NATIONAL COOPERATIVE HIGHWAY RESEARCH PROGRAM
REPORT **34**

EVALUATION OF CONSTRUCTION CONTROL PROCEDURES

INTERIM REPORT

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REPORT **34**

EVALUATION OF CONSTRUCTION CONTROL PROCEDURES

INTERIM REPORT

BY MILLER-WARDEN ASSOCIATES
RALEIGH, NORTH CAROLINA

RESEARCH SPONSORED BY THE AMERICAN ASSOCIATION
OF STATE HIGHWAY OFFICIALS IN COOPERATION
WITH THE BUREAU OF PUBLIC ROADS

SUBJECT CLASSIFICATION:
CONSTRUCTION
MINERAL AGGREGATES

HIGHWAY RESEARCH BOARD
DIVISION OF ENGINEERING NATIONAL RESEARCH COUNCIL
NATIONAL ACADEMY OF SCIENCES—NATIONAL ACADEMY OF ENGINEERING 1967

NATIONAL COOPERATIVE HIGHWAY RESEARCH PROGRAM

Systematic, well-designed research provides the most effective approach to the solution of many problems facing highway administrators and engineers. Often, highway problems are of local interest and can best be studied by highway departments individually or in cooperation with their state universities and others. However, the accelerating growth of highway transportation develops increasingly complex problems of wide interest to highway authorities. These problems are best studied through a coordinated program of cooperative research.

In recognition of these needs, the highway administrators of the American Association of State Highway Officials initiated in 1962 an objective national highway research program employing modern scientific techniques. This program is supported on a continuing basis by funds from participating member states of the Association and it receives the full cooperation and support of the Bureau of Public Roads, United States Department of Transportation.

The Highway Research Board of the National Academy of Sciences-National Research Council was requested by the Association to administer the research program because of the Board's recognized objectivity and understanding of modern research practices. The Board is uniquely suited for this purpose as: it maintains an extensive committee structure from which authorities on any highway transportation subject may be drawn; it possesses avenues of communications and cooperation with federal, state, and local governmental agencies, universities, and industry; its relationship to its parent organization, the National Academy of Sciences, a private, nonprofit institution, is an insurance of objectivity; it maintains a full-time research correlation staff of specialists in highway transportation matters to bring the findings of research directly to those who are in a position to use them.

The program is developed on the basis of research needs identified by chief administrators of the highway departments and by committees of AASHO. Each year, specific areas of research needs to be included in the program are proposed to the Academy and the Board by the American Association of State Highway Officials. Research projects to fulfill these needs are defined by the Board, and qualified research agencies are selected from those that have submitted proposals. Administration and surveillance of research contracts are responsibilities of the Academy and its Highway Research Board.

The needs for highway research are many, and the National Cooperative Highway Research Program can make significant contributions to the solution of highway transportation problems of mutual concern to many responsible groups. The program, however, is intended to complement rather than to substitute for or duplicate other highway research programs.

This report is one of a series of reports issued from a continuing research program conducted under a three-way agreement entered into in June 1962 by and among the National Academy of Sciences-National Research Council, the American Association of State Highway Officials, and the U. S. Bureau of Public Roads. Individual fiscal agreements are executed annually by the Academy-Research Council, the Bureau of Public Roads, and participating state highway departments, members of the American Association of State Highway Officials.

This report was prepared by the contracting research agency. It has been reviewed by the appropriate Advisory Panel for clarity, documentation, and fulfillment of the contract. It has been accepted by the Highway Research Board and published in the interest of an effectual dissemination of findings and their application in the formulation of policies, procedures, and practices in the subject problem area.

NCHRP Project 10-2 FY '64
NAS-NRC Publication 1479
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FOREWORD

By Staff

Highway Research Board

Highway materials and testing engineers, as well as specification writers and those involved in construction, will find this report of particular interest. Serious questions have often been raised concerning the degree of confidence that can be placed in the generally used methods of making acceptance decisions with respect to aggregate gradation, particularly when test results approach the specification limits. This report deals with a study of sampling and test procedures for determining the gradation of aggregates to be used in highway construction. It contains practical recommendations for improved aggregate sampling techniques, for a determination of the inherent variance of aggregate gradation, and for suggested specification changes.

Aggregates constitute more than 90 percent of the materials used in roadway and bridge construction; therefore, their quality and gradation are primary factors in ultimate performance. From an engineering standpoint, the intent of gradation specifications is to prevent the acceptance of improperly graded aggregates as well as to assure the acceptance of those that are suitable. The value of such specifications is closely associated with the method used to determine acceptance or rejection. Aggregates outside of the limits of gradation established by the specifications have been used without apparent detrimental effect, while aggregates that meet specifications have been known to produce disappointing results. Conjecture is thus raised as to whether or not present test methods provide an adequate basis for acceptance or rejection of aggregates, or whether or not specifications contain realistic gradation limits to provide for the variation inherent in materials, sampling procedures, and testing techniques.

The Miller-Warden Associates approach to the over-all problem of defining and evaluating the sources of variation that cause apparent or actual departure of aggregate gradations from those specified began with an analysis of the sources. These included such sources as inherent variation of the material, testing error, local segregation, etc. This was followed by field experiments at various locations of aggregate production and use. Statistical methods were employed, and a model was designed to incorporate the various sources of variation into the over-all variations in gradation expected among random samples. The individual sources of variation were evaluated from both a theoretical standpoint and practical operating conditions for a range of handling methods, sample sizes, and sampling methods. A nomograph was developed for estimating the minimum size of test portion required to obtain the true gradation of an aggregate within selected limits of error to account for the inherent variation due to the random arrangement of different sized particles in a pile or bin.

The field phase of the study consisted of collecting about 2,500 samples of coarse aggregates at the point of production, the point of use, and at least two other points in the process, and the measuring of the variations of gradation. Eight sampling locations were chosen in five widely separated geographical areas and included crushed stone, gravel and slag being used in portland cement concrete, bituminous concrete, and aggregate base. Handling procedures were those customarily employed in practice.

This is an interim report on the first phase of the research. Along with research conducted under NCHRP Project 10-3, "Effects of Different Methods of Stockpiling Aggregates," a more thorough understanding of the problem has resulted and will contribute to the development of practical approaches for improving the control of aggregates used in highway construction, including suggested specification changes. The final phase of the research is currently under way and has as its objective the extension of the knowledge gained in the first-phase work to a study of the effects of variations in gradations on the strength and workability of portland cement concrete and to a study of the variations in gradations in the hot bins of hot-mix bituminous paving plants. As they may be accommodated, related studies will involve effect of increment size in sampling of coarse aggregate and mathematical exploration of aggregate gradations.

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SUMMARY

The purpose of this study was to evaluate variations inherent in aggregate gradations. The research developed data relative to variations normal to coarse aggregate gradations at different points in the process stream at various types of plants at widely different geographical locations. Statistical techniques were used for determining the relative effect of testing accuracy, sampling methods, and segregation, with relation to the evaluation and acceptance of coarse aggregate used for highway construction. The work also included the development of a precision statement for the aggregate gradation test, and a method of drafting a realistic and adequate acceptance gradation specification for coarse aggregates.

In general, the research data indicate that testing accuracy, even under routine conditions, does not greatly influence acceptance or rejection of aggregates on the basis of gradation. Sampling methods do have a large effect on the reported gradation, and results based on a single test portion are not an adequate basis for determining compliance with specifications. Under normal conditions, as represented by the eight operating plants included in the program, variations in gradation due to segregation effects tend to increase as aggregates are moved from point of production to point of use. Consequently, sampling at source or at intermediate points in the process stream does not appear to provide reliable information as to the variations in gradation of the aggregate actually incorporated in the final product or construction. Large samplings consisting of a series of measurements and results indicate that overall variations in coarse aggregate gradation exceed the limits of many current specifications. This means that, if the requirements are to be rigidly enforced, realistic specifications must have wider tolerances, or changes in plant controls are required. The range of overall variation depends on a number of causes. The relative effects of these causes vary over wide limits, but the general order of magnitude found in this study was testing error 4, inherent variation 10, sampling error 30, and actual batch-to-batch variation 56.

The standard deviation for the repeatability of the gradation test on coarse aggregate, under routine conditions, is approximately 0.4 percent. The standard deviation for the reproducibility of this test depends on the weight and gradation of the test portion and must be computed for each particular case. A method is given for computing the required test portion or sample weight for any desired degree of accuracy.

Recommended methods of incorporating the results of this study into highway construction specifications and procedures are presented in the form of a complete model specification for graded coarse aggregate. Methods are given for drafting similar specifications, with an explanation of the reasoning and mathematical considerations involved.

Specifically, the major accomplishments and findings of this research are as follows:

1. A study was made to evaluate sampling and testing procedures associated with aggregate gradation by laboratory investigation, and by taking a large number of

test portions from the process streams of eight operating plants at widely separated geographical locations. The variability of the gradation of five types of coarse aggregate was assessed at the point of production, the point of use, and at intermediate points. Statistical techniques were selected and adapted for evaluating gradation test procedures, for methods of sampling aggregates for gradation tests, and for determining the variations inherent in aggregate gradations. The source sof variation of the gradation of aggregate were statistically analyzed and their relative importance assessed. The magnitude of variance components was determined and displayed in a schematic model.

The overall variation of the percentage passing a given sieve was hypothesized to consist of an inherent variance due to random arrangement of particle sizes, a testing error, a sampling error, and batch-to-batch variation.

This study included:

(a) Obtaining experimental proof of a theory of inherent variance of aggregate gradation expressed as a statistical parameter, and determining the effect of this variance on the gradation test. This was accomplished by securing a large number of test portions from a nearly perfectly mixed LOT of aggregate without introducing segregation effects.

A practical method of estimating minimum sample size for aggregate gradation tests was devised and is presented in the form of a nomograph. This nomograph largely eliminates the computations required to apply the principles developed in the study of the inherent variability of the aggregate gradation test.

(b) The variance due to testing error involved in the use of standard methods of testing for gradation, using different types and gradations of aggregates, was determined. This was accomplished by repeating the gradation test of each of a large number of test portions at several locations. The effect of simplified methods on the experimental error (inherent variation plus testing error) was also investigated, utilizing small test portions.

(c) The component of overall variance due to sampling error, using usual and special sampling tools, was measured.

The sampling error was found to be largely due to local or short-term segregation. In some cases it was so small as to be obscured by the estimates of other variances; in others it was a significant part of the overall variance. The effect of the use of special sampling tools did not prove to be significant.

2. Analysis of the data indicated that the variability of the gradation of different aggregates could be represented graphically by plotting the standard deviation of the percentages passing the individual sieves against $\sqrt{P(100-P)}$, where P is the percentage passing the sieve. An estimate of a standard deviation, independent of gradation, was obtained by projecting the best line through the individual points to the 50 percent level, which is the point of intersection with the maximum value of $\sqrt{P(100-P)}$. This estimated standard deviation was used in connection with the parameters called "degree of overall variability," "degree of segregation," and "segregation index," to compare the variability of aggregates having different gradations. The relative differences in these parameters as a function of the point of sampling and of aggregate type were determined.

3. The pattern of variation of gradation at each plant was studied by use of a quality history chart. This involves plotting the percentage passing a selected sieve for each test portion in sequential order. These patterns were analyzed with respect to handling and storing procedures.

4. A precision statement was developed for ASTM C136-63, "Sieve or Screen Analysis of Fine or Coarse Aggregate," with application to the percentages of coarse aggregate passing the ¾-in., ⅜-in., and No. 4 sieves. In terms of the ASTM precision statement format, the tentative testing error (difference of 2σ limits) was found to be about 1 percent. The tentative testing error for repeat tests on the same test portion was found to be 1.1 percent.

An equation was derived for estimating the testing error on replicate test portions from the same sample. A specific numerical value, applying generally, cannot be stated for this error, because it changes with different gradations, test portion weights, and other factors.

Testing error was found to contribute only a relatively small fraction to the overall variance.

5. Definite recommendations are made for incorporating the results of this study into highway construction specifications and procedures to provide a basis for acceptance and rejection of aggregates. These include the major items of (a) point and method of sampling, (b) minimum sample weight and number of test increments, and (c) graduated penalty system for noncompliance.

6. A model specification illustrating the method of incorporating the foregoing considerations has been drafted and is included in the report.

CHAPTER ONE

INTRODUCTION

OBJECTIVES

The overall objective of this study was to evaluate real or apparent variations of coarse aggregate gradation with respect to construction control procedures.

Specific objectives were (1) to study the theory and application of mathematical and statistical techniques for analyzing variations in gradation, and (2) to determine the reliability of customary aggregate sampling and test procedures applied to contract construction control and administration. Related objectives were to determine the repeatability of the gradation test and the requirements for developing a precision statement for the test method.

Of necessity, some rather involved statistical methods have been used in the analysis of certain portions of these data. Inasmuch as full exposition of these principles is beyond the scope of this report, users not versed in mathematical statistics are referred to the following:

1. "Development of Guidelines for Practical and Realistic Construction Specifications." *NCHRP Report 17* (1965) 109 pp.
2. "The Statistical Approach to Quality Control in Highway Construction." U.S. Bureau of Public Roads (Apr. 1965).
3. Military Standard 414.
4. "Facts from Figures," by M. J. Moroney, Pelican Books.
5. "Qualified Control and Industrial Statistics." Acheson J. Duncan, published by Richard D. Irvin, Inc.

In many cases, where no documented bases of comparison exist, the authors have expressed opinions that certain values were normal or satisfactory. These opinions are based on the personal experiences of the authors and may be subject to modification as more data are acquired.

IMPORTANCE

Aggregates account for more than 90 percent of most highways and structures and constitute one of the large elements of cost of highway construction. Information as to methods of control is basic to the formation of optimum acceptance standards for this important material. Unrealistic specifications, improper sampling methods, or inaccuracy of tests may result in the rejection of acceptable material (or the acceptance of unsuitable material), which will ultimately be reflected in increased highway construction costs. Some current sampling procedures place on the aggregate producer responsibility for variations in gradation resulting from causes not under his control. The most economical construction can be achieved only when control procedures insure that the PROBABILITY* of the Engineer accepting

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* Statistical terms and words not commonly used in highway engineering are given in small capitals the first few times they appear in the text and are defined in the Glossary, Appendix A.

unsuitable aggregate is minimized and, at the same time, does not subject the producer to an unfair hazard of rejection of properly graded aggregate.

THE PROBLEM

Most aggregates are purchased for use in specific types of construction and are subject to various specifications for gradation. The basic reasons for specifying gradation are to:

1. Control maximum size because of restricting dimensional considerations or to insure adequate shear strength.
2. Control quantity and size of aggregate voids.
3. Obtain suitable workability.
4. Minimize degradation in compacted courses.
5. Limit surface area.
6. Control texture of exposed surfaces.

The specified gradation requirement usually takes the following form:

Sieve Size	Percent Passing
1½ in.	95 - 100
¾ in.	35 - 70
⅜ in.	10 - 30
No. 4	0 - 5

This requirement establishes an allowable range for the percentage of aggregate passing each sieve size.

When a LOT of aggregate, such as a stockpile or a day's production, is presented for use, an ACCEPTANCE DECISION must be made as to whether the aggregate is of the specified gradation. Because it is obviously impossible to pass the entire LOT through the sieves, the acceptance decision is based on a SAMPLE taken from the LOT, and the percentages of the aggregate passing a series of small laboratory sieves are determined by means of a gradation test. The results of such a test might be as follows:

Sieve Size	Percent Passing
1½ in.	94.6
¾ in.	69.5
⅜ in.	32.2
No. 4	3.4

When the results of the test are compared with the gradation specification, it is apparent that the percentages passing some of the sieves are not within the required limits. In such an acceptance situation, a decision must be made as to whether the LOT of aggregate represented by the SAMPLE should be rejected, whether it substantially complies with the requirements, or whether another SAMPLE should be taken. If the chosen alternative is to reject the LOT of aggregate and if this decision should be disputed, an objective arbitrator might ask the following questions:

1. Does the fact that the SAMPLE did not meet the specifications mean that the aggregate in a load of concrete, or in all the loads of concrete, is not within the specified gradation limits?
2. What is the relationship of the gradation of the aggregate at the point in the process stream at which the SAMPLE was taken to the gradation of the same aggregate at the point of use?
3. How ACCURATE is the test method? If the same SAMPLE should be retested, how closely would the results correspond to those first reported?
4. What would be the effect with respect to the test results if a different SAMPLING PLAN were followed?
5. If another SAMPLE should be taken by the same method, would the test results be within ± 0.1, ± 1.0, or ± 10.0 percent of the percentages first reported?
6. Is the specification realistic? Can the results of tests on all SAMPLES be expected to fall within the limits?

Basically, the answer to all of these questions is tied to the word "SAMPLE." In assessing any results of tests on aggregates, it must be kept in mind that under practical conditions one can never measure the whole bulk of the material. Measurements of gradation are made on the SAMPLE, which is usually an extremely small part of the whole bulk or LOT of aggregate it is supposed to represent. The result of measurements on a SAMPLE is always an ESTIMATE of the true value and has a limited ACCURACY. In business, one can say that 6% interest on $100 for one year is $6.00, and be sure that this is the one and only right answer. However, when measurements are made on a SAMPLE, and the results indicate that 6% of the aggregate passes a certain sieve, the situation is quite different. The best that can be done is to say that the percentage of the whole LOT of aggregate that would pass the certain sieve is probably 6% ± *C*, where *C* is a constant associated with the uncertainty of obtaining a true value. Because the value of *C* is seldom known, it does not appear in the reported tabulation of test results; but visible or not, it is always there, because it is the inescapable consequence of making an ESTIMATE from a SAMPLE.

This situation is shown in Figure 1, in which the characteristics of the LOT are estimated from a sample. The scheme shown is also the plan used in the main series of field experiments in connection with this project. An additional purpose for presenting it as part of this introduction is to define the terms used and show their interrelation. The definitions of the terms LOT, BATCH, INCREMENT, TEST PORTION, GROUP, and SAMPLE are given in the Glossary (Appendix A), but the interrelationship of these subdivisions of the LOT are shown in Figure 1. It should be noted that TEST PORTION and SAMPLE are not synonymous. A test portion is the part of a sample actually tested and may consist of a single increment (shovelful), or a group (two or more increments from the same batch), or it may be obtained by reducing the sample or sample increments by quartering, riffling, or taking an aliquot quantity. The basic problem remains one of estimating the characteristics (average and variability) of the LOT from a limited number of measurements made on a sample.

THE APPROACH TO AN ANSWER

The research approach for this project was designed to separate and evaluate those factors which affect the value of *C* under various conditions of measuring the gradation of coarse aggregates used in highway construction. The report develops first the basic statistical concepts necessary for an understanding of VARIATION so that the deviations

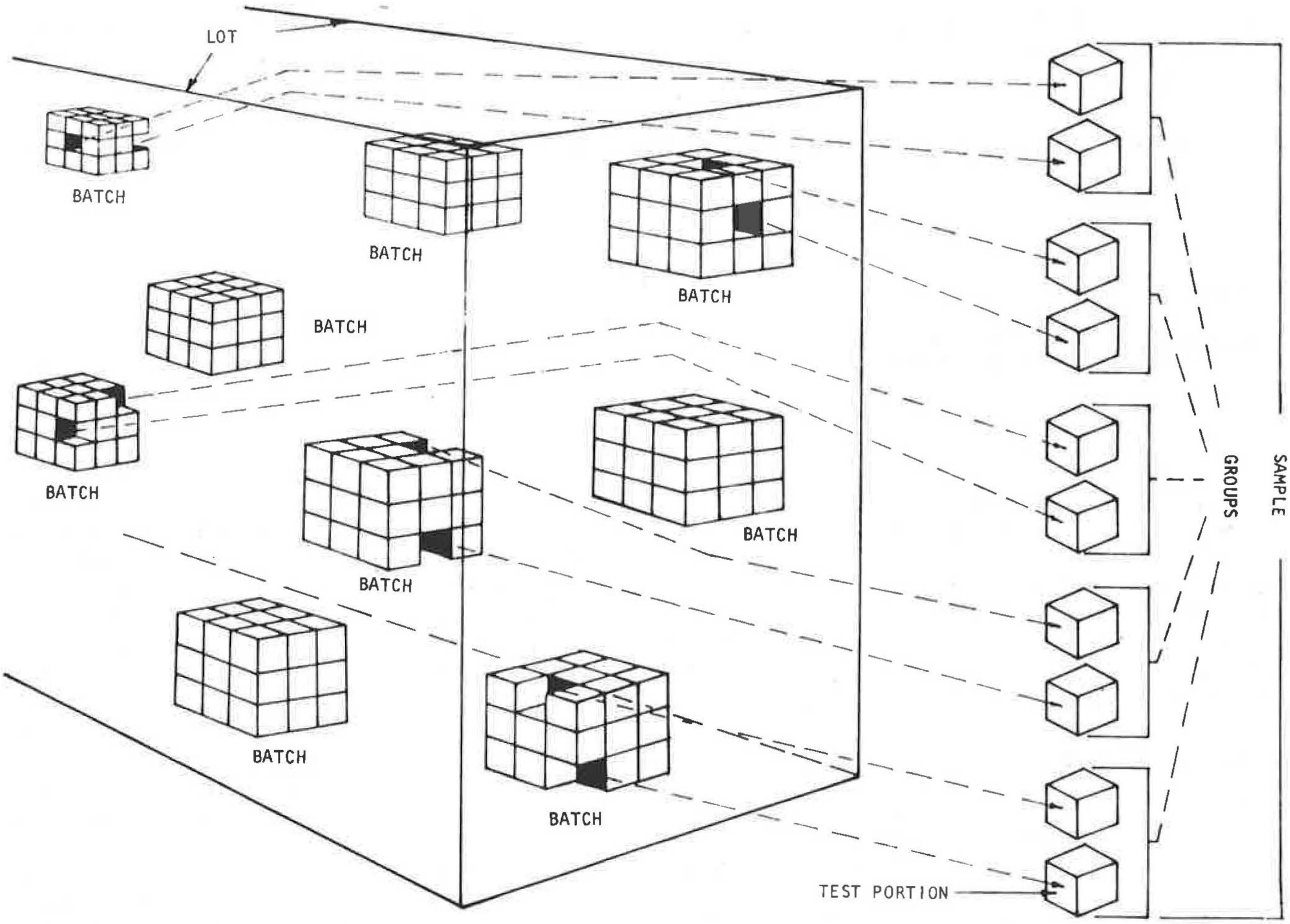

Figure 1. Sampling plan for determining average gradation, within-lot variation, and within-batch variation.

inherent to the methods of measurement and to the sampling and test procedures applied to contract construction control and administration can be defined and, where possible, quantified under typical operating conditions and with typical aggregates. In large part, coarse aggregates were selected for investigation because these are more prone to segregation during normal handling, sampling, and testing. One well-graded base course aggregate was included for comparative purposes. Typical operating plants and typical aggregates were purposely selected for study so that the relative measurements of variability obtained would approximate "normal" rather than "research" conditions. The main field investigation involved eight different plants, seven sampling points within the process stream (crushers, trucks, stockpiles, barges, hoppers, belt conveyors, and the roadway), five types of commercial aggregates (crushed stone, slag, and gravel), and seven different geographic locations (six States).

In addition to this main field investigation, some special experiments were conducted both in the laboratory and in the field to better define or to quantify the mathematical formula for inherent variance, the testing error or repeatability of the gradation test, a particle count experiment, and to a limited degree, the relative efficacy of different sampling tools. The raw data for all of the experimental work, both in the main field investigations and the special experiments, are not given here because they occupy 193 pages of some 3,000 individual test results and computations.

Briefly, the major steps undertaken to accomplish the objectives of this initial phase of the project consisted of:

1. Planning.
2. Review of literature and standards.
3. Statistical studies.
4. Planned laboratory experiment.
5. Field investigations.
6. Sampling and testing experiments.
7. Computations.
8. Analyses of data.
9. Drafting of report.
10. Recommendations for continuing research.

Particular attention is invited to the Glossary of Terms presented in Appendix A. An appreciation of the application of statistical concepts by most highway engineers is difficult enough without the added handicap of problems in semantics. For the first few pages of each chapter, those words that might be strange to the average reader and which are defined in the Glossary are given in small capitals. It is hoped that the reader will check the definitions of these words to assure a common understanding of their use within the context of the report.

CHAPTER TWO

STATISTICAL CONCEPTS

It is obviously beyond the scope of this report to present a textbook on statistics—nor is it necessary. On the other hand, certain statistical concepts are needed and are used in this study and in this report as tools for the definition and measurement of variability. These statistical concepts are essential to the breaking down of variability into its components so that the causes of variation can be identified, studied, and (hopefully) quantified or, at least, their relative magnitude estimated. The pertinent statistical fundamentals are very briefly reviewed in this chapter.

NORMAL DISTRIBUTION CURVE

STATISTICS is a scientific method that deals with the analyses of averages, and VARIATION around averages, as found in numerical DATA. By the use of proper statistical techniques, certain inferences can be drawn from limited data that would not otherwise be possible. In addition, optimum sampling and testing schedules can be developed that eliminate unnecessary expenditures of money, time, and effort by making only the number of tests necessary to evaluate a particular condition. One application of statistics used in this report is the concept of the NORMAL DISTRIBUTION. One of the properties of the NORMAL DISTRIBUTION CURVE is that, regardless of its shape, a definite percentage of the total area beneath the curve is defined by vertical lines spaced a definite number of STANDARD DEVIATION (σ) units from the centerline of the curve which represents the average value, $\overline{X}$, as shown in Figure 2.

The tails of the normal distribution curve approach the base line at approximately three standard deviation units on each side of the average. It should be noted, however, that about 68.2 percent of all possible test results would fall within $\pm 1\sigma$ limit from the average; 95 percent would fall within $\pm 2\sigma$ limits; and 99.7 percent of the results would fall within $\pm 3\sigma$ limits. Thus, under normal conditions, the number (or percentages) of measurements deviating from the average by any given amount can be predicted. When a very few samples are taken, this curve will often assume a shape other than that of a normal curve. This does not necessarily indicate that the parent distribution, consisting of all possible measurements, is not normally distributed.

In the case illustrated by Figure 2, SIGMA (σ) $= 2.3$, so $\pm 3\sigma = \pm 6.9$ and about 100 percent of the values are included in the RANGE 55.1 to 68.9. If these numbers represented the percentage of an aggregate passing a certain sieve and the results of a large number of tests indicated that the standard deviation, σ, of the measurements was in fact 2.3, it could be expected that few future measurements would normally exceed this range. Obviously, if σ was smaller the range would be narrower, while a large value of σ would correspond to a wider range of variation.

Sigma, then, is a means of expressing variation as a numerical value. For convenience, the VARIANCE, σ^2, which is the square of the standard deviation, is used instead of σ as a measure of variability in some parts of this report because variances can be added, whereas standard deviations cannot be directly treated arithmetically.

Accordingly, in this report both the standard deviation and the variance have been selected as the measures of variability. A relatively small value of either of these PARAMETERS indicates that essentially all measurements lie close to the average, while a relatively large value indicates that the measurements deviate from the average over a wider range.

SIGNIFICANCE OF VARIABILITY

When actual variations are compared with specification limits, there are three possible conditions (Figure 3), as follows:

(a) *A low variation with most results within specification limits.* This may indicate that the specifications are realistic and that the production process is in good control.

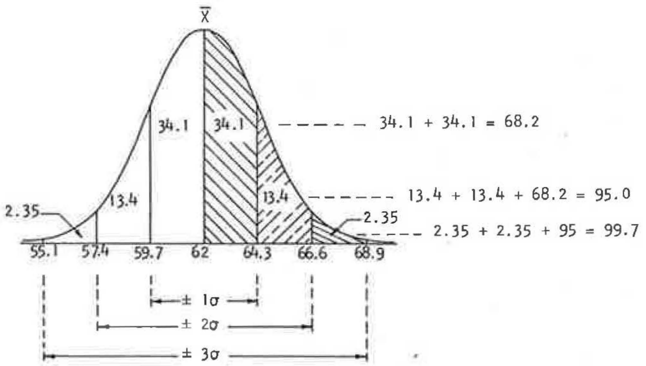

Figure 2. Percentages of area within given sigma limits.

However, if all results are within specification limits, the data may indicate that the sampling procedure is not entirely unbiased.

(b) *A relatively low variation with an average too close to the specification limit.* This may indicate that either the material production is offset with respect to the specification requirements, or that the specifications are offset with relation to current practice.

(c) *A high variation making it improbable that most results will fall within the specification limits most of the time.* This condition indicates that control needs to be tightened to reduce the variation to the uniformity required by the specification or that the specification tolerances are not realistic and need to be broadened.

These relationships can be profitably applied to the construction control of aggregates, as well as to many other materials, processes, test methods, and operations used in highway construction. They may be used for two main purposes: (1) to rate the compliance of a given aggregate, material, or process, with the specification requirements; or (2) to compare specification requirements with the variability of typical operations. This method of presentation by use of the NORMAL CURVE assists in visualizing the pertinent relationships between operating tolerances and specification limits, and also provides a logical means for selecting the more fruitful areas for additional detailed study and research, for determining whether there is a necessity for administrative investigation or improved control, and/or for indicating the need for a specification rewrite.

In the following sections, statistical methods based on the normal distribution curve have been used to analyze various problems, to treat the data, and to provide a means of measuring the relative sizes of the components of variation of the gradation of coarse aggregates at the critical points in the process stream of eight typical operating plants.

ANALYSIS OF VARIANCE

The means of isolating and measuring the relative magnitude of the individual components of variability is called ANALYSIS OF VARIANCE. The components to be isolated and defined will differ, depending on the system and on the objectives of the analysis. The statistical principles, however, are the same and, in general, involve a large number of replicate measurements on test portions selected in such a manner that the influence of other causes of variability are either eliminated or are capable of being otherwise estimated. Sometimes this involves some rather complicated interrelationships and occasionally some rather ingenious means of isolating and studying the individual components. The basic arithmetic, however, boils down to the fact that variances, σ^2, are additive.

Construction of Model

Early statistical studies made in connection with this project included the design of a model showing the sources of the overall variations in gradation expected among random samples of aggregate taken at a point in the process flow from source to the point where the aggregate was incorporated into the product or construction. It was concluded that the OVERALL VARIANCE, σ^2_o, of the gradation of aggregate samples taken from the same LOT such as a stockpile, railroad car, or bin, may be conveniently broken down into four basic components:

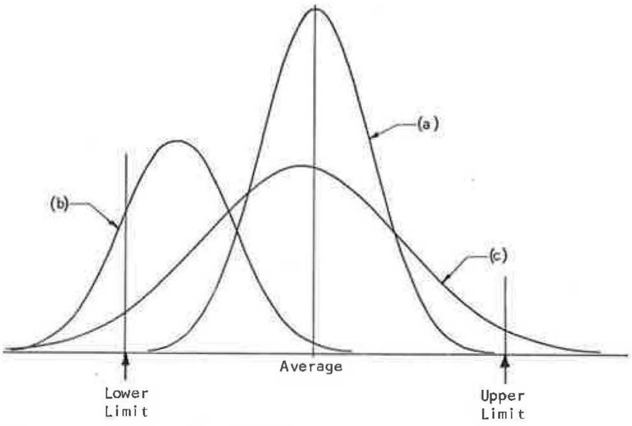

Figure 3. Effect of variability.

(a) σ^2_a — the inherent variation resulting from the random arrangement of particles of different sizes in a mixture;
(b) σ^2_t — a variance due to testing errors*;
(c) σ^2_s — a variance due to sampling errors*; and
(d) σ^2_l — the batch-to-batch variation within the lot.

$$\sigma^2_o = \sigma^2_a + \sigma^2_t + \sigma^2_s + \sigma^2_l \qquad (1)$$

Figure 4 shows the relationship of these variances, scaled roughly to the average size of the components of variance at the point of use. It shows how these components can be combined in various ways to construct a model germane to a given study.

In general, continuing research was directed to the evaluation of these variances, by both theoretical methods and by measurements on samples taken under practical operating conditions over a wide range of aggregate handling methods. The basic variance components and their pertinent combinations are now discussed individually.

Theoretical Maximum Variance

A theoretical variance not shown in Figure 4 is that represented by complete segregation (the condition illustrated in Fig. 5D). This theoretical maximum has no practical significance in real life, but it does provide a limiting parameter. As will be developed later, this variance acquires mathematical usefulness in the definition of a new parameter called DEGREE OF SEGREGATION, and another called DEGREE OF OVERALL VARIABILITY.

The theoretical maximum variance is designated in the report as σ^2_{max} and is derived from the binomial theorem as $P(100 - P)$, where P is the average percent passing a

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* These are not errors in the sense of someone making a mistake. They are random variations associated with the sampling and testing procedure.

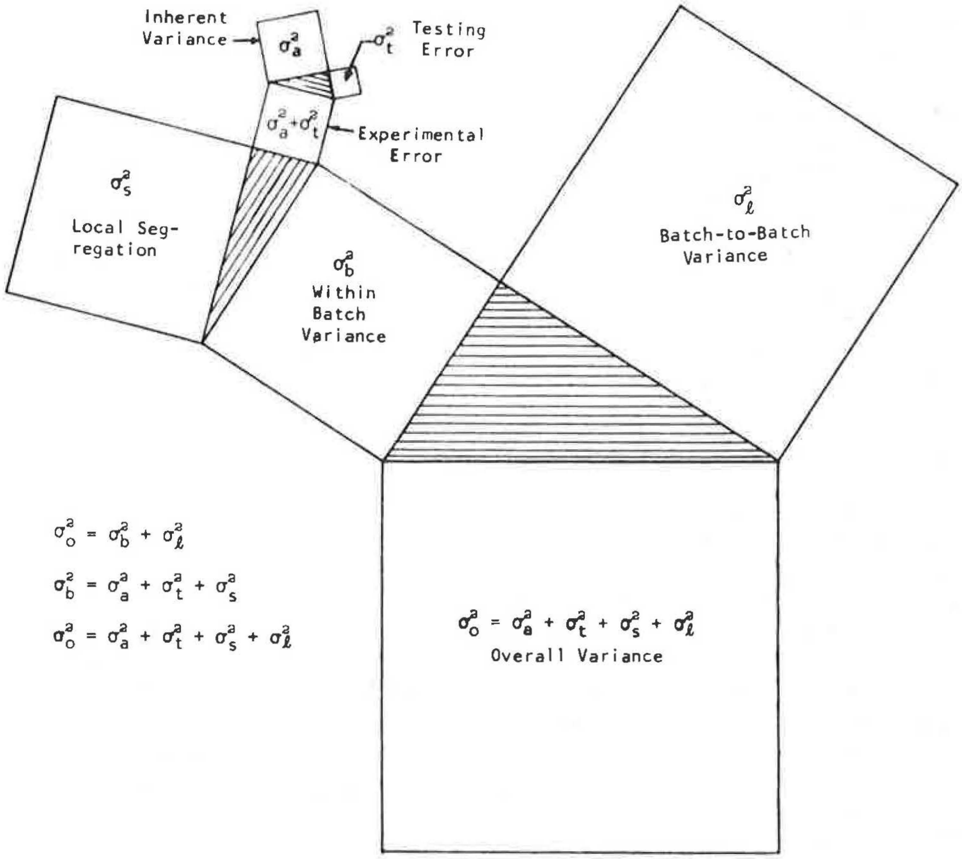

Figure 4. Sources of variance.

given sieve. Table 1 gives the relationship between P, the percent passing, and the theoretical maximum variability. The maximum standard deviation, $\sigma_{\max}$, lies between 30 and 50 percent over the range of $\bar{X}$ from 10 to 90 percent passing the particular sieve in question—below 10 percent and above 90 percent, $\sigma_{\max}$ drops off quite rapidly. This general trend seems to be a pattern followed by other sources or components of variance related to aggregate measurements. In general, variability seems to increase as the amount of material passing that particular sieve tends to approach the 50 percent level—conversely, the variability tends to decrease and correlations become poorer as $\bar{X}$ for that sieve approaches either zero or 100 percent passing.

Inherent Variance

An initial activity was to devise some method of estimating the INHERENT VARIANCE, σ^2_a, of the relative percentages of particle sizes due to the discrete nature and normal random distribution of aggregate particles.

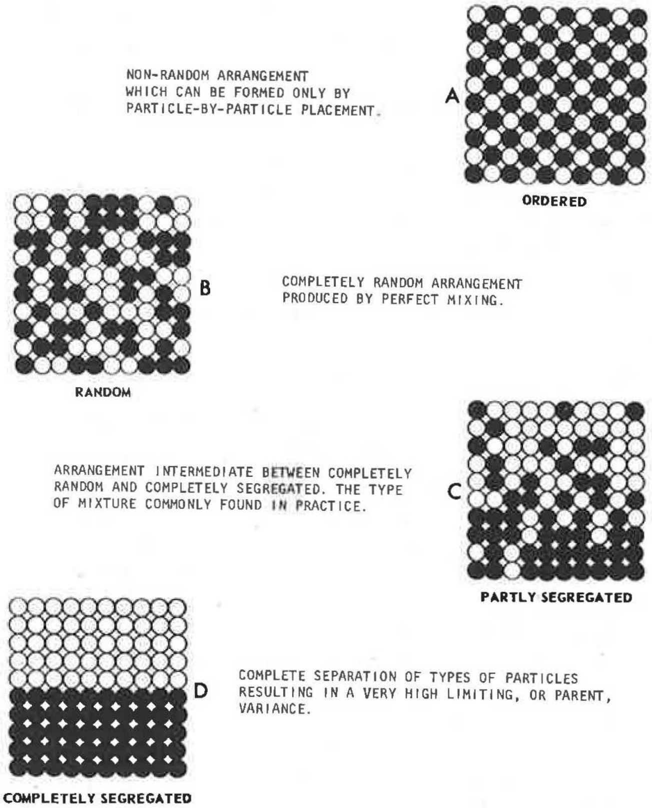

Figure 5. Particle arrangements. The white and black spots represent particles, or groups of particles, having unlike characteristics in an infinite population of combinations of such particles. The different arrangements represent the degree of dispersal of like particles throughout the mixture.

INHERENT VARIATION is due to the RANDOM arrangement of different sized particles in a collection of particles. This is illustrated by Figure 5, where the fine and coarse particles are represented by black and white spots. Although it may be thought that a well-mixed aggregate should have an ordered arrangement as in A, this is an unnatural condition which, if achieved, would disappear when the aggregate was moved or mixed. When particles of different sizes are thoroughly mixed, they are almost completely randomized, as in B; and this is as nearly a uniform distribution of sizes as can be expected.

It will be seen that if groups containing the same number of CONTIGUOUS spots are selected from B, some groups will contain more black spots than others, and the ratio of white spots to black spots will vary. This variance is symbolized by σ^2_a, and because the arrangement of B is truly random, it is possible to calculate the value of σ^2_a under various conditions. Also, if a large number of groups or INCREMENTS is selected, it is possible to predict from the normal distribution curve the percentage of times a certain number of black spots will occur in a group or increment. One peculiarity of this random distribution is that the variance depends on the size of the group or increment, and a collection of small increments.

Because σ^2_a is caused by non-homogeneity within the volume of aggregate actually tested, it is a basic variation in gradations that cannot be reduced by PROCESS CONTROL. Obviously, it would be impractical and uneconomical to modify any production process in an attempt to reduce the process level of variance below this inherent variance. Also, this basic variation must be considered when establishing numerical limits for gradation specifications. In addition, a method of estimating this variance is necessary to the development of a method of computing the minimum size of the sample or test portion required for a predetermined accuracy and degree of assurance.

Manning (*67*)[*], Buslik (*28*), and Visman (*106*) have devised formulas for computing the theoretical value of σ^2_a, but the data with which the theoretical values have been compared does not appear to be entirely satisfactory for the purpose of establishing the validity of the formulas for aggregate control. Because of lack of suitable data and disagreement among values obtained by their computations, a special experiment was designed to measure the inherent variance of two typical commercial coarse aggregates. As far as can be determined through a search of the literature, this experiment is the most comprehensive study ever undertaken on inherent variance using a practical aggregate gradation. The findings are in general agreement with equations based on the binomial distribution theory and, in particular, provide a reasonably good verification of Manning's equation. These raw data on which the findings are based (available on special request) should have special significance to the future researcher wishing to study this subject in greater depth.

TABLE 1
THEORETICAL MAXIMUM VARIABILITY

% PASSING, P	$\sigma^2_{max} = P(100-P)$	$\sigma_{max} = \sqrt{P(100-P)}$
50	2500	50
40 and 60	2400	49
30 and 70	2100	46
20 and 80	1600	40
10 and 90	900	30
5 and 95	475	22
2 and 98	196	14
1 and 99	99	10
0 and 100	0	0

The details of this experiment and associated computations are described in Appendix B. The results are summarized in Table 2. The theoretical inherent standard deviation with which the experimental values are compared is

$$\sigma_a = \sqrt{\frac{P(100-P)\bar{g}}{454\ W}} \qquad (2)$$

in which

$P =$ percent by weight of the aggregate passing a designated sieve;
$\sigma_a =$ the inherent standard deviation of that percentage;
$\bar{g} =$ the average particle weight, in grams, of all particles larger than the openings in the designated sieve[*]; and
$W =$ the total weight,[†] in pounds, of aggregate passed through the sieves.

Estimation of the average particle weight, $\bar{g}$, is not particularly easy. It was therefore desirable to develop a relationship between the average weight of aggregate particles passing one sieve and retained on the next smaller sieve and their effective diameter. To do this, it was necessary to count and weigh several thousand particles to determine an accurate estimate of the weight of the various sizes. This work was later extended to other aggregates and an equation and a nomograph were developed to simplify the estimation of $\bar{g}$ under normal or average conditions. The equation is developed and presented in Appendix B; the nomograph (Fig. 7) is explained later in this chapter.

As shown in Table 2, reasonable correlation between experimental and theoretical values of σ_a was obtained at the 95 percent confidence level, particularly with respect to the larger (¾-in.) particles in the gradation and when the weight of the test portion was in the order of 20 lb. On this basis, the values of σ_a obtained by the use of Eq. 2 were considered to be a sufficiently accurate estimate and values of σ_a so computed are used in the report. To show the relative magnitude of this source of variability, average σ_a values corresponding to a sample weight, W, of about 25 lb, are presented in Table 3 for the various sieve sizes of the gradation used in this study (1½ in. to No. 8).

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[*] Bibliography reference numbers (Appendix C).

[*] Note that this value is the average particle weight of *all* particles of *all* material that would be retained on the designated sieve if there were no coarser sieves in the stack. It is not the average particle weight of merely that material passing the next larger sieve and retained on the designated sieve as is customarily visualized in gradation considerations.

[†] Note also that W is the *total* weight and not merely the weight of aggregate passing the designated sieve.

TABLE 2

COMPARISON OF COMPUTED AND EXPERIMENTAL INHERENT STANDARD DEVIATION, σ_u

MATERIAL	REPETITIONS	AVG. WT. OF TEST PORTION (LB)	SIEVE SIZES	AVG. PARTICLE WGT. (GM)	PERCENT RETAINED	PERCENT PASSING	STD. DEVIATION, % PASS. THEOR.	EXPER.	95% C.L.
Crushed stone	100	19.0	1½"-¾"	29.0	13.3	100.0	—	—	—
			¾"-⅜"	4.3	51.2	86.7	2.0	1.9	1.66-2.20
			⅜"-No. 4	0.61	27.8	35.5	1.6	1.5	1.31-1.74
			No. 4-No. 8	0.09	7.7	7.7	0.56	0.53	0.46-0.61
Crushed stone	100	9.8	1½"-¾"	29.0	13.4	100.0	—	—	—
			¾"-⅜"	4.3	51.4	86.6	2.7	2.9	2.54-3.36
			⅜"-No. 4	0.61	27.5	35.2	2.2	1.7	1.49-1.97
			No. 4-No. 8	0.09	7.7	7.7	1.0	0.51	0.45-0.59
Crushed stone	100	5.0	1½"-¾"	29.0	13.8	100.0	—	—	—
			¾"-⅜"	4.3	52.0	86.2	3.9	3.8	3.33-4.40
			⅜"-No. 4	0.61	26.4	34.2	3.1	2.2	1.93-2.55
			No. 4-No. 8	0.09	7.8	7.8	1.5	0.65	0.57-0.75
Rounded gravel	200	24.4	1½"-¾"	16.2	16.4	100.0	—	—	—
			¾"-⅜"	4.3	57.1	83.6	1.4	1.4	1.24-1.64
			⅜"-No. 4	0.81	23.9	26.5	1.1	0.9	0.81-1.08
			No. 4-No. 8	0.11	2.2	2.6	0.35	0.26	0.23-0.30
			No. 8-Pan		0.4	0.4	0.14	0.16	0.14-0.19

Testing Error

The variance due to TESTING ERROR, denoted by $\sigma^2{}_t$, is the between-test-portion variance due to the lack of REPEATABILITY of the test procedure, which may include effects of reducing increments to TEST PORTION size, or other preparatory work. Even when the same sample is passed through the same sieves, results may differ. Aggregate particles are usually of irregular shape, and during one test may be favorably positioned for passing through a sieve opening, whereas during another test the same particles may not be so oriented. With some types of shaking equipment, particles that have passed through the openings of one sieve may even return to that sieve after prolonged shaking.

