644 ksi$
$f_y = 60 ksi$
$E = 29{,}000 ksi$
$f_{pu} = 270 ksi$
$E_{ps} = 28{,}500 ksi$

Prestress Forces

The cast-in-place post-tensioned box girder bridge has full length tendons extending along the entire bridge. These tendons are located in the webs and are draped to provide maximum eccentricity at critical locations with a smooth profile to minimize friction losses. The tendon is optimized to provide a minimum amount of prestress force to meet this condition at all sections.

The allowable tension stress is limited to 0.19 $\sqrt{f'c}$ (ksi) under Service III limit state loads, which includes permanent and live loads. The jacking force is designed under the Service III limit state and is estimated with the software CT Bridge to be 5,910 kips.

Section flexural capacity is compared to strength limit state moments. Reinforcement is placed in the top and bottom flanges to provide additional strength, where capacity of the section with the prestress strand alone is insufficient. Flexure reinforcement is typically located at the bents and middle of the span. This differs from precast girders, where it is more economical to provide additional prestress strand to resist strength moments.

Moment Profiles

The strength limit states shown in Figure B.4 are for the entire bridge, assuming a full-width design, where all girders within the section resist equal loads. The controlling moment envelope is shaded in this profile. Minimum reinforcement provisions control the strength limit states for Span 1 under positive bending and most of Span 1 under negative bending. As shown, the factored cracking moment increases with prestress tendon eccentricity.

The Proposed Method reduces the flexural cracking moment at all locations with the most significant reductions in the negative bending regions. Since minimum reinforcement does not control the peak negative moments at the support, the prime benefit of the Proposed Method over current AASHTO LRFD provisions is reduced bar cutoff lengths.

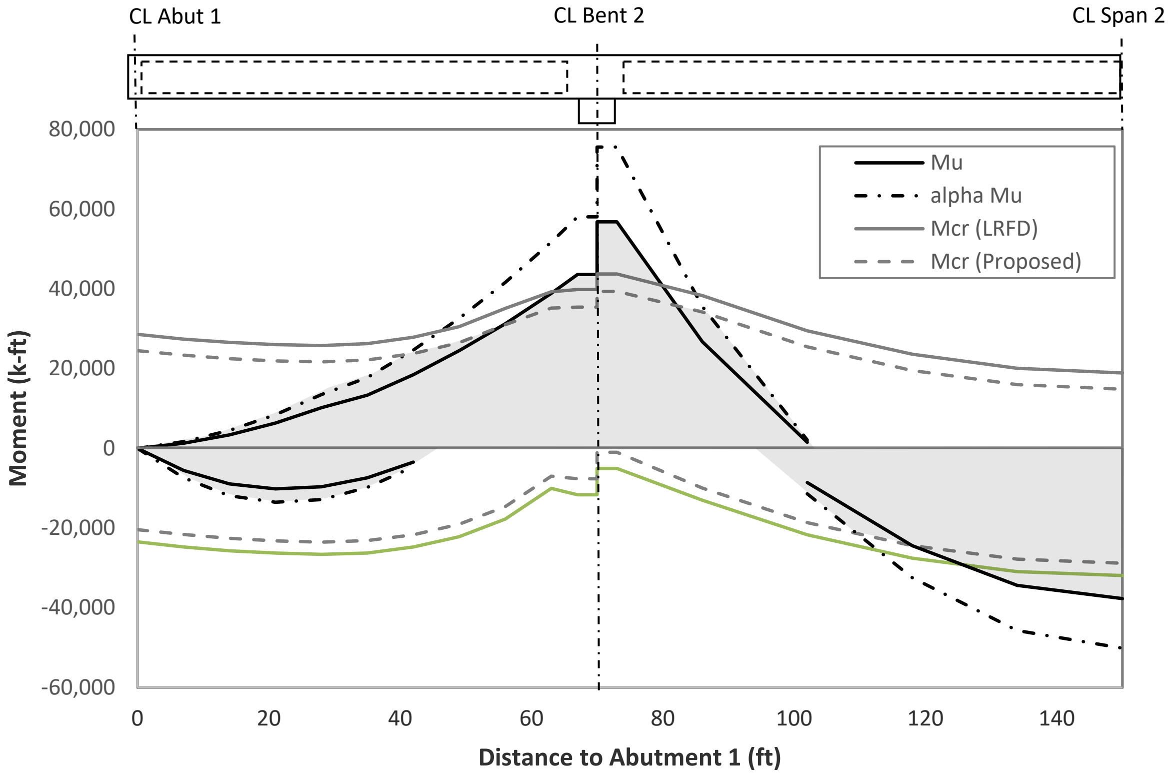

Figure B.4 – Precast Strength Limit State Moment Profiles with Minimum Flexural Reinforcement Provisions (Moments are plotted on the tension side)

Minimum Flexural Reinforcement Calculations

Section Properties

	Midspan	Cap face	
Area (A) =	10,500	13,600	in.^{2}
Depth (h) =	78.00	78.00	in.
Inertia (I) =	9,590,000	11,490,000	in.^{4}
Centroid (y_b) =	44.42	40.46	in.
Section Modulus (S_{top}) =	216,000	306,000	in.^{3}
Section Modulus (S_{bot}) =	286,000	284,000	in.^{3}
Strand centroid (x) =	36.0	66.0	in.
Top flange width (b_d) =	504	504	in.
Top flange thickness (t_d) =	8.75	8.75	in.
Bottom flange width (b_s) =	342	342	in.
Bottom Flange thickness (t_s) =	8.25	12.0	in.

Span 1 Maximum Positive Moment:

Design Moments

Factored Moment (M_u) = 9,650 kft (Strength I)

AASHTO LRFD

Flexural cracking variability factor (γ_1) =	1.6
Prestress variability factor (γ_2) =	1.1
Ratio of yield to ultimate reinforcement (γ_3) =	1.0
Area of prestress (A_{ps}) =	29.2 in.^{2} total 134 - 0.6 inch strands
Working prestress (f_{ps}) =	161 ksi (with all losses)
Effective prestress force (P_f) = A_{ps} f_{ps} =	4,725 k
Compressive stress (f_{cpe}) = P_f ($1/A+e/S$) =	0.633 ksi
Prestress eccentricity (e) = y_b - x =	8.42 in.
Modulus of rupture (f_r) = 0.24 $f'_c{}^{0.5}$ =	0.480 ksi
$M_{cr} = \gamma_3\left[\left(\gamma_1 f_r + \gamma_2 f_{cpe}\right)S\right]$ =	26,400 k-ft
1.33 M_u =	12,800 k-ft controls

Proposed Method

Flexural cracking variability factor (γ_1) 1.6 $h^{-0.15}$ =	1.24
Prestress variability factor (γ_2) =	1.1
Ratio of yield to ultimate reinforcement (γ_3) =	1.0
Area of prestress (A_{ps}) =	29.2 in.^{2} - total 134 - 0.6 inch strands
Working prestress (f_{ps}) =	161 ksi - with all losses
Effective prestress force (P_f) = A_{ps} f_{ps} =	4,725 k
Compressive stress (f_{cpe}) = P_f ($1/A+e/S$) =	0.633 ksi
Prestress eccentricity (e) = y_b - x =	8.42 in.
Modulus of rupture (f_r) = 0.24 $f'_c{}^{0.5}$ =	0.480 ksi
$M_{cr} = \gamma_3\left[\left(\gamma_1 f_r + \gamma_2 f_{cpe}\right)S\right]$ =	23,500 k-ft
Resistance factor (ϕ) =	1.0 - assume tension controlled
Moment modification factor (α) = 1.33 ϕ =	1.33
1.33 M_u =	12,800 k-ft controls

Moment Resistance

Prestress stress at ultimate (f_{ps}) = f_{pu} (1 - 0.28 c/d_p) =	261 ksi
Neutral axis depth (c) = a/0.85 =	5.22 in.
Flexural depth (d) = h - x =	42.0 in.
Compression flange width (b) =	504 in.
Stress block depth (a) = A_{ps} f_{ps}/(0.85 $f'c$ b) =	4.44 in.

Nominal Resistance (M_n) = $A_{ps}\, f_{ps}\, (d - a/2)$ = 25,200 k-ft
Resistance factor (ϕ) = 1.0
Moment Resistance (M_r) = $\phi\, M_n$ = 25,200 k-ft
Net tensile strain (ε_t) = $\frac{\varepsilon_c(d-c)}{c} + \frac{f_{cpe}}{E_g}$ = 0.020 > 0.005 Tension controlled

Negative Bending – Span 1 Cap Face

Factored Moment (M_u) = 43,700 kft (Strength I)

AASHTO LRFD

Flexural cracking variability factor (γ_1) = 1.6
Prestress variability factor (γ_2) = 1.1
Ratio of yield to ultimate reinforcement (γ_3) = 1.0
Area of prestress (A_{ps}) = 29.2 in.^{2} total 134 - 0.6 inch strands
Working prestress (f_{ps}) = 170 ksi (with all losses)
Effective prestress force (P_f) = $A_{ps}\, f_{ps}$ = 4,952 k
Compressive stress (f_{cpe}) = $P_f\,(1/A+e/S)$ = 0.729 ksi – top fiber compression
Prestress eccentricity (e) = $y_b - x$ = 22.54 in.
Modulus of rupture (f_r) = $0.24\, f'_c{}^{0.5}$ = 0.480 ksi
$M_{cr} = \gamma_3\left[\left(\gamma_1 f_r + \gamma_2 f_{cpe}\right)S\right]$ = 40,000 k-ft
1.33 M_u = 58,100 k-ft