Sources of variation between reported gradations, not usually considered a part of the testing error of the procedure, may include differences in sieving efficiency and actual errors, such as the loss of aggregate particles from the sample testing, inaccurate weighing of groups of separated particles, or incorrect observations or calculations.

Probably the chief cause of variation, in many cases, is the use of too large a test portion. If a thick bed of particles remains on a sieve at the end of the shaking period, it is probable that varying numbers of the smaller particles will not have had the opportunity to pass through the sieve openings. All of the foregoing factors are affected by differences among items of equipment and differences among operators. In situations where variation due to testing error is large, and cannot be reduced by using more precise test equipment or improved operator technique, it is necessary to average the results of a number of nearly identical test portions to obtain the desired accuracy.

As used in this report, σ_t is a measure of the repeatability of the gradation test using the same test portion, the same equipment, and with the same operator. It is computed from

$$\sigma^2{}_t = \frac{\Sigma(X_1 - X_2)^2}{2n} \tag{3}$$

in which

$\sigma^2{}_t$ = variance due to lack of repeatability of the test;
X_1 = result of first test on test portion;
X_2 = result of second test on same portions; and
n = number of test portions (two measurements or tests were made on each test portion).

The scope of the work to be accomplished under this project included experimental measurement of $\sigma^2{}_t$ for coarse aggregate only. Because some aggregates are subject to degradation during sieving, $\sigma^2{}_t$ was determined by retesting test portions taken at random from the various

TABLE 3

AVERAGE σ_u VALUES FOR $W = 25$ LB

SIEVE SIZE	THEORETICAL INHERENT VARIABILITY, σ_u, % PASSING
1½ in.	2.8
¾ in.	2.0
⅜ in.	1.4
No. 4	0.7
No. 8	0.6

samples, rather than making multiple tests with the same test portion.

The retests were made under such conditions that the results were not BIASED by those originally obtained. A total of 312 retests were made on six different aggregates having different gradations (the test portion size varied from about 15 to 45 lb). The tests were made in three different laboratories and two types of sieving equipment were used.

The standard deviations of the percentages passing the various sieves are given in Table 4.

The data in Table 4 were obtained in the three laboratories under practical operating conditions considered to be typical in the average State highway department or commercial testing laboratory. Precautions were taken that only normal care was exercised in making either the original or the repeat tests and that the results of the repeat tests were not influenced by those previously obtained. The result was the rather wide range of repeatability shown in the table, which probably represents realistic routine procedure. However, it is quite possible that if special precautions were taken to further eliminate ASSIGNABLE CAUSES, more uniform results would be obtained.

Pooling the data by combining variances yields a rounded figure for average standard deviation, $\bar{\sigma}_t$, of 0.4 percent under these given conditions. This value of $\bar{\sigma}_t$ includes normal weighing errors, but does not include other possible sources of testing error which could occur when replicate test portions are tested, such as variations due to splitting or quartering. Accordingly, values of testing error indicated by the data of this table must be considered to be minimum values, which can be expected to be exceeded when routine gradation tests are made on replicate portions of a sample of aggregate.

Assuming that sieves having standard openings are used and are in good condition, the relative amount of sieving error depends on several variables. The most important of these are the thickness of the bed of particles on the sieve, the shape of the particles, the length of time the material is sieved, and the efficiency of the sieving equipment. If the bed is not more than one particle thick and the completeness of sieving is tested by the hand method as outlined in ASTM Method C 136, 5(b), the sieving error will be extremely small. Under the practical conditions of routine tests, a bed of several particles in thickness may form on one or more sieves, depending on the gradation of the aggregate. In such a case, there may be appreciable sieving error associated with these overloaded sieves, because there will be less opportunity for particles to be oriented into position for passage through the sieve openings during the practical limits of sieving time.

The values shown in Table 4 represent differences to be expected when the same test portion is retested under practical routine conditions using a range of aggregate types and gradations normally encountered in typical highway construction. As cited, only normal care was taken to prevent overloading of the sieves, to sieve completely, and to avoid weighing errors. Under these conditions, differences were found in the standard deviation of the percentage passing the same size sieve among the different series. With an extended gradation, such as in Series No. 5, the lesser number of particles retained on the coarse aggregate sieves apparently led to a reduced testing error. The differences between Series No. 7 and Series No. 8 apparently reflect the effect of different technicians using the same equipment, and essentially the same aggregate. It should be noted, however, that these routine testing errors are small compared with other sources of variation which combine to determine the CONFIDENCE LIMITS for the estimate of the true gradation of the aggregate. In other words, the accuracy of the test result is affected more by the sampling procedure and sample weight than by the precision of the test method.

Experimental Error

The sum of the variances due to inherent variation and testing error ($\sigma^2_a + \sigma^2_t$) has been called EXPERIMENTAL ERROR (σ_e). Inasmuch as it is this combined variance that affects

TABLE 4

SUMMARY OF RESULTS OF REPEATABILITY TESTS

SERIES NO.	EQUIPMENT	NO. OF TEST PORTIONS, n	STANDARD DEVIATION, σ_t, OF PERCENTAGES PASSING					
			1½ IN.	¾ IN.	⅜ IN.	NO. 4	NO. 8	$\overline{A}$
1	Gilson	100	—	0.49	0.65	0.35	0.26	0.02
2	Gilson	100	—	0.28	0.50	0.33	0.20	0.02
3	Gilson	20	0.57	0.37	0.26	0.25	0.25	0.02
4	Gilson	12	—	0.45	0.51	0.14	0.12	0.02
5	Gilson	20	—	0.26	0.25	0.25	0.28	0.02
6	Gilson	20	—	0.30	0.46	0.55	0.45	0.03
7	Weston	20	—	0.54	0.43	0.36	0.04	0.02
8	Weston	20	—	0.09	0.14	0.20	0.17	0.01
Weighted avg., $\bar{\sigma}^2_t$			0.318	0.149	0.268	0.111	0.061	
Wtd. avg. among series, $\bar{\sigma}_t$			0.6	0.4	0.5	0.3	0.3	0.02

the repeatability and REPRODUCIBILITY of an aggregate gradation test on duplicate samples, the PRECISION STATEMENT for this test must be based on this sum of variances (see Chapter Seven).

Sampling Error

The source of the sampling error, σ^2_s, is the incomplete mixing of a small volume of aggregate, such as in a batch or unit of construction, so that the distribution of the particles of different sizes is not entirely random. As a result, an increment taken from one part of the batch will not show the same test values as one taken from another part of the batch. It is computed by first finding the total within-batch variance, σ^2_b, then subtracting the sum of the inherent variance and the testing error, or

$$\sigma^2_s = \sigma^2_b - (\sigma^2_a + \sigma^2_t) \tag{4}$$

In several cases, the experimental sampling error proved to be zero or a slightly negative value. This probably indicates that no sampling error existed (the batch itself was well mixed) or that the estimate for σ^2_a or σ^2_t was too large. It also illustrates a fact which might well be emphasized at this point; namely, that these variances are not exact numbers, but are themselves subject to errors of measurement or estimating. (In fact, the analysis of variance involves obtaining the best *estimate* possible of the individual components, but knowing full well that certain approximations are a necessary part of the analysis.) It is not surprising, therefore, that small negative estimated values will occasionally appear when the variances to be estimated are small. Again, it should be noted that essentially all of the zero or slightly negative σ_s values were obtained when X, the level of percent passing, was either very high (95%+) or very low (0 to 2%).

The general order of magnitude of the variability due to sampling errors or local segregation is illustrated in Table 5. Here, the first column shows the sampling error, σ_s, on the ⅜-inch sieve for various points of sampling in the process stream. It should be emphasized that these results are presented here primarily to illustrate the order of magnitude of this source of variance. A more detailed presentation of findings and the correlation between source of variance and point of sampling is presented later in the report.

Within-Batch Variance

The within-batch variance is found by taking two test portions or increments from suitably separated points in the same batch, making the specified tests, and substituting the results in

$$\sigma^2_b = \frac{\Sigma(X_A - X_B)^2}{2n} \tag{5}$$

in which

σ^2_b = total within-batch variance;
X_A = test result on first increment;
X_B = test result on duplicate increment; and
n = number of paired increments (one-half the total number of increments).

In many instances, such as in the case of an aggregate for use in concrete, within-batch variance, σ^2_b, is of least practical importance, because the cause of this variance will be removed by further mixing. However, if the batch is sufficiently segregated, the sampling error, σ^2_s, may lead to misinterpretation of test results unless test portions are taken by collecting multiple increments of aggregate from different parts of the batch. The exercise of engineering judgment in interpreting the relative importance of within-batch variance for a given aggregate use can have much practical significance.

TABLE 5

AVERAGE STANDARD DEVIATION [a] OF CONCRETE AGGREGATE PERCENT PASSING ⅜-IN. SIEVE

	AVG. STD. DEVIATION VALUE			
SAMPLING POINT	SAMPLING, σ_s	WITHIN-BATCH, σ_b	BATCH-TO-BATCH, σ_l	OVERALL, σ_o
Crusher	0.9	1.7	3.5	3.9
Belt to bin	1.3	2.1	4.2	4.9
Bin to discharge	3.4	3.8	8.0	9.1
Barge	7.6	7.7	9.4	12.3
Stockpile	4.4	4.6	8.2	9.5
Truck	3.6	3.8	8.0	9.1

[a] For comparative purposes, average testing error $\bar{\sigma}_t = 0.4$, and average inherent variability $\sigma_a = 1.4$, on the ⅜-in. sieve.

Again, for comparative purposes only, average within-batch standard deviation, σ_b, values are presented in Table 5.

Batch-to-Batch Variance

The batch-to-batch, or within-lot variance, σ^2_l, is the most significant, because it can cause actual differences in the performance of different batches.

The size of the variance depends almost entirely on the efficiency of the methods of handling, transporting, and storing aggregates, and the resulting degree of segregation. It is computed by difference, using

$$\sigma^2_l = \sigma^2_o - \sigma^2_b \tag{6}$$

The relative magnitude of the standard deviation associated with batch-to-batch segregation, σ_l, is also given in Table 5.

Overall Variance

The total overall variance among test portions taken from a LOT is symbolized by σ^2_o, which is equal to $\sigma^2_a + \sigma^2_t + \sigma^2_s + \sigma^2_l$, and is computed by

$$\sigma^2_o = \frac{\Sigma X^2 - (\Sigma X)^2/n}{n-1} \tag{7}$$

in which

σ^2_o = total overall variance;
X = test result on an increment or test portion; and
n = number of measurements or test results.

Comparative values of average overall standard deviation of percent passing the ⅜-in. sieve at the different sampling points are also included in Table 5.

Computation of σ^2_0 can be quite tedious. In this study, all computations were made on a high-speed electric desk calculator using forms specially devised for this purpose. Use of this type of form (Fig. 6) greatly simplifies the calculations involved.

A computer program was subsequently developed for these computations and has been successfully used for a later project (HR 10-3(1)). It is now evident that this program will prove to be of value for extensions of Project HR 10-2 work.

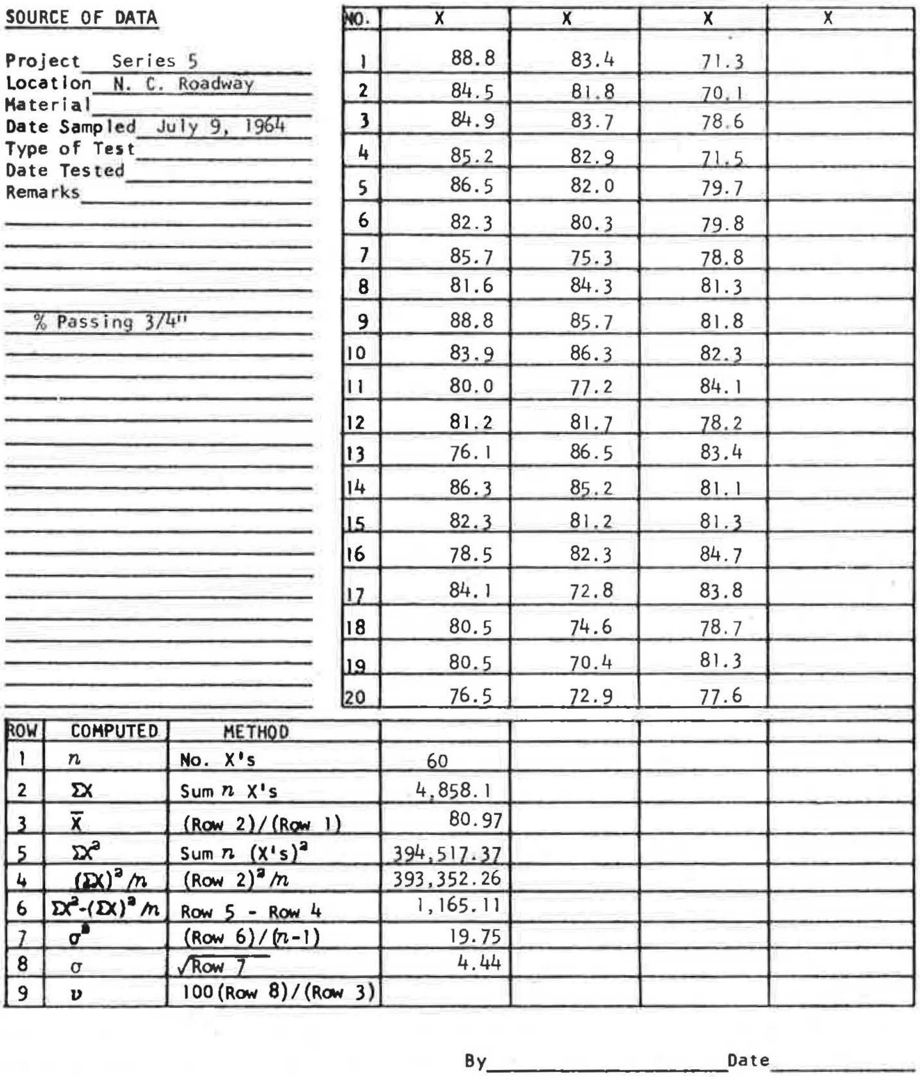
Summary of Variances

To sum up, the variations in gradation which may be expected among random samples taken from a LOT of aggregate stem from many causes, as shown in Figure 4 and Table 6.

The relative magnitudes of the corresponding standard deviations are summarized for comparative purposes in Table 5. A summary of the general observations regarding the factors which seem to influence these variations is presented in the following in order of relative size.

The testing error, σ_t, is the smallest of the group with an average of about 0.4 and a range of 0.2 to 0.6. The data indicate that there is a relation between σ_t and the amount of material on the sieve. For most of the aggregates studied, the percent passing the No. 4 and the No. 8 sieves was small and, in general, σ_t is correspondingly smaller than it is on the ¾-in. or ⅜-in. sieves. Series No. 5, on the other hand, is a well-graded aggregate with a relatively large percent passing the No. 4 and No. 8 sieves; the testing error on these sieves is correspondingly larger.

The inherent variability, σ_a, is next in line under the con-



SOURCE OF DATA

Project Series 5
Location N. C. Roadway
Material
Date Sampled July 9, 1964
Type of Test
Date Tested
Remarks

% Passing 3/4"

NO.	X	X	X	X
1	88.8	83.4	71.3	
2	84.5	81.8	70.1	
3	84.9	83.7	78.6	
4	85.2	82.9	71.5	
5	86.5	82.0	79.7	
6	82.3	80.3	79.8	
7	85.7	75.3	78.8	
8	81.6	84.3	81.3	
9	88.8	85.7	81.8	
10	83.9	86.3	82.3	
11	80.0	77.2	84.1	
12	81.2	81.7	78.2	
13	76.1	86.5	83.4	
14	86.3	85.2	81.1	
15	82.3	81.2	81.3	
16	78.5	82.3	84.7	
17	84.1	72.8	83.8	
18	80.5	74.6	78.7	
19	80.5	70.4	81.3	
20	76.5	72.9	77.6	

ROW	COMPUTED	METHOD				
1	n	No. X's	60			
2	ΣX	Sum n X's	4,858.1			
3	$\bar{X}$	(Row 2)/(Row 1)	80.97			
5	ΣX^2	Sum n (X's)^2	394,517.37			
4	$(\Sigma X)^2/n$	(Row 2)$^2/n$	393,352.26			
6	$\Sigma X^2-(\Sigma X)^2/n$	Row 5 - Row 4	1,165.11			
7	σ^2	(Row 6)/(n-1)	19.75			
8	σ	$\sqrt{\text{Row 7}}$	4.44			
9	v	100(Row 8)/(Row 3)				

By__________ Date__________

Figure 6. Example of worksheet for computing the standard deviation, σ, by use of a desk calculator.

TABLE 6

SUMMARY OF VARIANCES

VARIANCE	DESIGNATION	CAUSE	HOW ESTIMATED	EQUATION
Theoretical maximum	$\sigma^2_{\max}$	Complete segregation	Computed	$\sigma^2_{\max} = P(100 - P)$
Inherent (within-test portion)	σ^2_a	Inherent	Computed	$\sigma^2_a = \frac{P(100 - P)\bar{g}}{454\ W}$
Testing error (between tests)	σ^2_t	Testing error	By experiment	$\sigma^2_t = \frac{\Sigma(X_1 - X_2)^2}{2n}$
Sampling error (among increments)	σ^2_s	Sampling error	By difference	$\sigma^2_s = \sigma^2_b - (\sigma^2_a + \sigma^2_t)$
Within-batch	σ^2_b	Multiple (sum of σ^2_a, σ^2_t, σ^2_s)	By experiment	$\sigma^2_b = \frac{\Sigma(X_A - X_B)^2}{2n}$
Batch-to-batch (within lot)	σ^2_l	Segregation	By difference	$\sigma^2_l = \sigma^2_o - \sigma^2_b$
Overall	σ^2_o	Sum of variances	By experiment	$\sigma^2_o = \frac{\Sigma X^2 - (\Sigma X)^2/n}{n - 1}$

ditions of test (sample weight, $W = 25$ lb). Inherent variance is an inverse function of sample weight and is also related to particle size. Under the study conditions, σ_a varied from 0.6 for the No. 8 sieve to 2.8 for the 1½-in. sieve size.

The experimental error, σ_e, is a measure of the combined effects of testing error and inherent variation. It is the variance which will most closely approximate reproducibility of aggregate gradation tests on replicate samples, Thus, the implication is that a coarse aggregate gradation precision statement for among-different-laboratory testing may need to be related to sieve size, the quantity on that sieve, and sample weight, as well as the random variation in operators, equipment, etc.

Sampling error, σ_s, is a function of both the sampling method and the sampling point, ranging from an average of 0.9 at the crusher to 7.6 from the barges. Sampling error also varies with the amount of material on the sieve and frequently approximates zero when the percent passing is either very high (95%+) or very low (less than 5%). Although mentioned only briefly in this particular report, the measured variability due to local or within-batch segregation is also a function of the distance between the paired within-batch portions.

Within-batch variability, σ_b, at the 1.7 to 7.7 level, is next in line and is subject to the combined previously discussed effects.

Batch-to-batch variability, σ_l, is the measure of segregation which is usually the most important and which is largely a function of the processing or handling methods. In these studies, the average σ_l varied from 3.5 at the crusher to an average of 9.4 percent passing the ⅜-in. sieve on the barges.

Overall variability, σ_o, is the largest, because it is made up of the sum of all previously discussed sources of random variation. In these studies, the average σ_o varied from 3.9 at the crusher to 12.3 on the barges.

It is interesting to note that, even under the poorest conditions, the standard deviation values of random variation under typical conditions summarized in the foregoing for comparative purposes are in themselves small, relative to complete segregation, $\sigma_{\max}$.

ASSIGNABLE CAUSES OF VARIATION

In addition to the foregoing sources of variance due to random or chance causes, the influence of assignable causes of variation on aggregate gradation test results should not be overlooked. These are actual errors of omission or commission, such as intentional departure from specified proportions or methods, or malfunction of equipment. Assignable causes usually produce much larger variations than random causes, so they can be detected and eliminated by thorough inspection.

A summary of the more common errors encountered in determining the gradation of aggregates is presented in Table 7, in which both sampling and testing errors are analyzed with respect to source, type, cause, and suggested corrective action.

APPLICATIONS OF VARIANCES TO CONSTRUCTION CONTROLS AND SPECIFICATIONS

An understanding of the causes and sources of variability at the quarry site, in the transportation, processing and handling, and in the control of aggregates used in construction is obviously of importance to many segments of the highway industry. The aggregate supplier and the contractor must not only maintain satisfactory degrees of uniformity to meet specifications, but they also should be able to pinpoint the source of any undue variability easily and quickly so that positive corrective action can be taken. The responsibility for segregation throughout the full transportation and handling process should be capable of being placed where it rightfully belongs. The techniques and formulas developed herein for defining these sources of variation make it possible to design sampling plans which will pinpoint just where and to what degree the material is going out of specification or the uniformity is changing.

Another important application of potential value to the highway industry as a whole lies in the ability to better define just where the process needs help. For instance, it would be pointless for the instrument manufacturers to spend a lot of money developing a better means of control or of testing at a point in the process that would not significantly influence the overall variability, σ_o. Cutting the testing error in half at the $\sigma_t = 0.4$ level doesn't help much, if the sampling error, σ_s, is at the 5 to 7 level, or the batch-to-batch variability, σ_l, is in the 7 to 10 range. These observations, of course, apply to sources of *random* variation; any advancement in instrumentation, automation, or improved control of *assignable* causes of variation is an entirely different matter.

The equipment manufacturers might well find the techniques and formulas developed herein to be of value in the design of improved storage bins, silos, hoppers, etc. It is shown later that some of the test results obtained in the Philadelphia plants indicate that hopper design is the probable cause of a relatively large difference in segregation observed in two otherwise similar aggregate processing plants.

The last observation of a general nature before getting into the more specific applications of variances to construction controls and specifications lies in the improved ability to define realistic tolerances for both construction controls and material requirements.

TABLE 7

ERRORS IN DETERMINING THE GRADATION OF AGGREGATES

SOURCE	TYPE	CAUSE	REMARKS	CORRECTIVE ACTION
(*a*) SAMPLING ERRORS				
Chance	Random	Within-lot segregation	Gradation changes according to location in lot	Increase number of test portions
Chance	Random	Within-batch segregation	Gradation changes between adjacent sampling points	Increase number of increments per test portion
Chance	Random	Inherent variation	Function of gradation and maximum size of aggregate	Increase total weight of test portions
Equipment	Bias	Loss or exclusion of larger particles	Most likely when gradation includes wide range of particle sizes	Use appropriate sampling tool only
Equipment	Random	Disproportionate quantity of fine or coarse particles	Occurs when reducing aggregate to test portion size	Divide test portion and test all parts, or use riffle of adequate design
Technician	Bias	Biased sampling plan	Sample taken on "judgment" basis	Locate sampling points by use of random numbers
(*b*) TESTING ERRORS				
Equipment	Bias	Break or hole in sieve	Most likely in highest numbered sieves	Inspect, and run standard sample
Equipment	Bias	Plugged sieve openings	Most likely in highest numbered sieves	Inspect after each use. Clean openings
Equipment	Bias	Non-standard sieve openings	Worn or damaged sieves	Check sieve openings or use standard sample. Replace faulty sieves
Technician	Bias	Incomplete sieving	Most likely in highest numbered sieves	Reduce size of test portion, use intermediate sieves, increase sieving time, hand check for completeness
Equipment	Bias	Incorrect tare on scales	Constant error in one direction	Place known weight on pan
Equipment	Bias	Inaccurate scales	Usually proportional to weight	Repair or replace scales
Equipment	Random	Insensitive scales	Usually inversely proportional to weight	Repair or replace scales
Technician	Bias	Loss of part of fraction	Can be detected by repeat weighing of test portion	Improve technique
Technician	Random	Incorrect reading	Can de detected by repeating test	Improve technique
Technician	Random	Incorrect recording and/or computations	Can de detected by repeating test	Improve technique

General Applications

There are two broad applications worthy of note, as follows:

1. The ability to define and measure limiting tolerances beyond which further restrictions become impractical without some major change in the processing method or control. In other words, the defining of what is normal inherent variability associated with that particular operation or test method.
2. The ability to better define just what is good acceptable construction. Engineering judgment with respect to permissible variability (or conversely, to needed degree of uniformity) is based largely on the attempt to duplicate previous or known good experience. These statistical techniques provide the tools for measuring the variability of acceptable good construction, either in place or as it is being produced under normal control and conditions. Thus, these methods provide a valuable adjunct to engineering judgment.

Specific Applications

In addition to the general uses, only a few of which are discussed in the foregoing, there are certain specific applications of variances to construction controls and specifications pertinent to this study.

LIMITING MAXIMUM VARIANCE

As previously discussed, the limiting maximum variance of the percentage of aggregate passing a given sieve is given by $P(100 - P)$, where P is the average percentage passing. This means that as the 50 percent point is approached the limiting variance is 2,500 [= 50 × (100 − 50)], whereas for 1 percent the limit is 99. This effect is reflected in the range of specification limits which are practical for a graded aggregate. Due to the tendency for variations in the 40-60 range of percentages to be larger, the specifications limits in this area must necessarily be wider than for small percentages if specifications are to be consistently met.

This limiting variance may also be used, as it has been in this report, as a reference value. If different handling or stockpiling procedures are to be compared, using aggregates having different gradations, the "degree of segregation," obtained by dividing the batch-to-batch standard deviation, σ^2_b, by the limiting sigma, σ_{max}, to obtain an index, provides a better indication that one method actually resulted in less segregation than the other, and that the difference in segregation was not unduly influenced by the mathematical effect of the differences in gradation percentages.

LIMITING MINIMUM VARIANCE

As developed previously, the inherent variance, σ^2_a, due to nonhomogeneity of the discrete aggregate particles is a basic variation which cannot be further reduced by process control or by testing refinements. The magnitude of this basic variance is dependent on both the gradation, or average particle size, and the total weight of the aggregate sample that is actually tested. It thus becomes a limiting minimum value which can be tied directly to some definite limiting minimum weight of sample required to attain a given degree of accuracy or confidence in the test results. If only a small weight of an aggregate containing large particles is tested, σ^2_a will be large and the specification limits must be wide enough to accommodate $\pm 2\sigma^2_a$, plus variations due to normally good handling procedures. On the other hand, narrow specification limits require that the minimum weight of aggregate sampled and tested be sufficient to reduce σ^2_a to a value such that the gradation test has an acceptable degree of accuracy.

Eq. 2 may be used to estimate the minimum sample weight required to estimate the true gradation of an aggregate within allowable limits of error. Note that this is sample weight for the total weight of aggregate put through the sieves. How this total sample weight is divided up into the number of test portions, or the number of different batches, or the number of increments taken from each batch, is quite another matter involving sampling plan considerations discussed later. The number of test portions required to average out differences in the gradation of aggregate from different parts of the LOT depends on the size of the overall standard deviation, σ_o. To average out differences in gradation in different parts of the batch, a number of increments must be taken, the number depending on the size of the within-batch standard deviation, σ_b. Thus, there must be some minimum number of increments in each sample taken to represent the LOT. The size of these increments must be large enough so that the larger aggregate particles are not excluded, but the practical weight of increment taken is largely a matter of judgment and the sampling tool used. If the practical sample size determined by multiplying the practical increment weight by the number of increments is greater than shown by Eq. 2, the size is acceptable for the chosen degree of accuracy.

As a part of this study, a nomograph (Fig. 7) was devised to eliminate the computations required by Eq. 2. The scales on this nomograph may be used to estimate particle weight and the degree of accuracy associated with total sample weight under various conditions. The nomograph may also serve as a convenient way of judging alternate means for arriving at a practical balance of cost, increment size, and the number of test portions, in addition to providing a rational approach to estimating the minimum total sample weight required to attain a given degree of accuracy.

COMBINED VARIANCES

More effective sampling plans can be designed if estimates of the individual variances are available. For example, in some acceptance situations one objective is to estimate the average value of some characteristic of a LOT within some selected confidence limits, usually 95 percent, or $\pm 2\sigma_o$. In the case of aggregate gradations, the confidence limits attached to the average percentage passing a given sieve depend on the summation of several variables.

First, it is essential that all increments and test portions which form the sample representing the LOT be selected by the use of a random sampling plan. If this condition is met, the standard deviation associated with the confidence limits of each average gradation (σ_{CL}) will be the square root of the sum of four variance components; that is,

$$\sigma_{\mathrm{CL}} = \sqrt{\frac{\sigma^2_l}{b} + \frac{\sigma^2_t}{bt} + \frac{\sigma^2_s}{bti} + \frac{\sigma^2_a{}^*}{btiw}} \qquad (8)$$

in which

b = number of batches sampled;
t = number of test portions from each batch;
i = number of increments taken to form each test portion; and
w = weight, in pounds, of each increment.

The objective of any acceptance plan should be to reduce σ_{CL} to the smallest practical value. As previously discussed, the smallest value that can be expected is determined by the gross weight of the sample, *btiw*.

The most efficient way of splitting this total weight into the number of increments which will result in the desired confidence limits, CL, depends on the relative sizes of σ^2_l, σ^2_s, and σ^2_t, and the relative cost of acquiring increments and test portions. As shown by the investigations reported herein, many LOTS of aggregate have a relatively large batch-to-batch variance, σ^2_l. This means that b should usually be large with respect to i. In most acceptance sampling situations, only one test or portion is taken from each batch, in which case $t = 1$. The number of increments, i, taken from each batch to form the test portion depends on the size of the potential sampling error, σ^2_s, which in turn depends on the size of the within-batch variance, σ^2_b, under a given combination of circumstances. The currently reported investigations indicate that, under some sampling conditions, σ^2_s may be very small or nonexistent, and that σ^2_t is relatively small. However, for practical reasons, the number of increments, i, taken for each test portion should usually be greater than one. The size of the increments, w, must not be so small that a representative proportion of the larger aggregate particles is not included, but on the other hand should not be so large that they cannot be handled conveniently.

To sum up, accurate estimates of the average gradation of a LOT of coarse aggregate depend on the effects of many variables. To minimize these effects, a sampling plan must be designed for the individual acceptance situation so as to obtain the greatest accuracy consistent with limits of available funds and technician time requirements. The efficiency of the plan will depend largely on how much is known about the components of variance under the conditions of the particular situation.

OVERALL VARIANCE

It is the overall variance, σ^2_o, that directly affects the writing of practical specifications with realistic tolerances. The magnitude of σ^2_o also affects the sampling plan, because the number of test portions required to obtain a measured predetermined degree of assurance and accuracy is found from

$$n = \frac{t^2\,\sigma^2_o}{\Delta^2} \qquad (9)$$

in which

n = number of test portions;
t = the desired degree of assurance, or probability of success in obtaining a correct answer, measured in standard deviation units from the center of the t distribution curve;
σ_o = the overall standard deviation of the measurements; and
Δ = the maximum allowable difference between the computed average of the measurements and the true average.

For example, if it is known that the overall standard deviation of the percent by weight of aggregate passing the ¾-in. sieve is 4 percent, and it is desired to take enough test portions so that there is a 95 percent probability of obtaining an average value correct to ±1 percent, $n = [(2.00)^2 \times (4)^2/(1)^2 = 64$.

In the example, the value of $t = 2.00$ is for 60 DEGREES OF FREEDOM (d.f.) and 95 percent probability. This value of t must be found by ITERATION because d. f. = $(n - 1)$ and n is initially unknown. To simplify computations and reduce the number of iteration trials, a nomograph (Fig. 8) has been devised.

SUMMARY OF SPECIFIC APPLICATIONS

In summary:

(a) Complete segregation is represented by limiting maximum variance, $\sigma_{\max}$.

(b) The minimum total sample weight in pounds is equal to the factor W in Eq. 2 and may be estimated, for a desired degree of accuracy under various conditions of particle size and percent passing the designated sieve, from the nomograph (Fig. 7).

(c) The total sample weight as defined above may be made up of an infinite combination of increments of various sizes, of numbers of test portions per batch, and the number of batches to be sampled. The optimum balance to attain the standard deviation associated with given confidence limits is a function of the relative magnitude of σ^2_l, σ^2_t, σ^2_s, and σ^2_a expressed in Eq. 8.

(d) The total number of test portions or individual measurements which must be averaged to attain a given degree of assurance that the computed average lies within a given allowable range of the true average is a function of the overall variance, σ^2_o. This relationship is represented by Eq. 9 and the nomograph (Fig. 8). Overall variance is an important and useful parameter, whether or not the individual components or sources of variance are known. Although the results must be used with caution (checked to assure that the operations are comparable and the same sampling plan was involved), some values of overall variance are now well enough known that the normal random variability associated with certain phases of highway construction can be generally characterized. Average values from the so-called "sigma bank*" may, under properly

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* In this case, σ^2_a is the inherent variance associated with unit weight of aggregate.

* Established by Miller-Warden Associates, Raleigh, N.C. (see *NCHRP Rpt. 17*, p. 88).

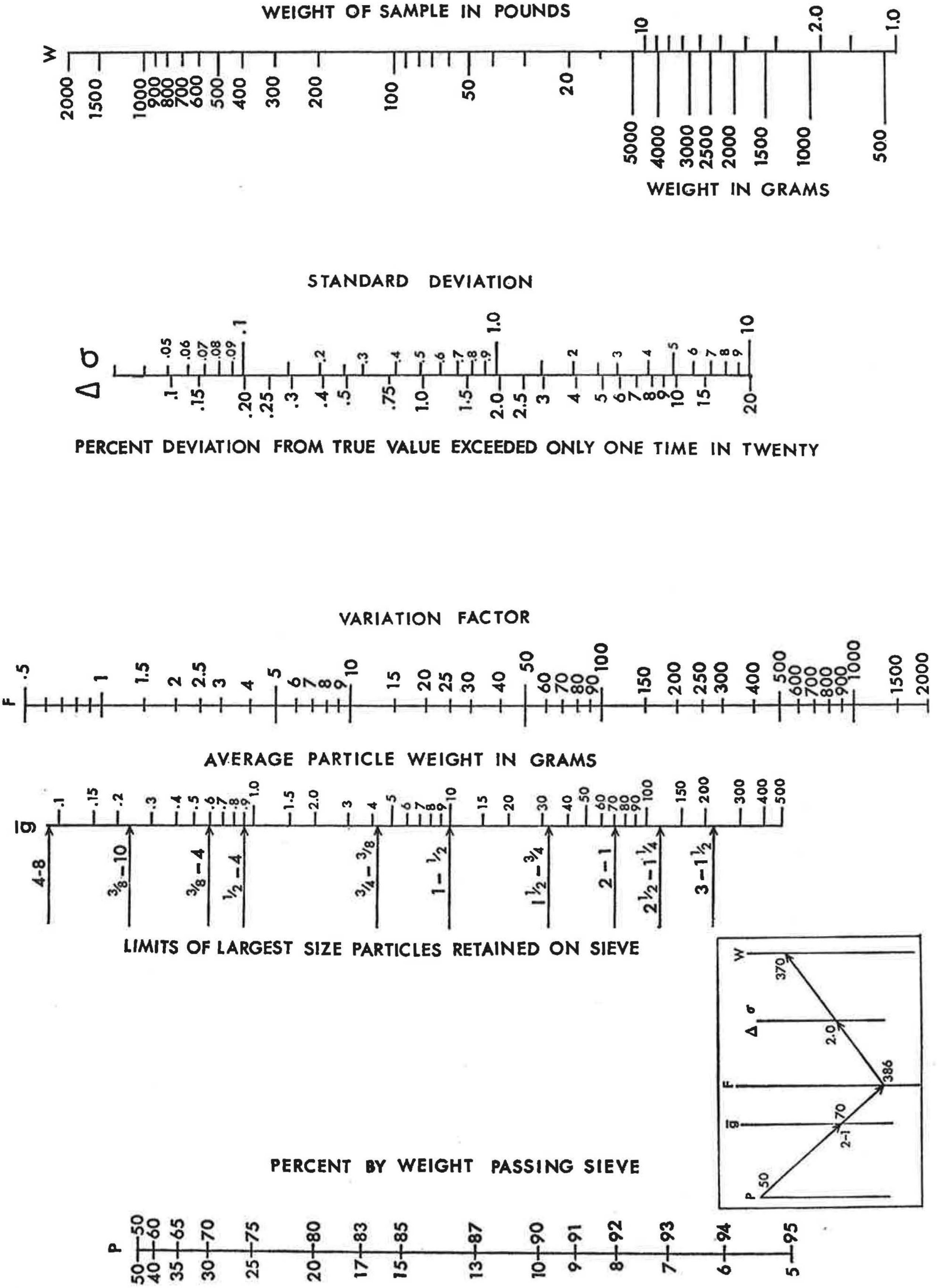
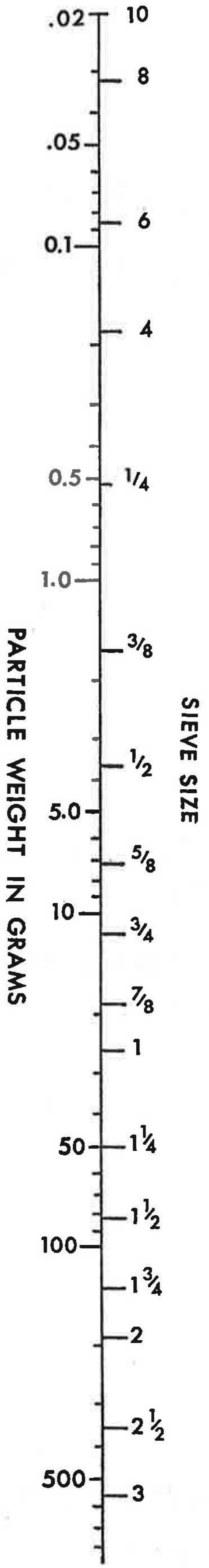


Figure 7. Nomograph for estimating weight of aggregate sample for gradation test.



USE OF NOMOGRAPH TO ESTIMATE SAMPLE WEIGHT

1. Select the critical sieve size. This is usually the size following that sieve which passes 90-100% of the aggregate.

   For Example: If 99% passes the 2" sieve and 50% passes the 1" sieve, the 1" sieve is the critical size.

2. Determine the average particle weight of all particles retained on the critical sieve. If this is unknown, it can be estimated roughly from the values shown on the left of the $\bar{g}$ scale.

3. To find $\bar{g}$ approximately by use of the scale on this page, find the weight opposite the mid-point of the distance between the sieve size that the particles pass and the sieve size on which they are retained. Then calculate a weighed average for the total material retained on the designated sieve.

   For Example:

Sieve Size	Percent Pass-Ret	Particle Weight
3/4-3/8	30	4. 1
3/8-4	50	0. 6
4-8	10	0. 09

$$\bar{g} = \frac{30\times4.1 + 50\times0.6 + 10\times0.09}{30 + 50 + 10} = 1.7 \text{ g}$$ (Av. wt. particles Ret on No. 8 sieve)

4. From the percentage passing the critical sieve on scale P, project a line through the average particle weight on scale $\bar{g}$ to scale F.

   For Example: If 50% passes the 1" sieve and the average particle weight of the aggregate retained on the sieve is 70 grams, project a line from 50 on scale P through 70 on scale $\bar{g}$ to 386 on scale F.

5. From the point on scale F, project a line through the desired degree of accuracy on scale Δ to the required total sample weight on scale W.

   For Example: With an F factor of 386 and a desired degree of accuracy of ± 2%, the line projected through these points indicates a required sample weight of 370 pounds on scale W.

6. The accuracy obtained by the use of a larger or smaller sample can be found by projecting a line from the F factor to the actual sample weight and reading the result on scale Δ.

   For Example: With an F factor of 386 and an actual sample weight of 50 pounds, the percent passing the 1" sieve will be correct to within ± 5%, 95 times in 100 determinations.

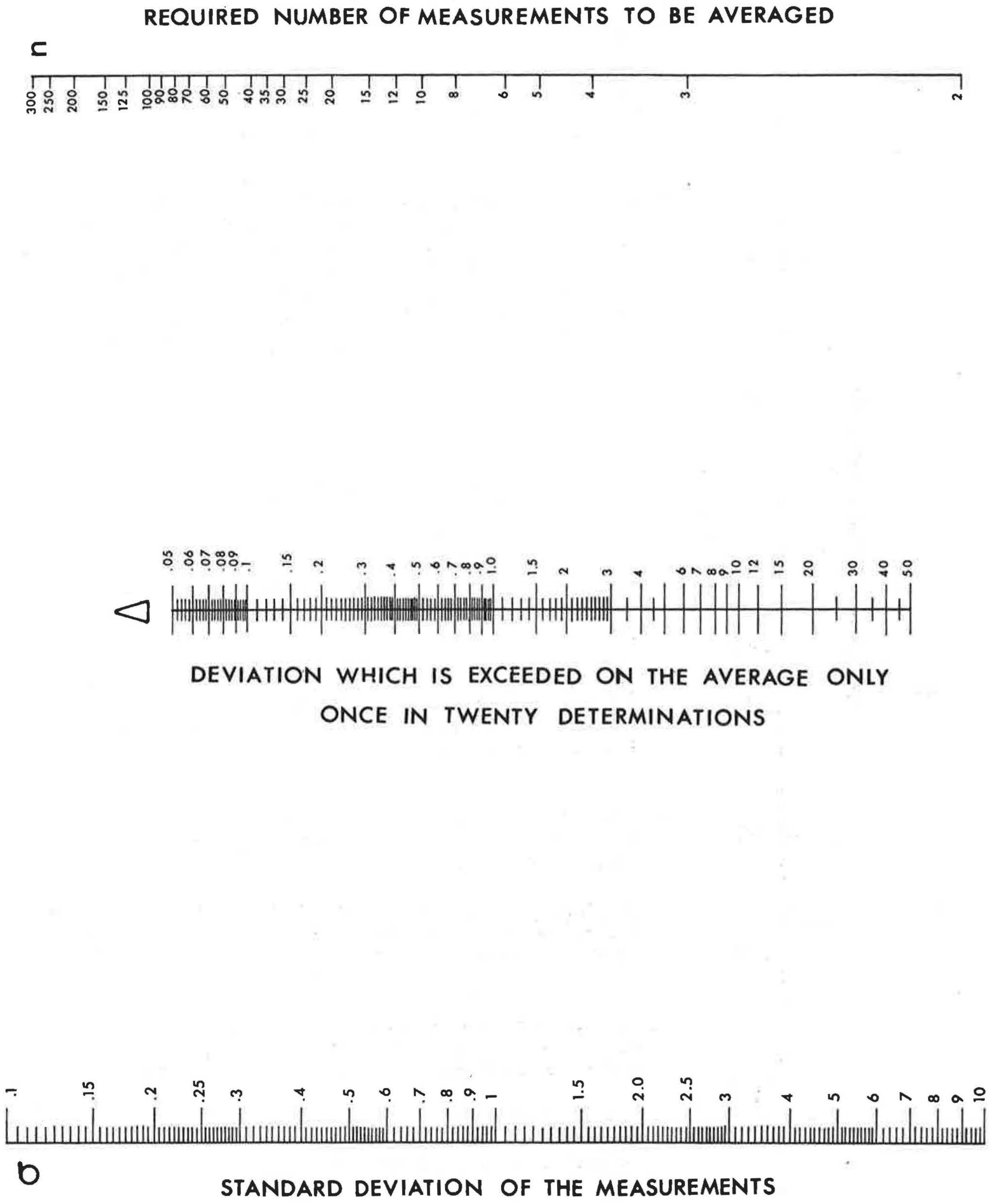


Figure 8. Nomograph for estimating required number of measurements.

defined conditions, provide useful preliminary estimates of the more common standard deviations.

GRADATION PARAMETERS

It is difficult to visualize or think of differences in gradation as a multiplicity of percentages passing different sieve sizes. Further, the experimental results obtained show that the magnitude of most of the sources of variance is a function of the average percent passing. It is therefore desirable to be able to express gradation as a single number and to either minimize or be able to define the effect of percent passing on the variance. Accordingly, this section describes the so-called HUDSON $\bar{A}$ parameter as a means of expressing gradation as a single number and the degree of overall variability (DOV), degree of segregation (D of S), and SEGREGATION INDEX (S) parameters as measures of relative variability.

Hudson $\bar{A}$

To measure and assess the effects of changes in gradation by the use of statistical methods, it is desirable to describe the gradation by a single number rather than a multiplicity of percentages. The FINENESS MODULUS (FM), originated by Abrams, is such a parameter and is useful when dealing with aggregates for portland cement concrete. However, the FM was intentionally designed to exclude the influence of the minus No. 200 aggregate in the gradation. This makes the FM unsuitable for use when dealing with aggregates for bituminous concrete or when other aggregate mixtures contain a significant quantity of minus No. 200 material.