Proposed Method

Flexural cracking variability factor (γ_1) $1.6\, h^{-0.15}$ = 1.24
Prestress variability factor (γ_2) = 1.1
Ratio of yield to ultimate reinforcement (γ_3) = 1.0
Area of prestress (A_{ps}) = 29.2 in.^{2} - total 134 - 0.6 inch strands
Working prestress (f_{ps}) = 161 ksi - with all losses
Effective prestress force (P_f) = $A_{ps}\, f_{ps}$ = 4,725 k
Compressive stress (f_{cpe}) = $P_f\,(1/A+e/S)$ = 0.633 ksi
Prestress eccentricity (e) = $y_b - x$ = 22.54 in.
Modulus of rupture (f_r) = $0.24\, f'_c{}^{0.5}$ = 0.480 ksi
$M_{cr} = \gamma_3\left[\left(\gamma_1 f_r + \gamma_2 f_{cpe}\right)S\right]$ = 33,400 k-ft
Resistance factor (ϕ) = 1.0 - assume tension controlled
Moment modification factor (α) = 1.33 ϕ = 1.33
1.33 M_u = 58,100 k-ft

Moment Resistance

Prestress stress at ultimate (f_{ps}) = f_{pu} (1 - 0.28 c/d_p) = 260 ksi

Neutral axis depth (c) = a/0.85 =	8.76 in.
Flexural depth (d) = x =	64.3 in. - effective centroid
Compression flange width (b) =	342 in.
Area of longitudinal reinforcement (A_s) =	18.0 in.^{2} - located in deck
Stress block depth (a) = (A_{ps} f_{ps} + A_s f_y)(0.85 $f'c$ b) =	7.45 in.
Nominal Resistance (M_n) = (A_{ps} f_{ps} + A_s f_y)(d - a/2) =	43,700 k-ft
Resistance factor (ϕ) =	1.00
Moment Resistance (M_r) = ϕ M_n =	43,700 k-ft - OK
Net tensile strain (ε_t) = $\frac{\varepsilon_c(d-c)}{c} + \frac{f_{cpe}}{E_g}$ =	0.019 > 0.005 Tension controlled

Summary

Both the current and proposed provisions for minimum flexural reinforcement control the required positive moment flexural strength in Span 1, and the prestress strand is sufficient to resist the positive demands. In the negative bending peak demands requires mild reinforcement in the deck in addition to the prestress tendons to provide required resistance. As shown minimum reinforcement demands control beyond 15 feet from the centerline of the bent. Consequently, the bar cutoff lengths are directly impacted by minimum reinforcement.

The proposed method reduces the minimum reinforcement from the current provisions in the LRFD. However, the savings are limited due because both the cracking strength and the resistance are dominated by the amount of prestress, and savings will be realized by reduced bar cutoff lengths of mild reinforcement placed in the deck over the caps to resist negative moment demands.

B.3 SPAN-BY-SPAN SEGMENTAL BRIDGE WITH EXTERNAL TENDONS

INTRODUCTION

A two-span precast segmental bridge is the subject of this design example. The bridge is built using the span-by-span construction method. The bridge chosen for this example is part of the I4/Lee Roy Selmon Expressway in Tampa, FL.

Each of the two spans in this bridge is simply supported. Only Span 2 of this bridge is the subject of this example. This represents a relatively large depth-to-span ratio bridge in which the minimum flexural reinforcement requirement could control the design. An elevation view of this bridge is shown in Figure B.5.

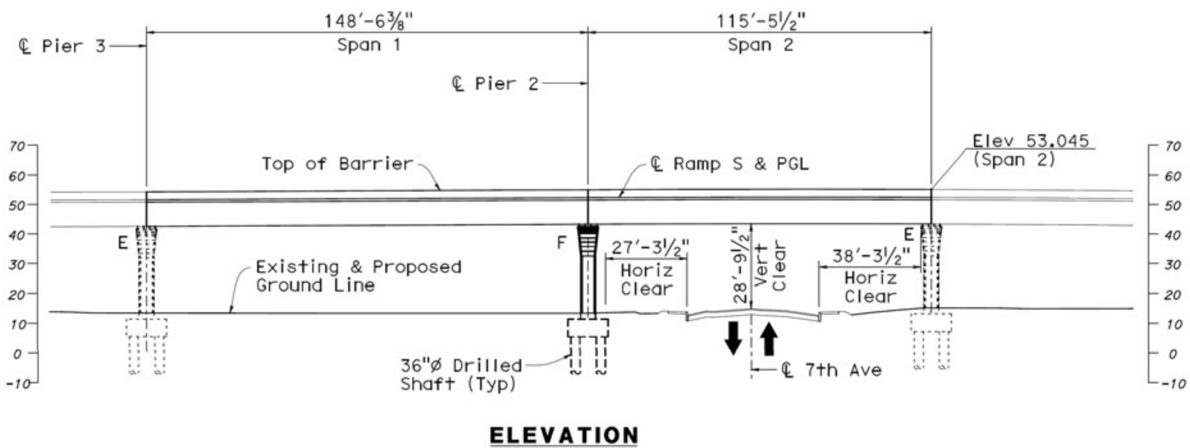

Figure B.5 – Segmental Span-By-Span Bridge Design Example

For Span 2, the cross section consists of a single-cell box section with long overhangs as shown in Figure B.6. The deck width is variable as indicated in Figure B.6. The length of Span 2 is approximately 115'-6" and the bridge is prestressed by means of external unbonded tendons as shown in the tendon layout in Figure B.7.

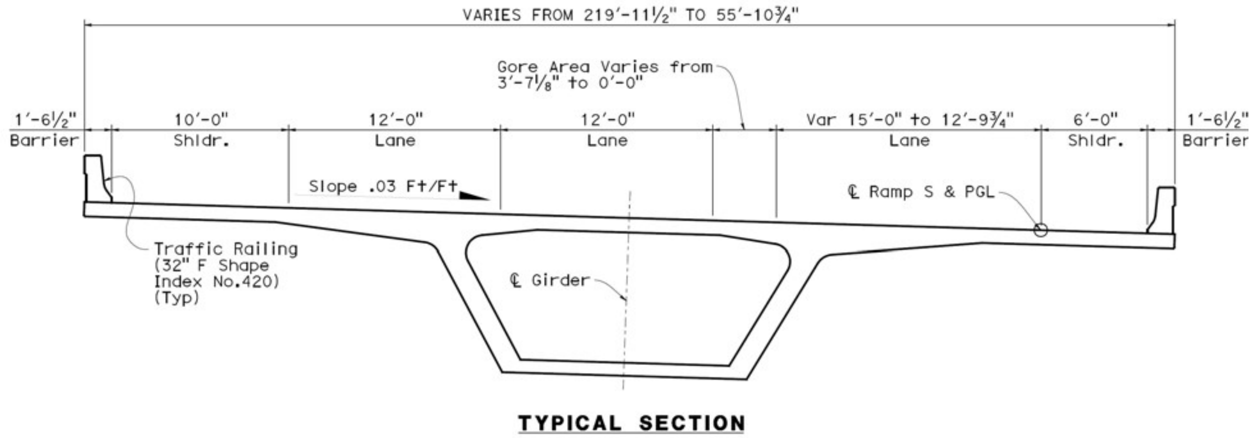

Figure B.6 – Cross Section (Span 2)

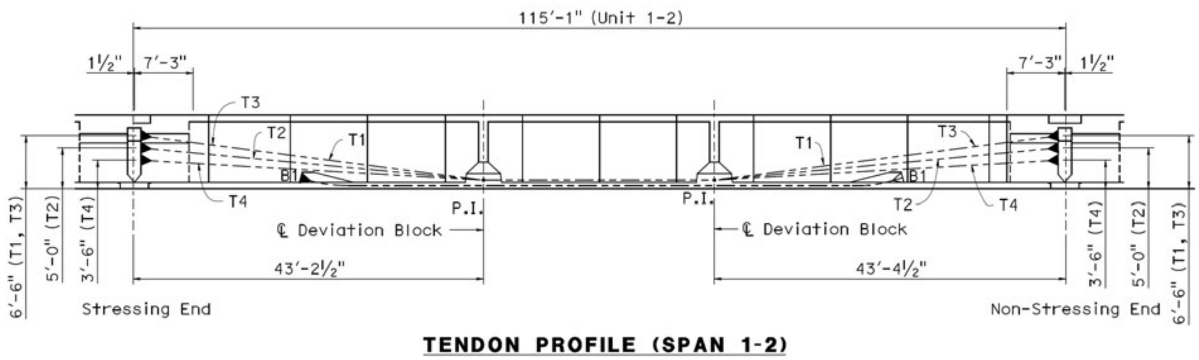

Figure B.7 – Prestressing Tendon Layout (Span 2)

SPECIFICATIONS

This example is designed based on the *AASHTO LRFD Bridge Design Specifications 8th Edition, 2017* and the Proposed method for computing minimum reinforcement.

MATERIAL PROPERTIES

$f'_c = 6.5\ ksi$

$E_c = 4{,}888\ ksi$

$f_y = 60\ ksi$

E_s = 29,000 *ksi*

f_{pu} = 270 *ksi*

E_{ps} = 28,500 *ksi*

PRESTRESS DESIGN

For precast segmental bridges with no bonded reinforcement or bonded tendons crossing the joints, no tensile stresses are allowed at all segment-to-segment joints under service loads. Longitudinal analysis and design of this bridge included concrete stresses under service loads, flexural capacity, shear capacity, principal stresses in the box girder webs and minimum flexural reinforcement requirements. Except for the minimum flexural reinforcement requirement, design is satisfactory with the use of four external tendons on each side of the box section; three of these tendons are composed of 19-0.6" ϕ strands and the fourth tendon is composed of 15-0.6" ϕ strands. Thus, the total number of external unbonded strands in this bridge is 144-0.6"ϕ.