Use of Figure 8

The purpose of this nomograph is to furnish an approximate solution of

$$n = \left[\frac{t\sigma}{\Delta}\right]^2$$

where t depends on the number of degrees of freedom ($n - 1$) associated with n.

1. To use, project a straight line from the standard deviation of the measurement on the left hand (σ) scale through the desired degree of accuracy on the center (Δ) scale. This line will intercept the right hand (n) scale at the approximate value of n indicated by the equation.

2. To obtain a more precise value of n, enter the t table with the number of degrees of freedom ($n - 1$) associated with the chart value, and opposite this value find t in the column which has $t = 1.96$ opposite d.f. $= \infty$.

Insert this t in the equation and solve for n. Use this value of n to find a new t, and continue to iterate until the value of n found by solving the equation is nearly the same as the value of n used to find t.

The UNIFORMITY COEFFICIENT, C_u, is another parameter that is useful in characterizing the properties of a gradation. This characteristic is defined as the ratio of the diameter of the 60 percent finer point to that at the 10 percent finer point on the gradation curve. Unfortunately, this parameter is difficult to derive and interpret.

For the reasons given, FM and C_u are not used in this report.

Recent studies have resulted in the concept of the so-called Hudson $\bar{A}$, which is simply one-hundredth of the sum of the percentages passing the ten STANDARD SIEVES startng with the 1½-in. and including the No. 200 sieves. The investigation of theoretical concepts, confirmed by limited experimental investigation, indicates that $\bar{A}$ is a fundamental constant, related to the relative surface area effects of the aggregate in any mixture of particle sizes. For example, the relationship of $\bar{A}$ to the asphalt demand in hot-mix bituminous pavement is shown in Figure 9, from which it can be seen that, with asphaltic concrete aggregates in the usual range of $\bar{A}$ from 4.00 to 6.00, a change of 0.50 in the value of $\bar{A}$ would change the asphalt demand by about 1 percent by volume, which is enough to affect the performance of the mixture. Thus, $\bar{A}$ appears to be a measure of relative coarseness of aggregate mixtures, sufficiently sensitive to reflect significant variations in aggregate gradation by the use of single numbers.

Segregation Index

An important parameter of relative variability is the ratio of overall variance, $\sigma^2{}_o$, to the within-batch variance, $\sigma^2{}_b$. This ratio has been termed "segregation index" and is designated by

$$S = \frac{\sigma^2{}_o}{\sigma^2{}_b} \qquad (10)$$

In a perfectly mixed stockpile, with no segregation, $\sigma^2{}_l = 0$ and $S = 1$. As segregation between separated portions of the LOT becomes more pronounced, $\sigma^2{}_l$ *increases*, as does S. Inasmuch as this is a ratio of variances, the F-test of Snedecor can be applied to estimate whether the ratio is sufficiently large for the degrees of freedom concerned to indicate a statistically significant degree of batch-to-batch segregation. Also, the variances, $\sigma^2{}_o$ and $\sigma^2{}_b$, can be expressed in terms of Hudson $\bar{A}$, thereby avoiding the problem of having to express gradation as percent passing a number of different sieves.

Variability as a Function of Percent Passing

As previously cited, the experimental results obtained in this study, as well as those from the HR 10-3 and HR 10-3(1) experiments in stockpiling, show that the magnitude of most of the components of variation is a function of the percent passing. Thus, it is difficult to compare the relative variability of different processes or handling procedures using aggregates of different gradations. It is therefore

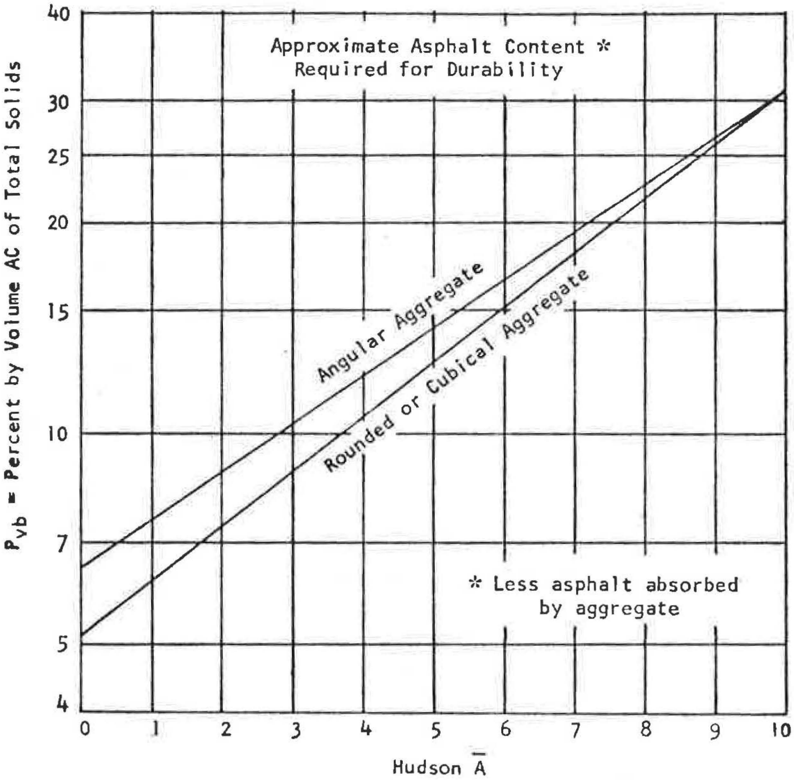


Figure 9. Relationship of $\bar{A}$ *to asphalt demand.*

desirable to find some means of relating variance with percent passing so that meaningful comparisons of relative variability can be made at some common point on the gradation curve. Although additional work is admittedly needed, two factors (degree of overall variability, DOV, and degree of segregation, D of S) are tentatively proposed herein as a means for placing a number on the relative overall variability and relative degree of segregation at the 50 percent passing point. This 50 percent passing point has been found, both theoretically and experimentally, to be the point of maximum variance for all coarse aggregate gradations studied to date.

THEORETICAL CONSIDERATIONS

Theoretically, both the inherent variance, σ^2_a, and the overall variance, σ^2_o, are a function of $P(100 - P)$. Experimental confirmation of the Manning adaption of the binomial theorem (Eq. 2) has been discussed. Further, the theoretical maximum variance, σ^2_{max}, is defined for purposes of this report as being equal to $P(100 - P)$, where P is the average percent passing. It is therefore logical to develop the desired relationship between variability and percent passing as a function of $P(100 - P)$. If the distribution was entirely binomial, as in the case of spots of equal size illustrated in Figure 5D, the mathematics would be reasonably straightforward.

In the case of actual aggregates used in construction, the situation is different in that particles are of different sizes and, consequently, the distribution is not exactly binomial. This Research Agency has done some work on the development of a mathematical model which will reflect the experimental findings from the combined studies of the variability of construction aggregates to date. However, this model is not ready for presentation as yet and further work is beyond the scope of this initial phase of the project reported herein. One of the major components of the model is $k[P(100 - P)]^t$, and the amount of segregation of any particle size in the gradation is expressed by the coefficient k and the exponent t. The value of t is 1 in the case of spots of equal size, but with real aggregates its value depends on the range and distribution of particle sizes in the gradation and possibly on other interactions or additional factors which have not as yet been evaluated. Some preliminary calculations of k and t by the method of least squares, using actual data, yielded values of $k = 0.02$ and $t = 1.7$. Knowing these factors, the relative variance (segregation) of any particle size can be calculated, provided the percentage, P, of that size is known. The variance or standard deviation of a fictitious size at which exactly 50 percent of the total aggregate would pass the sieve can be calculated. This may be a basic parameter for comparison purposes, because for large samples this 50 percent point is theoretically the point of maximum variability, regardless of aggregate type or gradation.

INTERIM GRAPHICAL ESTIMATIONS OF σ_{50}

Although further work on the mathematical model and further least-square method calculations could not be undertaken within the budget limitations of the initial contract, a graphical approach has been developed which seems to be practical for interim purposes. After trying a number of combinations and different ways of plotting the data, it was found that simple log-log plots of standard deviation versus $\sqrt{P(100 - P)}$ yield straight lines suitable for extrapolation to the 50 percent passing level ($P = 50$). Figure 10 shows log σ_o, σ_b, and σ_l, versus log $\sqrt{P(100 - P)}$ for the crusher (sampling point 1) of Series No. 1. Figure 11 shows log σ_o versus log $\sqrt{P(100 - P)}$ for all four sampling points of Series No. 1.

Similar log-log plots were drawn for all eight series and the corresponding σ_{o50}, σ_{b50}, σ_{l50} points read at the point of intersection of the best straight line drawn by eye with the $P = 50$ intercept. These values represent the standard deviations corresponding to overall variability, σ_{o50}, to within-batch variability, σ_{b50}, and to batch-to-batch variability, σ_{l50}, at the 50 percent passing level for each sampling point or process step for the various aggregates and plants tested. It is believed that comparisons at this 50 percent level are more meaningful because they overcome at least part of the difficulty due to differences in gradation.

Log-log graphs of σ_o, σ_b, and σ_l, versus $\sqrt{P(100 - P)}$ have been prepared for all of the aggregates and processing methods or sampling points tested to date for Projects HR 10-2, HR 10-3, and HR 10-3(1); all told, there are 53 sets of data. Essentially, all sets plotted up remarkably well and all but two types of aggregate gave good straight lines. The graphs are not shown here due to their substantial volume, coupled with the fact that most of the indicated results are of a fairly uniform nature. The relatively clean (washed) 1½-in. to ⅜-in. crushed stone from Gresham's Lake (Project HR 10-3(1)) showed an abnormal pattern. The well-graded crushed stone in Series No. 5 of Project HR 10-2 and the crushed aggregate base course materials used in the degradation studies of Project HR 10-3(1) showed some curvature and some deviation of the points greater than 50 percent to line up with the corresponding $P(100 - P)$ values on the minus 50 percent side of the gradation. The slope of the line obtained for most of the coarse aggregates was the same; exceptions were that, in some cases, aggregates sampled from stockpiles tended to show a steeper (greater) slope. The rounded gravels also plotted to a steeper (greater) slope. These graphs and the data are discussed further in Chapter Five.

The limitations of this technique are not as yet defined. It is hoped that further work on the mathematical model and further study of these various graphs and relationships in the continuing work on this project will result in better definition of the pertinent factors and their interaction. As is normally characteristic of successful research, this glimmer of a better understanding of the construction control procedures for aggregates opens up new vistas requiring further study and the need for more precise tools and techniques to still better define, assimilate, and use the new knowledge.

DEGREE OF OVERALL VARIABILITY AND DEGREE OF SEGREGATION

A new parameter called degree of overall variability, DOV, based on σ_{o50}, is proposed. The standard deviation is chosen instead of variance because most engineers are better equipped to think in terms of standard deviation, and the values have direct numerical significance as percent passing. Degree of overall variability is defined as

$$\text{DOV} = \frac{\sigma_{o50} \times 100}{50} \qquad (11)$$

The denominator, 50, is the standard deviation associated with theoretical maximum variance (complete segregation). It is the value of $\sqrt{P(100 - P)}$ when $P = 50$ percent. Thus, one way of visualizing the significance of the DOV parameter is to think of it as a percent of complete segregation. The right side of Eq. 11 is a measure of relative degree of overall variation based on standard deviation at the 50 percent level. A more meaningful significance to degree of overall variability, however, lies in the fact that the 100/50 part of the definition formula equals 2, therefore the degree of overall variability is also the $2\sigma_{o50}$ limit or percentage tolerance within which 95 percent of single measurements at the 50 percent passing level may be expected to fall.

Degree of segregation (D of S) is correspondingly defined as

$$\text{D of S} = \frac{\sigma_{l50} \times 100}{50} \qquad (12)$$

It is a measure of the relative batch-to-batch variability, and is indicative of the nonlocalized segregation most likely to result in nonuniform construction. Again, there is the percentage implication, because the denominator, 50, is theoretically the standard deviation associated with complete segregation.

Relative Significance of Variability Parameters

At this point, one might logically question the need for so many parameters, and it is interesting to compare possible significance of the three ratios, $\text{DOV} = \sigma_{o50}/50 \times 100$, $\text{D of S} = \sigma_{l50}/50 \times 100$, and $S = \sigma^2_o/\sigma^2_b$.

As previously discussed, the degree of overall variability, DOV, is a measure of relative degree of overall variability based on standard deviation at the 50 percent level, and as such is admittedly an arbitrary parameter, at least in the current stage of development and understanding of the basic theoretical concepts. As more is learned about these relationships, particularly about the degree of conformance to the binomial (or a multinomial) theorem, it may well be that this parameter (or its variance counterpart) will take on added significance. In any case, at this stage it provides a convenient means of comparing the relative overall variability of aggregates having different gradations. Further, the values are spread over a convenient range and the implication of percentage of the standard deviation associated with maximum variability or complete segregation may be helpful to the average materials engineer, who is not versed in visualizing statistical variance.

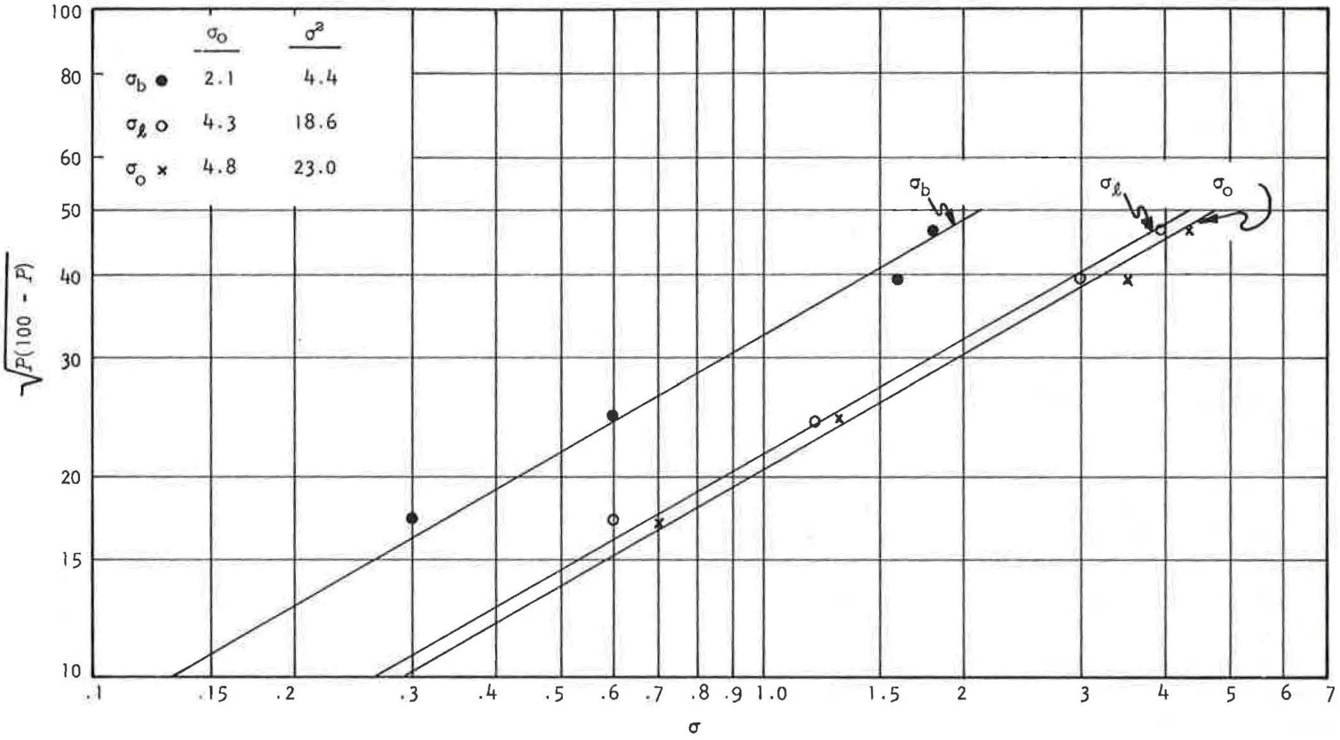


Figure 10. Statistical parameters for 50 percent passing level, Series No. 1, sampling point 1 (crusher).

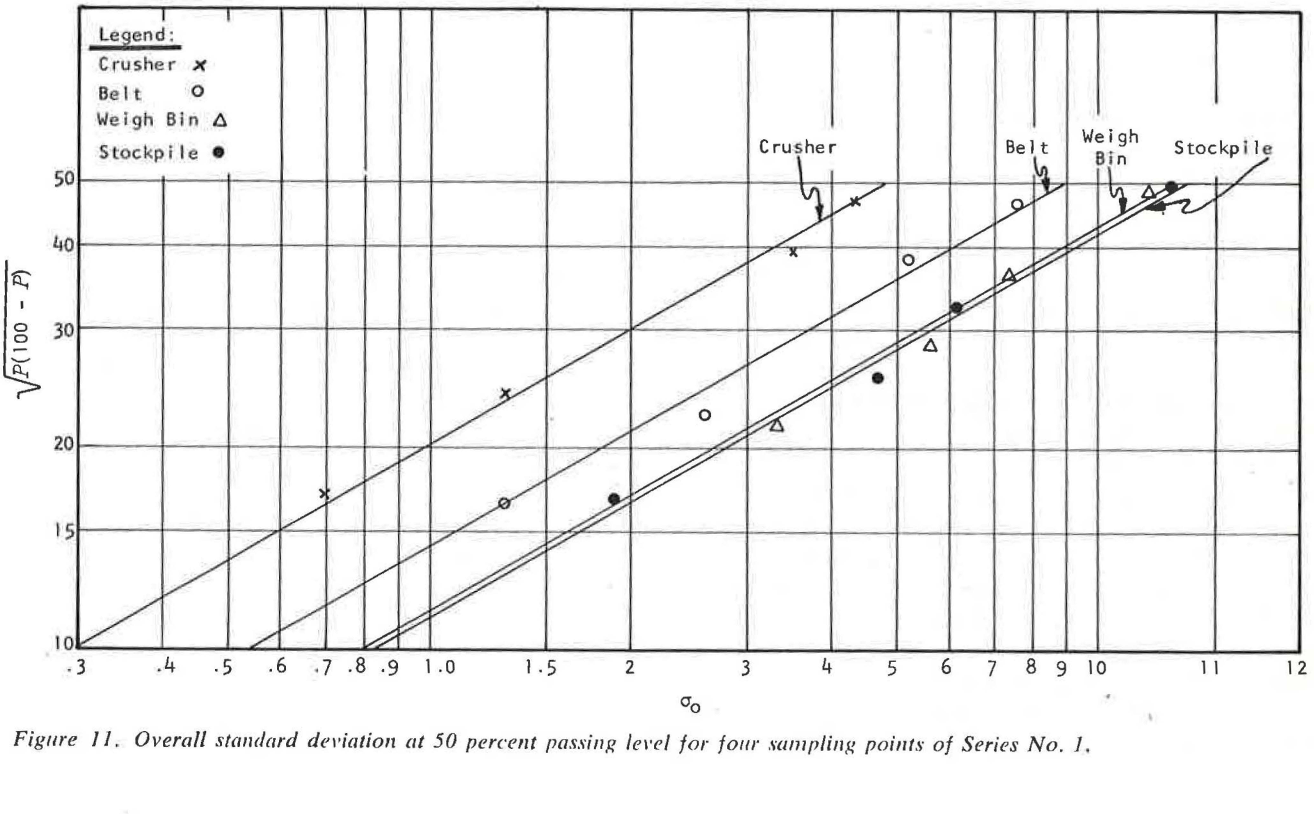


Figure 11. Overall standard deviation at 50 percent passing level for four sampling points of Series No. 1.

Degree of overall variability, DOV, does take on real significance when it is thought of as the $2\sigma_o$ limit, inasmuch as this is the gradation tolerance commonly used in construction aggregate specifications within which 95 percent of single measurements may be expected to fall. It has been recognized that wider tolerances are needed on those sieves at roughly the midpoint of the gradation (the 40 to 60 percent passing level), but the reason and the degree of difference has not been generally understood. The relative effect on gradation tolerances of percent (P) passing sieve is indicated in Table 8, which is based on a slope, t, of about 1.7, and σ_o of 5 at the 50 percent level. Other values of σ_o would be multiples of 5, but the relative effect on gradation tolerances would remain the same. The $2\sigma_o$ tolerances will, of course, decrease as the percent passing goes beyond 50 and P approaches 100 percent, because $P(100 - P)$ again approaches zero. In real life, there is some evidence that well-graded aggregates, in particular, do not regress along the same line but require somewhat higher $2\sigma_o$ tolerance limits over at least a portion of the coarse side of the gradation.

Although DOV thus has real significance as a reflection of overall variability and is the proper parameter for specifications and quality control purposes, it conveys nothing about the causes or nature of the variability. Degree of segregation (D of S) is the counterpart based on σ_{l50}, and it is this parameter which is a measure of batch-to-batch segregation within the LOT. It represents that type of variability (segregation) that is most likely to affect the quality of the construction or finished product. Further, it is the parameter that should motivate action, because it normally reflects variability that someone can do something about—a high value is usually associated with an assignable cause, such as improper stockpiling, poor hopper design, etc. In essentially all cases where the overall variability measured was really high, the σ_l component (segregation) was also high.

It should be repeated, however, that at this stage both DOV and D of S are more in the nature of practical parameters, with no implication that they are necessarily optimum units of measurement or comparison from the theoretical viewpoint. They are proposed as tools or units of measurement to assist the highway engineer in the evaluation of construction control procedures and in better visualizing the mechanism of segregation and other causes for lack of uniformity in aggregate gradation test results.

The segregation index, on the other hand, does have some theoretical basis for significance. The ratio of overall variance to within-batch variance is commonly used as a means of classifying the type or nature of the variability. It will be noted that the numerical value of S is small when either the overall variance, σ^2_o, is small or the within-batch variance, σ^2_b, is relatively large. Because σ^2_o is the sum of the within-batch variance and the variance caused by batch-to-batch segregation ($\sigma^2_o = \sigma^2_b + \sigma^2_l$), a high value of S means that there is a real or significant difference from place to place in the LOT. In many cases in actual road construction, this is the more important consideration because within-batch variability is partially negated by subsequent mixing.

TABLE 8

RELATIVE EFFECT ON GRADATION TOLERANCES OF PERCENT (P) PASSING SIEVE

PERCENT PASSING SIEVE	$2\sigma_o$ LIMITS (%)
5	±2
7	±3
10	±4
12	±5
15	±6
20	±7
25	±8
30	±9
50	±10

A look at degree of overall variability, degree of segregation, and segregation index together will often reveal the nature or pattern of variability. For instance, if DOV is large and S is small, there is a good bit of local segregation or difference between adjacent test portions, but relatively little difference from location to location within the LOT. This situation exists when aggregate is dredged with a clamshell and placed, a bucket at a time, onto a barge. Adjacent buckets may be quite different (one coarse and one fine), but the same pattern is repeated in about the same manner throughout the barge load. If adjacent test portions happen to be taken at the boundary between bucketloads, the overall variance, σ^2_o, the degree of segregation, D of S, and the within-batch variance, σ^2_b, are relatively large but the segregation index, S, is relatively small. In other words, local segregation (adjacent areas) may be quite large, but essentially the same condition exists at different locations on the barge. In the case of a coned stockpile, however, the pattern of variability is quite different. Here, adjacent test portions may be similar (local segregation very small), but the stockpile badly segregated from top to bottom; so, overall variance, σ^2_o, batch-to-batch variance, σ^2_l, degree of segregation, D of S, and segregation index, S, are all relatively large, with only the within-batch variance, σ^2_b, relatively small.

Particular caution must be used when comparing segregation index, S, values. It must be realized that, whereas this is a valuable ratio for some purposes, it is virtually independent of the overall variability at the moderate or low segregation levels. A high segregation index values does not necessarily mean a high degree of segregation; it merely means a low within-batch variance relative to the overall variance. By the same token, a relatively low degree of segregation, D of S, does not necessarily imply a superior construction control procedure; the within-batch variance may be high, resulting in sampling and testing difficulties. Finally, even the degree of overall variability, DOV, values must be interpreted in light of engineering judgment and prior experience or knowledge of the real needs for uniformity to economically duplicate known good construction and avoid known poor construction.

Thus, each of these parameters must be used to measure or to compare that property, or behavior pattern, or characteristic, or control procedure, for which it was designed. This is no more difficult than learning to properly use any other tools. Each contributes to a better understanding of some segment or aspect of the problem, but they must be used in combination, one with the other, to bring the overall picture into proper focus.

A NOTE TO THE PURIST

The purist may, with some justification, question the applicability of the binomial theorem concept to the dispersion of different-sized particles in an aggregate mixture. Whether or not further work will validate the binomial approach or show some other relationships to better fit the facts remains to be seen. At this stage, however, this experimental validation of the Manning formula for inherent variance would seem to lend significant support. Further, the $P(100 - P)$ factor seems to reflect observed behavior patterns with remarkable consistency.

CHAPTER THREE

EXPERIMENTAL WORK AND OBSERVATIONS

In the previous chapter, the statistical tools applicable to this study were described. This chapter presents the manner in which these tools were put to work in various field and laboratory experiments to accomplish the objectives of this initial phase of the study project.

GENERAL

Some of the experimental work was directed toward exploring or proving the suitability of certain statistical formulas, models, or hypotheses to research on the evaluation of aggregate construction control procedures. Other studies were directed toward variations inherent to measurement methods and testing and sampling.

The main field investigation, however, was oriented to measurement of the variability associated with the steps commonly encountered in the processing and handling of typical highway construction aggregates. Ideally, all materials should be sampled, and tested for variability, at the point where they are incorporated into the final product or construction. Because this is seldom practical, aggregates are often accepted or rejected at some intermediate processing or storage point. The primary objective of the main field investigation, therefore, was to explore the magnitude and, if possible, the nature of the variability of aggregates under routine process control at various customary sampling points from the crusher through to the final weigh hopper or point of end use. It should be noted that the primary objective was to gain some insight into the magnitude and nature of the variability associated with routine process control at customary sampling points, not to investigate the relative efficacy or merits of any particular transportation or handling technique. Nevertheless, a secondary benefit to be derived from this main field investigation is some supporting data on the relative degree of segregation associated with the stockpiling and handling procedures that happened to be in use in the eight commercial plants investigated.

Lastly, some of the experimental work was directed toward the evaluation of specific aggregate sampling tools. This work was largely qualitative in nature and the results are reported in the form of photographs and observations of the research engineer. Some work of a quantitative nature was accomplished; but, in general, comparative in-depth evaluation of different aggregate sampling tools was not considered to be within the scope of the initial study.

In addition to this general description of the experimental work, this chapter includes a summary list of the various experimental projects undertaken and the location within the report wherein the details of that particular study can be found. A discussion of sampling procedures applicable to all experiments is also presented.

SUMMARY LIST OF EXPERIMENTAL STUDIES

To enhance readability and maintain continuity, most of the various experiments conducted under this project are presented in other parts of the report or as appendices, depending on the nature and importance of the work. The purpose of this section is to list these experiments and to provide a reference guide to the detailed procedures and findings, as follows:

1. The main field investigation consisted of taking a relatively large number of test portions of aggregate at various sampling points from each of eight plants located in six states. The tests made at each plant are designated by a series number, and for each series there were three to four points of sampling. Normally, 100 test portions were taken at each sampling point, for a total of about 2,500 gradation tests made in the main field investigation.

In Chapter Four, a detailed description of process flow, the location of each sampling point, and the test results associated with each series are presented. Interpretation of the data and discussion of findings is covered in Chapter Five.

2. A rather sophisticated laboratory experiment was conducted to develop actual gradation test data for comparison with the theoretical binomial mathematical approach to computing inherent variance. The objective was to establish whether or not the Manning formula is valid and applicable for aggregate control considerations. The investigation included two types of aggregate and the mixing and testing of 500 test portions, as well as counting and weighing of several thousand aggregate particles to determine average size-weight relationships.

The details of this experiment are described in Appendix B. A discussion of inherent variance and its significance with respect to aggregate gradation sampling and testing, together with the Manning equation, is presented in Chapter Two. Data summaries are given in Tables 2 and 3. Finally, application of the Manning equation as a means for estimating limiting minimum variance is discussed in Chapter Two. Based on these considerations, the nomograph (Fig. 7) is developed as a rational approach to estimating the minimum total sample weight required to attain a given degree of accuracy for gradation testing.

3. To determine the repeatability of the gradation test method under practical routine operating conditions, tests were repeated on 312 test portions representing each type and gradation of aggregate investigated. These data are summarized in Table 4. Testing error is discussed in Chapter Two, and precision statements, generally, are discussed in Chapter Seven.

4. Several special sampling tools were developed for specific applications. The use of these tools is pictured in this chapter, with captions giving some qualitative observations as to their relative efficacy.

5. Results of hand sampling a coarse aggregate stockpile were compared with those obtained by belt sampling the same material. Results of the 84 tests made are summarized in Figure 21.

6. The results of hand sampling a stockpile of coarse aggregate, using a standard tool, were compared with results obtained with a specially designed tool. Fifty test portions were taken with each tool. The results are summarized in Table 11.

7. Results of hand sampling a coarse aggregate stockpile at four different levels on the pile (at the base, one-third and two-thirds up the pile, and at the top) are discussed later in this chapter and summarized in Table 10.

8. The possibility of increasing the efficiency of gradation testing by the use of a continuous sieving device was investigated. However, preliminary tests showed that this device was unsuitable for the intended purpose.

9. The complete raw data for all gradation tests performed under this project have been compiled; a limited number of copies are available on request to the research agency.

In addition, advantage has been taken of experimental data derived from the literature and from other research projects. For completeness, those germane to this summary list include:

10. Development of the so-called Hudson $\bar{A}$ as a gradation parameter, presented in Chapter Two, and the relationship of $\bar{A}$ to asphalt demand, as shown in Figure 9.

11. For comparative purposes, summary data have been taken from NCHRP Report 5, entitled, "The Effects of Different Methods of Stockpiling Aggregates—Interim Report."

SAMPLING AND TESTING CONSIDERATIONS

One of the most difficult and confusing problems facing the engineer-inspector is the selection of an unbiased sample from the LOT of aggregate in question. This sample or group of test portions is often used to check for compliance against gradation specifications or for design of bituminous, portland cement concrete, or other types of mixes. A countless amount of time and effort has been wasted in working with samples that were improperly secured and, consequently, resulted in conclusions that were erroneous.

It is not uncommon to start a project and quickly find that the aggregate being supplied does not conform to the gradation on which the design was based. At times, it is not possible to make even minor field adjustments without wasting undersized or oversized material. At other times, costly delays are involved while a new mix design is being developed based on a different gradation.

Even though highway and materials engineers have recognized these problems since the introduction of scientific concepts to roadway construction, no satisfactory solution has been provided to eliminate the difficulties. Too often, the inspector is given a shovel and instructed to get one or two bags of aggregate "representative" of the material in a stockpile containing 5,000 to 50,000 tons of aggregate (Fig. 12). It is only through sheer chance if he is able to obtain a sample that even approximates the aggregate sizes that would be obtained by putting the entire stockpile through a series of sieves. He will usually select a few (one, two, or three) locations that "look about average," insert the shovel a few times, and fill the bag or container to obtain a sample for testing. He may even select one portion of finer material and blend it with a second portion of coarser material such that the final sample is his estimate of what the gradation should be, based on visual inspection. It should be realized that, in many cases, there is no choice other than a hand-sampling technique. If the aggregate is already stockpiled at the time of sampling and if no mechanized equipment is available for moving the material, the inspector must use his best judgment as to the method of removing test portions to get a "representative" sample.

By the use of statistical procedures, the Engineer can determine a sampling pattern or system that reduces the human element or bias to a minimum. Such a system is known as a random sampling plan. A plan of this type makes use of a table of random numbers to locate the sampling points with respect to time or space. This table of numbers is generated by any one of several electronic or mechanical means which provide that every number has an equal chance of appearing. The specific application of a table of this type depends on the nature of the material

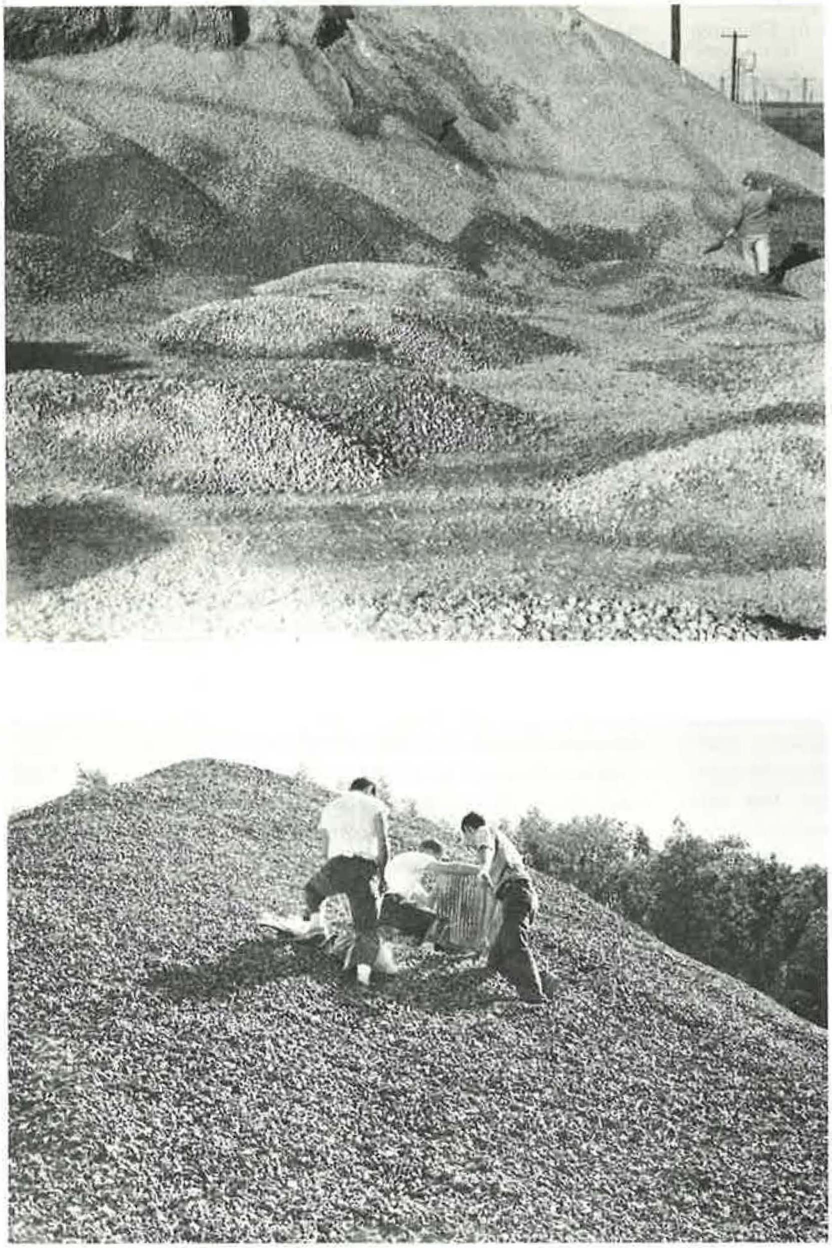


Figure 12. Some typical problems facing an inspector charged with the duty of taking a meaningful sample from a mountainous stockpile.

to be sampled; i.e., whether or not a given time element is to be the sampling control or whether the sampling control is to be expressed in units of area, volume, length, batch, or load number.

For example, if 50 test portions are to be taken from the production of an aggregate plant during a 10-hr day, it is only necessary to select 50 numbers from a table of random numbers and consider each number as a decimal. Each number is multiplied by 600 min to obtain a time in minutes from start-up. One test portion is taken from the plant output at each of these times, regardless of the apparent quality of the aggregate being produced at that moment.

A similar scheme is used when a definite number of batches of aggregate are to be weighed out during the course of the day. By taking an appropriate quantity of decimal random numbers from a table and multiplying each by the number of batches, the sequential number of each batch to be sampled can be found.

When random sampling locations are to be found on a surface, such as a barge load of aggregate, or a section of stabilized aggregate base, the coordinates of each sampling point are found by the use of two random numbers considered to be decimals. One of the pair is multiplied by the length and the other by the width of the area. One corner of the area is taken as a reference point and the exact location of each test portion is found by measurement from this point.

These sampling procedures eliminate the bias inherent in other methods and are essential if statistical methods are to be used to draw inferences from the results of tests on the sample. In connection with the work on this project, PROBABILITY SAMPLING was employed in all cases. The total amount of aggregate represented by the samples was chosen so as to be a rational LOT, such as a day's production. The location, or spacing, of test portions within the LOT was determined by the use of a table of random numbers and all test portions, so selected, were taken and tested regardless of appearance or any other judgment factor. In some cases, special sampling tools were devised and used to minimize mechanical bias during the act of acquiring the increments or test portions for the sample. In order to obtain data that would make it possible to distinguish between the less important (within-batch) segregation and the more significant (within-lot) segregation, duplicate increments were taken from each sampled batch or segment at sufficient distance apart to insure against correlation, or similarity because of continuity.

The main field investigation was designed so that some of the sampling procedures were common to each location. For example, belt sampling was accomplished at some point in seven out of the eight series. Two methods of belt sampling were used. One was by stopping the belt at a predetermined time, defining a specific area with metal templates shaped to fit the belt, and removing all aggregate between the templates (Fig. 13). The other belt sampling method was to cut the stream of aggregate flowing from the end of the belt with a suitable metal container (Fig. 14). Stockpiles and barges were sampled with a shovel from a cleared area behind a batter board at points randomly selected by a grid pattern (Figs. 15 and 16). In one case (Series No. 6), one face of a large slag stockpile was worked with a front-end loader, randomly selected front-end loader batches were dumped on the ground, and duplicate test portions were taken with a shovel from different parts of the batch. Flowing streams of aggregate from overhead storage bins, etc., were cut with different types of metal containers or high-sided sampling tools (Figs. 17 and 18). Special tools were also used for truck sampling (Fig. 19). In this case, it was the truck that was randomly selected, rather than the position or point of sampling within the truck. Test portions were obtained from the stabilized aggregate base using a sampling hoop (Fig. 20), at locations selected by the use of random numbers applied to a grid pattern.

In general, samples consisted of 50 groups, each made up of two replicated test portions taken at random locations in accordance with the research sampling plan (Fig. 1). However, except as otherwise noted, the technicians doing the sampling were purposely allowed to use the procedures and techniques with which they were accustomed. In other words, except for the special studies, it was desired that the variations measured include the sampling errors or variabilities associated with normal, routine construction control procedures.

For practical purposes, it was decided that the test portion was to be taken in one increment and the entire increment put through the sieves so that increment and test portion were one and the same. Most test portions were in the range of 20 to 30 lb in weight, and the entire portion was sieved.

At six of the plants in the main field investigation, a Gilson testing machine was used for sieving the aggregate to refusal, with screening time varying from 5 to 7 min, depending on the proportionate amount of minus No. 8 material in the test portion. A Weston rotary sieving device was used at the other two plants.

EXPERIMENTS IN THE HAND SAMPLING OF STOCKPILES

As previously discussed, there are construction control problems in the sampling of aggregate stockpiles. Hand sampling is often necessary, because it is not usually feasible to run the material across a sampling belt. It is therefore desirable to better define and, if possible, to quantify the nature and the magnitude of at least some of the stockpile sampling problems. This section presents the results of experiments and observations made during the course of this study which are related to this stockpile sampling problem.

Belt Sampling vs Hand Sampling

It was deemed advisable to explore first the order of magnitude of the difference in two sampling methods—belt versus hand. Armchair consideration of this question leads to the fact that the difference will be a function of the amount of segregation in the stockpile. Obviously, if there is no segregation all test portions will have essentially the same gradation, regardless of the sampling method. At the other extreme of complete segregation, the variability will depend on the method of reclaiming to place the material on the belt, and on the method and location of taking the hand sample. To some degree this will also be true, of course, at the intermediate stages of segregation encountered in real life. From a practical viewpoint, the problem reduces to defining the magnitude of the differences between hand and belt sampling, using normal average control techniques at the normal levels of segregation which may be encountered in actual practice.

One experiment was conducted at the Nello L. Teer Princeton Quarry, using the same 1-in. to No. 4 crushed stone aggregate that was used for the HR 10-3 studies. The stockpile sampled was the large parent stockpile, which, incidently, was quite uniform (very little segregation). Thirty-four test portions of aggregate were taken with a shovel from random locations on the surface of the stockpile and the gradations compared with those of 50 replicated samples from the same general area obtained by passing the material across a sampling belt.

In the case of shovel samples, the aggregate on the surface of the pile was removed for a depth of 10 to 12 in. A protective barrier was placed on the upper side of the sample area to prevent aggregate at a higher level from tumbling into the sample area. The shovel was inserted several times, such that a test increment of approximately 25 lb was obtained (Fig. 15). Belt samples were obtained in the manner pictured in Fig. 13; i.e., the belt was stopped, metal templates were inserted to define the sampling area, and all material between the plates was removed and tested for gradation.

The comparative results are presented in Figure 21. These charts show the gradation envelopes, wherein the

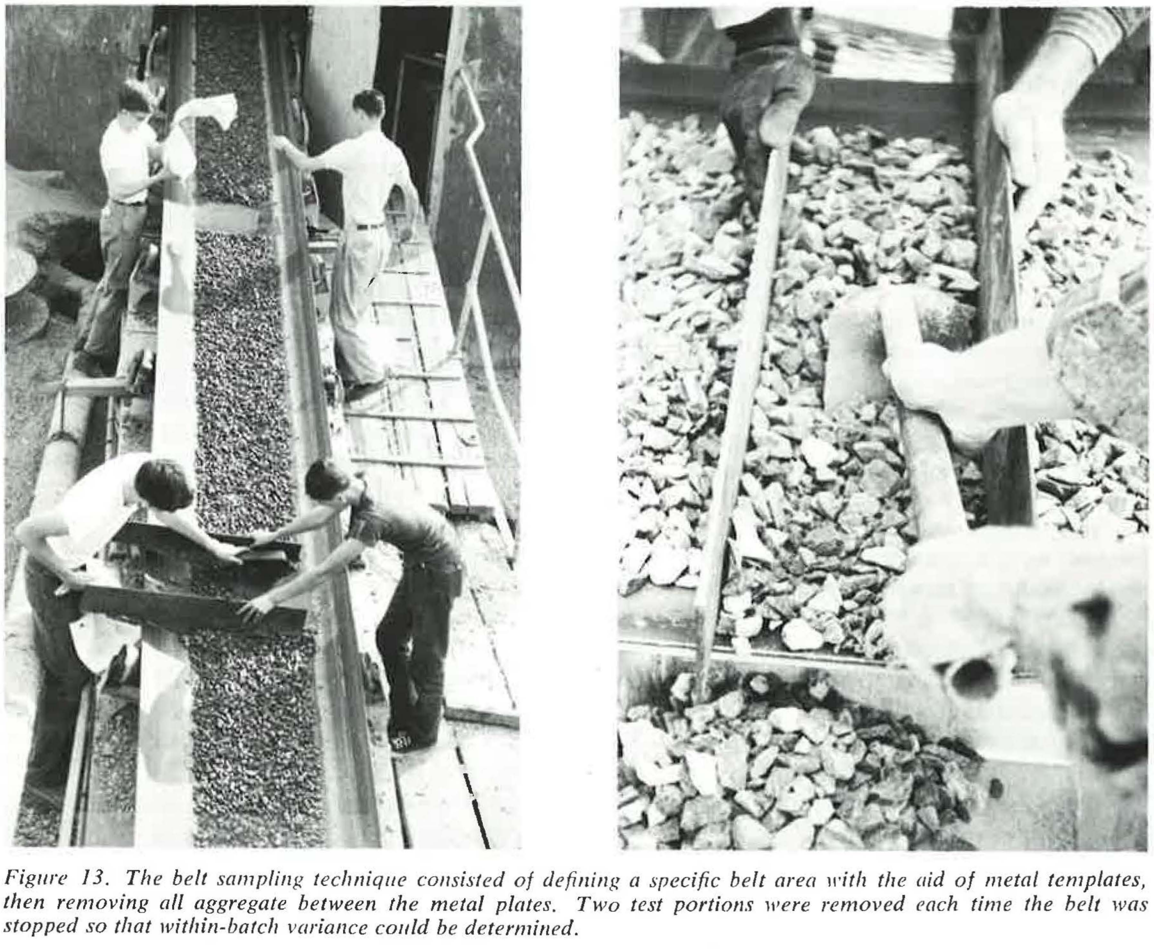


Figure 13. The belt sampling technique consisted of defining a specific belt area with the aid of metal templates, then removing all aggregate between the metal plates. Two test portions were removed each time the belt was stopped so that within-batch variance could be determined.