MOMENT DIAGRAMS

Figure B.8 shows the bending moments along the length of the single span bridge (Span 2). The figure shows the minimum design moments due to cracking according to the current AASHTO LRFD Specifications and based on the proposed method. The proposed provisions significantly reduce the factored cracking moments (M_{cr}). The controlling design envelop is shaded, which illustrates that the middle third of the span length, M_{cr} controls over $\alpha\ M_u$ for minimum reinforcement.

Figure B.8 also shows the factored flexural moment capacity is higher than the factored moment, M_u, at all sections. However, in the middle 80 ft of the span length, the LRFD minimum flexural reinforcement requirement is not satisfied. At midspan, the minimum reinforcement provisions are satisfied using the Proposed method. However, the provisions are not satisfied near the quarter span length locations, which are roughly 25 and 90 feet from the left support. A redesign could include moving the soffit tendon anchors one segment closer to the end spans.

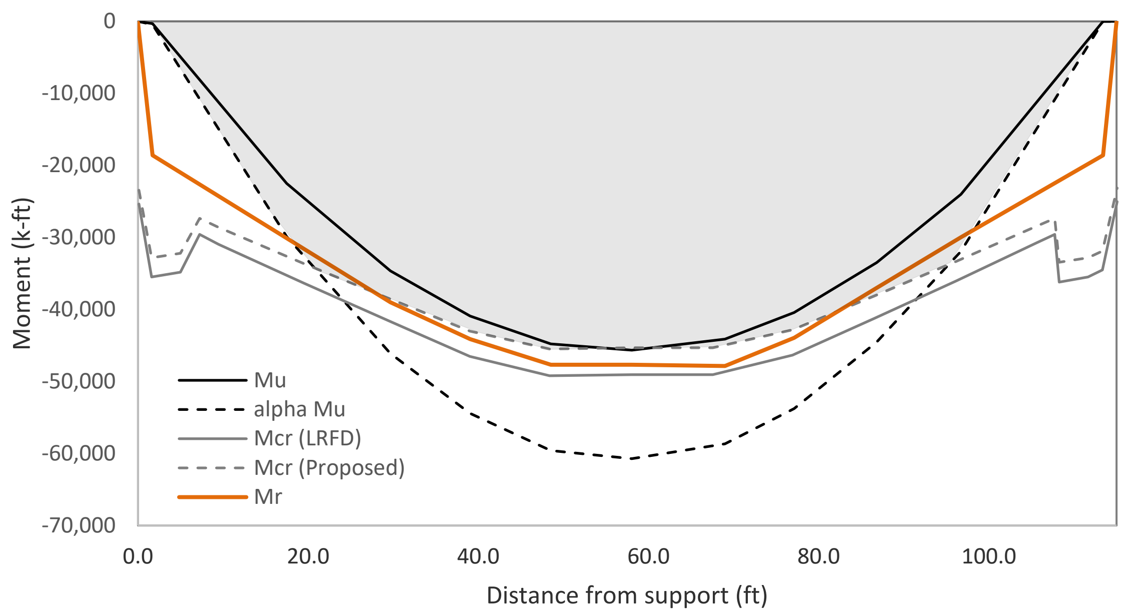

Figure B.8 – Cracking Moment, Factored Moment and Flexural Capacity of a Precast Segmental Span-By-Span Bridge Example

Analysis of this bridge was done using LARSA 4D. Construction stages and time-dependent effects were considered in the analysis. Below are hand calculations for the midspan section of the bridge.

Design moments:

Sign convention is positive for moment resulting in tensile stress at bottom surface (opposite to the sign shown in Figure B-8).

$M_{DC} = 24{,}677\ k\text{-}ft$ Self wt, ½" sacrificial wearing surface, diaphragms & barriers

$M_{DW} = 0\ k\text{-}ft$ No utilities or future wearing surface

$M_{SecP/S} = 0\ k\text{-}ft$ No secondary effects from prestressing for a single span bridge

$M_{TU} = 0\ k\text{-}ft$ No moments from uniform temperature rise for a single span bridge

$M_{TG} = 0\ k\text{-}ft$ No moments from temperature gradient for a single span bridge

$M_{HL\square 93\square I} = 8{,}560\ k\text{-}ft$

$$M_u = M_u^{StrengthI} = 1.25M_{DC} + 1.50M_{DW} + 1.00M_{\frac{Secp}{s}} + 1.75M_{HL-93+I} + 0.5M_{TU} + 0.5M_{TG}$$

$M_u = 1.25\text{x}(24{,}677) + 1.50\text{x}(0) + 1.00\text{x}(0) + 1.75\text{x}(8{,}560) + 0.50\text{x}(0) + 0.50\text{x}(0)$

$M_u = 45{,}826\ k\text{-}ft$

Section properties:

h = 9 ft =108 in The sacrificial surface is included as external load

only h_f = 9.5 in (minimum thickness of compression flange)

b_f = 58.625 ft = 703.5 in (compression flange width)

b_w = 30 in

I = 819.97 ft^4 =17,002,898 in^4

A = 91.02 ft^2 =13,106.88 in^2

$\overline{y_b}$ = 78.84 in (distance from section CG to bottom fiber)

$\overline{y_t}$ =29.16 in (distance from section CG to top fiber)

Calculation of ϕM_n from prestressing:

In Figure B-8, the moment capacity is calculated using LARSA 4D. At the midspan section, the factored flexural moment capacity is 47,631 kip-ft. The flexural capacity for the midspan section is calculated below using the AASHTO LRFD equations, which may result in slightly different values from those calculated by LARSA 4D.

A_{ps} = 144 x 0.217 in^2 = 31.248 in^2 Total of 144-0.6" ϕ strands (external unbonded)

d_p = 94.75 in (distance from P/S CG to top fiber)

$e = d_p - \overline{y_t}$ = 94.75- 29.16 = 65.59 in (tendon eccentricity)

β_1 = 0.85 - 0.05 (f_c' - 4) = 0.725

Effective prestressing force in external tendons (from LARSA 4D):

P_f = 5,247 $kips$

$$f_{pe} = \frac{P_f}{A_{ps}} = \frac{5{,}247 \text{ kips}}{31.248 \text{ in}^2} = 167.9 \text{ ksi}$$

Length of external tendon (approximate): l_i =114.83 ft

Number of support hinges crossed by external tendons (single span): N_i = 0

Effective length of external tendons:

$$l_e = 2 * \frac{l_i}{2 + N_i} = 114.83 \text{ ft}$$

Depth of compression zone: Assume $f_{ps} = 228\ ksi < f_{py} = 243\ ksi$

$$c = \frac{A_{ps} f_{ps}}{0.85 f_c' \ \beta_1 b_f} = 2.53 \text{ in}$$

Depth of neutral axis is smaller than deck thickness. Thus, use of equations for rectangular sections is justified.

Stress in external tendons at ultimate moment:

$$f_{ps} = f_{pe} + \frac{900(d_p - c)}{l_e} = 228 \text{ ksi} < f_{ps} = 243 \text{ ksi}$$

This stress is the same as assumed above. Thus, no iterations are needed.

Tensile force at ultimate moment: $T = A_{ps} f_{ps} = 7{,}124.5\ kips$

Depth of equivalent rectangular stress block: $a = \beta_1 c = 1.83\ in$

Resistance factor: $\phi = 0.90$ (segmental bridges with unbonded tendons)

Factored flexural moment capacity:

$$\phi M_n = \phi A_{ps} f_{ps} \left(d_p - \frac{a}{2}\right) = 50{,}139 \text{ kip} - \text{ft}$$

LARSA 4D calculated the factored moment capacity as 47,633 kip-ft (about 5% difference). It should be noted that the above-calculated factored moment capacity does not take into account the reduction in moment arm of the external tendons due to deflection of the superstructure. Thus, the predicted flexural capacity will be less than 50,139 kip-ft.

Minimum reinforcement by the proposed method (Modified LRFD):

$$\phi M_n \geq M_{fcr} \text{ or } \phi M_n \geq \alpha M_u; \text{ where } M_{fcr} = \gamma_3(\gamma_1 f_r + \gamma_2 f_{cpe})S$$

$$where\ \alpha = 1.0 + \frac{0.33(\varepsilon_t - \varepsilon_{cl})}{\varepsilon_{tl} - \varepsilon_{cl}}\ ; 1.0 \leq \alpha \leq 1.33$$

$\gamma_1 = 1.2h^{-0.15}$; where h = member depth (ft) (proposed for precast segmental bridges)

$$\gamma_1 = 1.2x(9)^{-0.15} = 0.863$$

$$f_r = 0.24\sqrt{f_c'} = 0.24x\sqrt{6.5} = 0.612 \text{ ksi}$$

$\gamma_2 = 1.0$ (proposed for bridges with only unbounded tendons)

$\gamma_3 = 1.0$ (tensile resistance is provided by prestressing steel)

Concrete compressive stress at bottom fiber due to prestressing (after losses):

$$f_{cpe} = \frac{P_f}{A} + \frac{P_f e \overline{y_b}}{I} = \frac{5{,}247}{13{,}106.88} + \frac{5{,}247x65.59x78.84}{17{,}002{,}898} = 2.000 \text{ ksi}$$

$$S = \frac{I}{\overline{y_b}} = \frac{17{,}002{,}898}{78.84} = 215{,}663 \text{ in}^3$$

$\gamma_3(\gamma_1 f_r + \gamma_2 f_{cpe})S = 1.0x(0.863x0.612 + 1.0x2.000)\left(\frac{215{,}663}{12}\right) = 45{,}435 \text{ k} - \text{ft} < M_u = 45{,}826 \text{ k} - \text{ft}$

$\gamma_3(\gamma_1 f_r + \gamma_2 f_{cpe})S = 45{,}435 \text{ k} - \text{ft}$ controls the design

$\phi M_n \geq \gamma_3(\gamma_1 f_{cr} + \gamma_2 f_{cpe})S$ (MFR Requirement)

$\phi M_n = 47{,}633$ k-ft $> \gamma_3(\gamma_1 f_r + \gamma_2 f_{cpe})S = 45{,}435$ k-ft

The factored flexural moment capacity calculated by LARSA 4D (used for the plot in Figure B.8) is smaller than the 50,139 kip-ft factored moment capacity calculated above. However, the actual factored moment capacity should be less than 50,139 k-ft as a result of the reduction in the internal moment arm of the section due to vertical downward deflection of the girder at midspan. The flexural moment capacity calculated by LARSA 4D is used in this example. Thus, the minimum flexural reinforcement requirement is satisfied for the Proposed method at midspan, as described herein.

SUMMARY

The example demonstrates how the minimum reinforcement provisions are incorporated in a span-by-span precast segmentally constructed bridge. As shown, this design satisfies minimum reinforcement provisions at the location of maximum moment using the Proposed method described in this research, while not satisfying the current AASHTO LRFD provisions.

B.4 BALANCED CANTILEVER BRIDGE WITH INTERNAL TENDONS

INTRODUCTION

A four-span precast segmental bridge is the subject of this design example. The bridge is built using the cantilever construction method. The bridge chosen for this example is part of the I-4/Lee Roy Selmon Expressway in Tampa, FL. Elevation view of the bridge is shown in Figure B.9. The approximate lengths of spans are 147'-3", 186'-1", 186'-9" and 145'-6" for Spans 1 through 4, respectively, with a total bridge length of 665'-7".

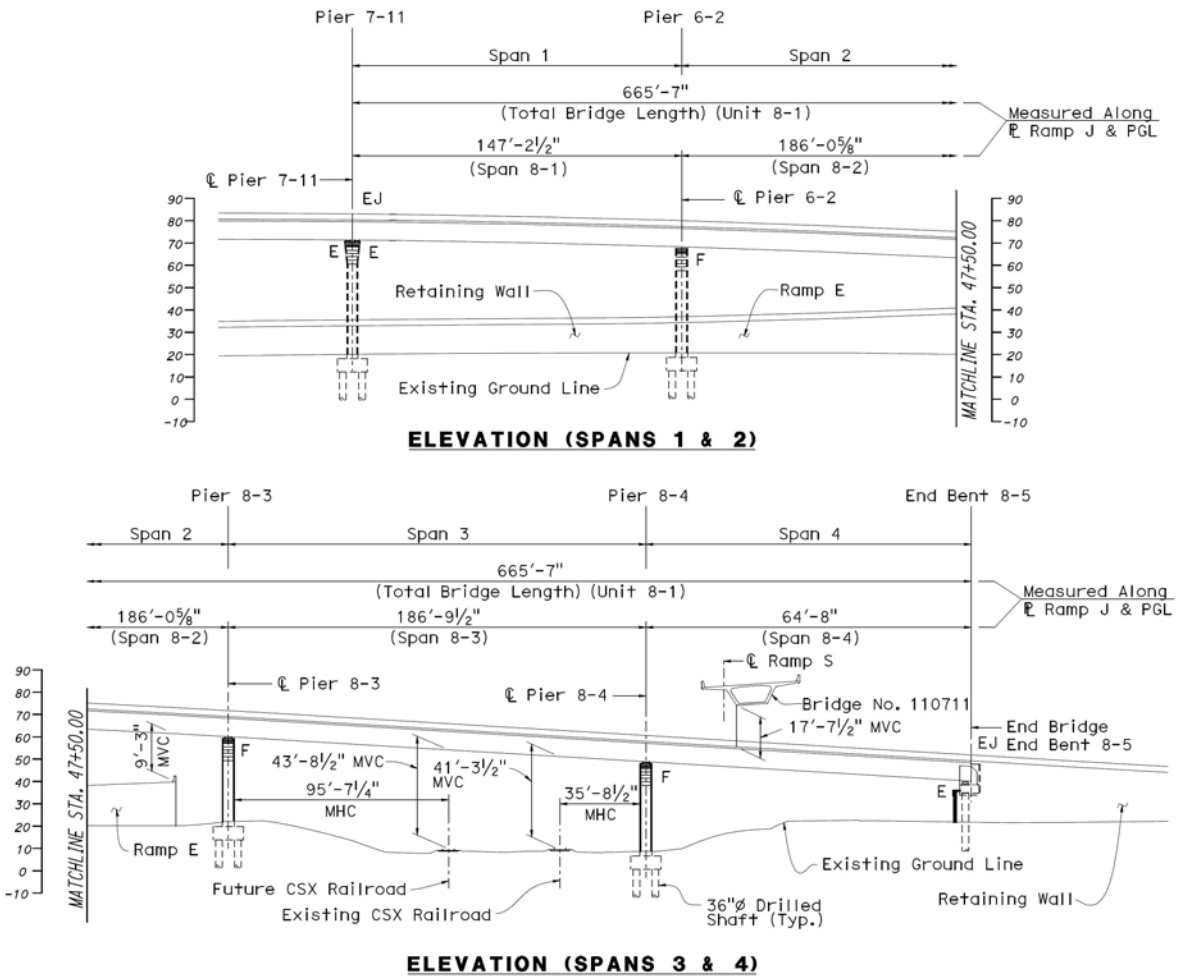

Figure B.9 – Precast Segmental Cantilever Bridge Design Example

The cross section consists of the single-cell box section shown in Figure B.10. The deck width is 30'-1" and is constant along the entire length of the bridge.

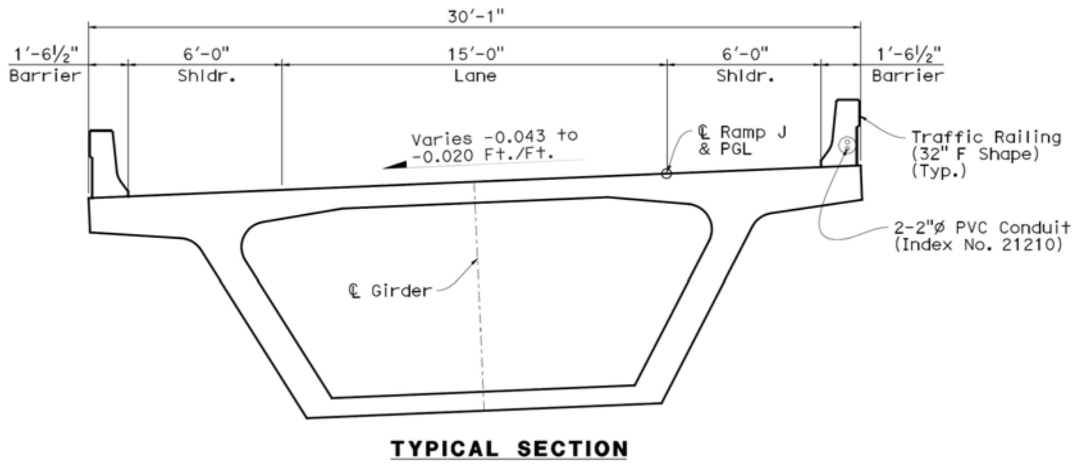

Figure B.10 – Cross section (Span 2)

The prestressing steel consists of typical internal (bonded) tendons in the deck slab. Continuity prestressing steel consists of external (unbonded) tendons as shown in Figure B.11 and Figure B.12. There are a total of three external tendons next to each of the two webs (Tendons T3, T4 & T5 in Figure B.11 and Figure B.12).

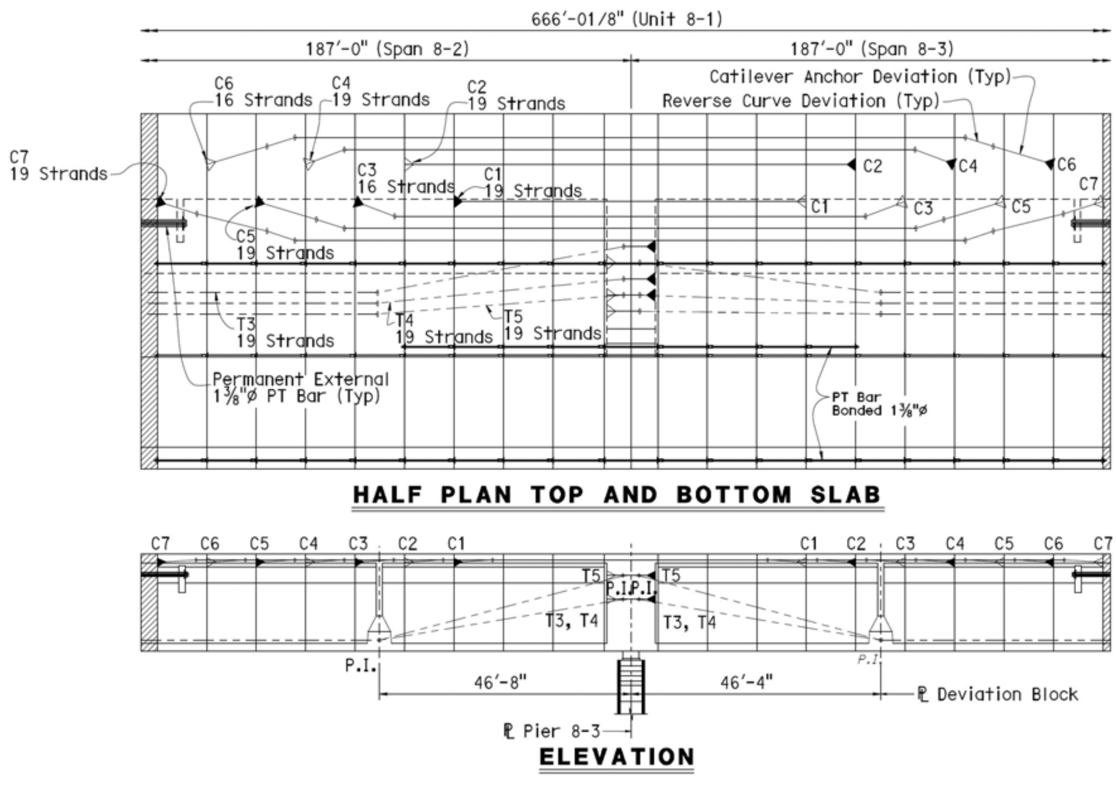

Figure B.11 – Tendon Layout for the Precast Segmental Cantilever Bridge Design Example