Figure 14. Belt from crusher to stockpile in Series No. 3. Test portions were obtained by passing a metal container through the stream of aggregate as in lower picture. This aggregate was subsequently hauled to an asphalt plant located about ¼ mile from the stockpile area.

width of the band is equal to $\pm 2\sigma_o$. The overall standard deviation, σ_o, for the belt method is about one-half of that obtained using the shovel sampling method for each sieve size and for Hudson $\bar{A}$. For instance, σ_o for percent passing the ¾-in. sieve was 3.3 for belt sampling and 6.3 for hand sampling. On the ⅜-in. sieve, the belt method looked even better; 2.1 for the belt versus 6.7 for the shovel.

It should again be emphasized that these results were obtained on a stockpile that was only slightly segregated. The corresponding standard deviation for percent passing the ¾-in. sieve on a coned pile, when tested by the belt method, was $\sigma_o = 11$ versus 3.3 for this parent stockpile.

To show the relative difference between belt and hand sampling for different aggregates and different degrees of segregation, the data in Table 9 are presented. The standard deviation at the 50 percent passing level, σ_{o50}, is shown for those series in the main investigation wherein aggregate en masse (either stockpiles or in big barges) was hand sampled and then subsequently belt sampled. In general, the same relationship seems to apply; namely, that the standard deviation for the belt method is roughly one-half that obtained by the hand (shovel) method.

It should be noted that this difference in standard deviation also reflects any mixing or segregation that might have resulted from the handling which occurred in transferring the aggregate from the stockpile or barge to the surge bin feeding the belt. It is not possible at this stage to distinguish between the change in standard deviation induced by handling and that due to the sampling method, although the latter is believed to be the dominant factor for these particular experiments.

In any case, it is evident that the greater variability of the shovel samples taken from stockpiled aggregates is a fact of life that should be understood and properly taken into account in the design of construction control procedures and in the writing of realistic specifications. Large risks are involved in taking one or two shovel samples from a stockpile and using the results for mix design and for acceptance or rejection decisions. For instance, if a nominal value of 10.5 is taken for the standard deviation associated with hand sampling, the futility of attempting to obtain an accurate estimate of gradation is illustrated by the fact that about 425 test increments of about 25 lb each would be required for an accuracy of ±1 percent at the 95 percent confidence level. In the usual case where only about five test increments are taken, the best that could be hoped for would be an accuracy of about ±13 percent for the amount passing the ⅜-in. sieve for the stockpiles hand sampled in this study. As discussed in Chapter Two, the statistical approach to construction control provides the Engineer with a numerical measure of these risks, whether they be large or small, so that he can use his judgment to better advantage in balancing the cost of further testing or increased control against the degree of accuracy and the confidence level warranted for the particular material or procedure under test.

TABLE 9

HAND (SHOVEL) VERSUS BELT SAMPLING OF AGGREGATE STOCKPILES

AGGREGATE			STD. DEV. AT 50% PASSING LEVEL, $\sigma_{0.50}$	
SERIES NO.	SIZE	TYPE	SHOVEL SAMPLING	BELT SAMPLING
–	1 in.-No. 4	Crushed stone	10.5	4.7
1	1 in.-No. 4	Crushed stone	13.6	8.3
2	¾ in.-No. 4	Crushed gravel	7.8	4.3
7	1 in.-No. 4	Rounded gravel	15.0	4.1
8	1 in.-No. 4	Rounded gravel	18.0	7.4

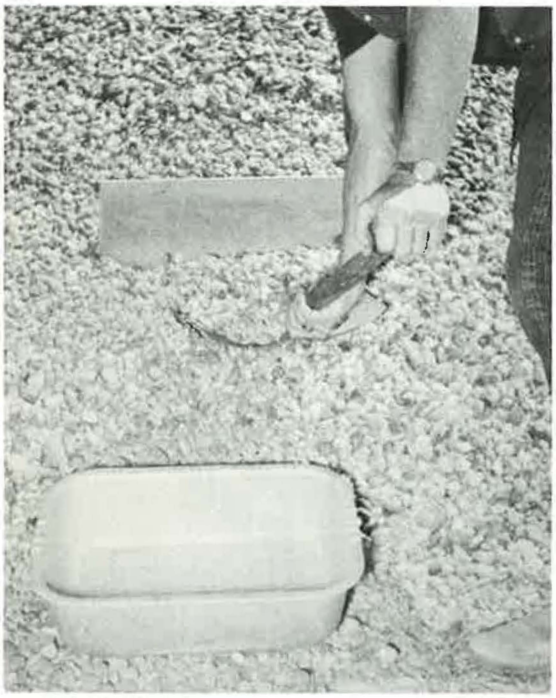


Figure 15. All stockpile samples were obtained with some type of shovel, using a technique similar to the method shown. In each case surface aggregate was cleared away and a barrier was used to prevent aggregate at a higher level from tumbling into the area to be sampled. The shovel was inserted a sufficient number of times to obtain test portions weighing 20 to 30 lb.

Segregation versus Sampling Level on a Stockpile

The mechanism of the segregation that occurs in a coned stockpile has been discussed qualitatively and it is generally known that the coarse aggregate tends to accumulate at the bottom of the pile. To better judge the hand sampling problem, however, it would be desirable to know the order of magnitude of this segregation under the most serious conditions likely to be encountered in the field. If the gradation obtained is a function of just how high up the pile the inspector climbs to take his sample, it is logical to question how high he should go and how much difference it does make.

To get a reading on this, hand (shovel) samples were taken from four different levels of a segregated coned stockpile made with a 1½-in. to ⅜-in. crushed stone aggregate. This is part of Series No. 3. Hand samples were taken at the base of the pile, about one-third up the pile, about two-thirds up the pile, and some close to the top. The average gradation, by percent passing ¾-in. sieve and Hudson $\bar{A}$, and the corresponding range at each location, is presented in Table 10.

This is admittedly a severe—probably extreme—condition, but it illustrates the potential magnitude of the problem. Less severe segregation would, of course, be reflected in less difference from top to bottom of the pile. Whether the same relative differences in gradation as a function of sampling location in the stockpile would prevail at varying degrees of segregation is unknown at this time.

In this instance, both the average percent passing the ¾-in. sieve and the average $\bar{A}$ values approximate a linear function of height of sampling location on the pile. The gradation of this 1½- to ⅜-in. aggregate, as produced at

Figure 16. In Series Nos. 7 and 8, the gravel aggregate was transported to the ready-mix concrete plant site in barges of 900- to 1,000-ton capacity. A grid pattern was projected on the surface of the aggregate pile and through a system of random numbers specific sampling areas were located. One hundred test increments were removed from each barge.

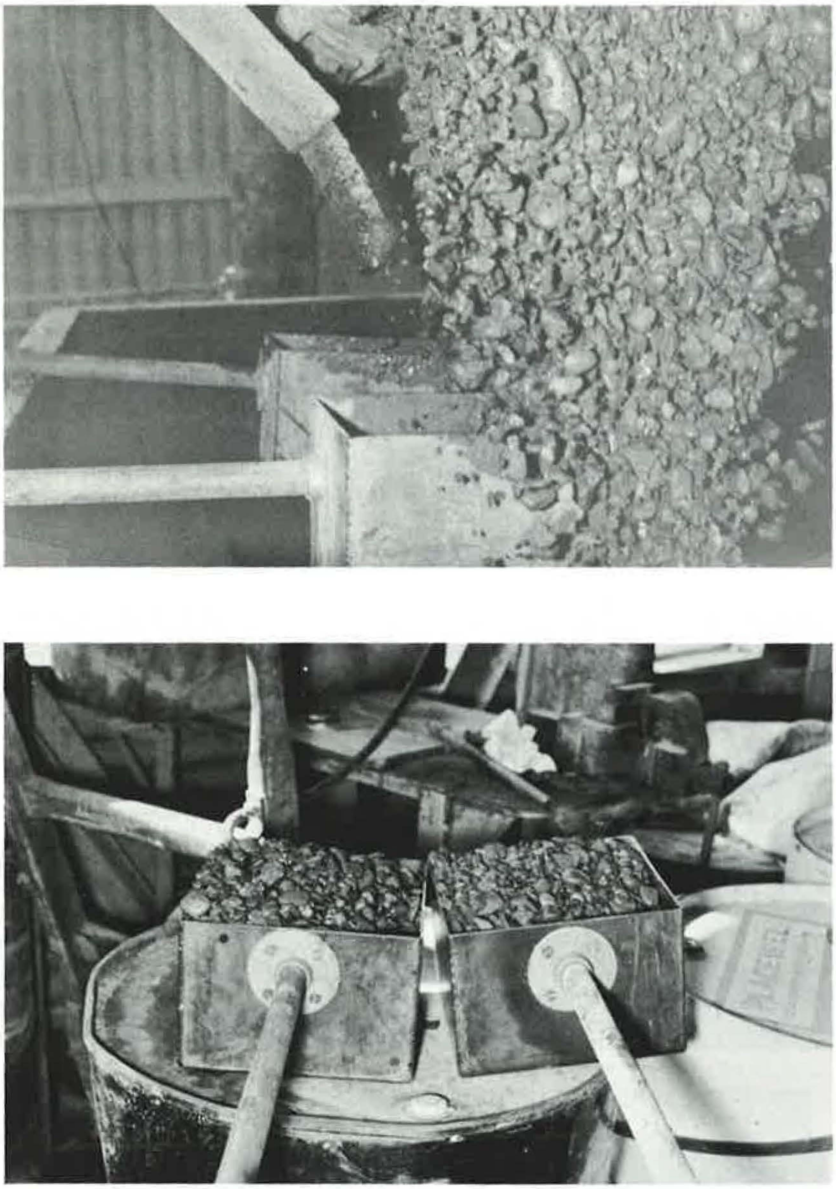


Figure 17. These sampling devices, used to cut the aggregate stream discharged into the weigh hopper in Series Nos. 7 and 8, are typical of those used to sample flowing streams of aggregate.

the crusher, averaged 37.6 percent passing the ¾-in. sieve and an average $\bar{A}$ of 1.362. Assuming a linear relationship, these numbers correspond to 50 percent up the pile to attain the passing ¾-in. average, and 51 percent up the pile to attain the Hudson $\bar{A}$ average. In other words, these very limited data indicate that the inspector would have a better chance of selecting an "average" sample, if he took material from a little over half-way up the pile, say between 50 and 60 percent up the side. However, there are a number of limitations to be applied to the implications of this observation in light of present knowledge.

First, this Research Agency definitely does *not* wish to imply that an inspector can, in fact, obtain an average sample from a segregated stockpile of aggregate by merely climbing a bit over 50 percent up the side of the pile and digging in. As previously stated, there are large risks involved in making any inference or decision based on a few shovel samples taken from any place in a segregated stockpile. The work of HR 10-3, HR 10-3(1), and the data and observations of this project clearly support this position and definitely show that a relatively large number of randomly selected test portions must be analyzed before any reliable inferences regarding either the average gradation or the variability can be drawn. Nevertheless, on many highway construction projects, inspectors are required to do the best they can in obtaining a "representative" sample from aggregate stockpiles. Therefore, any guidance that can be

TABLE 10

SEGREGATION VERSUS STOCKPILE SAMPLING LEVEL

SAMPLING LEVEL	NO. OF SAMPLES	% PASSING ¾-IN. SIEVE		HUDSON $\bar{A}$	
		AVERAGE	RANGE	AVERAGE	RANGE
Top	8	51.5	31.8-74.0	1.66	1.39-1.94
⅔ Up [a]	13	40.8	14.1-63.8	1.47	1.07-1.86
⅓ Up [a]	14	25.7	2.1-56.3	1.24	0.77-1.70
Base	15	9.3	1.8-23.4	1.01	0.81-1.31

[a] About one-third and two-thirds of the distance up the side of the stockpile.

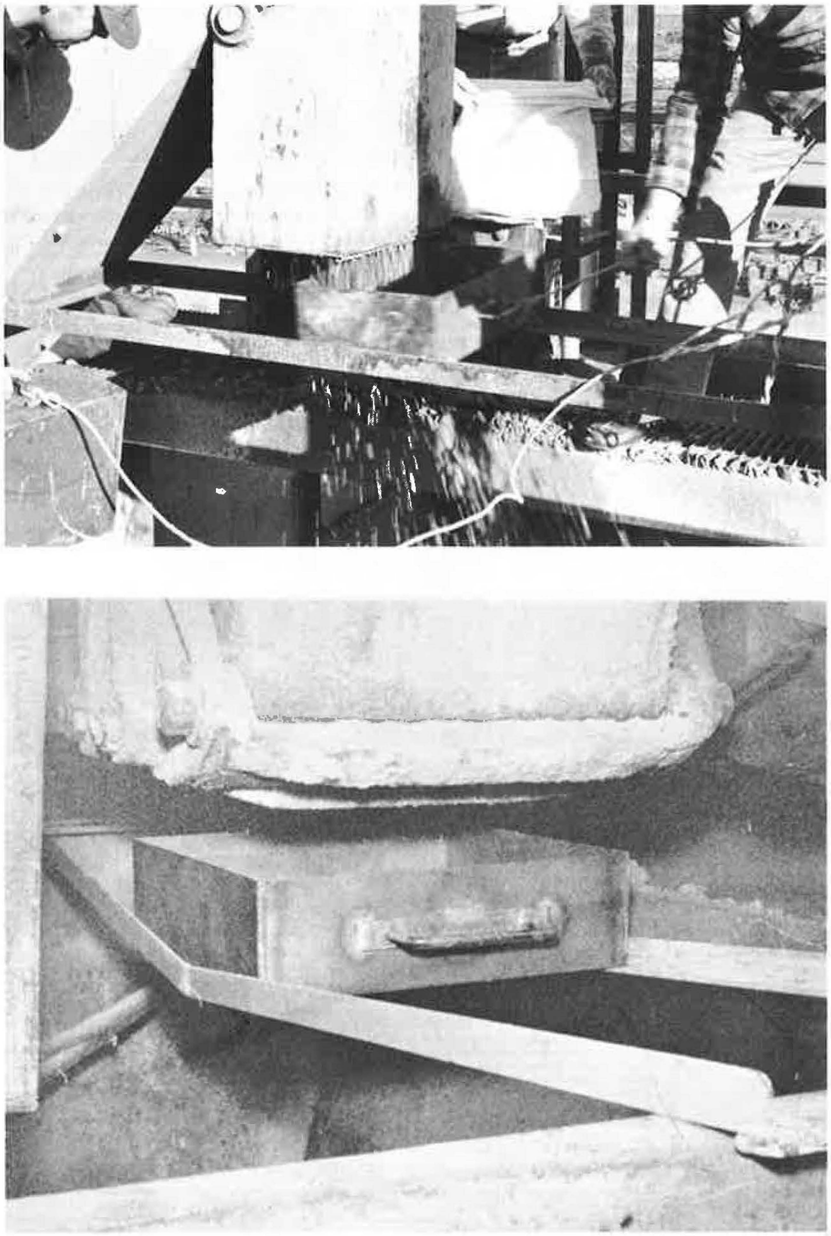


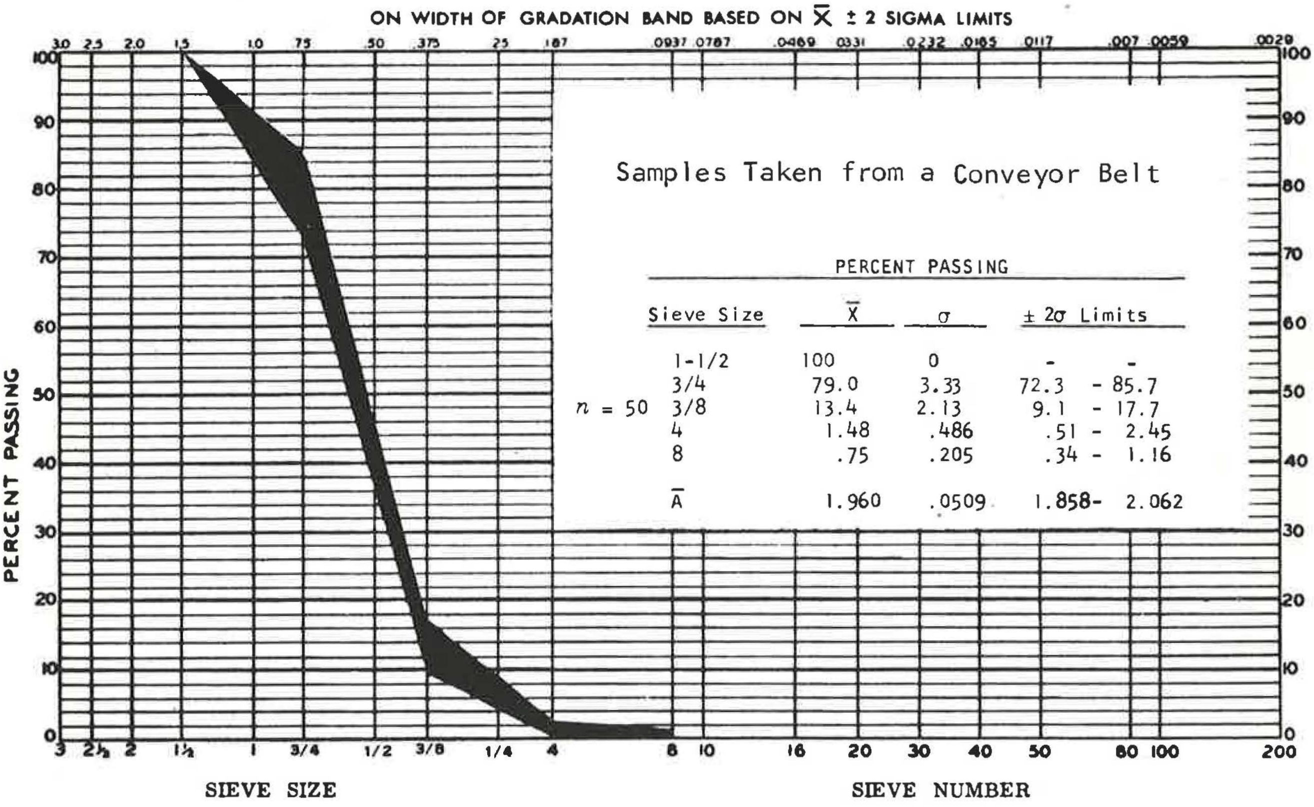
Figure 18. In sampling the vertical discharge chute at the head of the conveyor belt feeding the 50-ton overhead storage bins in Series No. 6 (upper photo), a track built of angle iron was used to support the sampling pan as it was pulled into and out of the aggregate stream with a rope. Lower photo shows a similar device used for sampling at the weigh hopper.

Figure 19. An aggregate used for construction of a stabilized aggregate base course was investigated in Series No. 5. The truck samples (after pugmill mixing) were secured with a sample thief. This device provided increment weights of approximately 15 lb each.

Figure 20. The roadway was sampled after the aggregate had been placed with a spreader and before compaction. A 10-in. diameter hoop defined the sampling areas and all material within the hoop was removed for testing.

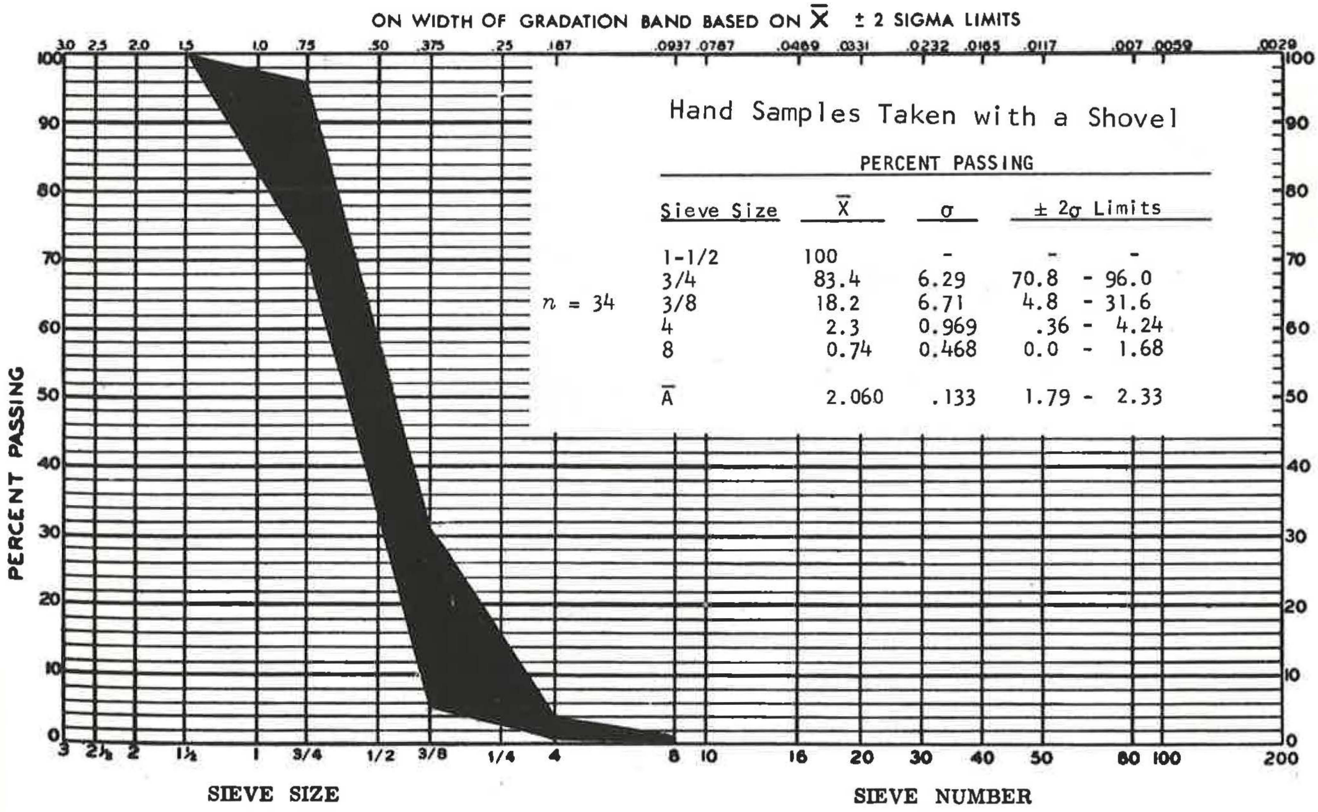
inferred from these limited observations is pertinent and potentially valuable under current construction control practices. However, the following limitations should be noted:

1. Although the average percent passing the ¾-in. sieve and the average Hudson $\bar{A}$ values presented in Table 10 line up pretty well, the ranges of the individual gradation test results vary widely. For instance, at the one-third up level, individual test results varied from 2.1 to 56.3; and from 14.1 to 63.8 on 25-lb samples taken at the two-thirds up level. Thus, the chance of an inspector getting an "average" sample, even at the same level up the pile, is very poor. A number of test increments must be taken and the results averaged in order to have any reasonable confidence



Samples Taken from a Conveyor Belt

	PERCENT PASSING			
	Sieve Size	$\bar{X}$	σ	± 2σ Limits
	1-1/2	100	0	- -
	3/4	79.0	3.33	72.3 - 85.7
n = 50	3/8	13.4	2.13	9.1 - 17.7
	4	1.48	.486	.51 - 2.45
	8	.75	.205	.34 - 1.16
	$\bar{A}$	1.960	.0509	1.858- 2.062



Hand Samples Taken with a Shovel

	PERCENT PASSING			
	Sieve Size	$\bar{X}$	σ	± 2σ Limits
	1-1/2	100	-	- -
	3/4	83.4	6.29	70.8 - 96.0
n = 34	3/8	18.2	6.71	4.8 - 31.6
	4	2.3	0.969	.36 - 4.24
	8	0.74	0.468	0.0 - 1.68
	$\bar{A}$	2.060	.133	1.79 - 2.33

Figure 21. Comparison of variation of gradation, belt sampling vs hand sampling.

in the gradation, even though all sampling is done at the same level on the pile.

2. This experiment was conducted with only one type and one size of aggregate, and on only one stockpile. Much additional work is necessary before valid generalizations can be made.

3. Even this limited experiment involved only a relatively few test portions and gradation results. As shown, total n was 50, with only 8 of these taken from the top.

In retrospect, it would have been desirable to obtain more data of this nature as part of this study. However, it was not part of the approved working plan and even this limited information was obtained, more or less, on an aside or "extra" basis. Additional experiments to better define segregation versus sampling level on a stockpile are recommended as part of the continuation studies on the project. It is a bit ironical to recommend additional research on a practice which is known to be inherently poor and unreliable at best. Nevertheless, as long as inspectors must sample stockpiles, they should be provided with the best and most complete guidance available that may improve their chances of obtaining a sample that more nearly represents the true characteristics of the LOT of aggregate.

EXPERIMENTS WITH DIFFERENT SAMPLING TOOLS

As previously noted, various sampling tools were tried in the course of this study. Most evaluations were made on a qualitative basis, but some quantitative comparisons are reported. The comparison of belt sampling versus hand sampling of stockpiles with a shovel was presented in the foregoing section.

Regular versus High-Sided Shovel Design

A special study was conducted to compare the performance of the two shovels shown in Figure 22. The narrow shovel is the standard sampling tool used by a State highway department. The other shovel, with built-up sides, was specially made for this investigation.

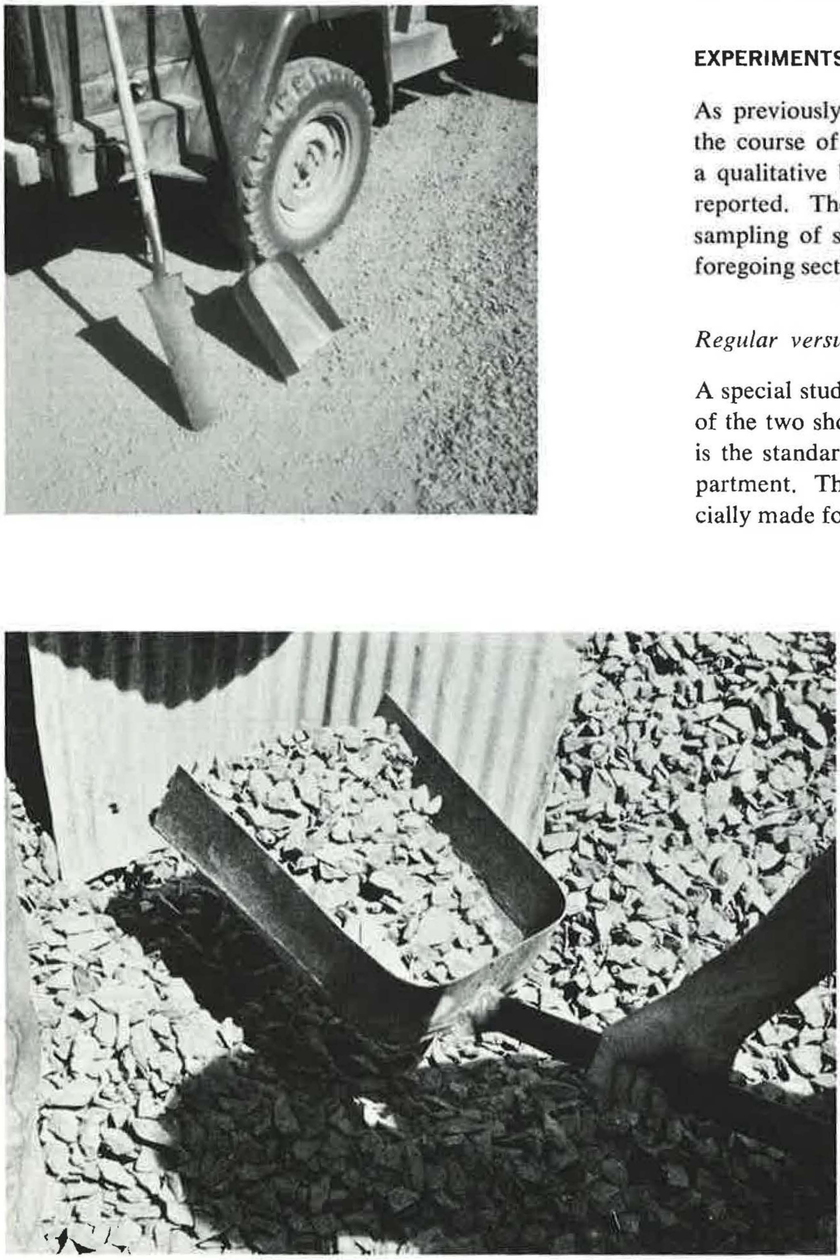


Figure 22. The modified shovel (lower photo) is generally recommended for hand sampling, particularly in the case of aggregates with a wide range of particle sizes. However, the narrow shovel (upper photo), a standard sampling tool used by some State highway departments, yielded identical results in the experimental sampling of the aggregate shown.

It was suspected that the narrow shovel would cause a portion of the larger aggregate particles to be lost from the shovel, resulting in a finer gradation—a difficulty that the wider and higher-sided shovel should alleviate. It was anticipated that this could be demonstrated and the magnitude of the larger particle loss measured. A total of 50 test increments was secured from a crushed stone stockpile with each tool. Increment weights averaged 25 to 30 lb and the entire portion was passed through the sieves.

Table 11 gives the values from a statistical analysis of the gradation test results.

This experiment failed to disclose any significant difference between samples of this aggregate secured with the two tools. In fact, the results are remarkably close to each other, both with respect to average percent passing, $\overline{X}$, and the standard deviation, σ, on all four sieves. The results checked more closely than might normally be expected if the 50 successive test increments had been taken with the same sampling tool, rather than attempting to show the difference between the two shovels.

No explanation is offered for the failure to measure a difference between these shovel designs. It is suspected, however, that this would not be the case with all aggregates. It is still believed that the logic of the built-up sides is sound and it is desired to repeat this experiment with other aggregates, particularly in light of results obtained with the truck sampling thieves discussed in the following.

Truck Sampling Results

One of the sampling points selected in two of the series of the main field investigation was the loaded truck. This is, of course, a common and reasonably convenient sampling point, but the results (Table 12) indicate that this may not be good practice from the construction control viewpoint. In each case, the average gradation, $\overline{X}$, was significantly finer than that obtained on samples of the same material taken at a sampling point before the trucks were loaded and again after they were discharged.

In Series No. 2, the aggregate is a ¾-in. to No. 4 crushed gravel used for portland cement concrete production in Colorado. Sampling point No. 1 is the belt conveyor taking the aggregate directly from the crusher discharge. The aggregate was then loaded by conveyor into trucks (sampling point No. 2) and moved to a large storage pile near the crusher site (sampling point No. 3). One hundred test increments weighing 25 to 30 lb each were taken at each sampling point and the full increment or test portion was put through the sieves. It will be noted that the

TABLE 11

COMPARISON OF SAMPLING RESULTS WITH TWO SHOVELS

SHOVEL TYPE	NO. OF TEST INCREM., n	¾-IN. SIEVE		⅜-IN. SIEVE		NO. 4 SIEVE		NO. 8 SIEVE	
		AVG. % PASS., $\overline{X}$	STD. DEV., σ	AVG. % PASS., $\overline{X}$	STD. DEV., σ	AVG. % PASS., $\overline{X}$	STD. DEV., σ	AVG. % PASS., $\overline{X}$	STD. DEV., σ
Std., narrow	50	63.05	15.54	3.15	3.31	0.76	0.63	0.60	0.59
Special[a]	50	63.64	15.57	2.78	3.92	0.75	0.72	0.56	0.57

[a] Wider shovel with built-up sides.

TABLE 12

COMPARISON OF TRUCK SAMPLING TEST RESULTS[a]

SAMPLING POINT	NO. OF TEST INCREM., n	½-IN. SIEVE				⅜-IN. SIEVE				NO. 4 SIEVE			
		$\overline{X}$	σ_o	σ_b	σ_t	$\overline{X}$	σ_o	σ_b	σ_t	$\overline{X}$	σ_o	σ_b	σ_t
(*a*) SERIES NO. 2													
No. 1, before	100	51.4	4.4	2.1	3.8	28.6	3.5	1.6	3.1	—	—	—	—
No. 2, truck	100	66.0	9.3	3.4	8.6	41.2	10.4	3.8	9.7	—	—	—	—
No. 3, after	100	56.4	7.2	5.9	4.2	33.1	7.0	5.5	4.3	—	—	—	—
(*b*) SERIES NO. 5													
No. 1, before	60	—	—	—	—	58.2	4.0	1.8	3.1	39.3	3.4	1.7	2.9
No. 2, truck	60	—	—	—	—	68.5	6.0	4.6	4.1	49.3	5.7	4.2	3.8
No. 3, after	60	—	—	—	—	54.7	6.6	3.7	5.4	36.3	5.7	3.1	4.8

[a] $\overline{X}$ = average percent passing; σ_o = overall standard deviation; σ_b = within-batch deviation; σ_t = batch-to-batch deviation.

average percent passing the ⅜-in. sieve, for instance, increased from 28.6 to 41.2, then dropped back to 33.1. The same pattern is shown by the average percent passing the ½-in. sieve.

Series No. 5 is a well-graded crushed aggregate (essentially 100 percent passing 1 in., down to about 6 percent minus 200) used for aggregate base course construction in North Carolina. It consists of 85 percent crusher run, to which 15 percent fines were added on the belt feeding a 2-ton pugmill used for mixing and for bringing the combined aggregate to optimum moisture content. Sampling point No. 1 was located on the belt feeding the pugmill, immediately after the fines had been added. After mixing, the aggregate was dropped into trucks (sampling point No. 2). The trucks fed a mechanical spreader, which put down an uncompacted course of about 8 in. After spreading, the aggregate was sampled by placing a 10-in. diameter hoop on the surface and removing all material within the area of the hoop (sampling point No. 3). Again, it is evident that the samples taken from the truck (sampling point No. 2) were significantly finer than those taken from the belt (sampling point No. 1) or, subsequently, from the roadway (sampling point No. 3).

Thus, in both cases there is evidence that sampling from the truck tended to give misleading results in that the average gradation was about 10 percent passing higher than results obtained on the same sieves, both before loading and after discharging from the truck. The reason for this is not readily apparent, although there are a number of possible causes, which are briefly as follows:

1. Segregation in the truck. It is well known that the coarse particles tend to segregate in the truck just as they do in a coned stockpile. In fact, the truckload is, in essence, a small coned pile, with the added handicap that the sides of the truck make it even more difficult to sample the bottom of the pile. Just how much segregation might have been enhanced by the method of loading is unknown, but it is probable that the belt conveyor method used to load the Series 2 aggregate resulted in some additional "throw" of the coarser particles.

2. The random sampling plan was applied in this case to truck selection, rather than to the location within the truck at which the sample was to be taken. It will be recalled that the object of this part of the research was to evaluate or compare various common sampling points using routine procedures normally employed in routine construction control. The technicians were therefore not guided as to what level or what position in the truck should be sampled, but were purposely allowed to use their customary procedure. It is probable, therefore, that one of the primary causes for the finer average gradation results obtained in the truck samples is that the sampling point was too high on the pile, and thus reflected the segregation discussed in item 1.

3. The sampling method used in Series 2 was essentially that shown in Figures 15 and 16; namely, shoveling from behind a barrier or batter board used to prevent aggregate at a higher level from tumbling into the sampling zone. This is the usual method used for sampling stockpiles, barges, etc., and there is no consistent evidence that it necessarily contributes one way or the other to sampling errors that would result in finer or coarser average gradation.

For Series No. 5, however, a sampling tool or thief (Fig. 19) was used. These sample thieves were easy to use and could be readily inserted into the aggregate mixture. They were too small (6-in. diameter by 8½-in. length), however, in that the test portion obtained was only 15 lb, which is less than is normally desired. Also, it is probable that when the edge of the thief came in contact with the larger particles, mechanical sampling error was induced by the act of pushing the tool into a mass of aggregate which was trapped or confined toward the inside as the tube filled, but with the particles freer to move away from the edge of the tool toward the outside or unconfined areas. This mechanical action could be observed during the qualitative experiments shown in Figure 23, in which an 8-in. diameter pipe 10 ft long was forced into a stockpile. This pipe method was also found to be unsatisfactory because it was too heavy and unwieldy, as well as because of the apparent tendency for the intruding end of the pipe to deflect the larger particles outward.

Regardless of the cause, or the combination of causes, these experiments clearly illustrate the potential dangers of sampling from a truck—a common practice, particularly in the sampling of hot-mix asphaltic concrete. Additional data are needed before these effects can be properly quantified, but the implications from the construction control viewpoint are obvious. Asphalt content, for instance, is directly related to gradation and obtaining a sample that is too fine could result in a satisfactory test on a mix that was really too lean. Another critical point is crushed aggregate base course, wherein control of the minus 200 material is important. Sampling too high on a stockpile or getting a fine test portion from a truck could result in unnecessary rejection of a borderline material.

Qualitative Results Using Other Sampling Tools

Although time and money did not permit full quantitative evaluation, some other types of sampling tools were designed and used on an experimental basis. These are described briefly in the following.

Mention has already been made of the pipe sampling tool that was tried as a thief method of obtaining samples from a stockpile. Figure 23 shows forcing of this 8-in. diameter by 10-ft long pipe into a stockpile. No practical method was apparent for handling this much weight with equipment available around highway construction. A special adapter for a fork lift truck or front-end loader could probably be devised, if warranted. However, based on limited data, but largely on observation, the method was found to be unsatisfactory anyway because of a tendency for the intruding end of the pipe to deflect the larger aggregate particles outward.

The 10-in. hoop shown in Figure 20 proved to be practical and satisfactory in every respect. It was used to take the roadway samples of crushed aggregate base course after spreading, but before compaction.

The bin sampling devices shown in Figure 17 provided

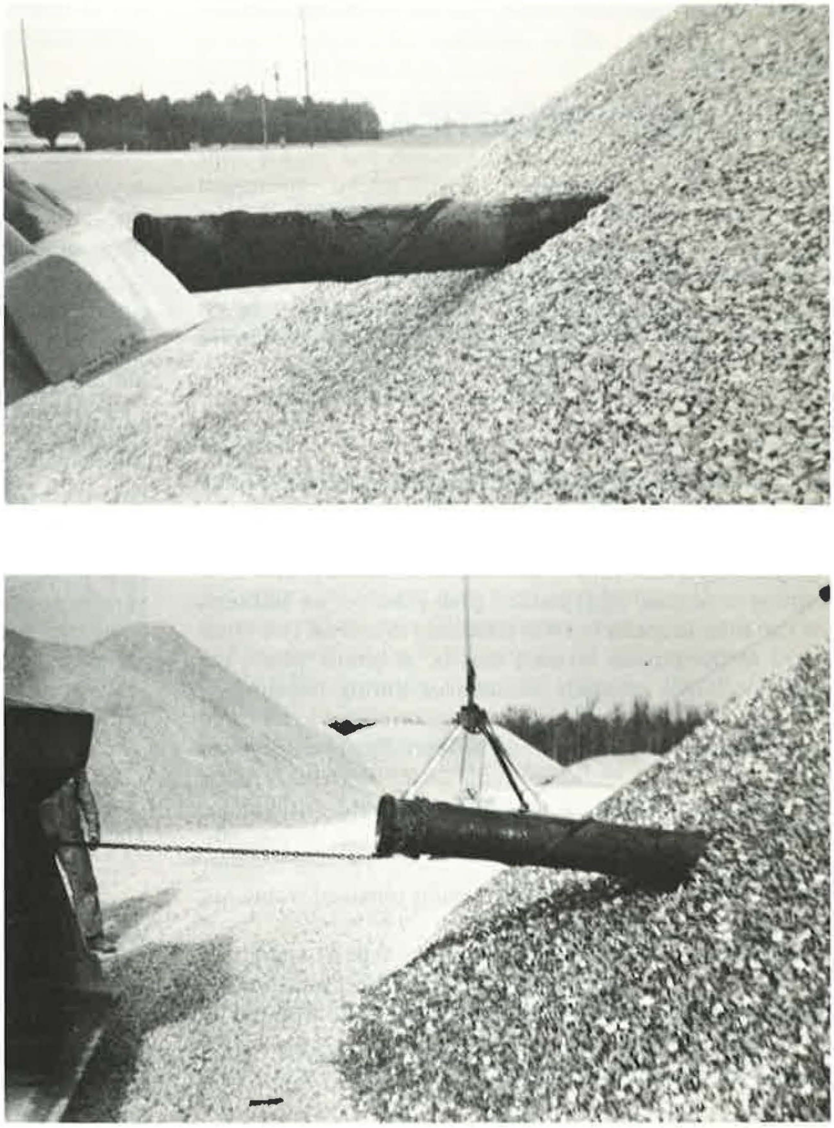


Figure 23. Special-type sampling devices used experimentally included this 8-in. diameter, 10-ft long pipe shown being forced into a stockpile. This method proved unsatisfactory because there was a tendency for the intruding end of the pipe to deflect the larger particles outward.

a safe and satisfactory method for sampling aggregates from a bin discharge. This was also true of the sliding sampling pan that could be pulled through an aggregate stream when supported and guided on a track built of angle iron (Fig. 18).

The belt sampling technique shown in Figure 13 is probably the best of all the sampling procedures used. It is easy and simple to obtain the recommended randomization by stopping the belt at preselected times. The technicians soon become proficient in rapidly inserting the templates, designed to fit the particular belt in use, and in scraping and brushing the full sample into a container. There is no personal judgment or bias involved and each segment of the LOT has an equal chance of being tested. This is the sampling technique and construction control procedure recommended whenever possible.

CHAPTER FOUR

MAIN FIELD INVESTIGATION

In this chapter each of the eight series in the main field investigation is described and the test data obtained are summarized. Although some brief discussion of the findings germane to each individual series is included, analysis of the test results as a whole and discussion of comparative trends or relationships is reserved for Chapter Five.

GENERAL

The general approach for obtaining the results reported herein was, in all cases, to measure the variation of gradation of aggregates at the point where they were actually being incorporated into a product or a construction conforming to normal (acceptable) good practice. In addition, the variation in gradation was measured in at least two other places in the process stream; that is, at points where the aggregate might normally be sampled during handling or storage. Also, regular routine procedures were followed throughout so that results would be representative of typical process variations found in commercial aggregate plants and in plants producing ready-mix concrete, bituminous mixtures, and aggregate bases. In each case, it was emphasized that none other than day-to-day routine controls were to be exercised in order that results obtained would not be affected by special refinements.

The general location of each plant, the type of aggregate, the point of sampling, and the number of test portions for each of the eight series are given in Table 13. Because of advance agreement, the specific company furnishing aggregate in each case is not identified.

CHARTS AND TABLES

The format for presentation of the data is the same for all series. Following the description of the process flow and identification of sampling points, the results of the analysis of variance (Chapter Two) are given in each case as outlined in the following.

Variation of Gradation at Point of Use

Plus and minus 2σ limits are compared with the specification limits in current use for that particular aggregate and State. Both the overall variation limits, $\pm 2\sigma_o$, and the batch-to-batch variation limits, $\pm 2\sigma_l$, are given. The overall variation shows the limits within which 95 percent of single test results could be expected to fall. The batch-to-batch variation shows the limits within which the gradation of 95 percent of the batches would fall, if the entire batch was put through the sieves. The $\pm 2\sigma_o$ limits, based on overall variance, are significant with respect to establishing realistic tolerances because they include the random testing errors and other within-batch variances. The $\pm 2\sigma_l$ limits, based on batch-to-batch variance, are of significance with respect to the actual effects of variability (segregation) on the road or structure. Because σ^2_l is the significant variance that is responsible for nonuniformity of products or construction, the comparison of $\pm 2\sigma_o$ with the specification limits is meaningful from the viewpoint of deciding whether the LOT should be accepted or rejected, whereas the $\pm 2\sigma_l$ limits reflect that portion of the overall measured variability which is actually meaningful with respect to the life or serviceability of the road or structure.

Variation of Gradation at Other Sampling Points

Again, the $\pm 2\sigma_o$ limits obtained on samples taken from various points in the process back as far as the crusher are compared with the specification limits. The batch-to-batch variance loses significance with relation to the specification limits at these other points in the process, but σ_l is presented and discussed in other parts of the report.

Individual Series Summary Charts

The write-up for each series contains the following:

1. Summary tables showing the average percent passing, $\overline{X}$, the variances, and the standard deviations, for each sieve size at each sampling point and for Hudson $\overline{A}$.
2. A bar chart showing the relative variance of $\overline{A}$ at different points in the process stream. Each bar is divided to delineate within-batch variance and batch-to-batch variance, as well as the overall variance.
3. Aggregate grading charts, one for each sampling point, to show the $\pm 2\sigma_o$ range envelope.
4. Line graphs showing a plot of the individual test results of percent passing some convenient sized sieve at each sampling point within each series. These data are plotted in the order in which the samples were taken and therefore reflect changes in gradation with time and as a function of the processing or handling method. The $\pm 2\sigma_o$ limits are also shown on these line graphs. About 5 percent of the individual test results should fall outside of the $\pm 2\sigma_o$ limits. In the overall aggregate stockpiling and construction control studies this 5 percent has been observed with rather remarkable consistency, indicating close approximation to the Gaussian normal distribution.
5. A summary table of the gradation and statistical parameters, including pertinent standard deviations at the 50 percent passing level and for Hudson $\overline{A}$; segregation index based on both the 50 percent passing level, S_{50}, and the $\overline{A}$ variances, $S_{\overline{A}}$; degree of overall variability, DOV; and degree of segregation, D of S.