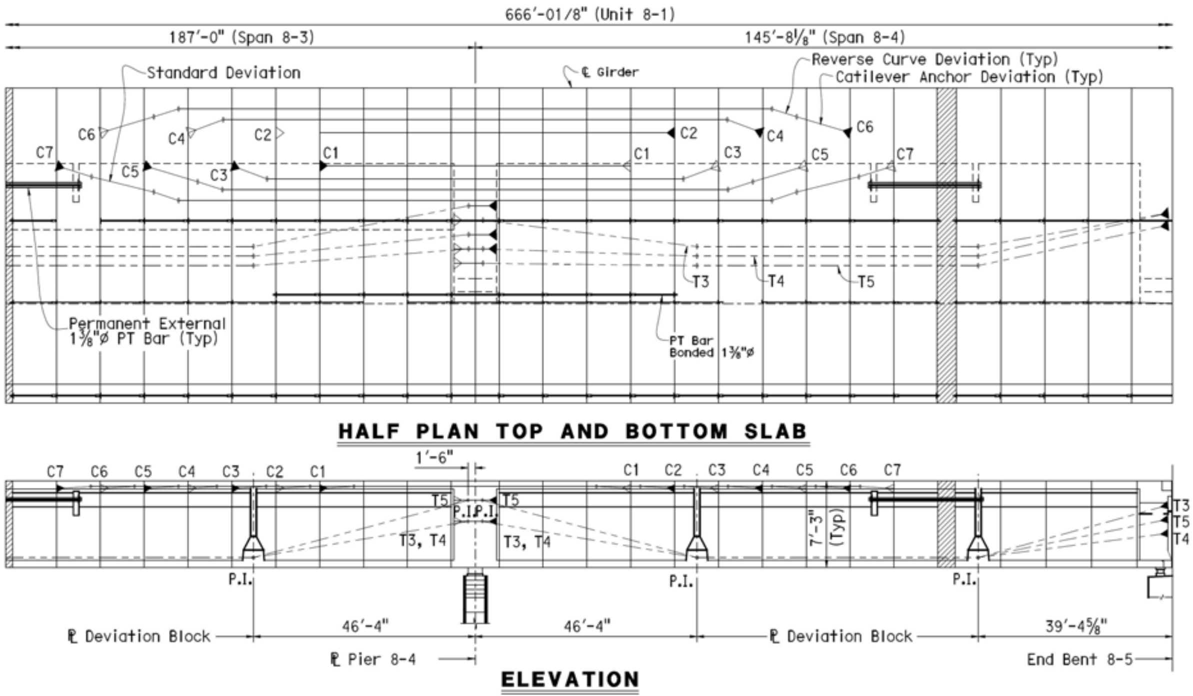

Figure B.12 – Tendon Layout for the Precast Segmental Cantilever Bridge Design Example

SPECIFICATIONS

This example is designed based on the *AASHTO LRFD Bridge Design Specifications 8th Edition, 2017.*

Chapter 1 **MATERIAL PROPERTIES**

$$f'_c = 8.5\ ksi$$
$$E_c = 5{,}589\ ksi$$
$$f_y = 60\ ksi$$
$$E_s = 29{,}000\ ksi$$
$$f_{pu} = 270\ ksi$$
$$E_{ps} = 28{,}500\ ksi$$

PRESTRESS DESIGN

For precast segmental bridges, no tensile stresses are allowed at all segment-to-segment joints under service loads. Longitudinal analysis and design of this bridge included concrete stresses under service loads, flexural capacity, shear capacity, principal stresses in the box girder webs and minimum flexural reinforcement requirements. At the first segment-to-segment joint next to Pier 8-3 in Span 4 (most critical section for negative moment), there are a total of 254-0.6" ϕ internal (bonded) strands and 114"-0.6 ϕ unbonded strands (external tendons). In the positive moment region in Span 4 (most critical section for positive moment), the only prestressing is provided by the continuity external tendons and the total number of strands is 114.

MOMENT DIAGRAMS

The bridge is almost symmetric about centerline of Pier 8-3, and therefore, moment diagrams for only one half of the bridge are shown. Figure B.13 shows the negative bending moments along the length of Spans 3 & 4 (from Pier 8-3 to End Bent 8-5). Negative moment results in tensile stresses at top surface of the superstructure. The figure shows the minimum design moments due to cracking according to the current AASHTO LRFD Specifications and based on the proposed method. It is clear that the proposed provisions considerably reduce the requirements for minimum reinforcement. Under negative bending, $1.33M_u$ is less than M_{cr} (AASHTO LRFD Specifications) or the cracking moment based on the proposed method. Further, the proposed method reduces the required negative bending moments because the section is not tensioned controlled, and α is less than 1.33.

Figure B.13, shows variation of the positive bending moments (bending moments resulting in tensile stresses at bottom surface of the superstructure). Again, use of the proposed method significantly reduces the required design moments as compared to the current AASHTO LRFD provisions. Near midspan, the M_{cr} moment profile controls over αM_u, whereas αM_u controls MFR for sections near the supports. The factored flexural moment resistance (M_r) are exceeded by a negligible margin near the middle of Span 8-3.

Analysis of this bridge was done using LARSA 4D. Construction stages and time-dependent effects were considered in the analysis. The following are hand calculations for the section at first segment-to-segment joint in Span 4 (joint at Pier 8-4) as well as maximum positive moment section in Span 4 of the bridge.

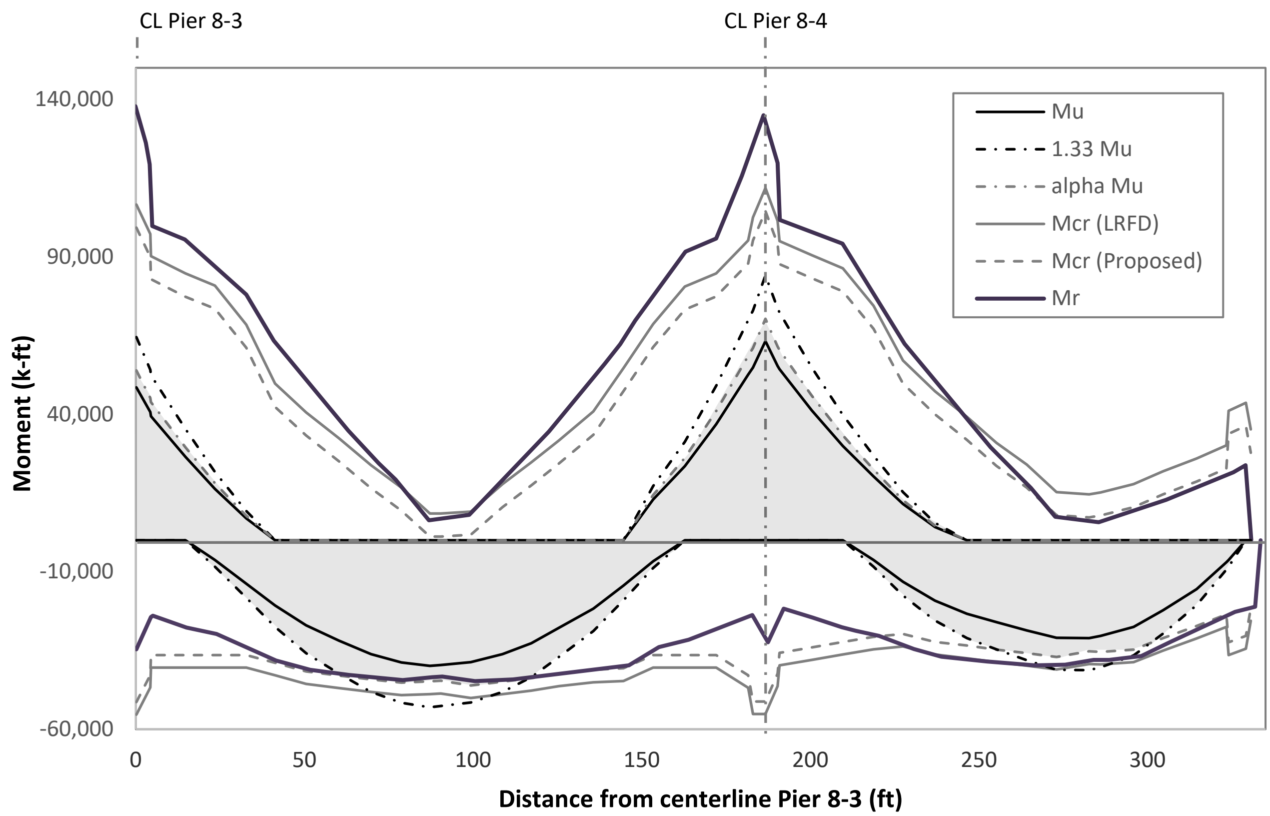

Figure B.13 – Cracking Moment, Factored Moment and Flexural Capacity of a Precast Segmental Cantilever Bridge Example

Design moments:

Sign convention is positive for moment resulting in tensile stress at bottom surface (opposite to the sign shown in Figure B.13).