There is also a limited discussion of the results from the individual series as they apply to the primary objectives and scope of this project; namely, "Evaluation of Construction Control Procedures." Interrelations among the series and findings, or trends which are not directly related to the construction control aspects, are taken up in Chapter Five.

SUMMARY OF DATA FOR INDIVIDUAL SERIES

Series No. 1—Crushed Limestone (1 in.-No. 4) Aggregate for PC Concrete

PLANT DESCRIPTION AND PROCESS FLOW

This sampling was from the process stream (crusher, stockpile, bin feeder belt, weigh hopper*) of the aggregate supplied to a large central-mix concrete plant in Nebraska. The nominal ¾-in. crushed limestone comprised about 30 percent of the total aggregate in concrete used for structural, municipal paving, and State highway paving purposes, as well as for the production of prestressed structural members.

The material from the crusher screens fed a small surge pile by belt conveyor. Aggregate was then drawn through a gate opening beneath this pile onto a belt (sampling point No. 1), which transferred the material to railroad cars. The aggregate was shipped by rail to the concrete plant, where it was bottom dumped into a small surge pile beneath the tracks and over an opening through which the material could be drawn onto a belt which filled the plant bins. Due to the limited capacity of the surge pile, coupled with the limited capacity of the belt delivery-hopper storage system, it was necessary for aggregate in the surge pile to be rehandled by front-end loader into trucks which dumped into a large plant stockpile (sampling point No. 2). Subsequently, this stockpile was reloaded on trucks by a clamshell bucket and dumped back on the original surge pile beneath the tracks. From an opening at the bottom of this pile, the aggregate was drawn onto a tunnel belt beneath the surge pile. The belt (sampling point No. 3) discharged the aggregate into the plant bins. From the coarse aggregate bin, the aggregate was drawn through a gate (sampling point No. 4), into the weigh hopper.

VARIATION IN GRADATION AT POINT OF USE

Sample increments of aggregate were taken by sliding a sample container into the gravity feed from the overhead storage bins into the weigh hopper. Two increments having an average weight of 26 lb each were taken from each 5-cu yd batch selected by the use of a table of random numbers. The entire increment was used as a test portion, and was sieved to refusal on a Gilson shaker.

The specification requirements for the gradation of the crushed limestone are compared with the overall variation and batch-to-batch variation in Table 14. The overall variation shows the limits within which 95 percent of results of tests on single increments, selected at random, could be expected to fall. The batch-to-batch variation shows the

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* In most cases, weigh hopper samples were taken from the discharge stream of the bin feeding the weigh hopper.

TABLE 13

MAIN FIELD INVESTIGATION, AGGREGATES AND SAMPLING LOCATIONS

SERIES NO.	LOCATION	AGGREGATE		SAMPLING POINT	NO. OF TEST PORTIONS, *n*
		TYPE	SIZE		
1	Nebraska	Crushed limestone	1 in.-No. 4	Crusher	100
				Stockpile	100
				Belt charging plant bins	100
				Bin discharge to weigh hopper	100
2	Colorado	Crushed gravel	¾ in.-No. 4	Crusher	100
				Truck	100
				Stockpile	100
				Belt to weigh hopper	100
3	N. Carolina	Crushed stone	1½ in.-⅜ in.	Crusher	100
				Stockpile	50
				Cold feed bin discharge	100
4	Wyoming	Uncrushed gravel	1 in.-No. 4	Screening plant, bin discharge	100
				Screening plant, stockpile	100
				Concrete plant, bin discharge	100
5	N. Carolina	Cr. aggreg. base course	1 in.-No. 200	Plant, belt feeding pugmill mixer	60
				Trucks, after disch. from pugmill mixer	60
				Roadway, after spreading	60
6	Maryland	Crushed slag	1 in.-No. 8	Stockpile[a]	100
				Chute, belt disch. to storage bin	82
				Bin discharge to weigh hopper	88
7	Pennsylvania (Plant 1)	Uncrushed gravel	1 in.-No. 4	Barge	100
				Transfer belt	100
				Bin discharge to weigh hopper	100
8	Pennsylvania (Plant 2)	Uncrushed gravel	1 in.-No. 4	Barge	100
				Transfer belt	100
				Bin discharge to weigh hopper	100

[a] Front-end loader samplings from stockpile.

TABLE 14

GRADATION OF SERIES 1 AGGREGATE AT POINT OF USE

SIEVE SIZE	PERCENT PASSING SIEVE		
	SPEC. LIMITS	OVERALL VARIATION, $\pm 2\sigma_o$ LIMITS	BATCH-TO-BATCH VARIATION, $\pm 2\sigma_l$ LIMITS
1½ in.	100	100	100
¾ in.	70-100	70-99	70-99
⅜ in.	20-50	13-61	14-60
No. 4	0-10	0-20	0-20
No. 8	—	0-12	0-11

TABLE 15

GRADATION OF SERIES 1 AGGREGATE AT OTHER SAMPLING POINTS

SIEVE SIZE	PERCENT PASSING SIEVE, $\pm 2\sigma_o$ LIMITS			
	SPEC. LIMITS	CRUSHER	PLANT STOCKPILE	BELT
1½ in.	100	100	100	100
¾ in.	70-100	74-88	76-100	72-93
⅜ in.	20-50	23-40	14-66	17-47
No. 4	0-10	3-9	0-16	0-10
No. 8	—	2-4	0-7	0-5

probable limits within which the gradation of the coarse aggregate in 95 percent of the batches would fall, if the entire batch was put through the sieves, and is obtained by applying $\pm 2\sigma_l$ limits to the average percentage, $\bar{X}$, passing the sieves.

VARIATION IN GRADATION AT OTHER SAMPLING POINTS

To compare the specified gradation with the gradation at the source and at intermediate sampling points, random sample increments were taken, in duplicate, at the crusher, at the plant stockpile, and from the belt leading to the bins. The overall variation found at these points is compared with the specification limits in Table 15.

SERIES 1 SUMMARY CHARTS

The relative variances and standard deviations of the percentage passing the different sieves are summarized in Table 16. In general, it may be noted that within-batch variance is relatively small and, as shown in Table 14, correcting for this component of variance does not greatly affect the observed variations in gradation. At the point of use the variations in gradation extend beyond both the upper and lower specification limits for the percentage passing the ⅜-in. and the No. 4 sieves. Nevertheless, the concrete from this plant has a satisfactory performance record; therefore, a more realistic specification, with wider limits, is indicated.

From Table 15 it may be noted that at the time it left the crusher the aggregate was well within the specification limits on all sieves. As the aggregate was handled and stored, there was a tendency for the variability to become greater, so that the variation at the point of use was greater than at any previous point. This trend toward nonuniformity is shown in Figure 24, which shows the variance of the coarseness factor, $\bar{A}$, at the point of use and at the various sampling points.

An aggregate gradation chart for each sampling point, showing the $\pm 2\sigma$ envelope, is presented in Figure 25. The individual values for percent passing the ⅜-in. sieve at each of the sampling points are shown in Figure 26.

There is a strong indication that considerable segregation occurs in the plant storage bins. If it should be considered economically expedient to do so, reducing the magnitude of the variation in gradation would require some type of remixing operation immediately before the material was placed in the bins, plus an improved bin design to minimize segregation.

Table 17 summarizes the various statistical parameters for each of the four sampling points. In general, these parameters support the observations previously made; namely, that this aggregate was reasonably uniform as produced at the crusher, became segregated in the stockpile, showed improved uniformity again after rehandling into the tunnel belt, and, finally, the variability markedly increased due to segregation occurring in the plant storage bins. Attention is invited to the unusually low within-batch variability ($\sigma_{b50} = 1.6$, and $\sigma_{\bar{A}50} = 0.030$) obtained on the belt samples taken at sampling point No. 3. It is probable that these paired test increments were taken a bit too close together. In any case, this low within-batch variability results in a high segregation index value ($S_{\bar{A}}$ and S_{50}). A better representation of the loss in uniformity as this aggregate was processed from the crusher, to the stockpile, to the belt, to the weigh hopper, is reflected by the degree of overall variability, DOV, values, which are 9.6, 27.2, 16.6, and 26.0, respectively.

Further interpretation and discussion of these data is presented in Chapter Five.

Series No. 2—Crushed Gravel (¾ in.-No. 4) Aggregate for PC Concrete

PLANT DESCRIPTION AND PROCESS FLOW

This nominal ¾-in. crushed gravel aggregate was part of the supply to a Colorado batch plant for transit-mix trucks producing concrete for use in local municipal paving and for building construction.

At this plant the process stream stemmed from the crusher discharge. The material was delivered from the crusher by belt conveyor (sampling point No. 1). It was then loaded by belt conveyor into trucks (sampling point No. 2), and moved to a large storage pile near the crusher site (sampling point No. 3). From this storage pile the aggregate was again loaded into trucks by front-end loader and transported to the concrete plant. At the plant the material was dumped through a grizzly into a surge hopper, which discharged onto a belt (sampling point No. 4) that discharged directly into a weigh hopper without passing

TABLE 16

SUMMARY OF GRADATION AVERAGES, VARIANCES, AND STANDARD DEVIATIONS, SERIES 1 AGGREGATE[a]

			VARIANCE					STANDARD DEVIATION				
SIEVE SIZE	SAMPLING POINT	AVG. % PASSING, $\bar{X}$	TESTING, σ^2_t	INHERENT σ^2_a	OVERALL, σ^2_o	WITHIN-BATCH, σ^2_b	BATCH-TO-BATCH, σ^2_l	TESTING, σ_t	INHERENT σ_a	OVERALL, σ_o	WITHIN-BATCH, σ_b	BATCH-TO-BATCH, σ_l
¾ in.	Crusher	80.9	0.24	4.00	11.9	2.7	9.2	0.5	2.0	3.5	1.6	3.0
	Stockpile	87.8	0.24	4.00	36.9	3.5	33.4	0.5	2.0	6.1	1.9	5.8
	Belt	82.3	0.24	4.00	26.4	2.1	24.3	0.5	2.0	5.2	1.5	4.9
	Bin discharge[b]	84.4	0.24	4.00	55.0	2.7	52.3	0.5	2.0	7.4	1.6	7.2
⅜ in.	Crusher	31.3	0.43	2.25	18.6	3.2	15.4	0.6	1.5	4.3	1.8	3.9
	Stockpile	40.2	0.43	2.25	168.3	16.7	151.6	0.6	1.5	13.0	4.1	12.4
	Belt	31.7	0.43	2.25	57.0	1.7	55.3	0.6	1.5	7.6	1.3	7.5
	Bin discharge[b]	37.2	0.43	2.25	143.0	7.4	135.6	0.6	1.5	12.0	2.7	11.6
No. 4	Crusher	6.2	0.12	0.49	1.8	0.3	1.5	0.4	0.7	1.3	0.6	1.2
	Stockpile	7.0	0.12	0.49	21.8	2.4	19.4	0.4	0.7	4.7	1.6	4.4
	Belt	5.3	0.12	0.49	6.6	0.2	6.4	0.4	0.7	2.6	0.4	2.5
	Bin discharge[b]	8.9	0.12	0.49	31.2	1.9	29.3	0.4	0.7	5.6	1.4	5.4
No. 8	Crusher	3.0	0.07	0.25	0.5	0.1	0.4	0.3	0.5	0.7	0.3	0.6
	Stockpile	2.9	0.07	0.25	3.7	0.4	3.3	0.3	0.5	1.9	0.6	1.8
	Belt	2.8	0.07	0.25	1.6	0.1	1.5	0.3	0.5	1.3	0.2	1.2
	Bin discharge[b]	5.0	0.07	0.25	10.8	0.7	10.1	0.3	0.5	3.3	0.8	3.2
$\bar{A}$	Crusher	2.263	0.0004	—	0.0096	0.0017	0.0079	0.02	—	0.098	0.041	0.089
	Stockpile	2.435	0.0004	—	0.0667	0.0063	0.0604	0.02	—	0.258	0.079	0.246
	Belt	2.276	0.0004	—	0.0263	0.0009	0.0254	0.02	—	0.162	0.030	0.159
	Bin discharge[b]	2.456	0.0004	—	0.1022	0.0043	0.0979	0.02	—	0.320	0.066	0.313

[a] See Table 13 for description of aggregate. [b] To weigh hopper.

TABLE 17

SUMMARY OF GRADATION AND STATISTICAL PARAMETERS, SERIES 1 AGGREGATE[a]

SAMPLING POINT		AVERAGE SIZE, $\bar{X}$			50% LEVEL STD. DEVIATION				HUDSON $\bar{A}$ STD. DEVIATION			SEGREGATION INDEX		DEG. OF OVERALL VARIABILITY, DOV	DEG. OF SEGREGATION, D OF S
NO.	LOCATION	% PASS. ¾ IN.	% PASS. ⅜ IN.	HUDSON $\bar{A}$	OVERALL, σ_{o-50}	WITHIN-BATCH, σ_{b-50}	BATCH-TO-BATCH, σ_{l-50}	SLOPE	OVERALL, $\sigma_{o-\bar{A}}$	WITHIN-BATCH, $\sigma_{b-\bar{A}}$	BATCH-TO-BATCH, $\sigma_{l-\bar{A}}$	BASED ON $\bar{A}$, $S_{\bar{A}}$	BASED ON 50% P, S_{50}		
1	Crusher	80.9	31.3	2.263	4.8	2.1	4.3	1.73	0.098	0.041	0.089	5.6	4.3	9.6	8.6
2	Stockpile	87.8	40.2	2.435	13.6	4.4	12.9	1.73	0.258	0.079	0.246	10.6	9.5	27.2	25.8
3	Belt	82.3	31.7	2.276	8.3	1.6	8.1	1.73	0.162	0.030	0.159	28.3	27.5	16.6	16.2
4	Bin discharge[b]	84.4	37.2	2.456	13.0	2.9	12.3	1.73	0.320	0.066	0.313	23.8	20.1	26.0	24.6

[a] See Table 13 for description of aggregate. [b] To weigh hopper.

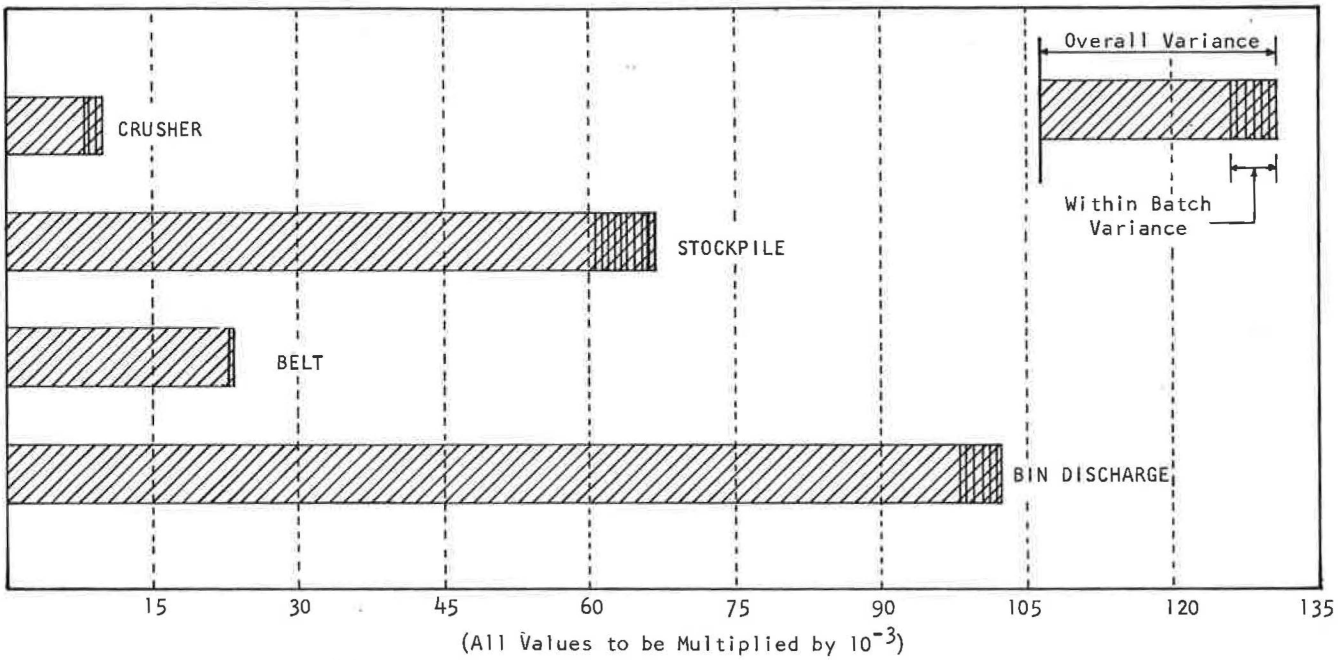


Figure 24. Relative variance of $\overline{A}$ at different points in process stream, Series No. 1, crushed limestone (1 in.-No. 4), Nebraska.

TABLE 18

GRADATION OF SERIES 2 AGGREGATE AT POINT OF USE

	PERCENT PASSING SIEVE		
SIEVE SIZE	SPEC. LIMITS	OVERALL VARIATION, $\pm 2\sigma_o$ LIMITS	BATCH-TO-BATCH VARIATION, $\pm 2\sigma_t$ LIMITS
1½ in.	100	100	100
1 in.	95-100	—	—
¾ in.	—	96-99	96-98
½ in.	25-60	47-64	47-64
⅜ in.	—	25-41	25-41
No. 4	0-10	4-8	4-8
No. 8	—	2-5	2-4

TABLE 19

GRADATION OF SERIES 2 AGGREGATE AT OTHER SAMPLING POINTS

	PERCENT PASSING SIEVE, $\pm 2\sigma_o$ LIMITS			
SIEVE SIZE	SPEC. LIMITS	CRUSHER	TRUCKS	STOCK-PILE
1½ in.	100	100	100	100
1 in.	95-100	—	—	—
¾ in.	—	94-98	96-100	96-100
½ in.	25-60	43-60	47-85	42-71
⅜ in.	—	22-36	20-62	19-47
No. 4	0-10	1-6	1-7	1-8
No. 8	—	0-4	1-3	1-4

through an intermediate bin. For this reason, the weigh hopper sampling was taken from the aggregate discharge stream from this belt instead of from the aggregate stream from the bin gates, as was the case at several other locations.

VARIATION IN GRADATION AT POINT OF USE

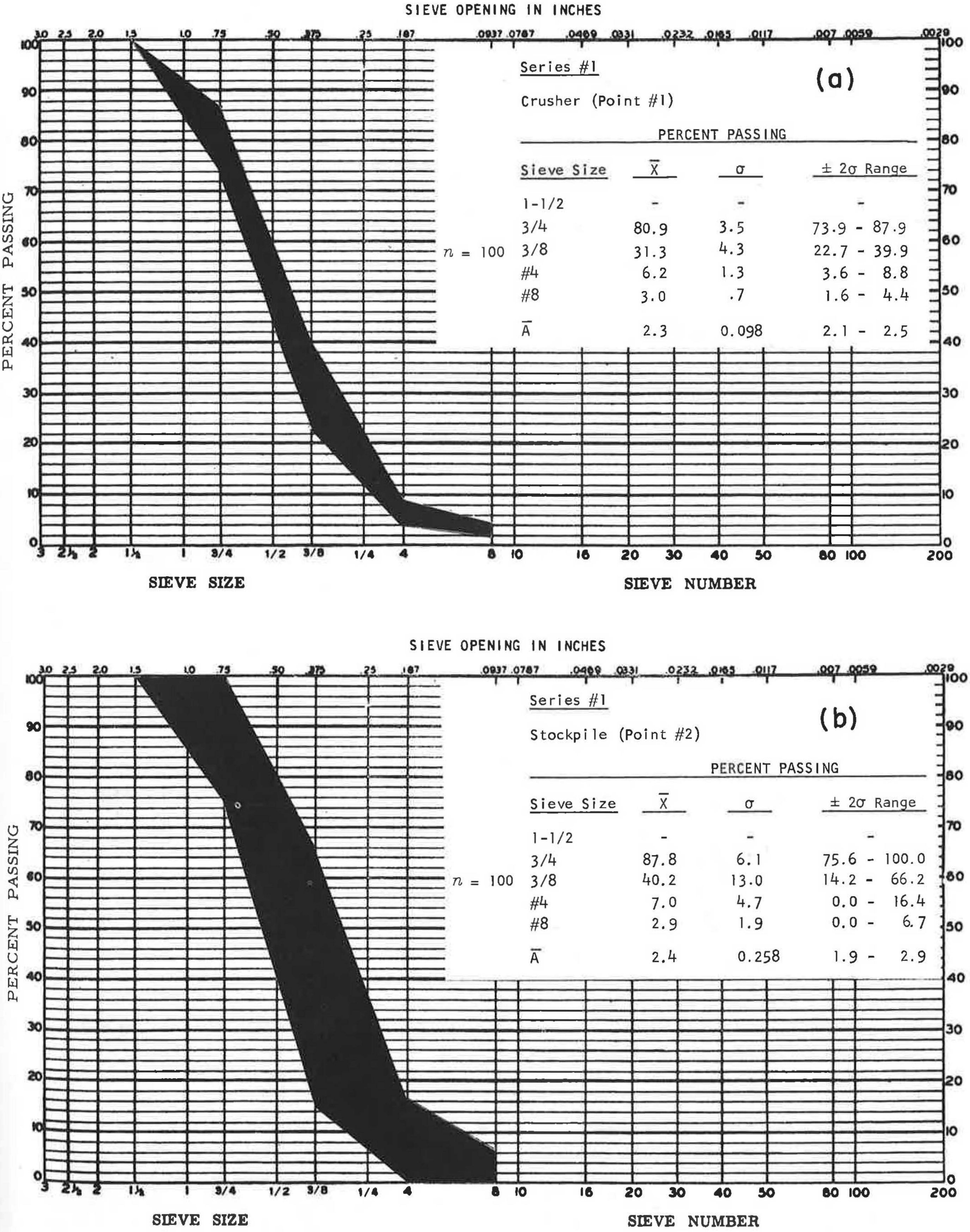
Sample increments of aggregate were taken by removing a measured portion of the aggregate from the belt (sampling point No. 4), which discharged directly into the weigh hopper. Two increments having an average weight of 24 lb each were taken at random locations from the belt. The entire increment was used as a test portion and was sieved to refusal on a Gilson shaker.

The specification requirements for the gradation of the crushed gravel are compared with the overall variation and batch-to-batch variation in Table 18. The overall variation shows the limits within which 95 percent of results of tests on single increments, selected at random, could be expected to fall. The batch-to-batch variation shows the probable limits within which the gradation of the coarse aggregate in 95 percent of the batches would fall, if the entire batch was put through the sieves.

VARIATION IN GRADATION AT OTHER SAMPLING POINTS

To compare the specified gradation with the gradation at the source and at intermediate sampling points, increments were taken by probability sampling, in duplicate, at the crusher (sampling point No. 1), from the trucks (sampling point No. 2), and from the stockpile (sampling point No. 3).

The overall variation found at these points is compared with the specification limits in Table 19.



Series #1

Crusher (Point #1)

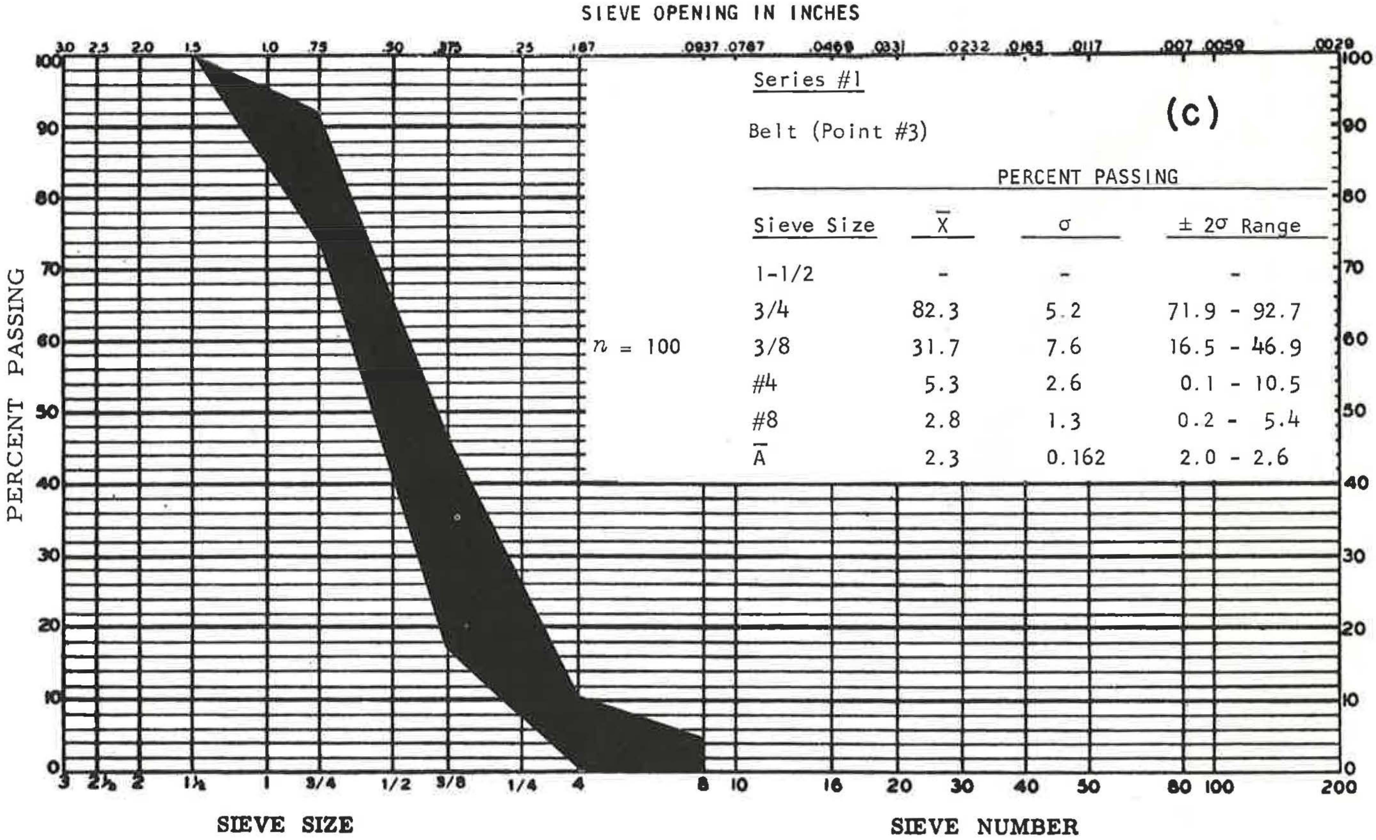
	Sieve Size	$\overline{X}$	σ	± 2σ Range
		PERCENT PASSING		
	1-1/2	-	-	-
	3/4	80.9	3.5	73.9 - 87.9
n = 100	3/8	31.3	4.3	22.7 - 39.9
	#4	6.2	1.3	3.6 - 8.8
	#8	3.0	.7	1.6 - 4.4
	$\overline{A}$	2.3	0.098	2.1 - 2.5

Series #1

Stockpile (Point #2)

	Sieve Size	$\overline{X}$	σ	± 2σ Range
		PERCENT PASSING		
	1-1/2	-	-	-
	3/4	87.8	6.1	75.6 - 100.0
n = 100	3/8	40.2	13.0	14.2 - 66.2
	#4	7.0	4.7	0.0 - 16.4
	#8	2.9	1.9	0.0 - 6.7
	$\overline{A}$	2.4	0.258	1.9 - 2.9

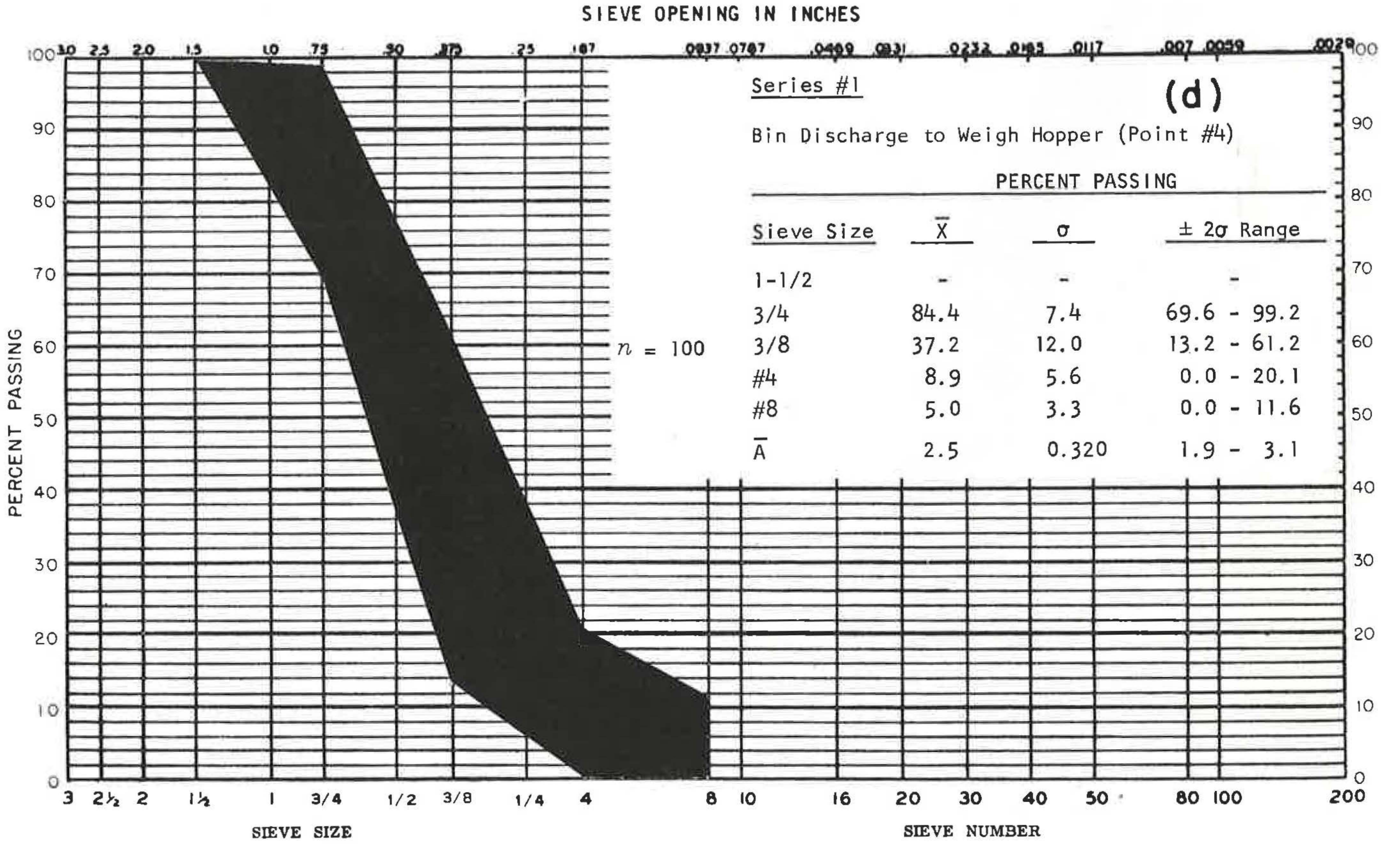
Figure 25. Aggregate gradation charts for Series No. 1 samples taken at (a) crusher, (b) stockpile, (c) belt, and (d) bin discharge to weigh hopper.



Series #1

Belt (Point #3)

	PERCENT PASSING		
Sieve Size	$\overline{X}$	σ	± 2σ Range
1-1/2	-	-	-
3/4	82.3	5.2	71.9 - 92.7
3/8	31.7	7.6	16.5 - 46.9
#4	5.3	2.6	0.1 - 10.5
#8	2.8	1.3	0.2 - 5.4
$\overline{A}$	2.3	0.162	2.0 - 2.6



Series #1

Bin Discharge to Weigh Hopper (Point #4)

	PERCENT PASSING		
Sieve Size	$\overline{X}$	σ	± 2σ Range
1-1/2	-	-	-
3/4	84.4	7.4	69.6 - 99.2
3/8	37.2	12.0	13.2 - 61.2
#4	8.9	5.6	0.0 - 20.1
#8	5.0	3.3	0.0 - 11.6
$\overline{A}$	2.5	0.320	1.9 - 3.1

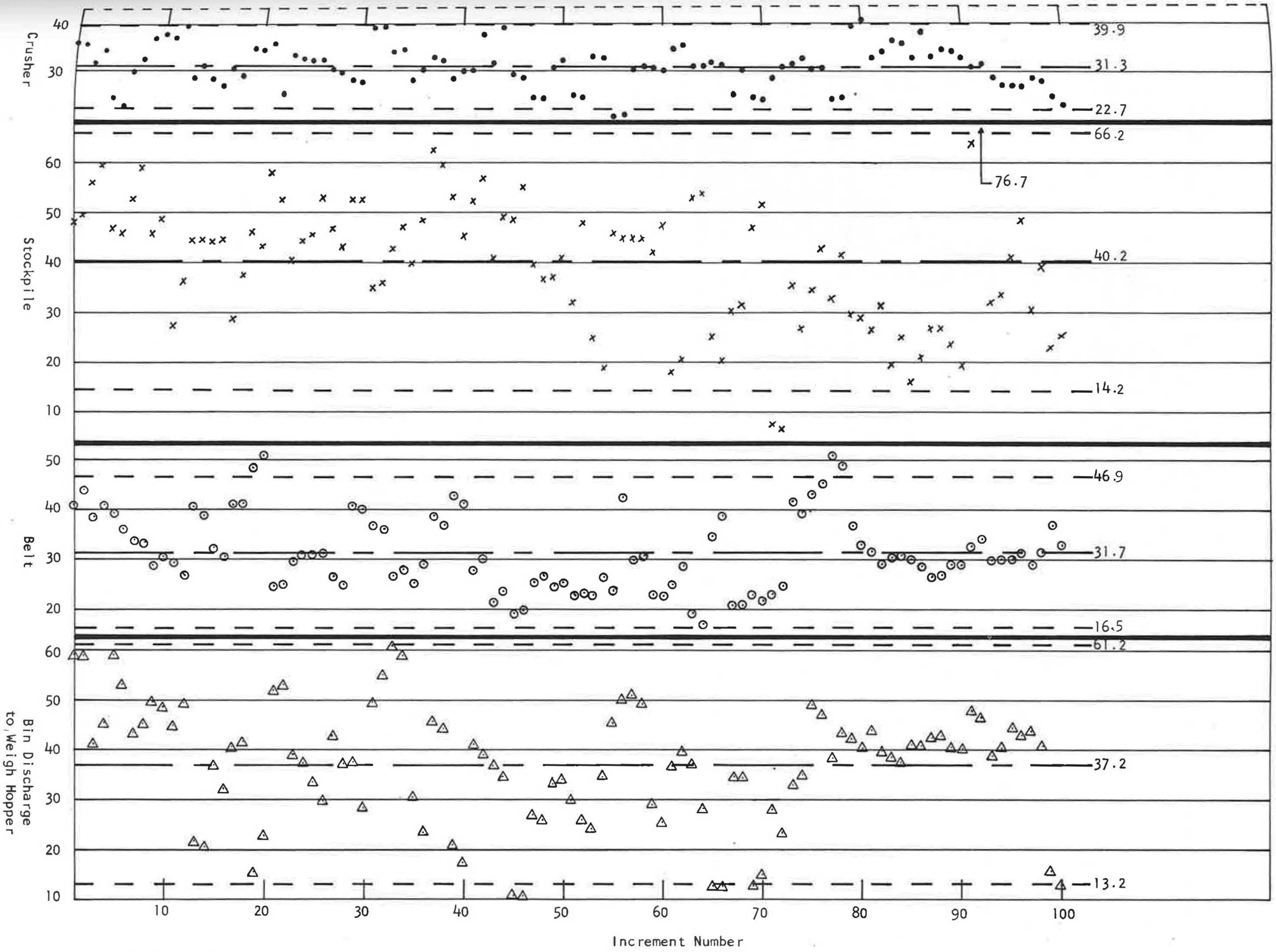


Figure 26. Individual test values, percent passing 3/8-in. sieve, Series No. 1.

SERIES 2 SUMMARY CHARTS

The relative variances and standard deviations of the percentages passing the different sieves are summarized in Table 20. Even though the ½-in. sieve is not one of the standard sieves used in calculating $\bar{A}$ and normally recommended for these studies, it has been included in this case because of the relatively large amount of material passing the ¾-in. and retained on the ⅜-in. sieves.

Variability was not large, relative to the other aggregates studied, at any of the four sampling points. Although a full explanation of the reasons for this greater uniformity is not known at this time, contributing factors are probably associated with smaller size (¾ in. versus 1 in. or 1½ in.) and the fact that it was crushed, largely one-sized, aggregate. Also, the processing steps and handling methods used at this location were not particularly conducive to severe segregation.

This good behavior is also reflected in Figure 27, showing the relative variance of $\bar{A}$ at different points in the process stream. The same pattern is reflected also by the aggregate grading charts (Figure 28). The plots of the individual values passing the ½-in. sieve at each sampling point are shown in Figure 29.

Attention is invited to the fact that the truck sample results are significantly finer (higher percent passing) than those taken at the other sampling points. This same tendency was noted in the truck samples taken in Series No. 5, and it is the data from these two series that are presented and discussed in Chapter Three, under "Experiments with Different Sampling Tools, Truck Sampling Results."

Attention is also invited to the pattern of the individual test results obtained at sampling point No. 4 (Fig. 29). This sampling is from a belt fed from a surge hopper and discharging directly into the weigh hopper at the concrete plant. An occasional surge of coarse aggregate may be noted. Taken alone, this slight surging would probably not be significant, or would be overlooked entirely. However, combined with all of the other data on samples taken from aggregate streams emitting from surge or storage hoppers, there is ample evidence that this surging is a very real thing. Although admittedly minor in this instance, attention is invited to the fact because it does appear to be an important factor in construction controls in some of the subsequent series.

The gradation and statistical summary parameters for this series are presented in Table 21. In general, these parameters are consistent within themselves and reflect the relatively good uniformity previously observed. One exception is that with the segregation index both $S_{\bar{A}}$ and S_{50} are high at sampling point No. 4 ($S_{\bar{A}} = 17.1$, and $S_{50} = 10.9$). This is entirely due to an abnormally low within-batch variance, however, because the overall variability as reflected by both σ_o and $\bar{A}$ for σ_{o50} is low—in fact, essentially equal to the standard deviations measured at the crusher, sampling point No. 1. No explanation is apparent as to why the within-batch variability was so low at sampling point No. 4. Taken at their face value, these data would indicate that the rehandling from the relatively unsegregated stockpile with the front-end loader and truck dumping through a grizzly and a properly designed surge hopper had not only slightly reduced the degree of batch-to-batch segregation, but somehow had remixed the aggregate so as to materially reduce the within-batch variability.

In any case, this situation illustrates the danger of relying on trends indicated by any one of these parameters, without a careful look at the overall data.

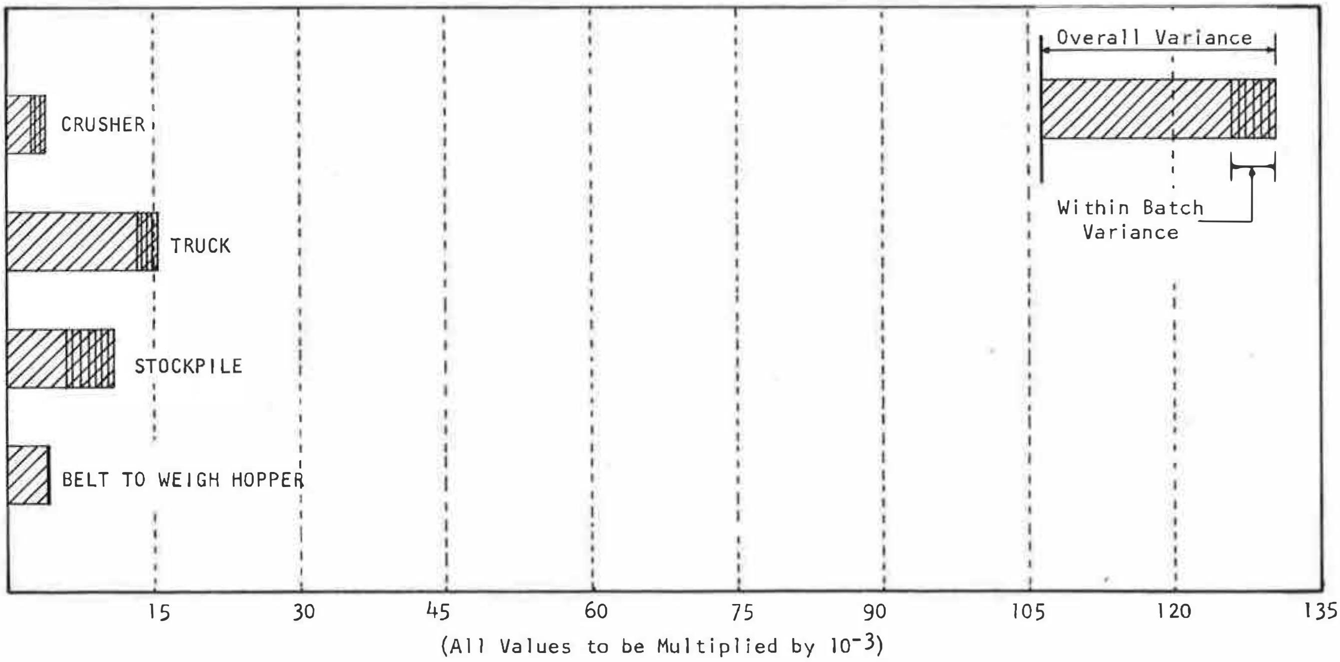


Figure 27. Relative variance of $\bar{A}$ at different points in process stream, Series No. 2, crushed gravel (¾ in.-No. 4), Colorado.

TABLE 20

SUMMARY OF GRADATION AVERAGES, VARIANCES, AND STANDARD DEVIATIONS, SERIES 2 AGGREGATE [a]

			VARIANCE					STANDARD DEVIATION				
SIEVE SIZE	SAMPLING POINT	AVG. % PASSING, $\bar{X}$	TESTING, σ^2_t	INHERENT σ^2_a	OVERALL, σ^2_o	WITHIN-BATCH, σ^2_b	BATCH-TO-BATCH, σ^2_l	TESTING, σ_t	INHERENT σ_a	OVERALL, σ_o	WITHIN-BATCH, σ_b	BATCH-TO-BATCH, σ_l
¾ in.	Crusher	96.1	0.08	0.81	1.2	0.5	0.7	0.3	0.9	1.1	0.7	0.8
	Truck	98.1	0.08	0.81	0.9	0.3	0.6	0.3	0.9	0.9	0.5	0.8
	Stockpile	98.1	0.08	0.81	0.8	0.7	0.1	0.3	0.9	0.9	0.8	0.3
	Belt [b]	97.1	0.08	0.81	0.6	0.3	0.3	0.3	0.9	0.8	0.5	0.5
½ in.	Crusher	51.4	0.41	1.80	19.3	4.6	14.7	0.6	1.3	4.4	2.1	3.8
	Truck	66.0	0.41	1.80	86.1	11.6	74.5	0.6	1.3	9.3	3.4	8.6
	Stockpile	56.4	0.41	1.80	52.3	34.5	17.8	0.6	1.3	7.2	5.9	4.2
	Belt [b]	55.6	0.41	1.80	18.4	1.6	16.8	0.6	1.3	4.3	1.3	4.1
⅜ in.	Crusher	28.6	0.26	1.00	12.2	2.4	9.8	0.5	1.0	3.5	1.6	3.1
	Truck	41.2	0.26	1.00	108.2	14.5	93.7	0.5	1.0	10.4	3.8	9.7
	Stockpile	33.1	0.26	1.00	48.8	30.3	18.5	0.5	1.0	7.0	5.5	4.3
	Belt [b]	33.3	0.26	1.00	16.0	0.7	15.3	0.5	1.0	4.0	0.8	3.9
No. 4	Crusher	3.5	0.11	0.16	1.7	0.8	0.9	0.3	0.4	1.3	0.9	0.9
	Truck	3.8	0.11	0.16	2.1	0.2	1.9	0.3	0.4	1.4	0.4	1.4
	Stockpile	4.6	0.11	0.16	2.9	0.7	2.2	0.3	0.4	1.7	0.8	1.5
	Belt [b]	5.9	0.11	0.16	1.2	0.1	1.1	0.3	0.4	1.1	0.3	1.1
No. 8	Crusher	2.0	0.04	0.09	0.6	0.4	0.2	0.2	0.3	0.8	0.6	0.4
	Truck	2.0	0.04	0.09	0.4	0.2	0.2	0.2	0.3	0.6	0.4	0.4
	Stockpile	2.6	0.04	0.09	0.9	0.2	0.7	0.2	0.3	0.9	0.4	0.8
	Belt [b]	3.4	0.04	0.09	0.4	0.1	0.3	0.2	0.3	0.6	0.3	0.5
A	Crusher	2.341	0.0004	—	0.0040	0.0015	0.0025	0.02	—	0.063	0.039	0.050
	Truck	2.490	0.0004	—	0.0153	0.0021	0.0132	0.02	—	0.124	0.046	0.115
	Stockpile	2.426	0.0004	—	0.0109	0.0050	0.0055	0.02	—	0.104	0.071	0.074
	Belt [b]	2.465	0.0004	—	0.0042	0.0002	0.0040	0.02	—	0.065	0.016	0.063

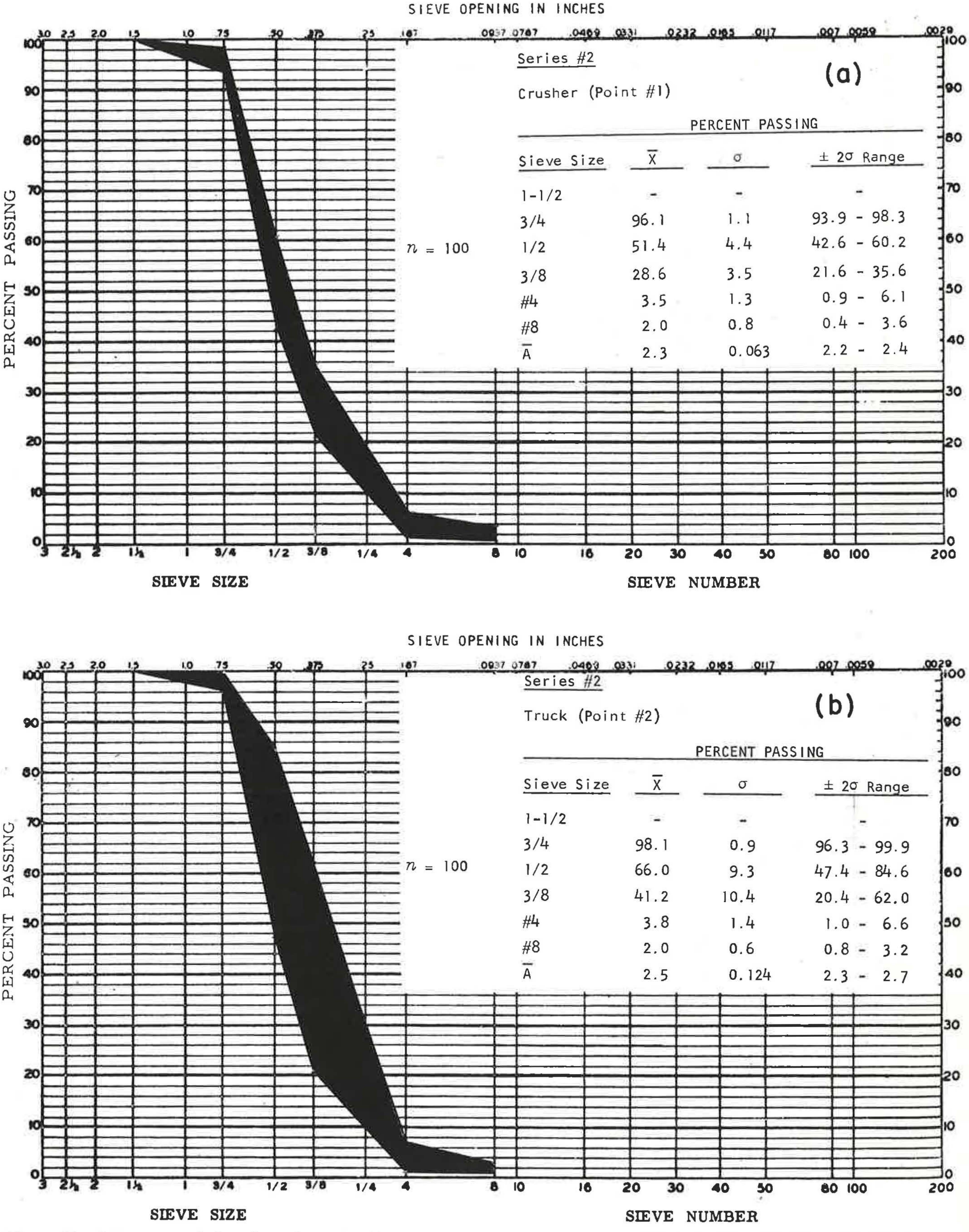
[a] See Table 13 for description of aggregate. [b] To weigh hopper.

TABLE 21

SUMMARY OF GRADATION AND STATISTICAL PARAMETERS, SERIES 2 AGGREGATE [a]

SAMPLING POINT		AVERAGE SIZE, $\bar{X}$			50% LEVEL STD. DEVIATION				HUDSON $\bar{A}$ STD. DEVIATION			SEGREGATION INDEX		DEG. OF OVERALL VARIABILITY,	DEG. OF SEGREGATION,
NO.	LOCATION	% PASS. ¾ IN.	% PASS. ⅜ IN.	HUDSON $\bar{A}$	OVERALL, σ_{o-50}	WITHIN-BATCH, σ_{b-50}	BATCH-TO-BATCH, σ_{l-50}	SLOPE	OVERALL, $\sigma_{o-\bar{A}}$	WITHIN-BATCH, $\sigma_{b-\bar{A}}$	BATCH-TO-BATCH, $\sigma_{l-\bar{A}}$	BASED ON $\bar{A}$, $S_{\bar{A}}$	BASED ON 50% *P*, S_{50}	DOV	D OF S
1	Crusher	96.1	28.6	2.341	4.5	2.4	3.8	1.73	0.063	0.039	0.050	2.7	3.5	9.0	7.6
2	Truck	98.1	41.2	2.490	10.0	4.0	9.3	1.73	0.124	0.046	0.115	7.3	6.3	20.0	18.6
3	Stockpile	98.1	33.1	2.426	7.8	5.9	5.1	1.73	0.104	0.071	0.074	2.2	1.8	15.6	10.2
4	Belt [b]	97.1	33.3	2.465	4.3	1.3	4.1	1.73	0.065	0.016	0.063	17.1	10.9	8.6	8.2

[a] See Table 13 for description of aggregate. [b] To weigh hopper.



Series #2

Crusher (Point #1)

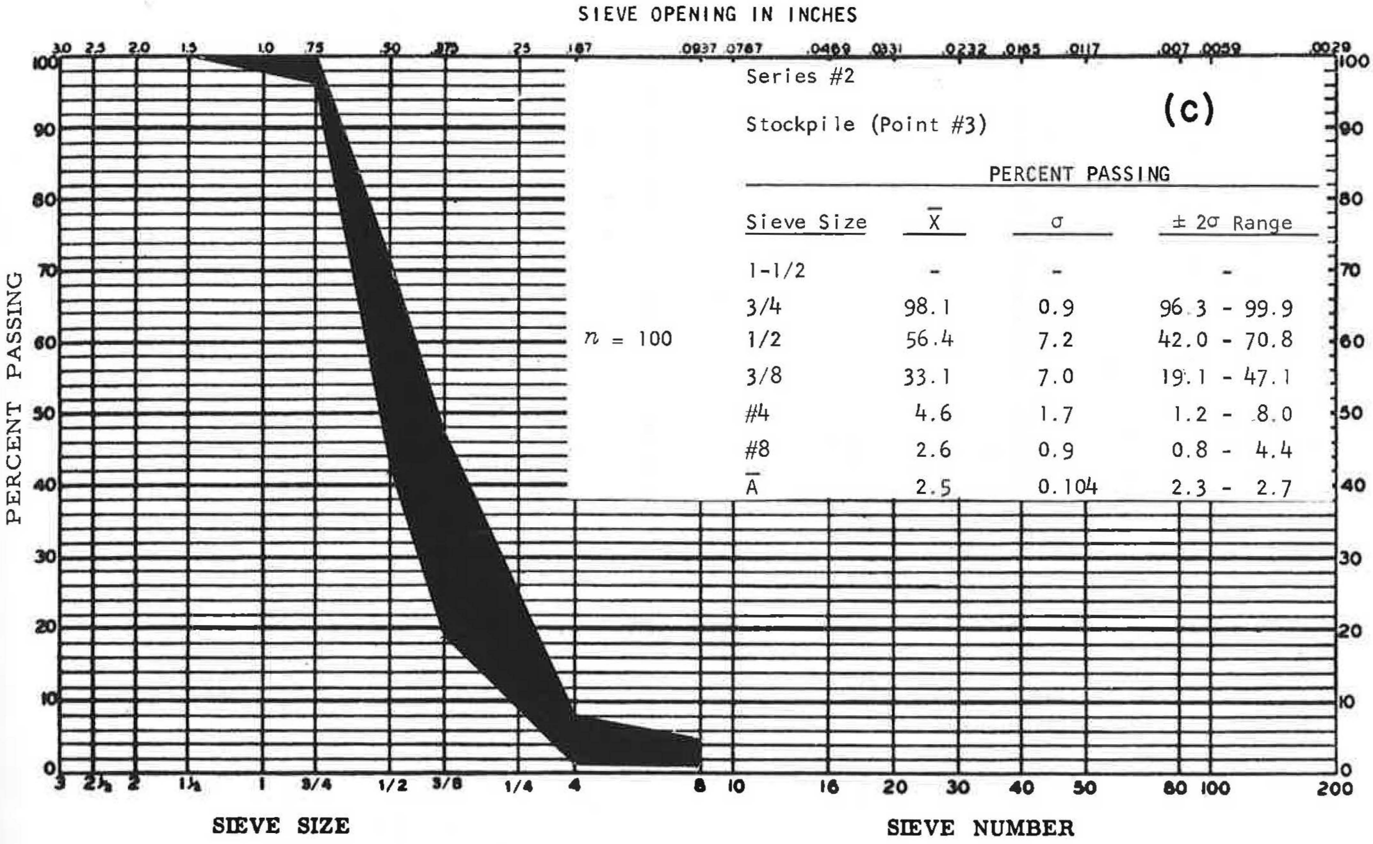
Sieve Size	PERCENT PASSING $\bar{X}$	σ	± 2σ Range
1-1/2	-	-	-
3/4	96.1	1.1	93.9 - 98.3
1/2	51.4	4.4	42.6 - 60.2
3/8	28.6	3.5	21.6 - 35.6
#4	3.5	1.3	0.9 - 6.1
#8	2.0	0.8	0.4 - 3.6
$\bar{A}$	2.3	0.063	2.2 - 2.4

Series #2

Truck (Point #2)

Sieve Size	PERCENT PASSING $\bar{X}$	σ	± 2σ Range
1-1/2	-	-	-
3/4	98.1	0.9	96.3 - 99.9
1/2	66.0	9.3	47.4 - 84.6
3/8	41.2	10.4	20.4 - 62.0
#4	3.8	1.4	1.0 - 6.6
#8	2.0	0.6	0.8 - 3.2
$\bar{A}$	2.5	0.124	2.3 - 2.7

Figure 28. Aggregate gradation charts for Series No. 2 samples taken at (a) crusher, (b) truck, (c) stockpile, and (d) belt to weigh hopper.

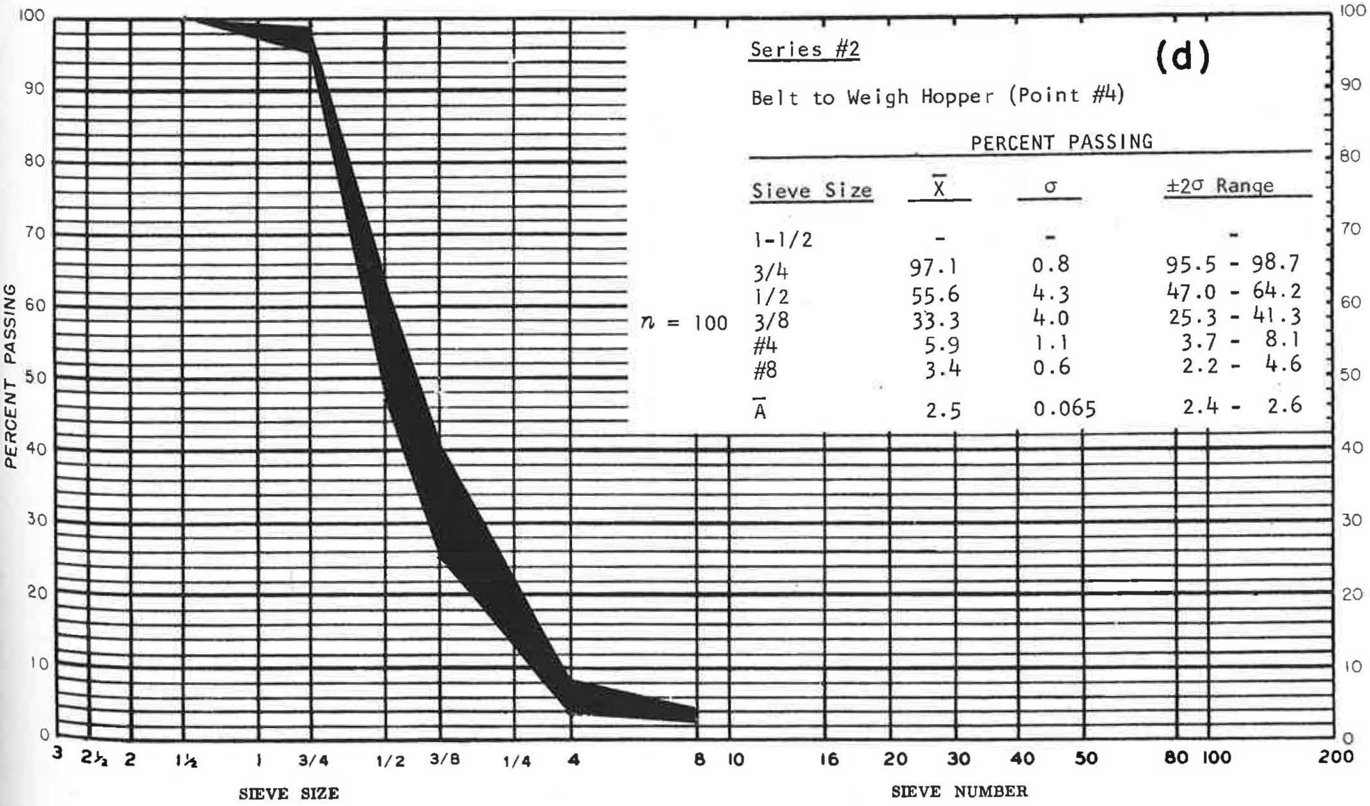


Series #2

Stockpile (Point #3)

n = 100

	PERCENT PASSING		
Sieve Size	$\overline{X}$	σ	± 2σ Range
1-1/2	-	-	-
3/4	98.1	0.9	96.3 - 99.9
1/2	56.4	7.2	42.0 - 70.8
3/8	33.1	7.0	19.1 - 47.1
#4	4.6	1.7	1.2 - 8.0
#8	2.6	0.9	0.8 - 4.4
$\overline{A}$	2.5	0.104	2.3 - 2.7



Series #2

Belt to Weigh Hopper (Point #4)

n = 100

	PERCENT PASSING		
Sieve Size	$\overline{X}$	σ	±2σ Range
1-1/2	-	-	-
3/4	97.1	0.8	95.5 - 98.7
1/2	55.6	4.3	47.0 - 64.2
3/8	33.3	4.0	25.3 - 41.3
#4	5.9	1.1	3.7 - 8.1
#8	3.4	0.6	2.2 - 4.6
$\overline{A}$	2.5	0.065	2.4 - 2.6

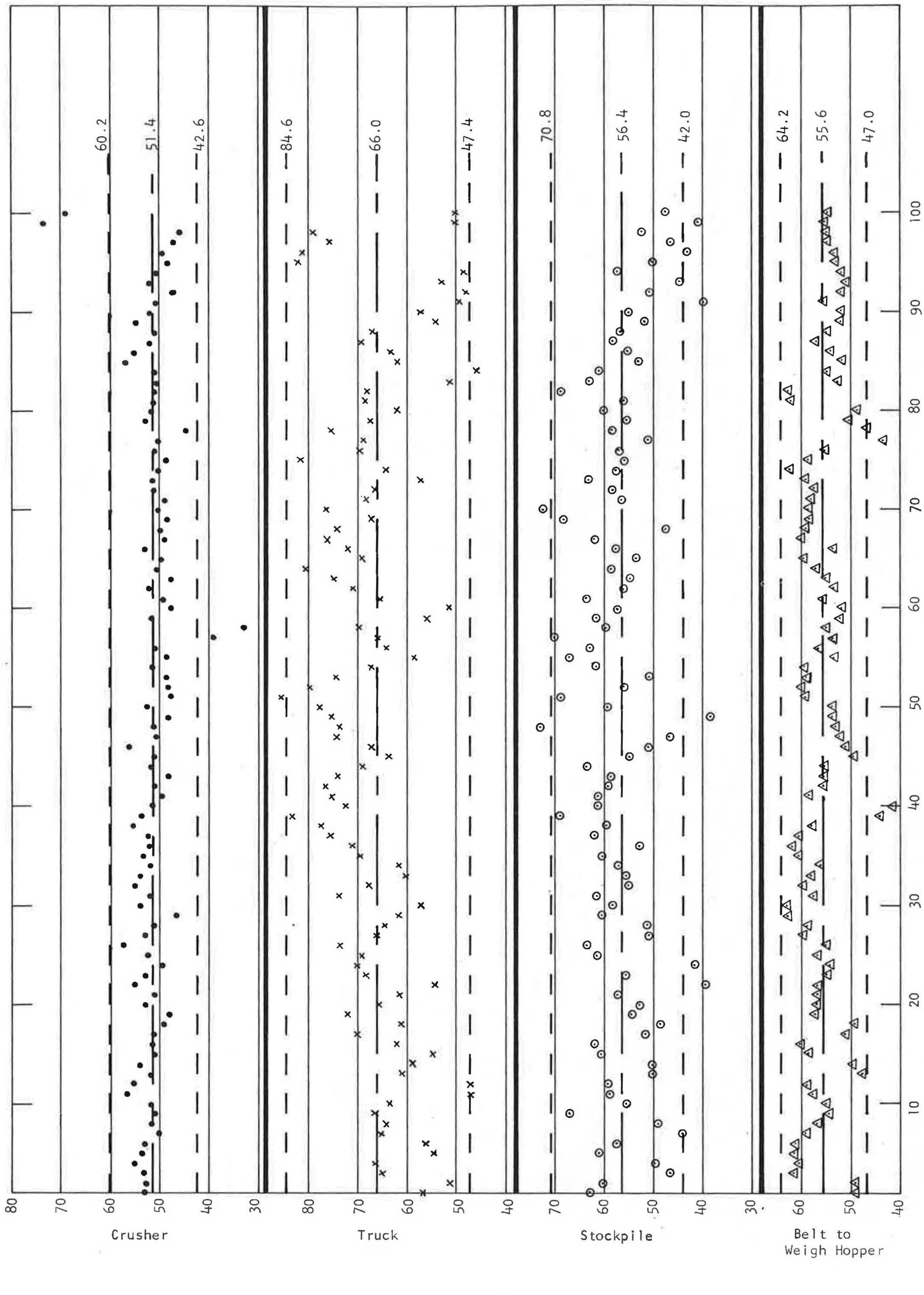


Figure 29. Individual test values, percent passing ½-in. sieve, Series No. 2.

Series No. 3—Crushed Stone (1½ in.-⅜ in.) Aggregate for Black Base Asphaltic Concrete

PLANT DESCRIPTOIN AND PROCESS FLOW

This nominal 1½-in. coarse aggregate is crushed granite used in the production of hot-mix black base (HB mix) for use in State highway work in North Carolina.

Crusher discharge was fed into a screening unit, which removed virtually all of the minus No. 4 material. The coarse aggregate was washed and discharged from the screening unit onto a conveyor belt (sampling point No. 1), which fed a second conveyor belt, which dumped the aggregate directly onto a surge stockpile. As sufficient material built up beneath the belt discharge, a clamshell bucket picked up the aggregate and deposited it on a large storage stockpile (sampling point No. 2). It was then loaded into trucks (as required) by front-end loader and transported to an asphalt plant stockpile approximately ¼ to ½ mile away, where it was dumped at the base of a relatively large working pile. At the plant site, a clambucket was used to keep this pile built up and also to charge the raw aggregate bins from which the material dropped onto a belt feeding the drier. Sampling point No. 3 was the coarse aggregate cold feed bin discharge.

VARIATION IN GRADATION AT POINT OF USE

Sample increments were taken by passing a large scoop shovel through the stream of aggregate at the point where the coarse aggregate was discharged from the cold bin onto the belt carrying the total aggregate to the dryer. Two increments having an average weight of 42 lb each were taken at intervals selected by the use of a table of random numbers. The entire increment was used as a test portion and was sieved to refusal on a Gilson shaker.

The specification requirements for the gradation of the crushed stone are compared with the overall variation and batch-to-batch variation in Table 22. The overall variation shows the limits within which 95 percent of the results of tests on single increments, selected at random, could be expected to fall. The batch-to-batch variation shows the probable limits within which the coarse aggregate in 95 percent of the batches would fall, if the aggregate went directly into the batch, and the entire batch was put through the sieves. In this case, the uniformity of gradation of the coarse aggregate in the asphaltic concrete was undoubtedly better than indicated by the estimated variation, because the aggregate was rescreened and binned after passing the sampling point.

VARIATION IN GRADATION AT OTHER SAMPLING POINTS

To compare the specified gradation with the gradation at the source and at an intermediate point, random sample increments were taken in duplicate from the belt carrying the crusher discharge and from the quarry stockpile. The overall variation found at all points is compared with the specification limits in Table 23.

SERIES 3 SUMMARY CHARTS

The relative variances and standard deviations of the percentages passing the different sieves are summarized in Table 24. In this case, it is evident that the crusher was not properly set to produce the specified gradation, because all sampling showed an average of 29 to 38 percent passing the ¾-in. sieve, as compared with the specification limits of 15 to 30 percent. Also, the specification limits were not sufficiently wide to allow for variations measured in the percent passing the ¾-in. sieve at the crusher.

The relative variability at the three sampling points—crusher, stockpile, and plant cold feed bin—is shown graphically in Figure 30, and the aggregate gradation 2σ band envelopes are presented in Figure 31. It is apparent that this aggregate is reasonably uniform as produced, but that stockpiling with a clamshell resulted in a very high degree of segregation. This segregation in the stockpile was reflected in both relatively high localized or within-batch segregation, σ_b, and in relatively high batch-to-batch segregation, σ_l, at different locations in the pile. Localized segregation (high σ_b) means, of course, that there are sampling difficulties which greatly increase both the buyer's and seller's risks in making an acceptance or rejection decision based on samples taken from such a stockpile.

The condition at the three sampling points for this series is further shown in Figure 32, a plot of the percent passing

TABLE 22

GRADATION OF SERIES 3 AGGREGATE AT COLD FEED

SIEVE SIZE	PERCENT PASSING SIEVE SPEC. LIMITS	 OVERALL VARIATION, $\pm 2\sigma_o$ LIMITS	 BATCH-TO-BATCH VARIATION, $\pm 2\sigma_l$ LIMITS
2 in.	100	—	—
1½ in.	80-100	80-100	81-99
¾ in.	15-30	8-58	9-57
⅜ in.	—	0-16	0-16
No. 4	0-5	0-9	0-9
No. 8	—	0-7	0-7

TABLE 23

GRADATION OF SERIES 3 AGGREGATE AT OTHER SAMPLING POINTS

SIEVE SIZE	PERCENT PASSING SIEVE, $\pm 2\sigma_o$ LIMITS SPEC. LIMITS	 CRUSHER	 PLANT STOCKPILE	 COLD FEED
2 in.	100	100	100	100
1½ in.	80-100	86-97	75-100	80-100
¾ in.	15-30	24-51	0-69	8-58
⅜ in.	—	1-6	0-12	0-16
No. 4	0-5	1-2	0-5	0-9
No. 8	—	0-1	0-4	0-7

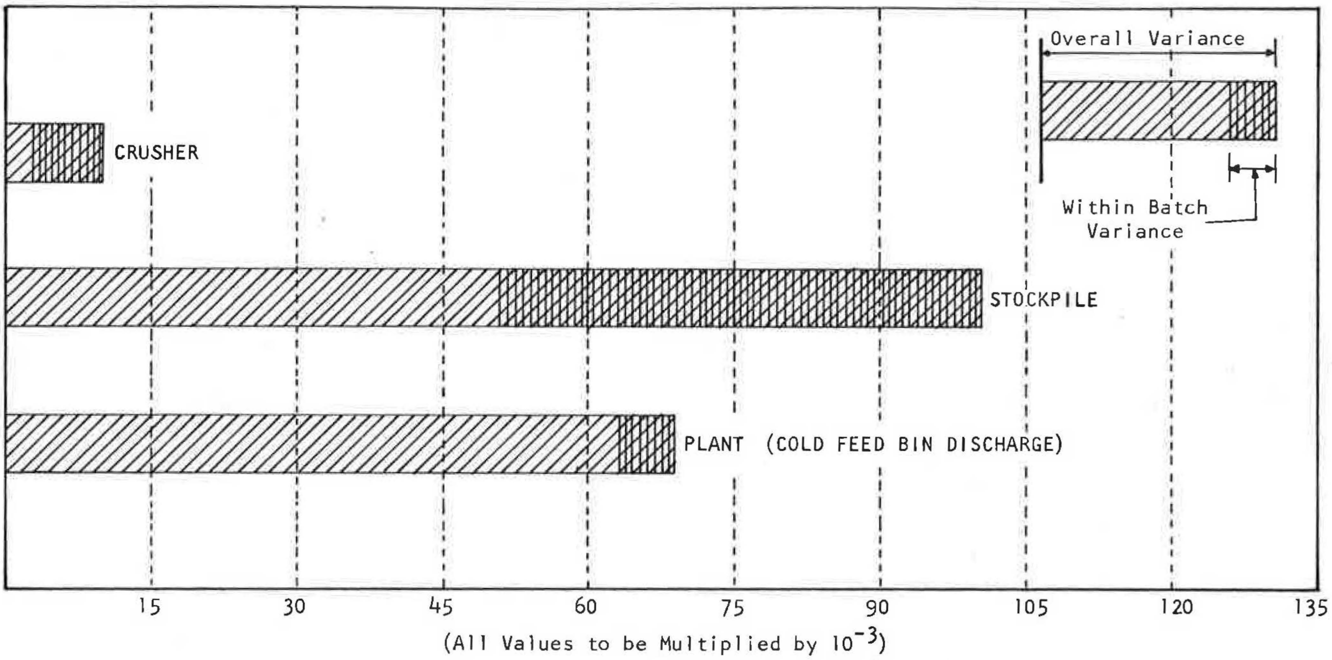


Figure 30. Relative variance of $\overline{A}$ *at different points in process stream, Series No. 3, crushed stone (1½ in.-⅜ in.), North Carolina.*

the ¾-in. sieve for each successive test portion. In this instance, only 50, rather than 100, samples were taken of the stockpile; however, the pattern is very evident. Test portions taken from this stockpile range from as low as 2 percent to as high as 74 percent passing the ¾-in. sieve, with an average of 28.8 percent. This average is about 9 points lower (coarser) than the average of the same material measured at the crusher (37.6 at the crusher, versus 28.8 percent passing the ¾-in. sieve at the stockpile).

The condition at the cold feed bin is of particular interest. The highly segregated stockpile was taken down with a front-end loader and the pattern of successive samples taken at the cold feed bin discharge reflects the carry-over of both segregation and the method of handling. For instance, the first 20 samples have an average of 17.3 percent passing the ¾-in. sieve, and a range from 10.3 to 28.9 percent. These represent the coarse aggregate picked up by the front-end loader at the base of the pile. Then, as the loader continued to eat into the side, the pile slipped or slid at about sample 21, carrying the finer material, from higher up on the pile, down into the loading area. The next 10 samples average about 40 percent passing the ¾-in. sieve. The front-end loader worked through this fine material and again bit into the coarse aggregate at the base of the pile from about sample 33 to 44, during which time the passing ¾-in. material gradually dropped from the 40 percent level to 12 percent passing. Then, the pile again slipped, bringing down fine material, and the pattern is repeated, but with decreased amplitude, as the stockpile becomes depleted. The last 35 samples do not reflect as definite a trend, although there is a tendency for the curve to peak around sample 70 with a high value of 62.3 percent passing. The last 13 results are finer than the average gradation, averaging 47.7 percent passing the ¾-in. sieve. Thus, the aggregate fed to the coarse cold feed bin at this asphalt plant varied from about 17 percent to 48 percent passing the ¾-in. sieve over a 9-hr period, with repeated surges reflecting segregation in the stockpile.

At an asphalt plant, this lack of uniformity is largely corrected by rescreening prior to proportioning into the mix. Even so, it is fair to ask, "how much does this variability cost the producer in either overrun or waiting for material in his No. 3 or No. 4 hot bin?" If this is a manual plant, how many times does the operator make up the shortage from his No. 2 bin? Lastly, "What cold feed sample does the inspector or the contractor's technician take to send to the laboratory for a mix design or as a basis for a change in job-mix formula?"

The gradation and statistical summary parameters for this series are presented in Table 25. The degree of overall variability (DOV), 45.0, is the highest of any of the stockpiles constructed and statistically analyzed by this Research Agency in the combined HR 10-2, HR 10-3, and HR 10-3(1) studies. The only other one that comes close, DOV = 42.0, is the coned pile built with 1 in. to No. 4 rounded gravel at Baltimore for HR 10-3(1). This means that the standard deviation at the 50 percent passing level, σ_{o50}, is 45 percent of the standard deviation theoretically associated with complete segregation, σ_{max}.

The corresponding degree of segregation, D of S, values for both the stockpile and the cold feed bin sampling points are also large, indicating a relatively high degree of batch-to-batch segregation. It is interesting to note, however, that the corresponding segregation index values, $S_{\overline{A}}$ and S_{50}, for the stockpile samples do not reflect this high degree of either overall variability or segregation, because of the influence

TABLE 24

SUMMARY OF GRADATION AVERAGES, VARIANCES, AND STANDARD DEVIATIONS, SERIES 3 AGGREGATE[a]

SIEVE SIZE	SAMPLING POINT	AVG. % PASSING, $\bar{X}$	VARIANCE					STANDARD DEVIATION				
			TESTING, σ^2_t	INHERENT σ^2_a	OVERALL, σ^2_o	WITHIN-BATCH, σ^2_b	BATCH-TO-BATCH, σ^2_l	TESTING, σ_t	INHERENT σ_a	OVERALL, σ_o	WITHIN-BATCH, σ_b	BATCH-TO-BATCH, σ_l
1½ in.	Crusher	91.6	0.32	10.65	8.6	5.6	3.0	0.6	3.4	2.9	2.4	1.7
	Stockpile	90.0	0.32	10.65	54.2	31.6	22.6	0.6	3.4	7.4	5.6	4.8
	Bin discharge[b]	90.0	0.32	10.65	26.5	5.6	20.9	0.6	3.4	5.1	2.4	4.6
¾ in.	Crusher	37.6	0.14	7.24	44.7	37.9	6.8	0.4	2.7	6.7	6.2	2.6
	Stockpile	28.8	0.14	7.24	409.3	190.8	218.5	0.4	2.7	20.2	13.8	14.8
	Bin discharge[b]	32.9	0.14	7.24	158.4	13.2	145.2	0.4	2.7	12.6	3.6	12.0
⅜ in.	Crusher	3.5	0.07	1.13	2.0	1.1	0.9	0.3	1.1	1.4	1.0	0.9
	Stockpile	4.2	0.07	1.13	14.1	9.7	4.4	0.3	1.1	3.8	3.1	2.1
	Bin discharge[b]	7.8	0.07	1.13	18.4	2.4	16.0	0.3	1.1	4.3	1.6	4.0
No. 4	Crusher	1.1	0.06	0.58	0.1	0.0	0.0	0.3	0.8	0.3	0.0	0.0
	Stockpile	2.2	0.06	0.58	2.5	1.8	0.7	0.3	0.8	1.6	1.3	0.8
	Bin discharge[b]	4.4	0.06	0.58	5.2	0.6	4.6	0.3	0.8	2.3	0.8	2.1
No. 8	Crusher	0.9	0.06	0.36	0.0	0.0	0.0	0.3	0.6	0.2	0.0	0.0
	Stockpile	1.8	0.06	0.36	1.2	0.8	0.4	0.3	0.6	1.1	0.9	0.6
	Bin discharge[b]	3.6	0.06	0.36	2.8	0.3	2.5	0.3	0.6	1.7	0.5	1.6
$\bar{A}$	Crusher	1.362	0.0006	—	0.0099	0.0071	0.0028	0.03	—	0.100	0.084	0.053
	Stockpile	1.299	0.0006	—	0.1005	0.0496	0.0509	0.03	—	0.316	0.223	0.226
	Bin discharge[b]	1.441	0.0006	—	0.0688	0.0055	0.0633	0.03	—	0.262	0.074	0.251

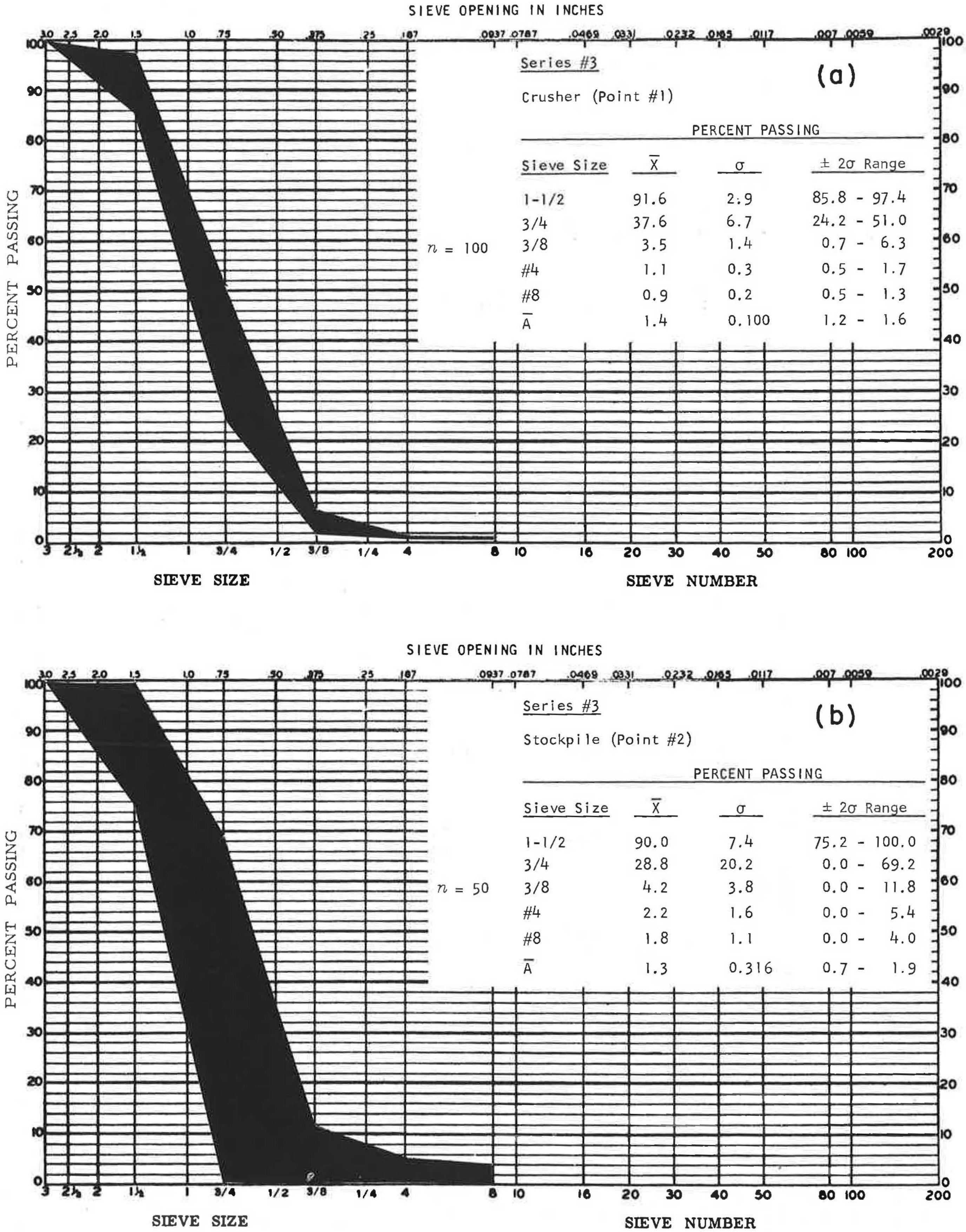
[a] See Table 13 for description of aggregate. [b] Cold feed bin discharge.

TABLE 25

SUMMARY OF GRADATION AND STATISTICAL PARAMETERS, SERIES 3 AGGREGATE[a]

SAMPLING POINT		AVERAGE SIZE, $\bar{X}$				50% LEVEL STD. DEVIATION				HUDSON $\bar{A}$ STD. DEVIATION			SEGREGATION INDEX		DEG. OF OVERALL VARIA-BILITY, DOV	DEG. OF SEGRE-GATION, D OF S
NO.	LOCATION	% PASS. 1½ IN.	% PASS. ¾ IN.	% PASS. ⅜ IN.	HUDSON $\bar{A}$	OVERALL, σ_{o-50}	WITHIN-BATCH, σ_{b-50}	BATCH-TO-BATCH, σ_{l-50}	SLOPE	OVERALL, $\sigma_{o-\bar{A}}$	WITHIN-BATCH, $\sigma_{b-\bar{A}}$	BATCH-TO-BATCH, $\sigma_{l-\bar{A}}$	BASED ON $\bar{A}$, $S_{\bar{A}}$	BASED ON 50% P, S_{50}		
1	Crusher	91.6	37.6	3.5	1.362	7.1	6.0	4.3	1.73	0.100	0.084	0.053	1.4	1.4	14.1	8.0
2	Stockpile	90.0	28.8	4.2	1.299	22.5	15.5	16.4	2.14	0.316	0.223	0.226	2.0	2.1	45.0	32.8
3	Bin discharge[b]	90.0	32.9	7.8	1.441	14.5	5.0	13.6	1.73	0.262	0.074	0.251	12.5	8.4	29.0	27.2

[a] See Table 13 for description of aggregate. [b] Cold feed bin discharge.



Series #3

Crusher (Point #1)

Sieve Size	PERCENT PASSING $\overline{X}$	σ	± 2σ Range
1-1/2	91.6	2.9	85.8 - 97.4
3/4	37.6	6.7	24.2 - 51.0
3/8	3.5	1.4	0.7 - 6.3
#4	1.1	0.3	0.5 - 1.7
#8	0.9	0.2	0.5 - 1.3
$\overline{A}$	1.4	0.100	1.2 - 1.6

Series #3

Stockpile (Point #2)

Sieve Size	PERCENT PASSING $\overline{X}$	σ	± 2σ Range
1-1/2	90.0	7.4	75.2 - 100.0
3/4	28.8	20.2	0.0 - 69.2
3/8	4.2	3.8	0.0 - 11.8
#4	2.2	1.6	0.0 - 5.4
#8	1.8	1.1	0.0 - 4.0
$\overline{A}$	1.3	0.316	0.7 - 1.9

Figure 31. Aggregate gradation charts for Series No. 3 samples taken at (a) crusher, (b) stockpile, and (c) cold feed bin discharge.

of the high within-batch variance, σ^2_b, on this ratio. Again, the point is illustrated that no one parameter thus far developed is capable of reflecting a complete picture of the factors involved in the control of construction aggregates. On the other hand, consideration of these parameters as a whole, and their relation one to the other, does provide some insight into the mechanisms involved and a better understanding of the construction control procedures needed.

Series No. 4—Uncrushed Gravel (1 in.-No. 4) Aggregate for PC Concrete

PLANT DESCRIPTION AND PROCESS FLOW

This sampling was from the process stream (screens, storage pile, plant stockpile, weigh hopper) of the aggregate supply to a batch plant for transit-mix trucks in Wyoming. The resulting concrete is used for municipal paving and for structures.

Raw aggregate at this production set-up was fed through a screening and washing unit, from which it was discharged into a bin (sampling point No. 1). The aggregate was then stockpiled into a large pile at the production plant (sampling point No. 2). It was then transported to the concrete plant, where it was again built into a large stockpile (sampling point No. 3). From the plant stockpile, it was conveyed to the concrete plant bins (sampling point No. 4), from which it was drawn into the weigh hopper.

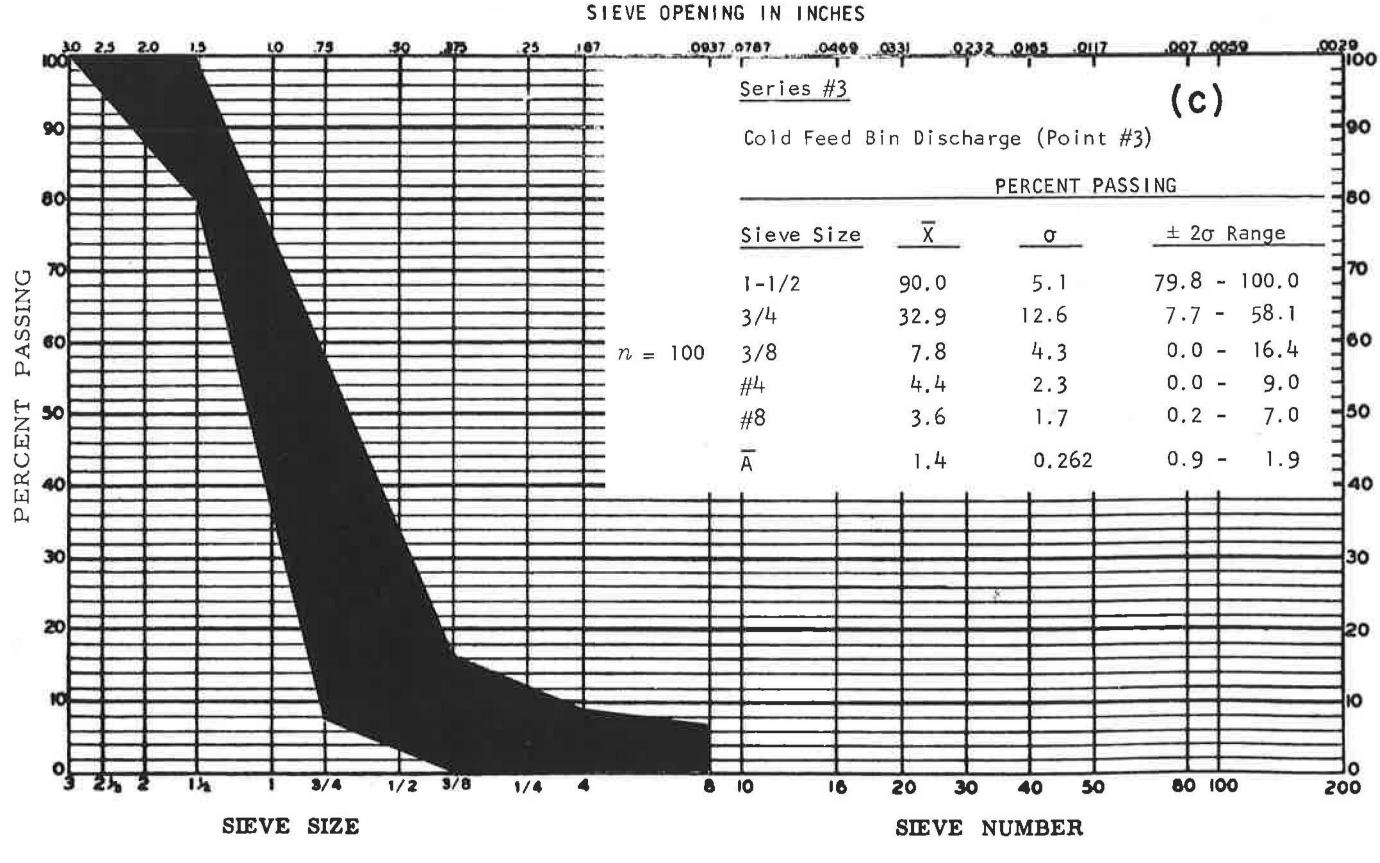
VARIATION IN GRADATION AT POINT OF USE

Sample increments of aggregate were taken by sliding a sample container into the gravity feed (sampling point No. 4) from the overhead storage bin into the weigh hopper. Two increments, having an average weight of 27 lb each, were taken from each 1½-cu yd batch selected by the use of a table of random numbers. The entire increment was used as a test portion and was sieved to refusal on a Gilson shaker.