<u>Section A: Section at First Joint (Pier Segment) in Span 4:</u>

$M_{DC} = -53{,}167\ k-\ ft$ Self wt, ½" sacrificial wearing surface, diaphragms & barriers

$M_{DW} = 0\ k-\ ft$ No utilities or future wearing surface

$M_{LT} = 1{,}842\ k-$ ft Long-term effects (concrete creep & shrinkage and relaxation of prestressing steel)

$M_{SecP/S} = 25{,}000\ k-\ ft$ Secondary effects from prestressing

$M_{TU} = -2\ k-\ ft$ Uniform temperature rise

$M_{TG} = -1{,}628\ k-\ ft$ Temperature gradient

$M_{HL-93+I} = -7{,}727\ k-\ ft$

$$M_u = M_{u\ StrengthI} = 1.25M_{DC} + 1.50M_{DW} + 0.50M_{LT} + 1.00M_{SecP/S} + 1.75M_{HL-93+I}$$

$+0.50M_{TU} + 0.50M_{TG}$

$$M_u = 1.25\times(-53{,}167) + 1.50\times(0) + 0.50\times(1{,}842) + 1.00\times(25{,}000) + 1.75\times(-7{,}727)$$
$$+ 0.50\times(-2) + 0.50\times(-1{,}628)$$

$$M_u = 54{,}875\ k-ft$$

<u>Section B: Section at Location of Maximum Positive Moment in Span 4:</u>

$M_{DC} = 6{,}792\ k-ft$ Self wt, ½" sacrificial wearing surface, diaphragms & barriers

$M_{DW} = 0\ k-ft$ No utilities or future wearing surface

$M_{LT} = 628\ k-ft$ Long-term effects (concrete creep & shrinkage and relaxation of prestressing steel)

$M_{SecP/S} = 8{,}667\ k-ft$ Secondary effects from prestressing

$M_{TU} = -1\ k-ft$ Uniform temperature rise

$M_{TG} = 1{,}842\ k-ft$ Temperature gradient

$M_{HL-93+I} = 7{,}209\ k-ft$

$$M_u = M_{u\ StrengthI} = 1.25M_{DC} + 1.50M_{DW} + 0.50M_{LT} + 1.00M_{SecP/S} + 1.75M_{HL-93+I}$$
$$+ 0.50M_{TU} + 0.50M_{TG}$$
$$M_u = 1.25\times(6{,}792) + 1.50\times(0) + 0.50\times(628) + 1.00\times(8{,}667) + 1.75\times(7{,}209)$$
$$+ 0.50\times(-1) + 0.50\times(1{,}842)$$
$$M_u = 31{,}008\ k-ft$$

Section properties:

Section properties for both Sections A & B are similar.

$h = 9\ ft = 108\ in$ The sacrificial surface is included as external load only

$h_f = 9.0\ in$ (minimum thickness of compression flange for negative moment section)

$h_f = 9.5\ in$ (minimum thickness of compression flange for positive moment section)

$b_f = 13.83\ ft = 166\ in$ compression flange width for negative moment section

$b_f = 30.08\ ft = 361\ in$ compression flange width for positive moment section

$b_w = 30\ in$

$I = 684.20\ ft^4 = 14{,}187{,}571\ in^4$

$A = 63.33\ ft^2 = 9{,}119.52\ in^2$

$y_b = 69.96\ in$ distance from section CG to bottom fiber

$y_t = 38.04\ in$ distance from section CG to top fiber

Calculation of factored moment resistance - $M_r = \phi M_n$**:**

In Figure B.13, the moment capacity is calculated using LARSA 4D. The factored flexural moment capacities are 101,729 kip-ft and 37,958 kip-ft at Sections A & B, respectively. The flexural capacities for both sections are calculated below using the AASHTO LRFD equations, which may result in slightly different values from those calculated by LARSA 4D.

Section A: Section at First Joint (Pier Segment) in Span 4:

$A_{ps1} = 254 \times 0.217\, in^2 = 55.118\, in^2$ Total of 254-0.6" ϕ strands (internal bonded)

$A_{ps2} = 114 \times 0.217\, in^2 = 24.738\, in^2$ Total of 114-0.6" ϕ strands (external unbonded)

$A = 3 \times 1.58\, in^2 = 4.74\, in^2$ 3-1.58" ϕ strands high-strength bars in the deck slab

Yield strength for high-strength bars:

$f_y = 120\, ksi$

$d_{p1} = 100.50\, in$ (distance from bottom fiber to C.G. of bonded tendons)

$d_{p2} = 66.83\, in$ (distance from bottom fiber to C.G. of external tendons)

$d_s = 102\, in$ (distance from bottom fiber to C.G. of high strength bars)

$\beta_1 = 0.85 - 0.05\,(f_c - 4) = 0.625$

Effective prestressing force in external tendons (from LARSA 4D):

$P_{f2} = 4{,}427\, kips$

$A_{ps} = 24.738\, in^2$

f_{pe} = 4,427 kips/24.73 in^2 = 178.9 ksi

Length of external tendon (approximate): $l_i = 150.50\, ft$

Number of support hinges crossed by external tendons (end span): $N_i = 1$

Effective length of external tendons: $l_e = \frac{l_i}{(2-N_i)} = 100.3\, ft$

Depth of compression zone (assume f_{ps} = 182 ksi):

$$c = \frac{A_{ps1} f_{pu} + A_{ps2} f_{ps2} + A_s f_y - 0.85 f'_c (b_f - b_w) h_f}{0.85 f'_c \beta_1 + k A_{ps1} \frac{f_{pu}}{d_{p1}}} = 49.2\, in$$

Stress in the bonded tendons at ultimate:

$$f_{ps1} = f_{pu}\left(1 - k\frac{c}{d_{p1}}\right) = 229\, ksi$$

Stress in the external tendons at ultimate:

$$f_{ps2} = f_{pe} + 900\frac{(d_{p2} - c)}{l_e} = 210\, ksi$$

This stress is the same as assumed above. Thus, no iterations are needed.

Tensile force at ultimate moment:

$$T = A_{ps1}f_{ps1} + A_{ps2}f_{ps2} + A_s f_y = 18{,}400\ kips$$

Depth of equivalent rectangular stress block:

$$a = \beta_1\ c = 41.8\ in$$

Net tensile strain:

$$\varepsilon_t = \varepsilon_c \frac{(d-c)}{c} + \frac{f_{cpe}}{E} = 0.0030$$

where d is the effective depth of all prestress at ultimate, fcpe is the effective prestress with all losses and E is the concrete modulus of elasticity.

Since the net tensile strain is in between 0.002 and 0.005, the section is in the transition zone. Therefore the resistance factor is:

$$\phi = 0.75 + 0.25\frac{(\varepsilon_t - \varepsilon_{cl})}{(\varepsilon_{tl} - \varepsilon_{cl})} = 0.833$$

Factored moment resistance:

$$M_r = \phi\left(\mathrm{T}\left(d - \frac{a}{2}\right) + 0.85f'_c(b - b_w)h_f\left(\frac{a}{2} - \frac{h_f}{2}\right)\right) = 100{,}000\ k - ft$$

The factored moment capacity calculated by LARSA 4D is 101,729 kip-ft (less than 2% difference).

Required minimum reinforcement requirements

AASHTO LRFD Article 5.6.3.3:

$\phi M_n \geq 1.33M_u$ or $M_{cr} = \gamma_3\ (\gamma_1 f_{cr} + \gamma_2 f_{cpe})S$

$\gamma_1 = 1.20$ (precast segmental bridges)

$f_r = 0.24\sqrt{f'_c} = 0.24\sqrt{8.5} = 0.700\ ksi$

$\gamma_2 = 1.1$ (bonded tendons)

$\gamma_3 = 1.00$ (tensile resistance is provided by prestressing steel)

Concrete compressive stress at top fiber due to prestressing (after losses):

$$f_{cpe} = \sum P_f\left(\frac{1}{A} - \frac{e}{S}\right) = 2.45\ ksi$$

$M_{cr} = \gamma_3\ (\gamma_1 f_r + \gamma_2 f_{cpe})S = 110{,}400$ *k-ft*

$1.33M_u = 1.33 \times 54{,}900 = 72{,}900$ *k-ft* - controls minimum flexural reinforcement requirements.

Proposed Method

$\phi M_n \geq \alpha M_u$ or $M_{cr} = \gamma_3\ (\gamma_1 f_{cr} + \gamma_2 f_{cpe})S$

where:

$$\alpha = 1 + 0.33\frac{(\varepsilon_t - \varepsilon_{cl})}{(\varepsilon_{tl} - \varepsilon_{cl})} = 1.11$$

It should be noted that since the section is not tension controlled, α is not 1.33 because the reduction due to reduced ductility is already accounted for in reduced resistance factor ϕ.