The specification requirements for the gradation of the gravel are compared with the overall variation and batch-to-batch variations in Table 26. The overall variation shows the limits within which 95 percent of the results of tests on single increments, selected at random, could be expected to fall. The batch-to-batch variation shows the probable limits within which the gradation of the coarse aggregate in 95 percent of the batches would fall, if the entire batch was put through the sieves.

VARIATION IN GRADATION AT OTHER SAMPLING POINTS

To compare the specified gradation with the gradation at the source and at intermediate sampling points, random sample increments were taken, in duplicate, from the screen discharge (sampling point No. 1), from the storage stockpile at the screening plant (sampling point No. 2), and from the stockpile at the batching plant (sampling point



Series #3

Cold Feed Bin Discharge (Point #3)

n = 100

	PERCENT PASSING		
Sieve Size	$\overline{X}$	σ	± 2σ Range
1-1/2	90.0	5.1	79.8 - 100.0
3/4	32.9	12.6	7.7 - 58.1
3/8	7.8	4.3	0.0 - 16.4
#4	4.4	2.3	0.0 - 9.0
#8	3.6	1.7	0.2 - 7.0
$\overline{A}$	1.4	0.262	0.9 - 1.9

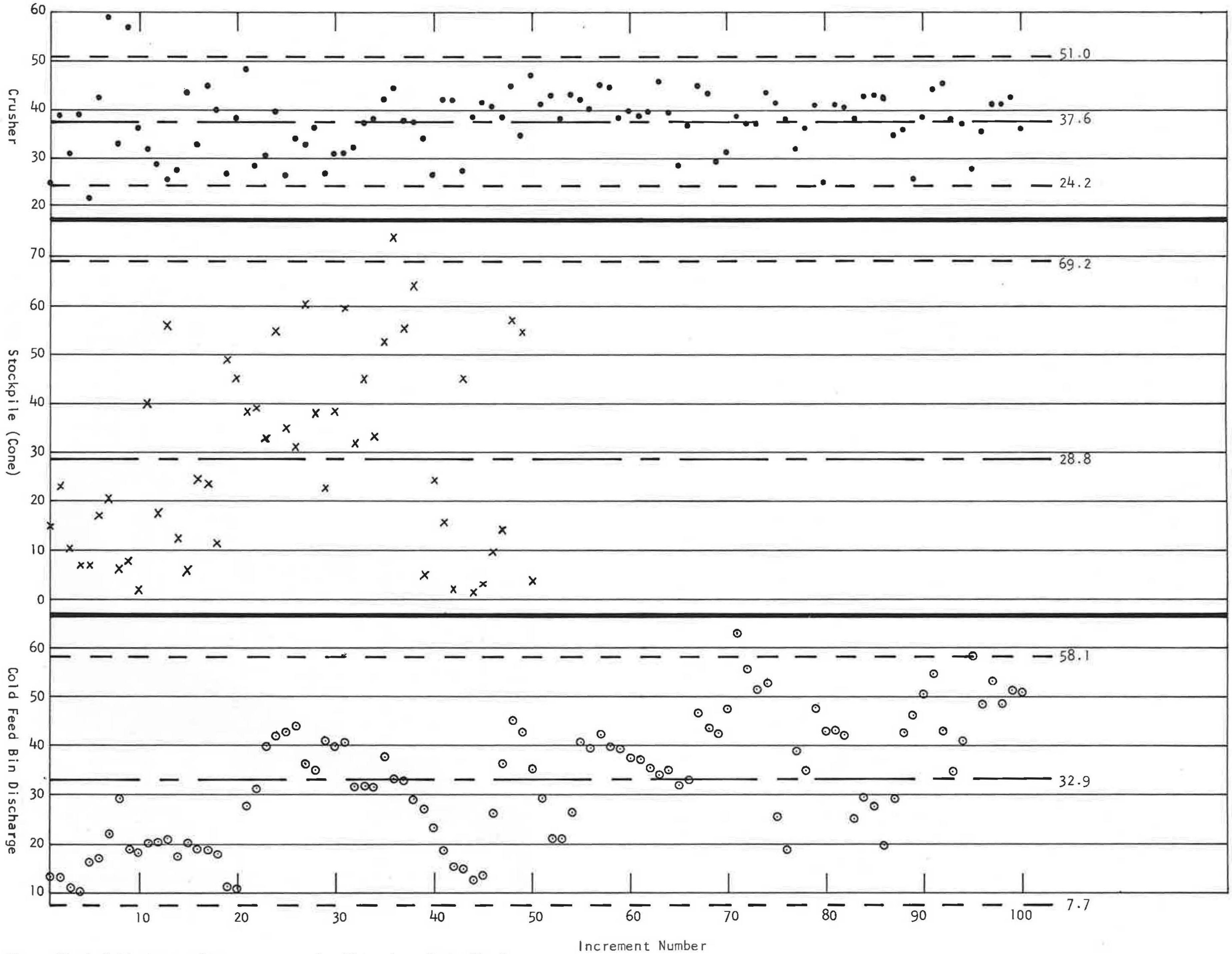


Figure 32. Individual test values, percent passing ¾-in. sieve, Series No. 3.

TABLE 26

GRADATION OF SERIES 4 GRAVEL AT POINT OF USE

SIEVE SIZE	PERCENT PASSING SIEVE SPEC. LIMITS	OVERALL VARIATION, $\pm 2\sigma_o$ LIMITS	BATCH-TO-BATCH VARIATION, $\pm 2\sigma_l$ LIMITS
1 in.	100	—	—
¾ in.	—	79-95	80-94
3/3 in.	15-35	23-46	24-45
No. 4	0-10	2-10	2-9
No. 8	—	0-7	0-6

TABLE 27

GRADATION OF SERIES 4 GRAVEL AT OTHER SAMPLING POINTS

SIEVE SIZE	PERCENT PASSING SIEVE, $\pm 2\sigma_o$ LIMITS SPEC. LIMITS	SCREEN DISCHARGE	STORAGE PILE	PLANT STOCK-PILE
1 in.	100	—	—	—
¾ in.	—	72-91	72-100	73-95
⅜ in.	15-35	14-37	8-52	12-40
No. 4	0-10	1-3	0-3	0-2
No. 8	—	0-1	0-1	0-1

No. 3). The overall variations found at these points are compared with the specification limits in Table 27.

SERIES 4 SUMMARY CHARTS

The relative variances and standard deviations of the percentages passing the different sieves are summarized in Table 28.

As might be anticipated, rounded gravels have a greater tendency toward segregation than do crushed aggregates. In general, the variability associated with this Series 4 operation is considered to be about normal for rounded gravel processing. The amount of segregation increases moderately due to stockpiling at the screening plant, but the subsequent transportation and handling again reduce the variability to about the level of the screening plant bin discharge. Both at the source and at the point of use, the overall variability is sufficient to cause the percent passing the ⅜-in. sieve to exceed the specification limits at the $\pm 2\sigma$ level. Because the performance of the concrete produced from this aggregate is presumably satisfactory, wider specification limits for the ⅜-in. sieve are indicated.

The relative variance of $\bar{A}$ at the different sampling points is shown in Figure 33. The aggregate grading charts showing the $\pm 2\sigma$ envelopes at each of the four sampling points are presented in Figure 34. These charts reflect the moderately high, but reasonably consistent, segregation previously noted.

Figure 35 is a plot of the individual values for percent passing the ⅜-in. sieve at each of the four sampling points.

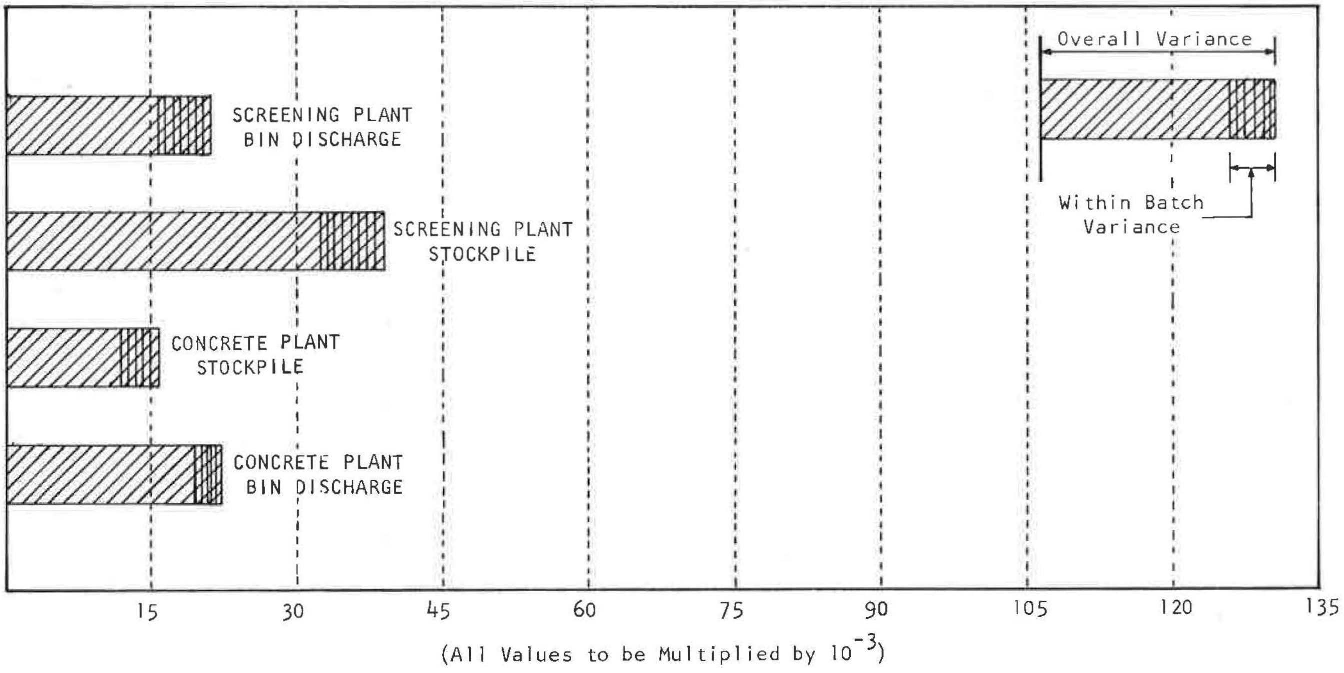
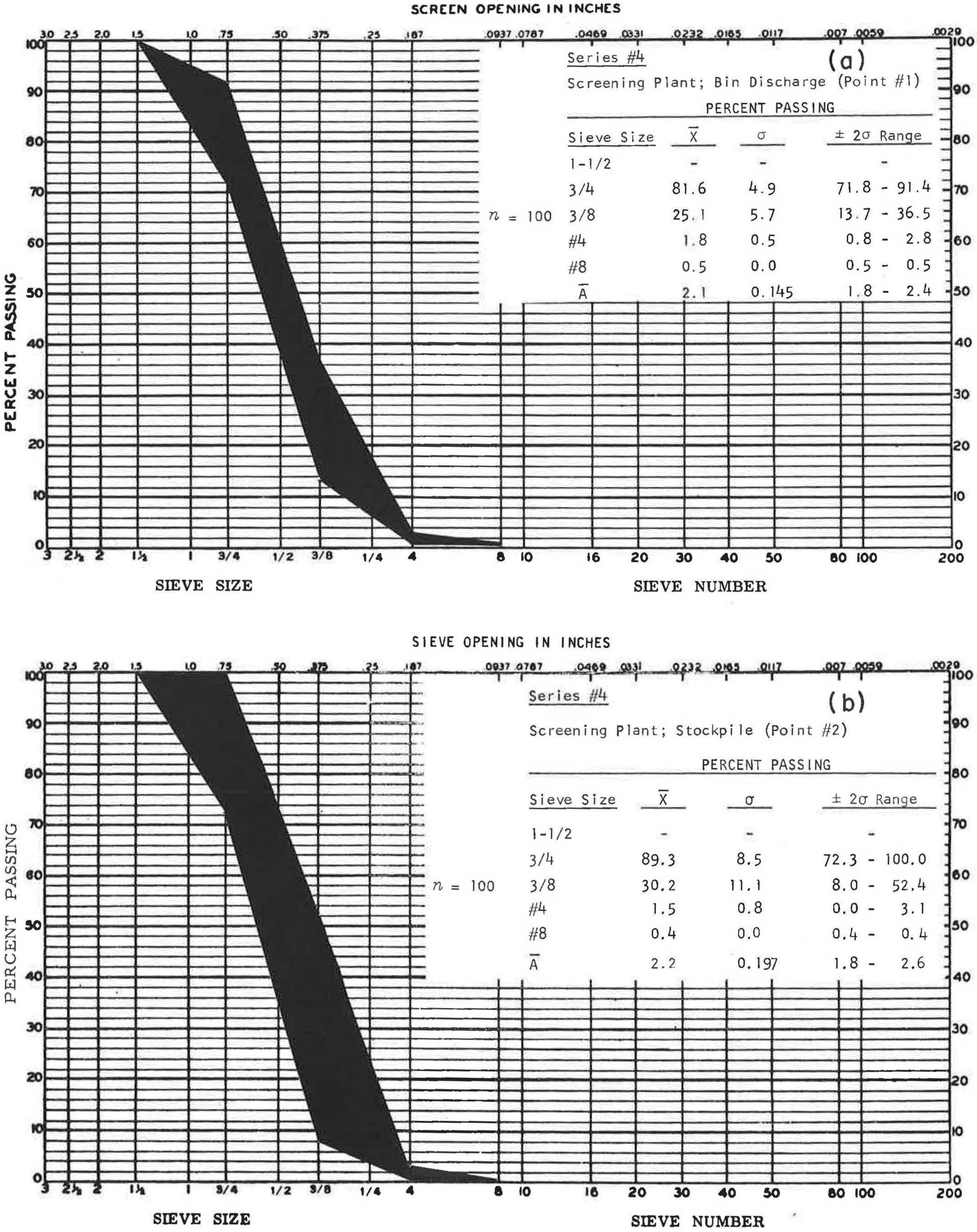


Figure 33. Relative variance of $\bar{A}$ at different points in processstream, Series No. 4, rounded gravel (1 in.-No. 4), Wyoming.



Series #4

Screening Plant; Bin Discharge (Point #1)

n = 100

	PERCENT PASSING		
Sieve Size	X̄	σ	± 2σ Range
1-1/2	-	-	-
3/4	81.6	4.9	71.8 - 91.4
3/8	25.1	5.7	13.7 - 36.5
#4	1.8	0.5	0.8 - 2.8
#8	0.5	0.0	0.5 - 0.5
Ā	2.1	0.145	1.8 - 2.4

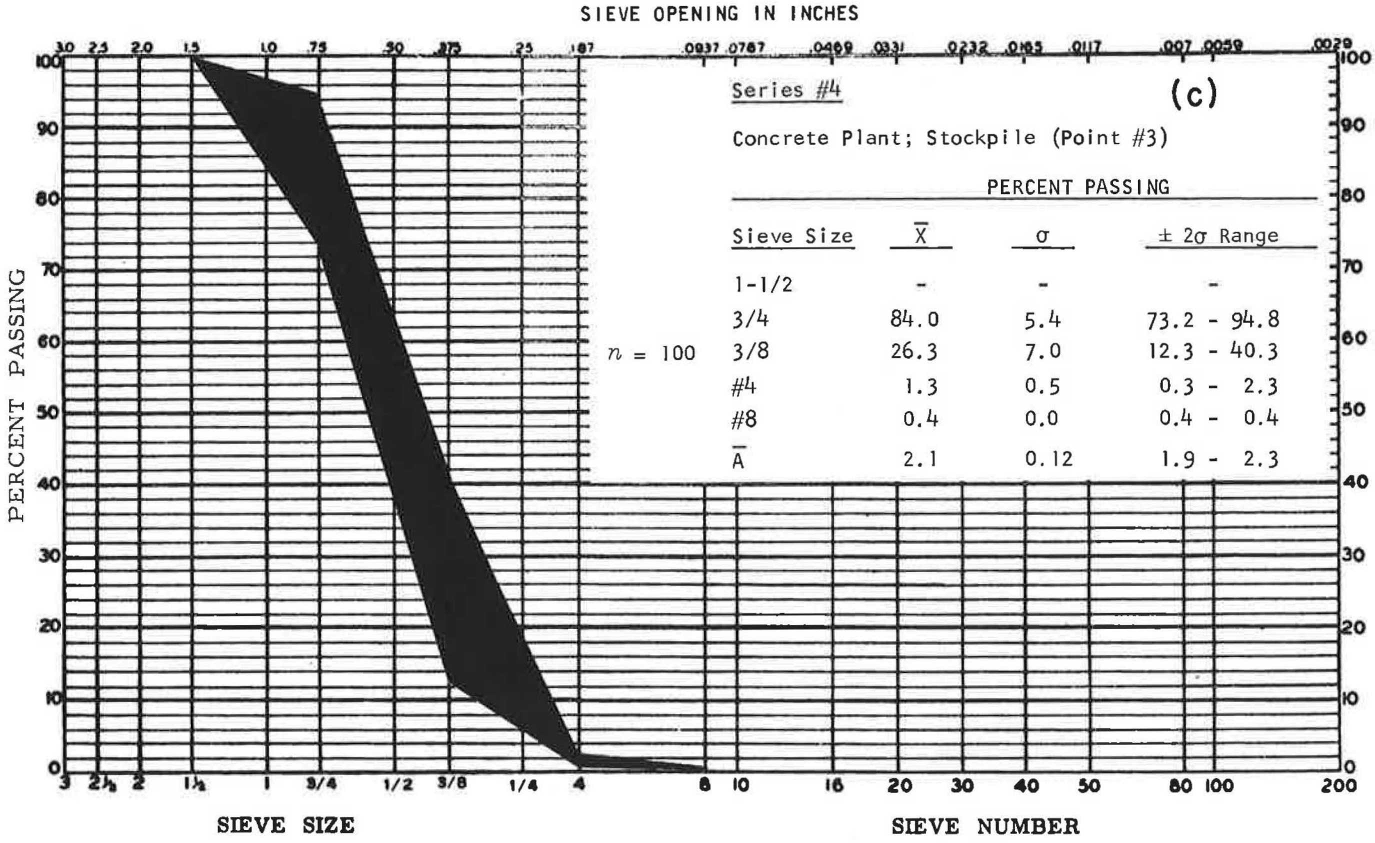
Series #4

Screening Plant; Stockpile (Point #2)

n = 100

	PERCENT PASSING		
Sieve Size	X̄	σ	± 2σ Range
1-1/2	-	-	-
3/4	89.3	8.5	72.3 - 100.0
3/8	30.2	11.1	8.0 - 52.4
#4	1.5	0.8	0.0 - 3.1
#8	0.4	0.0	0.4 - 0.4
Ā	2.2	0.197	1.8 - 2.6

Figure 34. Aggregate gradation charts for Series No. 4 samples taken at (a) screening plant bin discharge, (b) screening plant stockpile, (c) concrete plant stockpile, and (d) concrete plant bin discharge.



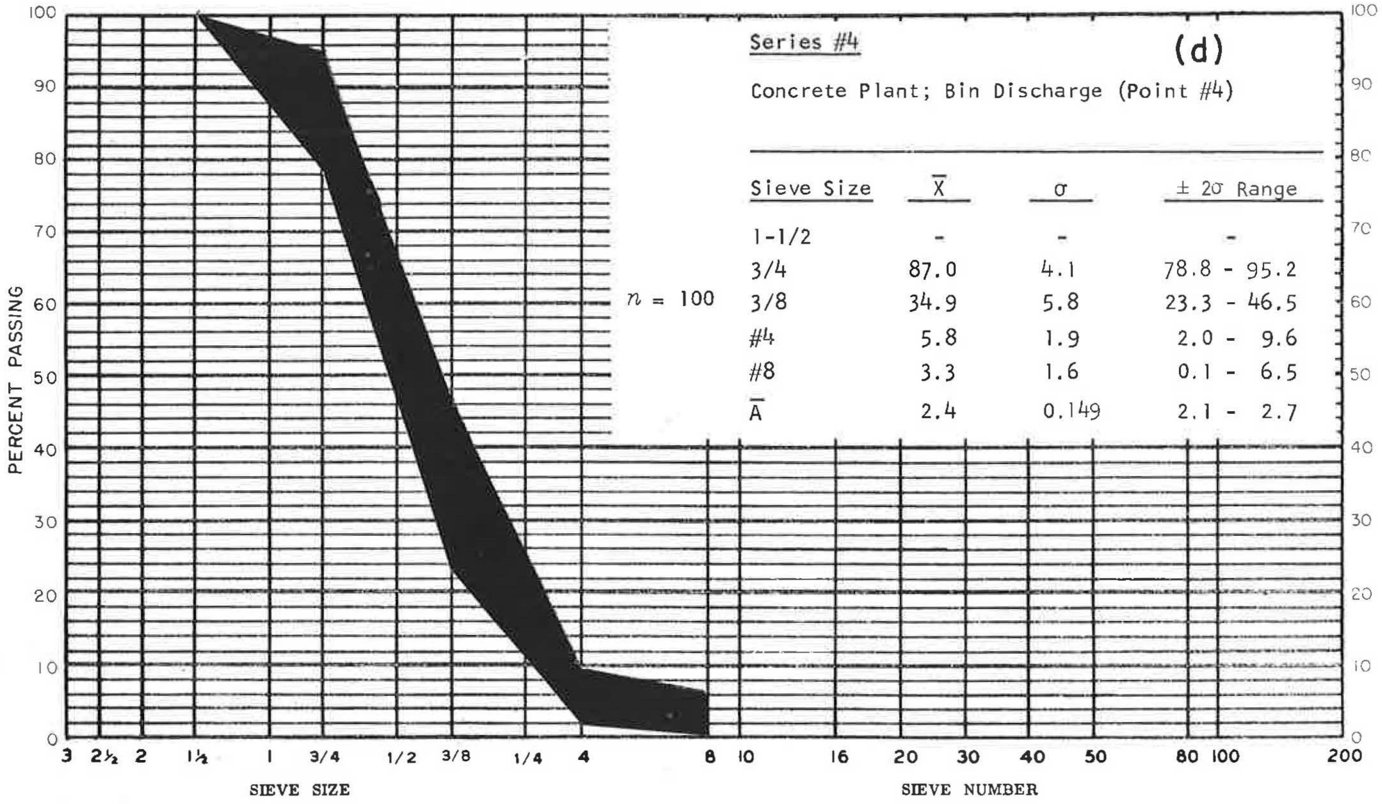
Series #4

Concrete Plant; Stockpile (Point #3)

PERCENT PASSING

n = 100

Sieve Size	$\overline{X}$	σ	± 2σ Range
1-1/2	-	-	-
3/4	84.0	5.4	73.2 - 94.8
3/8	26.3	7.0	12.3 - 40.3
#4	1.3	0.5	0.3 - 2.3
#8	0.4	0.0	0.4 - 0.4
$\overline{A}$	2.1	0.12	1.9 - 2.3

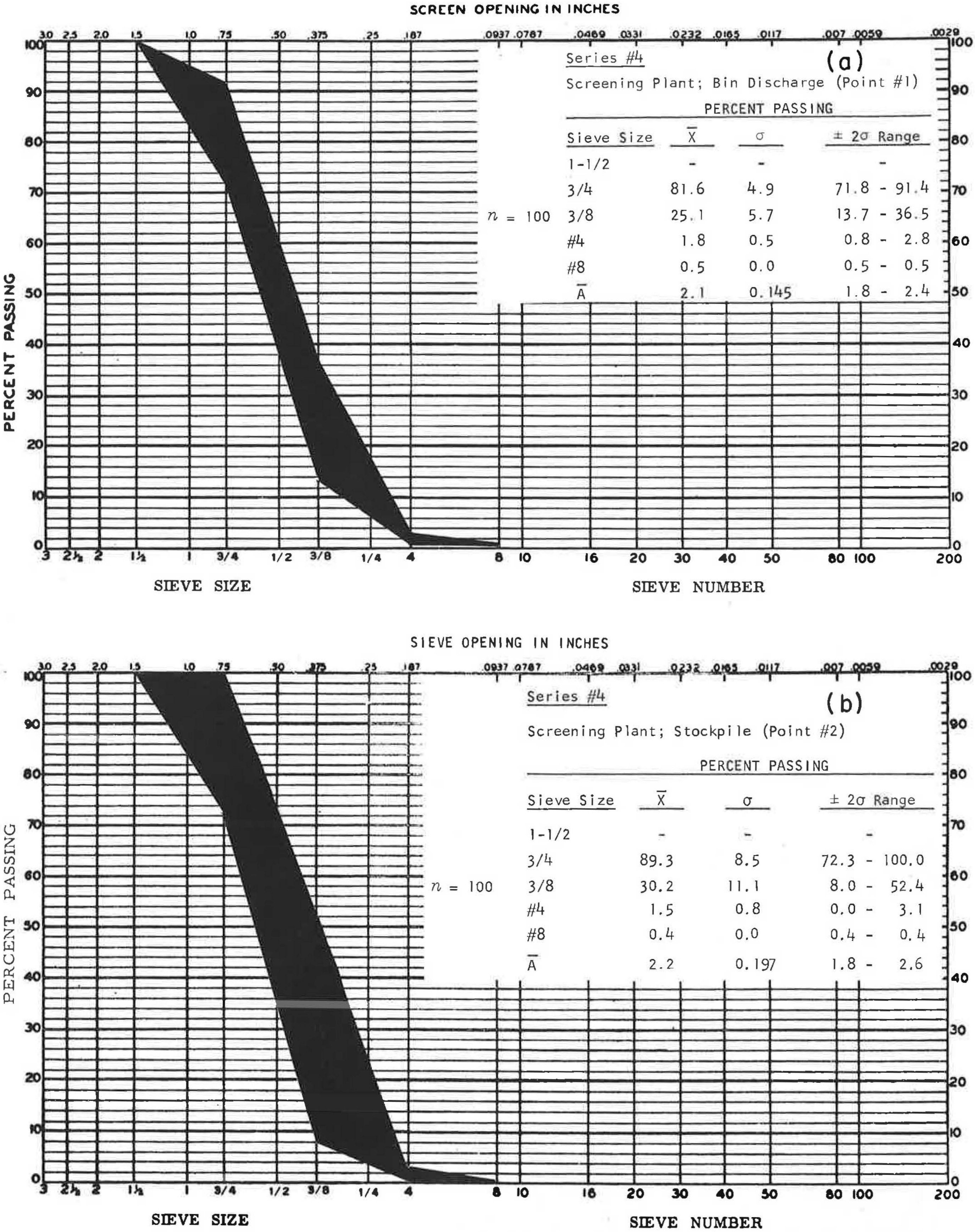


Series #4

Concrete Plant; Bin Discharge (Point #4)

n = 100

Sieve Size	$\overline{X}$	σ	± 2σ Range
1-1/2	-	-	-
3/4	87.0	4.1	78.8 - 95.2
3/8	34.9	5.8	23.3 - 46.5
#4	5.8	1.9	2.0 - 9.6
#8	3.3	1.6	0.1 - 6.5
$\overline{A}$	2.4	0.149	2.1 - 2.7



Series #4

Screening Plant; Bin Discharge (Point #1)

n = 100

Sieve Size	$\overline{X}$	σ	± 2σ Range
	PERCENT PASSING		
1-1/2	-	-	-
3/4	81.6	4.9	71.8 - 91.4
3/8	25.1	5.7	13.7 - 36.5
#4	1.8	0.5	0.8 - 2.8
#8	0.5	0.0	0.5 - 0.5
$\overline{A}$	2.1	0.145	1.8 - 2.4

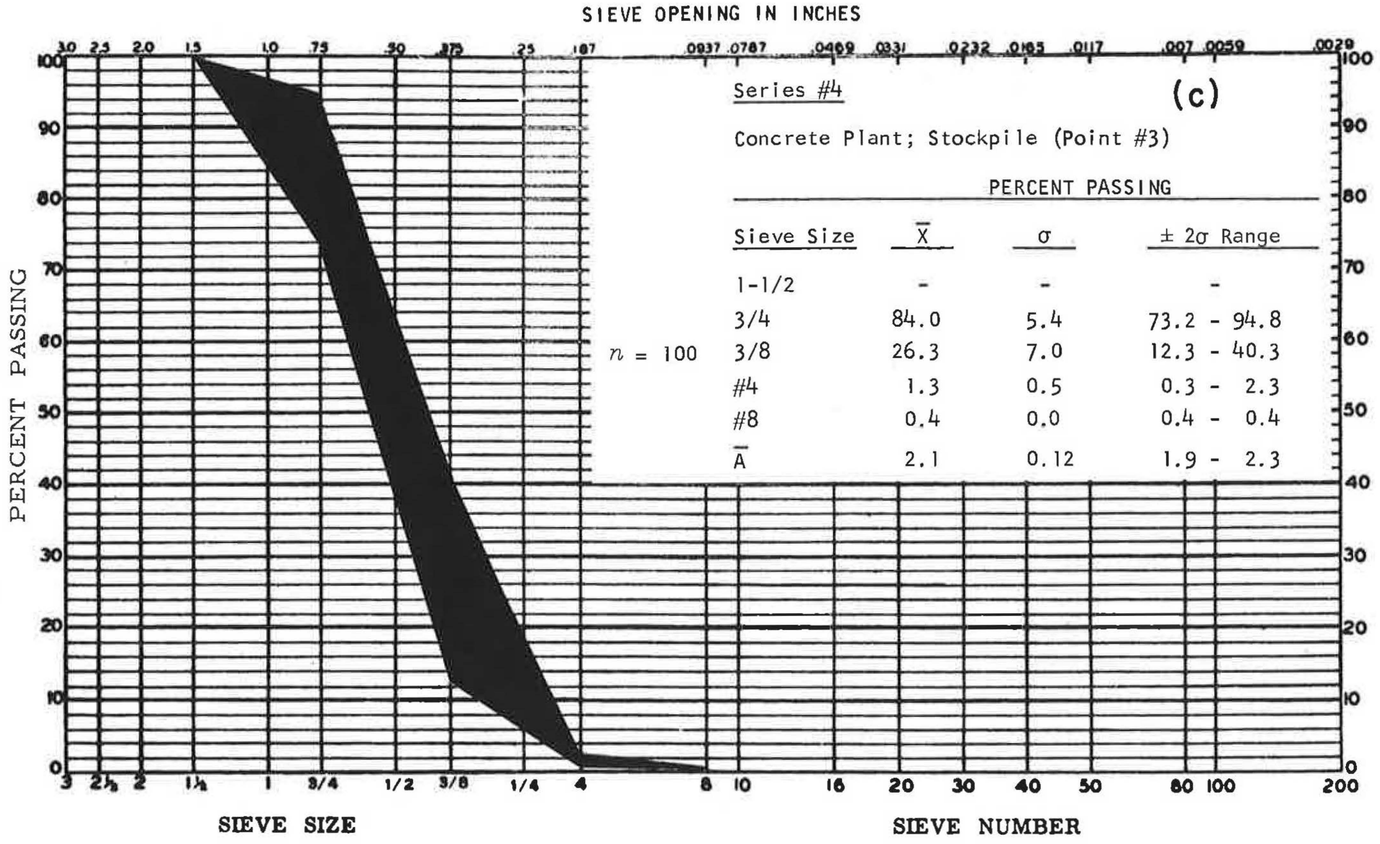
Series #4

Screening Plant; Stockpile (Point #2)

n = 100

Sieve Size	$\overline{X}$	σ	± 2σ Range
	PERCENT PASSING		
1-1/2	-	-	-
3/4	89.3	8.5	72.3 - 100.0
3/8	30.2	11.1	8.0 - 52.4
#4	1.5	0.8	0.0 - 3.1
#8	0.4	0.0	0.4 - 0.4
$\overline{A}$	2.2	0.197	1.8 - 2.6

Figure 34. Aggregate gradation charts for Series No. 4 samples taken at (a) screening plant bin discharge, (b) screening plant stockpile, (c) concrete plant stockpile, and (d) concrete plant bin discharge.

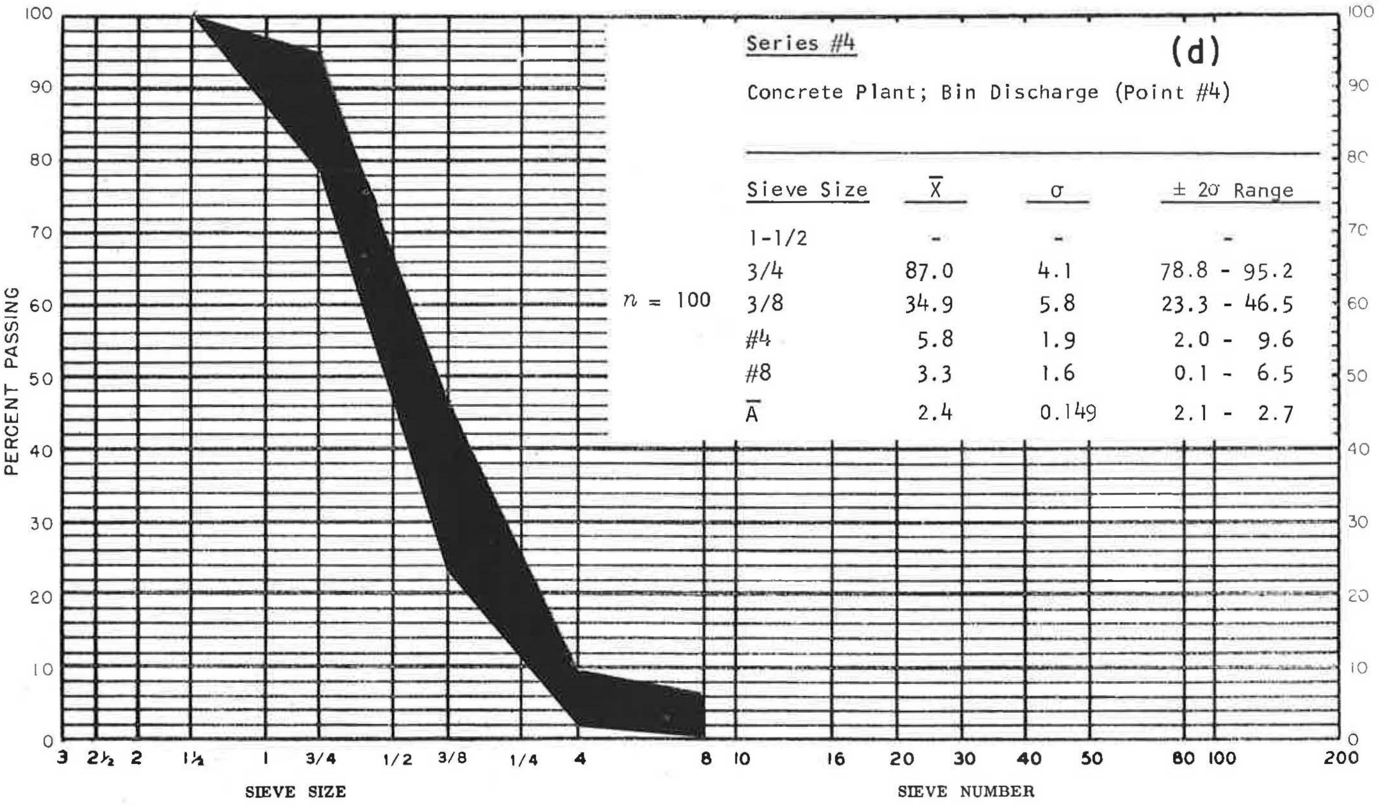


Series #4

Concrete Plant; Stockpile (Point #3)

$n = 100$

	PERCENT PASSING		
Sieve Size	$\overline{X}$	σ	± 2σ Range
1-1/2	-	-	-
3/4	84.0	5.4	73.2 - 94.8
3/8	26.3	7.0	12.3 - 40.3
#4	1.3	0.5	0.3 - 2.3
#8	0.4	0.0	0.4 - 0.4
$\overline{A}$	2.1	0.12	1.9 - 2.3



Series #4

Concrete Plant; Bin Discharge (Point #4)

$n = 100$

Sieve Size	$\overline{X}$	σ	± 2σ Range
1-1/2	-	-	-
3/4	87.0	4.1	78.8 - 95.2
3/8	34.9	5.8	23.3 - 46.5
#4	5.8	1.9	2.0 - 9.6
#8	3.3	1.6	0.1 - 6.5
$\overline{A}$	2.4	0.149	2.1 - 2.7

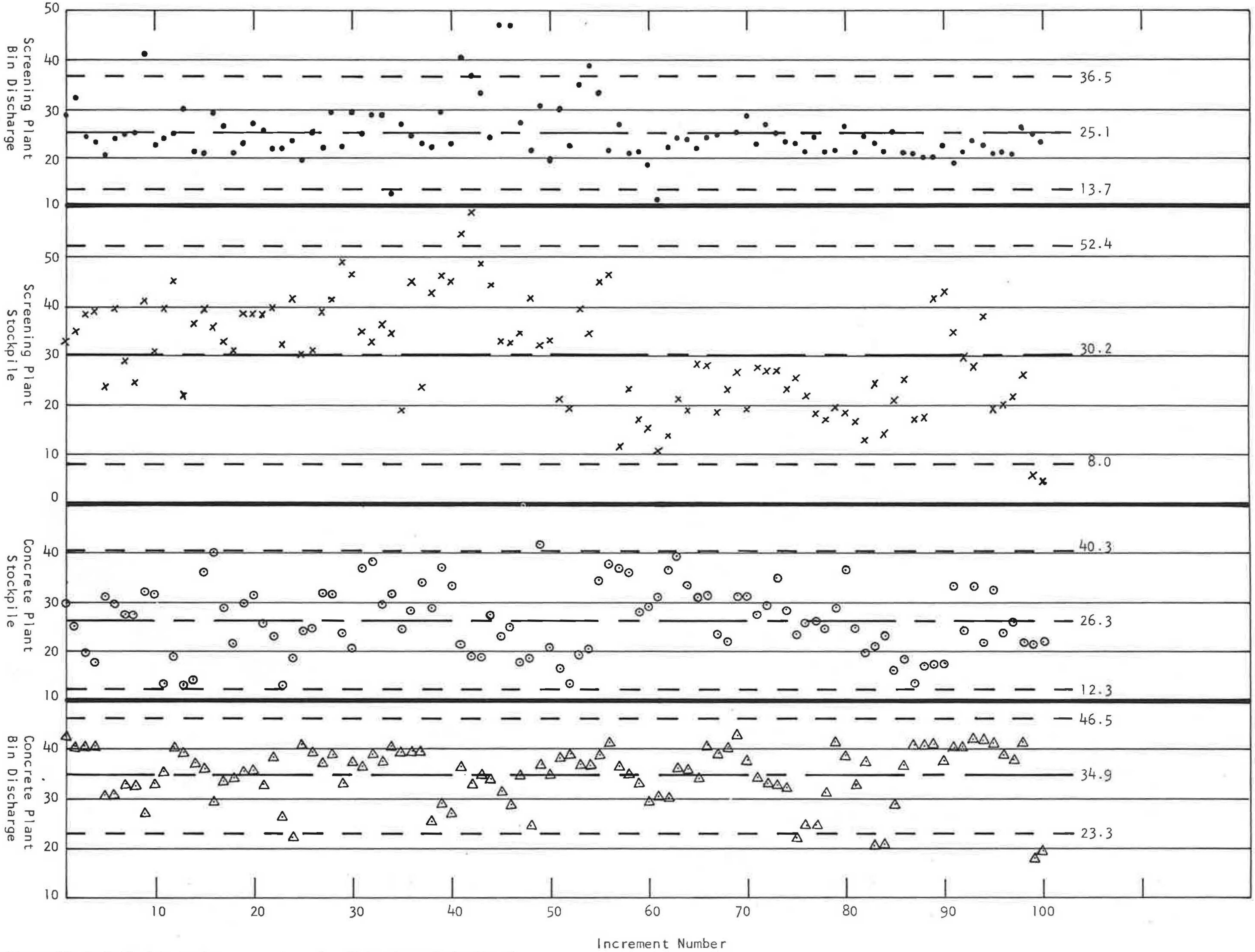


Figure 35. Individual test values, percent passing 3/8-in. sieve, Series No. 4.

Again, the pattern at the bin discharge sampling points is of interest. At the screening plant, the bin discharge samples do not reflect a systematic surging or periodic change in gradation. Something happened between samples 40 and 60 which resulted in a greater variability, but, in general, the pattern is reasonably consistent. However, at the concrete plant bin discharge feeding the weigh hopper (sampling point No. 4), the situation is quite different. Here, there is a clear indication of a surge or interval during which appreciably coarser material is discharged. Although the pattern in this case is not regular in the sense of recurring after a specific time or number of samples, it does appear that the percent passing the ⅜-in. sieve will ride along at a 30 to 40 level, and then, suddenly, there will be two or three samples down around the 20 to 25 percent passing level.

Again, from a construction control viewpoint, it is fair to ask what effect this difference might have on the quality of the concrete produced at this plant; and the related question of which sample the inspector should use for purposes of specification compliance; and which sample should be used to judge the need for and the extent of any adjustment in mix proportions that might be contemplated.

The gradation and statistical summary parameters for this series are presented in Table 29. There is nothing worthy of further comment at this point, but these data are used for comparisons with other series in Chapter Five.

Series No. 5—Crushed Aggregate (1 in.-No. 200) Base Course

PLANT DESCRIPTION AND PROCESS FLOW

This sampling was from the process stream (feed to mixer, truckloads of mixed material, and material in place on the roadway) of a continuous pugmill producing aggregate base course for State highway construction in North Carolina.

This base course aggregate was produced from a crusher-run material, which was stockpiled by belt conveyor into a very large pile located over a tunnel belt. By opening one or more of several gates located beneath the pile, the quantity of material deposited on the belt could be varied. The aggregate was moved by this belt to the mixer unit, where water was added. This belt was intersected at right angles by a second belt at about its midpoint between the stockpile and the mixer unit. The second belt fed a continuous stream of fine material (stone screenings) onto the main belt. The fine material was deposited on top of the crusher-run material in proportions of approximately 15 percent fines and 85 percent crusher run. Sampling point No. 1 was located on the belt immediately after the fines had been added. After pugmill mixing at optimum moisture (2-ton batches), the base course aggregate was dropped into trucks for transport to the roadway. Sampling point No. 2 was from the trucks using a sample thief. The trucks deposited the aggregate into a mechanical spreader, which put down a shoulder adjacent to concrete pavement in a single course having an uncompacted thickness of about 8 in. After spreading, the aggregate was sampled by forcing a 10-in. diameter hoop into the course, and removing all material within the hoop for the full depth of the course (sampling point No. 3).

VARIATION IN GRADATION AT POINT OF USE

As mentioned, sample increments were taken by use of a hoop sampling device from the spread, but uncompacted, base material in place on the roadway (sampling point No. 3). Two increments having an average weight of 23 lb each were taken at each sampling point. The clear distance between the two test portions was approximately 1 ft, and the sampling points were chosen by the use of a table of random numbers. The entire increment, or quantity of material removed from the hoop, was used as a test portion for testing the gradation of the coarse aggregate. The gradation of the minus No. 8 material was determined by taking a test portion of about 200 g from the part of each increment that passed the No. 8 sieve. The coarse aggregate was tested on a Gilson, and the fine aggregate on a Newark, sieve shaker.

Inasmuch as the specifications contain requirements based on nonstandard sieves, the results cannot be compared directly with the specifications. However, the specified gradation range and the tabulated values for overall and actual variation are shown in Table 30. The overall variation shows the limits within which 95 percent of the results of tests on single test portions taken from the roadway could be expected to fall. The batch-to-batch variation shows the probable limits within which the gradation of the material in 95 percent of the base would fall if an entire square yard of material was put through the sieves.

VARIATION IN GRADATION AT OTHER SAMPLING POINTS

To compare the variation in gradations of the unmixed material with that found when sampling the truckloads of mixed material and the material in place on the roadway, increments were taken in duplicate, by probability sampling from the belt feeding the pugmill (sampling point No. 1) and from the trucks immediately after loading (sampling point No. 2).

The overall variations found at the three process points are compared in Table 31.