$\gamma_1 = 1.20\ h^{-0.15} = 0.863$ (precast segmental bridges)

$f_r = 0.24\sqrt{f'_c} = 0.24\sqrt{8.5} = 0.700\ ksi$

$\gamma_2 = 1.1$ (bonded prestress tendons)

$\gamma_3 = 1.00$ (tensile resistance is provided by prestressing steel)

Concrete compressive stress at top fiber due to prestressing (after losses):

$$f_{cpe} = \sum P_f\left(\frac{1}{A} + \frac{e}{S}\right) = 2.45\ ksi$$

$M_{cr} = \gamma_3\ (\gamma_1 f_r + \gamma_2 f_{cpe})S = 103{,}000\ k - ft$

$\alpha M_u = 1.11 \times 54{,}900 = 61{,}000\ k\text{ - }ft$ controls minimum flexural reinforcement requirements.

The factored moment resistance far exceeds the required moment capacity for both methods at this section. This design example demonstrates that the proposed method reduces M_{cr} and the 1.33 M_u ($\alpha\ M_u$) values over the current AASHTO LRFD provisions.

Section B: Section at Maximum Positive Moment in Span 4:

Prestressing tendons at this section is composed of external (unbonded) tendons only.

$A_{ps} = 114 \times 0.217\ in^2 = 24.738\ in^2$

Total of 114-0.6" ϕ strands (external unbonded)

$d_p = 86.49\ in$ (distance from P/S CG to top fiber)

$e = d_p - y_t = 86.49 - 38.04 = 48.45$ in (tendon eccentricity)

$\beta_1 = 0.85 - 0.05\ (f_c - 4) = 0.625$

Effective prestressing force in external tendons (from LARSA 4D): $P_f = 4{,}427\ kips$

$f_{pe} = P_f/A_{ps} = 178.9$ ksi

Length of external tendon (approximate): $l_i = 150.50\ ft$

Number of support hinges crossed by external tendons (end span): $N_i = 1$

Effective length of external tendons: $l_e = \frac{l_i}{(2-N_i)} = 100.3\ ft$

Stress in the external tendons at ultimate:

$$f_{ps2} = f_{pe} + 900\frac{(d_{p2} - c)}{l_e} = 241\ ksi$$

This stress is the same as assumed above. Thus, no iterations are needed.

Tensile force at ultimate moment:

$$T = A_{ps} f_{ps} = 5{,}960\ kips$$

Depth of equivalent rectangular stress block:

$$a = \beta_1\ c = 2.30\ in$$

Net tensile strain:

$$\varepsilon_t = \varepsilon_c \frac{(d-c)}{c} + \frac{f_{cpe}}{E} = 0.068$$

where d is the effective depth of all prestress at ultimate, fcpe is the effective prestress with all losses and E is the concrete modulus of elasticity.

Since the net tensile strain is greater than 0.005, the section is in the tension controlled. Therefore the resistance factor is:

$$\phi = 0.9$$

Factored moment resistance:

$$M_r = \phi T\left(d - \frac{a}{2}\right) = 38{,}200\ k - ft$$

LARSA 4D calculated the factored moment resistance as 37,960 kip-ft (less than 1% difference). It should be noted that the above-calculated factored moment capacity does not take into account the reduction in moment arm of the external tendons due to deflection of the superstructure. Thus, the predicted flexural capacity will be slightly less than 38,473 kip-ft.

Required minimum reinforcement requirements

AASHTO LRFD Article 5.6.3.3:

$\phi M_n \geq 1.33 M_u$ or $M_{cr} = \gamma_3\ (\gamma_1 f_{cr} + \gamma_2 f_{cpe})S$

$\gamma_1 = 1.20$ (precast segmental bridges)

$f_r = 0.24\sqrt{f'_c} = 0.24\sqrt{8.5} = 0.700\ ksi$

$\gamma_2 = 1.0$ (external tendons)

$\gamma_3 = 1.0$ (tensile resistance is provided by prestressing steel)

Concrete compressive stress at top fiber due to prestressing (after losses):

$$f_{cpe} = \sum P_f\left(\frac{1}{A} + \frac{e}{S}\right) = 1.54\ ksi$$

$M_{cr} = \gamma_3\ (\gamma_1 f_r + \gamma_2 f_{cpe})S = 40{,}300\ k\text{-}ft$ - controls minimum flexural reinforcement requirements.

$1.33 M_u = 1.33 \times 54{,}900 = 41{,}260\ k\text{-}ft$

Proposed Method

$\phi M_n \geq \alpha M_u$ or $M_{cr} = \gamma_3\ (\gamma_1 f_{cr} + \gamma_2 f_{cpe})S$

Since the section is tensioned controlled, $\alpha = 1.33$.

$\gamma_1 = 1.20\ h^{-0.15} = 0.863$ (precast segmental bridges)

$f_r = 0.24\sqrt{f'_c} = 0.24\sqrt{8.5} = 0.700\ ksi$

$\gamma_2 = 1.1$ (bonded prestress tendons)

$\gamma_3 = 1.00$ (tensile resistance is provided by prestressing steel)

Concrete compressive stress at top fiber due to prestressing (after losses):

$$f_{cpe} = \sum P_f \left(\frac{1}{A} + \frac{e}{S}\right) = 1.54\ ksi$$

$M_{cr} = \gamma_3 (\gamma_1 f_r + \gamma_2 f_{cpe})S = 38{,}900\ k\text{-}ft$ - controls minimum flexural reinforcement requirements.

$\alpha M_u = 1.11 \times 54{,}900 = 41{,}300\ k\text{-}ft$

The calculated factored moment resistance of 38,200 k-ft is within 2%, and redesign is not necessary. As shown, minimum reinforcement requirements control the flexural design capacity at this section. This design example demonstrates a significant reduction in the minimum flexural moment capacity requirements.

B.5 CAP BEAM

DESCRIPTION OF CAP

The cap beam shown in Figure B.14 has a main span of 23.0 ft and two cantilever spans of 12.5 ft, each. The cap is 6.5 ft wide by 6.0 ft deep. The columns are square with section dimensions of 6.0 ft by 6.0 ft.

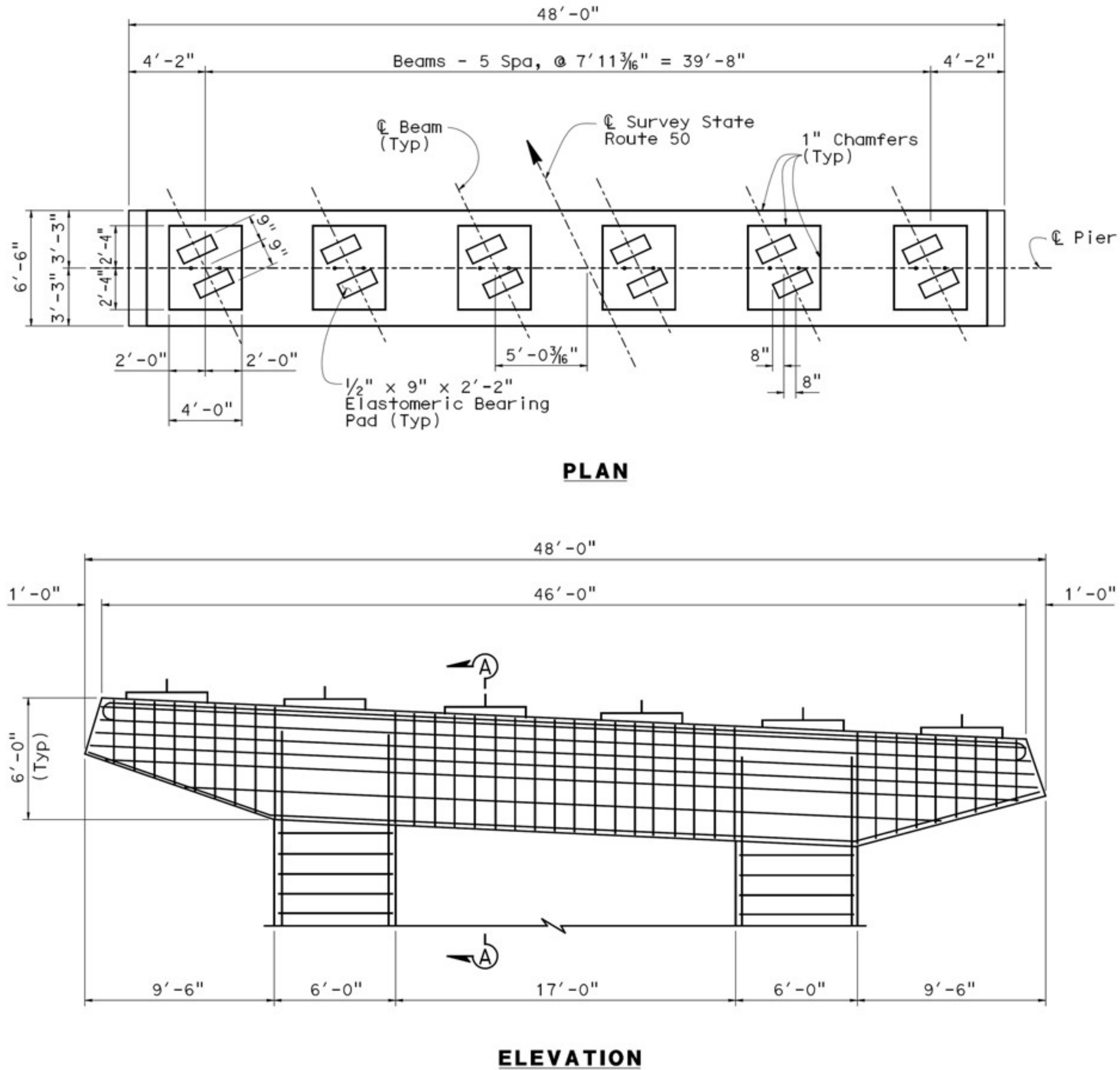

Figure B.14 – Cap Beam Design Example Schematics

MATERIAL PROPERTIES

$f'_c = 4\ ksi$

$E_c = 3{,}644\ ksi$

$f_y = 60\ ksi$

$E_s = 29{,}000\ ksi$

MOMENT DIAGRAMS

The strength limit state moments, as shown in Figure B.15, are for half of the cap, since the moments are symmetric about the centerline of the span. The controlling moment envelope is shaded in this figure. Minimum reinforcement provisions control the strength limit states for positive bending moments.

The Proposed Method significantly reduces the flexural cracking moment at all locations. For positive bending, the Proposed M_{cr} controls the amount of reinforcement, which is slightly less than $\alpha\ M_u$, where α is equal to 1.33. Minimum reinforcement does not control the peak negative moments at the face of the supports. Therefore, the prime benefit of the Proposed Method over current AASHTO LRFD provisions in this example is reduced bar cutoff lengths.

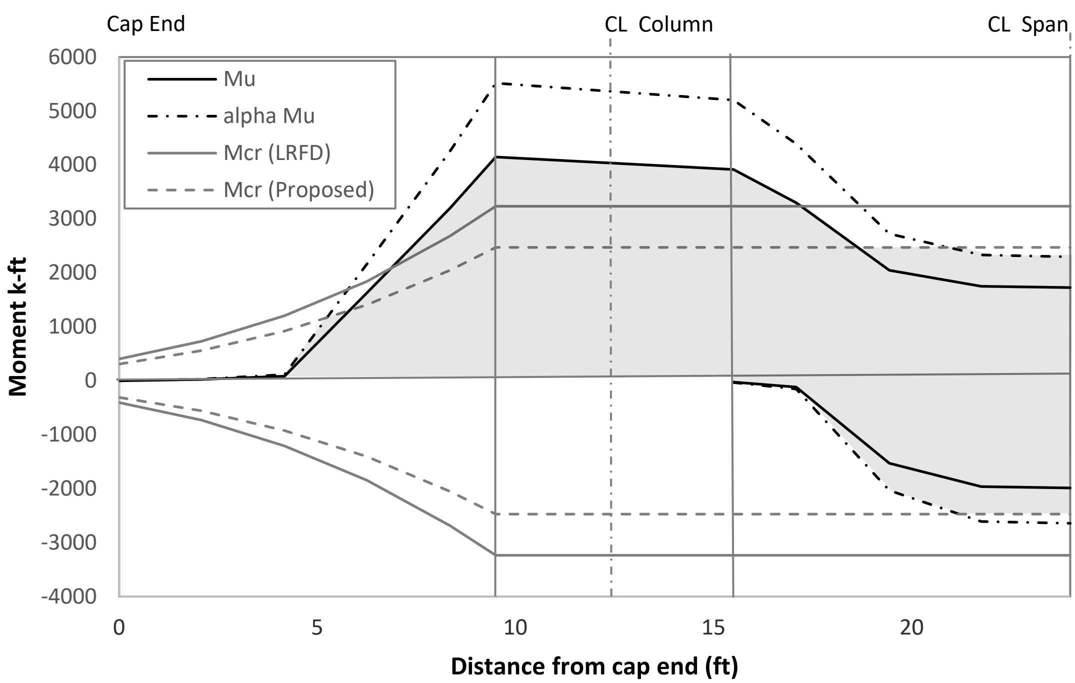

Figure B.15 – Strength Limit Bending Moments Shown on the Tension Side

At the inside face of support (negative moment):

Design moments:

$M_{DC} = -1{,}381 k\text{-}ft$

$M_{DW} = -183 k\text{-}ft$

$M_{HL\text{-}93} = -1{,}093 k\text{-}ft$

$M_{u\ StrengthI} = 1.25 M_{DC} + 1.50 M_{DW} + 1.75 M_{HL\text{-}93}$

$M_u^{StrengthI} = 1.25\text{x}(-1{,}381) + 1.50\text{x}(-183) + 1.75\text{x}(-1{,}093)$

$M_{u\ StrengthI} = -3{,}914 k\text{-}ft$

Section properties:

$h = 6\ ft = 72 in$

$b = 6.5\ ft = 78 in$

$I = 117\ ft^4$

$A = 39\ ft^2$

$y_t = 36 in$ (distance from section CG to top fiber)

Minimum required flexural reinforcement:

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right) = \phi A_s f_y \left(d - \frac{A_s f_y}{2x0.85 f'\ cb}\right)$$

The section is tension-controlled and $\phi = 0.90$

$d = 68.7 in$ assuming #11 mild steel reinforcement

$$3{,}914\ x\ 12 = 0.90\ x\ A_s\ x\ 60 \left(68.7 - \frac{A_s\ x\ 60}{2\ x\ 0.85\ x\ 4\ x\ 78}\right)$$

Solve the quadratic equation for $A_s = 12.94\ in^2$.

The net tensile strain is:

$$\varepsilon_s = 0.003 \left(\frac{d-c}{c}\right)\ where\ c = \left(\frac{A_s f_y}{0.85 \beta f'_c b}\right) = \left(\frac{12.94\ x\ 60}{0.85^2\ x\ 4\ x\ 78}\right) = 3.44\ in$$

Therefore, $\varepsilon_s = 0.003 \left(\frac{68.7-3.44}{3.44}\right) = 0.057$, which is greater than 0.0075. *Hence, requirements* *of Section 5.7.3.5 are met for redistribution, and minimum flexure reinforcement per proposed revised Article 5.7.3.3.2 is not required for negative bending between the columns.*

Summary:

$A_s = 12.94 in^2$ mild steel reinforcement is required at the top of cap.

<u>At 0.5 Span 2 (positive moment):</u>

Design moments:

$M_{DC} = -59k - ft$

$M_{DW} = -20k - ft$

$M_{HL\text{-}93} = 1{,}138k = ft$

$M_{u\ StrengthI} = 0.9M_{DC} + 0.65M_{DW} + 1.75M_{HL\text{-}93}$

$M_u^{StrengthI} = 0.9\text{x}(-59) + 0.65\text{x}(-20) + 1.75\text{x}(1{,}138)$

$M_{u\ StrengthI} = 1{,}925k - ft$

Minimum reinforcement by the proposed method:

$\phi M_n \geq M_{fcr}\ where\ M_{fcr} = \gamma_3[(\gamma_1 f_r + \gamma_2 f_{cpe})]S\ and\ \phi M_n \geq\ \alpha M_u$,

$$where\ 1.0 \leq\ \alpha = 1.0 + \frac{0.33(\varepsilon_t - \varepsilon_{cl})}{\varepsilon_{tl} - \varepsilon_{cl}} \leq 1.33$$

$\gamma_3 = 0.75$ for A706 Grade 60 reinforcement, assumed for this example.

$\gamma_1 = 1.6h^{-0.15}\ where\ h = member\ depth\ (ft)$ for non p/c segmental members

$\gamma_1 = 1.6\ x\ 6^{-0.15} = 1.22$

$f_r = 0.24\sqrt{f_c\ '}\ = 0.24\sqrt{4} = 0.48\ ksi$

$f_{cpe} = 0\ ksi$

$S = \frac{I}{y_b} = \frac{117\ x\ 12^4}{36} = 67{,}392\ in^3$

$M_{fcr} = 0.75[(1.22\ x\ 0.48)]\left(\frac{67{,}392}{12}\right) = 2{,}472\ k - ft \geq M_u^{StrengthI} = 1{,}925\ k - ft$

$M_{fcr} = 2{,}470\ k - ft \geq\ \alpha M_u\ , assume\ \alpha = 1.33$

$\alpha M_u = 1.33\ x\ 1{,}925 = 2{,}560\ k - ft\ , so$

$M_{fcr} = 2{,}470\ k - ft\ controls\ the\ design$

$\phi M_n =\ \phi A_s f_y \left(d - \frac{a}{2}\right) =\ \phi A_s f_y (d - \frac{A_s f_y}{2x0.85f\ '\ cb})$

The section is tension-controlled and $\phi = 0.90$

d = 68.7in assuming #11 mild steel reinforcement

$$2{,}470\ x\ 12 = 0.90\ x\ A_s\ x\ 60\left(68.7 - \frac{A_s\ x\ 60}{2\ x\ 0.85\ x\ 4\ x\ 78}\right)$$

Solve the quadratic equation for A_s =8.40 in^2 .

Net tensile strain assumption check:

$$\varepsilon_s = 0.003\left(\frac{d-c}{c}\right)\ where\ c = \left(\frac{A_s f_y}{0.85\beta f_c' b}\right) = \left(\frac{8.40\ x\ 60}{0.85^2\ x\ 4\ x\ 78}\right) = 2.24\ in$$

Therefore, $\varepsilon_s = 0.003\left(\frac{68.7-2.24}{2.24}\right) = 0.089$ and the tension controlled assumption is correct.

Summary:

Both the current and proposed provisions for minimum flexural reinforcement control the required positive moment flexural strength. In the negative bending regions, minimum reinforcement provisions do not control at the face of support. However, the bar termination lengths could be reduced with the incorporation of these provisions.

The proposed method reduces the minimum reinforcement from the current provisions in the LRFD. This savings is manifest in the factored cracking strength formula, were the reduction is 25% to account for depth adjustment.