SERIES 5 SUMMARY CHARTS

The relative variances and standard deviations of the percentage passing the different sieves are summarized in Table 32. In this case there was a relatively small amount of segregation of the mixed material, probably due to the moist fine aggregate in the mixture. However, as shown in Figure 36, there was considerable variation in the relative coarseness of the mixture in the trucks and in the actual base as constructed. In general, the material was much coarser than required by the specifications. Inasmuch as the

TABLE 28

SUMMARY OF GRADATION AVERAGES, VARIANCES, AND STANDARD DEVIATIONS, SERIES 4 AGGREGATE [a]

			VARIANCE					STANDARD DEVIATION				
SIEVE SIZE	SAMPLING POINT	AVG. % PASSING, $\bar{X}$	TESTING, σ^2_t	INHERENT σ^2_a	OVERALL, σ^2_o	WITHIN-BATCH, σ^2_b	BATCH-TO-BATCH, σ^2_l	TESTING, σ_t	INHERENT σ_a	OVERALL, σ_o	WITHIN-BATCH, σ_b	BATCH-TO-BATCH, σ_l
¾ in.	Scr. pl. bin disch.	81.6	0.20	3.24	24.4	12.5	11.9	0.5	1.8	4.9	3.5	3.5
	Scr. pl. stockpile	89.3	0.20	3.24	73.0	12.0	61.0	0.5	1.8	8.5	3.5	7.8
	Conc. pl. stockpile	84.0	0.20	3.24	29.6	6.9	22.7	0.5	1.8	5.4	2.6	4.8
	Conc. pl. bin disch.[b]	87.0	0.20	3.24	16.5	2.7	13.8	0.5	1.8	4.1	1.7	3.7
⅜ in.	Scr. pl. bin disch.	25.1	0.26	1.44	33.1	16.1	17.0	0.5	1.2	5.7	4.0	4.1
	Scr. pl. stockpile	30.2	0.26	1.44	122.5	24.6	97.9	0.5	1.2	11.1	5.0	9.9
	Conc. pl. stockpile	26.3	0.26	1.44	49.6	13.3	36.3	0.5	1.2	7.0	3.6	6.0
	Conc. pl. bin disch.[b]	34.9	0.26	1.44	33.5	6.1	27.4	0.5	1.2	5.8	2.5	5.3
No. 4	Scr. pl. bin disch.	1.8	0.02	0.16	0.3	0.2	0.1	0.1	0.4	0.5	0.4	0.3
	Scr. pl. stockpile	1.5	0.02	0.16	0.6	0.2	0.4	0.1	0.4	0.8	0.4	0.6
	Conc. pl. stockpile	1.3	0.02	0.16	0.3	0.1	0.2	0.1	0.4	0.5	0.3	0.4
	Conc. pl. bin disch.[b]	5.8	0.02	0.16	3.6	0.5	3.1	0.1	0.4	1.9	0.7	1.8
No. 8	Scr. pl. bin disch.	0.5	0.01	0.09	0.0	0.0	0.0	0.1	0.3	0.0	0.0	0.0
	Scr. pl. stockpile	0.4	0.01	0.09	0.0	0.0	0.0	0.1	0.3	0.0	0.0	0.0
	Conc. pl. stockpile	0.4	0.01	0.09	0.0	0.0	0.0	0.1	0.3	0.0	0.0	0.0
	Conc. pl. bin disch.[b]	3.3	0.01	0.09	2.7	0.4	2.3	0.1	0.3	1.6	0.6	1.5
$\bar{A}$	Scr. pl. bin disch.	2.111	0.0003	—	0.0210	0.0054	0.0156	0.02	—	0.145	0.073	0.125
	Scr. pl. stockpile	2.219	0.0003	—	0.0387	0.0064	0.0323	0.02	—	0.197	0.080	0.180
	Conc. pl. stockpile	2.127	0.0003	—	0.0156	0.0038	0.0118	0.02	—	0.125	0.062	0.109
	Conc. pl. bin disch.[b]	3.372	0.0003	—	0.0220	0.0028	0.0192	0.02	—	0.149	0.053	0.139

[a] See Table 13 for description of aggregate. [b] To weigh hopper.

TABLE 29

SUMMARY OF GRADATION AND STATISTICAL PARAMETERS, SERIES 4 AGGREGATE [a]

SAMPLING POINT		AVERAGE SIZE, $\bar{X}$			50% LEVEL STD. DEVIATION				HUDSON $\bar{A}$ STD. DEVIATION			SEGREGATION INDEX		DEG. OF OVERALL VARIABILITY, DOV	DEG. OF SEGREGATION, D OF S
NO.	LOCATION	% PASS. ¾ IN.	% PASS. ⅜ IN.	HUDSON $\bar{A}$	OVERALL, σ_{o-50}	WITHIN-BATCH, σ_{b-50}	BATCH-TO-BATCH, σ_{l-50}	SLOPE	OVERALL, $\sigma_{o-\bar{A}}$	WITHIN-BATCH, $\sigma_{b-\bar{A}}$	BATCH-TO-BATCH, $\sigma_{l-\bar{A}}$	BASED ON $\bar{A}$, $S_{\bar{A}}$	BASED ON 50% P, S_{50}		
1	Scr. pl. bin disch.	81.6	25.1	2.111	8.5	5.0	6.9	2.14	0.145	0.073	0.125	3.9	2.9	17.0	13.8
2	Scr. pl. stockpile	89.3	30.2	2.219	14.5	6.0	13.2	2.14	0.197	0.080	0.180	6.1	5.8	29.0	26.4
3	Conc. pl. stockpile	84.0	26.3	2.127	9.4	5.0	8.0	2.14	0.155	0.062	0.109	4.1	3.5	18.8	16.0
4	Conc. pl. bin disch.[b]	87.0	34.9	2.372	8.0	3.2	7.3	2.14	0.149	0.053	0.139	7.8	6.4	16.0	14.6

[a] See Table 13 for description of aggregate. [b] To weigh hopper.

material is presumably giving satisfactory service, wider specification limits are indicated.

As shown in Figure 36, a large part of the overall variation of the truck sample was due to within-batch variance. It should also be noted that there are statistically significant differences in gradation between the truck samples and the samples taken from the plant and the roadway. These differences are discussed, together with similar data from Series 2, in Chapter Three, under "Experiments with Different Sampling Tools, Truck Sampling Results."

The aggregate grading charts, showing the $\pm 2\sigma$ envelopes at each of the three sampling points, are presented in Figure 37. The individual test results for percent passing the No. 4 sieve are plotted in Figure 38.

The gradation and statistical summary parameters for this series are presented in Table 33. The relatively high within-batch variability results in a correspondingly low segregation index for both $S_{\bar{A}}$ and S_{50}. The degree of overall variability, DOV, is essentially the same (12.8 and 13.2) for the truck and the roadway samples, indicating that the spreading and manipulating on the roadway has not resulted in increased overall variability. The degree of segregation has about doubled (roadway versus truck samples), but it is probable that, in this case, the difference in segregation pattern—σ_b versus σ_l—is influenced by the sampling method and sampling tools used, rather than reflecting a really significant difference in aggregate behavior. In any case, it is apparent that well-graded aggregates have less tendency to segregate than those having a deficiency of fines.

There is another difference in the pattern of variability

TABLE 30
GRADATION OF SERIES 5 AGGREGATE BASE ON ROADWAY

	PERCENT PASSING SIEVE		
SIEVE SIZE	SPEC. LIMITS	OVERALL VARIATION, $\pm 2\sigma_o$ LIMITS	BATCH-TO-BATCH VARIATION, $\pm 2\sigma_l$ LIMITS
1½ in.	100	—	—
1 in.	80-95	—	—
¾ in.	—	72-90	74-88
½ in.	60-75	—	—
⅜ in.	—	42-68	44-66
No. 4	40-55	25-48	27-46
No. 8	—	17-34	18-33
No. 10	28-43	—	—
No. 200	—	5-8	5-8

TABLE 31

GRADATION OF SERIES 5 AGGREGRATE AT OTHER SAMPLING POINTS

	PERCENT PASSING SIEVE, $\pm 2\sigma_o$ LIMITS		
SIEVE SIZE	FEEDER BELT	LOADED TRUCKS	IN-PLACE BASE
¾ in.	78-89	82-95	72-90
⅜ in.	50-66	56-80	42-68
No. 4	32-46	38-61	25-48
No. 8	22-33	27-43	17-34

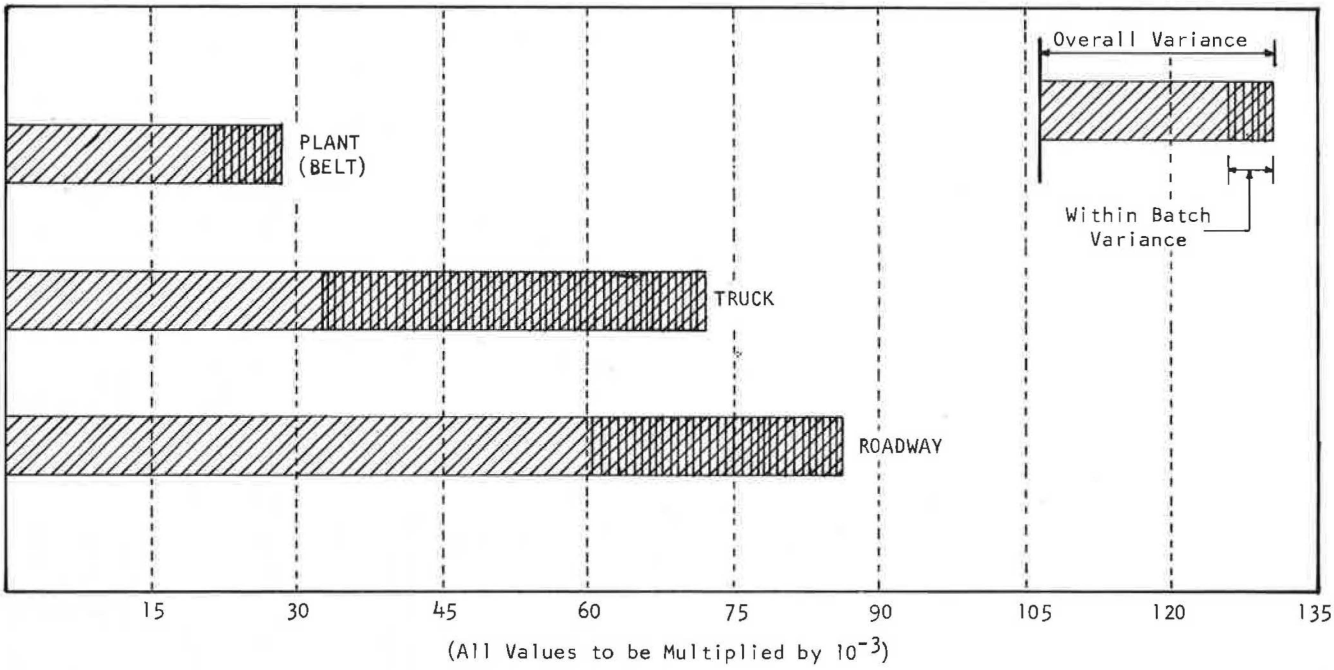


Figure 36. Relative variance of $\bar{A}$ at different points in process stream, Series No. 5, crushed aggregate base course (1 in.-No. 200), North Carolina.

TABLE 32

SUMMARY OF GRADATION AVERAGES, VARIANCES, AND STANDARD DEVIATIONS, SERIES 5 AGGREGATE[a]

			VARIANCE					STANDARD DEVIATION				
SIEVE SIZE	SAMPLING POINT	AVG. % PASSING, $\bar{X}$	TESTING, σ^2_t	INHERENT σ^2_a	OVERALL, σ^2_o	WITHIN-BATCH, σ^2_b	BATCH-TO-BATCH, σ^2_l	TESTING, σ_t	INHERENT σ_a	OVERALL, σ_o	WITHIN-BATCH, σ_b	BATCH-TO-BATCH, σ_l
¾ in.	Plant, belt	83.5	0.07	4.41	8.3	2.5	5.8	0.3	2.1	2.9	1.6	2.4
	Truck	88.8	0.07	4.41	10.4	7.0	3.4	0.3	2.1	3.2	2.6	1.8
	Roadway	81.0	0.07	4.41	19.8	8.3	11.5	0.3	2.1	4.4	2.9	3.4
⅜ in.	Plant, belt	58.2	0.06	3.62	16.0	3.3	12.7	0.3	1.9	4.0	1.8	3.6
	Truck	68.5	0.06	3.62	37.4	20.8	16.6	0.3	1.9	6.0	4.6	4.1
	Roadway	54.7	0.06	3.62	43.1	13.5	29.6	0.3	1.9	6.6	3.7	5.4
No. 4	Plant, belt	39.3	0.06	2.57	11.5	2.9	8.6	0.3	1.6	3.4	1.7	2.9
	Truck	49.3	0.06	2.57	32.3	17.3	15.0	0.3	1.6	5.7	4.2	3.8
	Roadway	36.3	0.06	2.57	32.6	9.5	23.1	0.3	1.6	5.7	3.1	4.8
No. 8	Plant, belt	27.9	0.08	1.96	7.5	1.7	5.8	0.3	1.4	2.7	1.3	2.4
	Truck	35.2	0.08	1.96	17.0	8.9	8.1	0.3	1.4	4.1	3.0	2.8
	Roadway	25.4	0.08	1.96	19.1	4.0	15.1	0.3	1.4	4.4	2.0	3.9
No. 16	Plant, belt	18.8	0.06	—	3.3	0.8	2.5	0.3	—	1.8	0.9	1.6
	Truck	23.8	0.06	—	7.8	4.3	3.5	0.3	—	2.8	2.1	1.9
	Roadway	17.7	0.06	—	8.6	1.9	6.7	0.3	—	2.9	1.4	2.6
No. 30	Plant, belt	14.8	0.06	—	2.0	0.5	1.5	0.3	—	1.4	0.7	1.2
	Truck	18.8	0.06	—	5.1	2.8	2.3	0.3	—	2.2	1.7	1.5
	Roadway	14.3	0.06	—	5.2	1.3	3.9	0.3	—	2.3	1.1	2.0
No. 50	Plant, belt	11.4	0.05	—	1.1	0.3	0.8	0.2	—	1.0	0.5	0.9
	Truck	14.6	0.05	—	3.3	1.7	1.6	0.2	—	1.8	1.3	1.3
	Roadway	11.2	0.05	—	2.8	0.9	1.9	0.2	—	1.7	0.9	1.4
No. 100	Plant, belt	8.5	0.05	—	0.6	0.1	0.5	0.2	—	0.8	0.3	0.7
	Truck	11.1	0.05	—	2.4	1.0	1.4	0.2	—	1.5	1.0	1.2
	Roadway	8.9	0.05	—	1.6	0.5	1.1	0.2	—	1.3	0.7	1.0
No. 200	Plant, belt	5.5	0.04	—	0.3	0.1	0.2	0.2	—	0.5	1.3	0.9
	Truck	7.8	0.04	—	1.6	0.4	1.2	0.2	—	0.3	0.7	0.5
	Roadway	6.5	0.04	—	0.8	0.3	0.5	0.2	—	0.4	1.1	0.7
$\bar{A}$	Plant, belt	2.677	0.0006	—	0.0285	0.0071	0.0214	0.3	—	0.169	0.085	0.146
	Truck	3.179	0.0006	—	0.0723	0.0397	0.0326	0.3	—	0.269	0.200	0.164
	Roadway	2.561	0.0006	—	0.0861	0.0257	0.0604	0.3	—	0.293	0.160	0.246

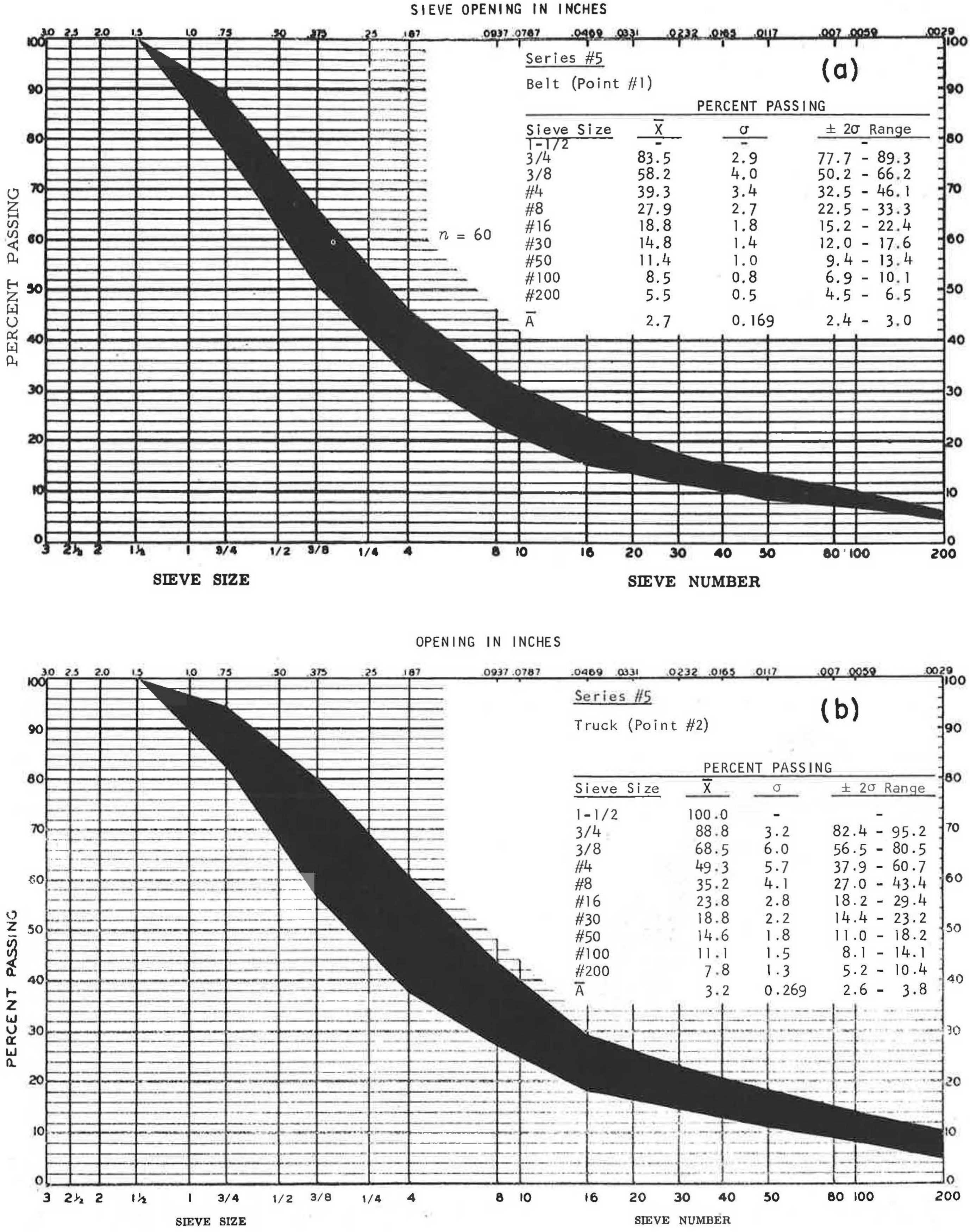
[a] See Table 13 for description of aggregate.

TABLE 33

SUMMARY OF GRADATION AND STATISTICAL PARAMETERS, SERIES 5 AGGREGATE[a]

SAMPLING POINT		AVERAGE SIZE, $\bar{X}$			50% LEVEL STD. DEVIATION				HUDSON $\bar{A}$ STD. DEVIATION			SEGREGATION INDEX		DEG. OF OVERALL VARIABILITY, DOV	DEG. OF SEGREGATION, D OF S
NO.	LOCATION	% PASS. ¾ IN.	% PASS. ⅜ IN.	HUDSON $\bar{A}$	OVERALL, σ_{o-50}	WITHIN-BATCH, σ_{b-50}	BATCH-TO-BATCH, σ_{l-50}	SLOPE	OVERALL, $\sigma_{o-\bar{A}}$	WITHIN-BATCH, $\sigma_{b-\bar{A}}$	BATCH-TO-BATCH, $\sigma_{l-\bar{A}}$	BASED ON $\bar{A}$, $S_{\bar{A}}$	BASED ON 50% P, S_{50}		
1	Plant, belt	83.5	58.2	2.677	4.1	1.9	3.6	(3.1)	0.169	0.085	0.146	4.0	4.6	8.2	7.2
2	Truck	88.8	68.5	3.179	6.4	4.6	4.4	(3.1)	0.269	0.200	0.164	1.8	2.0	12.8	8.8
3	Roadway	81.0	54.7	2.561	6.6	3.7	5.4	(3.1)	0.293	0.160	0.246	3.4	3.2	13.2	10.8

[a] See Table 13 for description of aggregate.



Series #5

Belt (Point #1)

	PERCENT PASSING		
Sieve Size	$\overline{X}$	σ	± 2σ Range
1-1/2	-	-	-
3/4	83.5	2.9	77.7 - 89.3
3/8	58.2	4.0	50.2 - 66.2
#4	39.3	3.4	32.5 - 46.1
#8	27.9	2.7	22.5 - 33.3
#16	18.8	1.8	15.2 - 22.4
#30	14.8	1.4	12.0 - 17.6
#50	11.4	1.0	9.4 - 13.4
#100	8.5	0.8	6.9 - 10.1
#200	5.5	0.5	4.5 - 6.5
$\overline{A}$	2.7	0.169	2.4 - 3.0

Series #5

Truck (Point #2)

	PERCENT PASSING		
Sieve Size	$\overline{X}$	σ	± 2σ Range
1-1/2	100.0	-	-
3/4	88.8	3.2	82.4 - 95.2
3/8	68.5	6.0	56.5 - 80.5
#4	49.3	5.7	37.9 - 60.7
#8	35.2	4.1	27.0 - 43.4
#16	23.8	2.8	18.2 - 29.4
#30	18.8	2.2	14.4 - 23.2
#50	14.6	1.8	11.0 - 18.2
#100	11.1	1.5	8.1 - 14.1
#200	7.8	1.3	5.2 - 10.4
$\overline{A}$	3.2	0.269	2.6 - 3.8

Figure 37. Aggregate gradation charts for Series No. 5 samples taken at (a) feeder belt, (b) loaded trucks, and (c) in-place base.

of the well-graded aggregate that should be noted at this time. All of the other series gave reasonably straight lines when log σ_o was plotted against log $\sqrt{P(100 - P)}$. These are the lines that are projected to the 50 percent passing level, at which point the σ_{o50}, σ_{b50}, and σ_{t50} parameters are read (see Chapter Two, under "Gradation Parameters.") However, this was not the case for the well-graded aggregate, Series 5, where the plots resulted in smooth curves instead of straight lines for points below 50 percent passing, and then the points for over 50 percent passing did not follow the same curve back down; i.e., the standard deviation for the coarser sieve sizes was higher than would be expected from the relationship fitting the data from all other series. In Table 33 the slope value of 3.1 is given in parenthesis, as this is a very rough estimate of what the slope might be if a straight line was used to approximate the curve actually obtained for this series.

The significance of this observed difference between well-graded and the other coarse aggregates is unknown at this time. Some theories have been discussed, and there is basis for hypothesizing a different behavior pattern for the well-graded aggregate. These considerations are beyond the scope of this report; however, it is hoped they might be made part of continuation studies. The fact that there was a distinct difference in behavior patterns observed in the current studies is reported herein for the record.

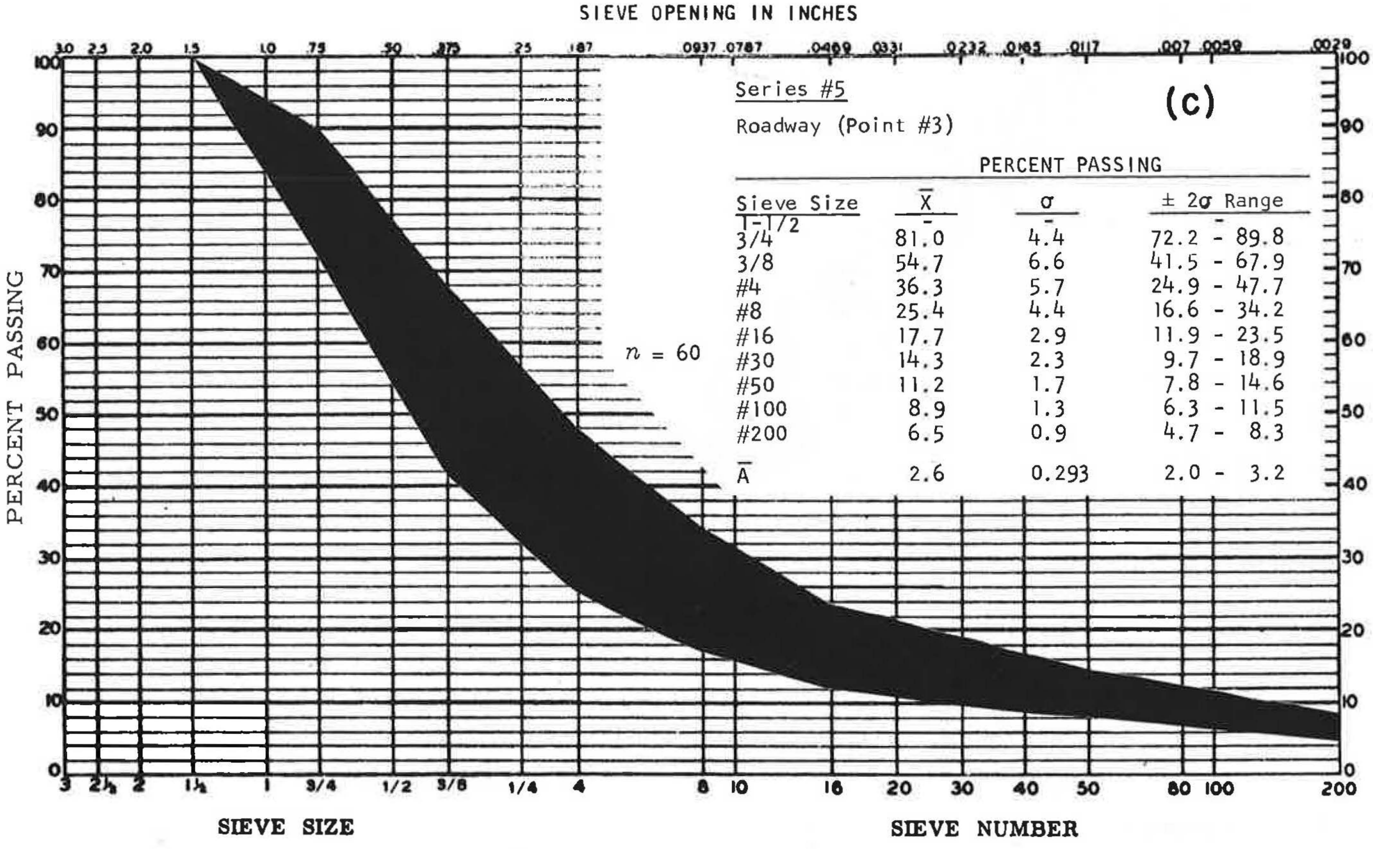
Series No. 6—Slag Coarse (1 in.-No. 8) Aggregate for PC Concrete

PLANT DESCRIPTION AND PROCESS FLOW

The slag sampling program was set up to investigate three points in the production of these aggregates at the steel mill and their handling to the point of use in a transit-mix concrete batch plant in Maryland.

The crushed slag aggregate is obtained as a by-product from a local steel mill. It is produced to meet Maryland No. 6 gradation specifications, which are equivalent to Simplified Practice Recommendation Grading No. 57. The molten slag is deposited in large pits and allowed to cool and solidify for about one month. After cooling and solidification, it is excavated by electric shovel, loaded in rail cars, and delivered to the crusher. It is crushed in the conventional manner, screened, and sized. When processing is complete, it is stored in bins at the crusher site and discharged for shipment into either rail cars or 15-ton Euclid trucks.

In addition, large storage stockpiles of concrete aggregate are built either by clamshell or by dump trucks and front-end loader, depending on the time of year and the relative consumption versus production rates. At the time of sampling for this study (October 1964) a spread-out pile,



Series #5

Roadway (Point #3)

Sieve Size	PERCENT PASSING $\bar{X}$	σ	$\pm 2\sigma$ Range
1-1/2	-	-	-
3/4	81.0	4.4	72.2 - 89.8
3/8	54.7	6.6	41.5 - 67.9
#4	36.3	5.7	24.9 - 47.7
#8	25.4	4.4	16.6 - 34.2
#16	17.7	2.9	11.9 - 23.5
#30	14.3	2.3	9.7 - 18.9
#50	11.2	1.7	7.8 - 14.6
#100	8.9	1.3	6.3 - 11.5
#200	6.5	0.9	4.7 - 8.3
$\bar{A}$	2.6	0.293	2.0 - 3.2

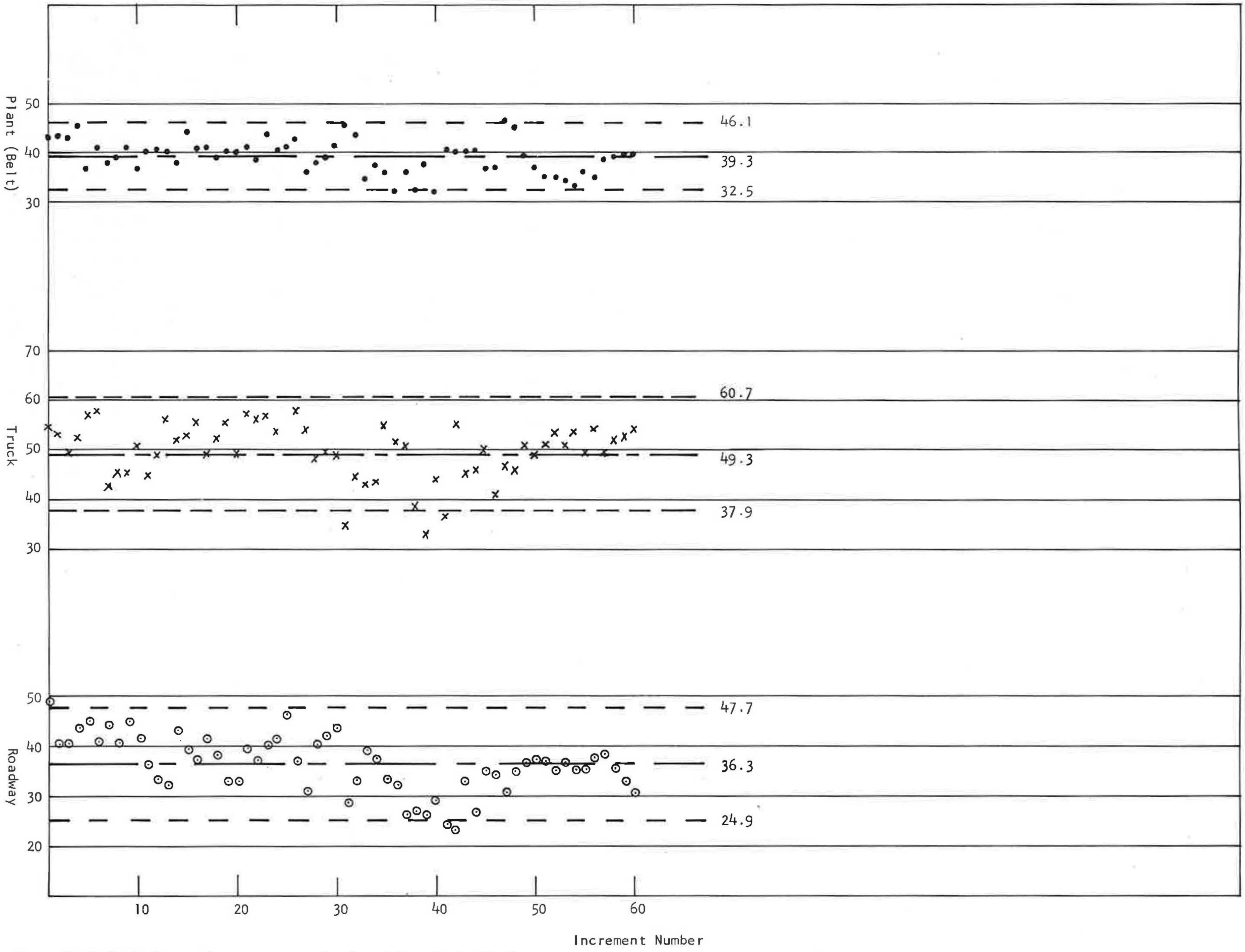


Figure 38. Individual test values, percent passing No. 4 sieve, Series No. 5.

about 15 ft high and containing about 30,000 tons, was being transferred as needed with a 3-cu yd front-end loader to 15-ton Euclid trucks for transport to the concrete plant. In this case, it was deemed more desirable to measure the characteristics of the aggregate actually being loaded to the trucks, rather than the variability or segregation that might have been built into the large spread-out stockpile. Accordingly, sampling point No. 1 is at the point of transfer from the stockpile to the trucks with a 3-cu yd front-end loader. Randomly selected batches (about 3 cu yd) were dumped on the ground, rather than into the truck, and two test portions were taken, with the high-sided shovel, about 5 to 6 ft apart. This spacing and the sample location within the 3-cu yd batch was also varied at random, but no two test portions were taken immediately adjacent to each other.

At the concrete plant, the 15-ton Euclids discharged through an aggregate feed hopper onto a belt which conveyed the aggregate to overhead storage bins of about 50-ton capacity. It was not feasible to stop this belt for sampling purposes, so sampling point No. 2 was from a vertical discharge chute between the end of the belt and the overhead storage bins. Belt flow was determined to be 2 tons per minute; therefore, belt time for one 3-ton batch was 1½ min. A sampling scheme based on belt running time was devised, and 50 random batches were selected for sampling. Two test portions were removed from each batch selected; the first at 5 sec from the beginning of the batch, and the second at 30 sec from the first test portion. Samples were obtained by pulling a pan through the stream of aggregate, completely cutting the cross-section of flow (see Fig. 18). Maintenance of a uniform sample size with this procedure was difficult, as the sample size was influenced by the speed with which the pan was pulled through the stream. Altogether, 41 duplicate samples were obtained in one day's production. A check with the production personnel revealed that this number of samples agreed well with the yardage for that day's production.

When needed to make a batch of concrete, the slag aggregate was gravity fed from the overhead storage bin to the aggregate weigh hopper (sampling point No. 3). Test portions were taken by inserting a sampling pan in the flow from overhead storage to the weigh hopper. It took 16 sec to discharge 3 cu yd. As usual, to measure within-batch variability, σ_b, two test portions, average weight of 20 lb each, were taken for each 3-cu yd batch. The first test portion was taken immediately when the gate was opened, and the second portion 10 sec later. The second portion was taken by shutting the flow off, inserting the sampling pan, and opening the discharge gate again. In all, 44 duplicate samples were taken at this point for the day's production. Sampling point No. 3 is the point of inspection of the Maryland State Roads Commission for this plant.

VARIATION AT THE POINT OF USE

The specification requirements for the gradation of the slag, as well as the overall variation and batch-to-batch variation at sampling point No. 3, are given in Table 34. Because some of the specification limits are based on non-standard sieves, a direct comparison of the results with specifications cannot be made, although the results, in this case, indicated that the aggregate gradation varied over a greater range than allowed by the specifications, and a widening of tolerances is indicated if specifications are to be met.

The overall variation shows the limits within which 95 percent of results of tests on single increments, selected at random, could be expected to fall. The batch-to-batch variation shows the probable limits within which the gradation of the coarse aggregate in 95 percent of the batches would fall, if the entire batch was put through the sieves.

TABLE 34
GRADATION OF SERIES 6 SLAG AT POINT OF USE

SIEVE SIZE	PERCENT PASSING SIEVE		
	SPEC. LIMITS	OVERALL VARIATION, $\pm 2\sigma_o$ LIMITS	BATCH-TO-BATCH VARIATION, $\pm 2\sigma_l$ LIMITS
1½ in.	100	100	100
1 in.	90-100	—	—
¾ in.	—	63-100	63-100
½ in.	25-60	25-92	26-92
⅜ in.	—	8-70	8-70
No. 4	0-10	0-22	0-22
No. 8	0-5	0-8	0-8

TABLE 35
GRADATION OF SERIES 6 SLAG AT OTHER SAMPLING POINTS

SIEVE SIZE	PERCENT PASSING SIEVE, $\pm 2\sigma_o$ LIMITS			
	SPEC. LIMITS	3-CU YD BATCHES FROM STOCKPILE	BELT CHARGING STORAGE BINS	FEED TO WEIGH HOPPER
1½ in.	100	—	—	—
1 in.	90-100	—	—	—
¾ in.	—	70-93	76-91	63-100
½ in.	25-60	34-74	47-70	25-92
⅜ in.	—	14-53	28-49	8-70
No. 4	0-10	1-14	6-14	0-22
No. 8	0-5	0-6	2-5	0-8

VARIATION IN GRADATION AT OTHER SAMPLING POINTS

To measure the variations in gradation at other sampling points in the process stream, random sample increments were taken of 3-cu yd batches from the stockpile (sampling point No. 1), from the belt discharge into the plant storage bins (sampling point No. 2), as well as the feed to the weigh hopper. The overall variance at all of these points is given in Table 35.

SERIES 6 SUMMARY CHARTS

The relative variances and standard deviations of the percentages passing the different sieves are summarized in Table 36. The relative variance of the Hudson $\bar{A}$ values is shown in Figure 39. The aggregate grading charts showing the $\pm 2\sigma$ envelopes at each of the three sampling points are presented in Figure 40. Figure 41 is a plot of the individual test results of percent passing the ½-in. sieve, in the order of sampling from the stockpile batches, the belt discharge, and the gravity feed from the overhead storage bin to the weigh hopper of the transit-mix concrete plant. The gradation and statistical summary parameters for this series are presented in Table 37.

These various figures and tables are discussed collectively. The within-batch variance and standard deviation values obtained on duplicate test portions taken at the stockpile are nearly the same as the overall variance. For instance, $\sigma_o = 0.203$ and $\sigma_b = 0.197$ for Hudson $\bar{A}$; the corresponding numbers for the percent passing the ½-in. sieve are $\sigma_o = 10.3$ and $\sigma_b = 9.8$. These values are abnormally close together and indicate that the gradation of adjacent portions of the slag aggregate taken from the same 3-cu yd front-end loader batch were greatly different. On the other hand, the batch-to-batch variability was relatively low. This is understandable and, in fact, expected, because the front-end loader was working the same face of the stockpile. This means, of course, that the parameters for batch-to-batch segregation will all be relatively low; i.e., segregation index, $S_{\bar{A}} = 1.1$ and $S_{50} = 1.1$, $\sigma_l = 0.050$ for Hudson $\bar{A}$, and $\sigma_l = 3.2$ for percent passing the ½-in. sieve. These values are not comparable to those normally obtained for stockpiled aggregate, because the test portions were not taken from different parts of the stockpile; i.e., they do not reflect normal stockpile segregation, but they do reflect the batch-to-batch variation of slag taken from the same face of the pile during one working day.

Referring now to the belt end samples (sampling point No. 2), it will be noted that the within-batch variance and standard deviations are more normal relative to the other values. Reference to Table 37, for instance, shows $\sigma_{o50} = 5.9$, $\sigma_{b50} = 3.6$, $\sigma_{l50} = 4.6$, resulting in corresponding ratio parameters of segregation index, $S_{\bar{A}} = 2.6$ and $S_{50} = 2.7$; degree of overall variability, DOV, = 11.8; and degree of segregation, D of S, = 9.2. These values are quite similar to the comparable parameters for crushed stone and gravel aggregate.

Referring now to sampling point No. 3, the storage bin discharge into the weigh hopper, anomolous behavior is again apparent. This time, the within-batch variability is small relative to the overall variability (i.e., $\sigma_o = 17.4$, $\sigma_b = 2.6$, $\sigma_l = 17.2$ for percent passing the ½-in. sieve); corresponding $\bar{A}$ values for $\sigma_o = 0.354$, $\sigma_b = 0.062$, and $\sigma_l = 0.350$. These measurements result in very high values for segregation index, $S_{\bar{A}} = 33.1$ and $S_{50} = 44.5$; for degree of overall variability, DOV, = 34.8; and for degree of segregation, D of S, = 34.4 Here, also; there seems to be a logical explanation for the observed behavior.

Attention is invited first to the pattern of the plot of individual test results obtained on percent passing the ½-in. sieve (Fig. 41). Here, the surging previously noted on samples from bins or hoppers is very definite and very severe. The first 16 samples taken of this bin discharge were all quite uniform and showed an average of about 68 percent passing the ½-in. sieve, with one pair of results down as low as 25 percent minus ½-in. slag particles. Then,

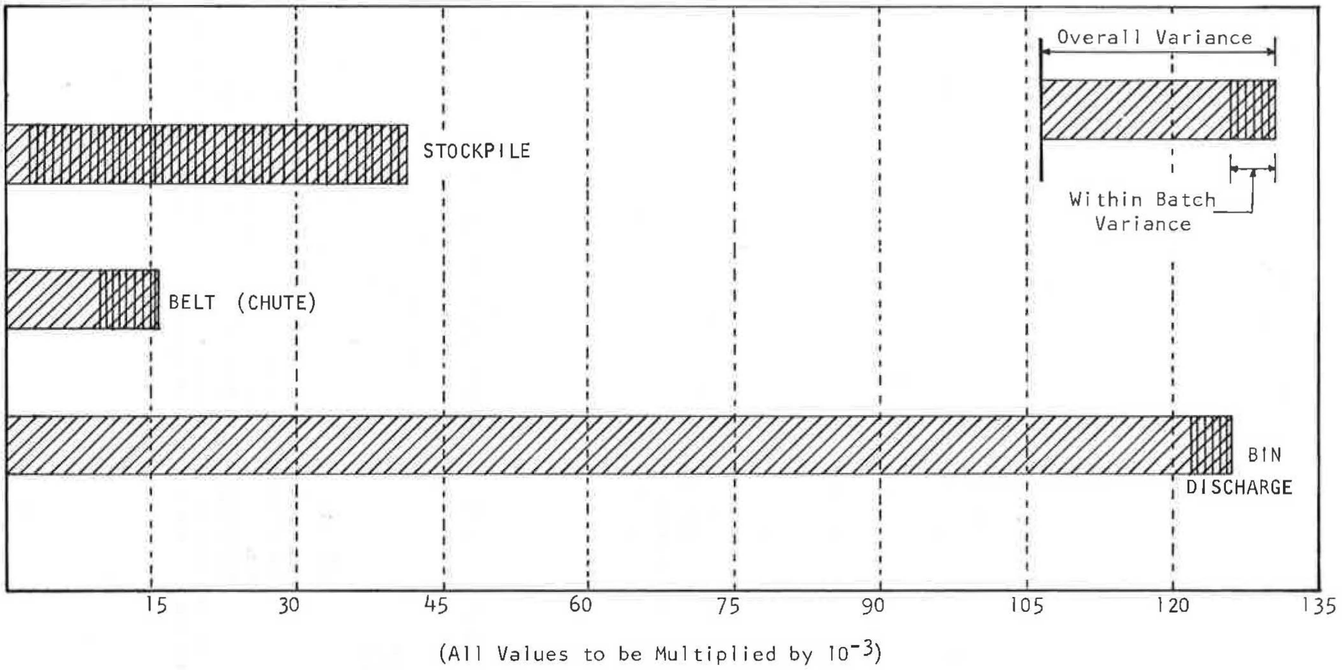


Figure 39. Relative variance of $\bar{A}$ at different points in process stream, Series No. 6, crushed slag (1 in.-No. 8), Maryland.

TABLE 36

SUMMARY OF GRADATION AVERAGES, VARIANCES, AND STANDARD DEVIATIONS, SERIES 6 AGGREGATE [a]

SIEVE SIZE	SAMPLING POINT	AVG. % PASSING, $\overline{X}$	VARIANCE TESTING, σ^2_t	INHERENT σ^2_a	OVERALL, σ^2_o	WITHIN-BATCH, σ^2_b	BATCH-TO-BATCH, σ^2_l	STANDARD DEVIATION TESTING, σ_t	INHERENT σ_a	OVERALL, σ_o	WITHIN-BATCH, σ_b	BATCH-TO-BATCH, σ_l
¾ in.	Stockpile	81.6	0.09	4.40	33.8	27.4	6.4	0.3	2.1	5.8	5.2	2.5
	Belt	83.7	0.09	4.40	12.9	7.6	5.3	0.3	2.1	3.6	2.7	2.3
	Bin discharge [b]	83.8	0.09	4.40	104.9	1.9	103.0	0.3	2.1	10.3	1.4	10.2
½ in.	Stockpile	53.9	0.26	4.00	103.7	95.5	8.2	0.5	2.0	10.2	9.8	2.9
	Belt	58.1	0.26	4.00	32.4	12.3	20.1	0.5	2.0	5.7	3.5	4.5
	Bin discharge [b]	58.6	0.26	4.00	277.5	6.1	271.4	0.5	2.0	16.7	2.5	16.5
⅜ in.	Stockpile	33.7	0.21	2.90	93.5	90.3	3.2	0.5	1.7	9.7	9.5	1.8
	Belt	38.5	0.21	2.90	26.1	11.5	14.6	0.5	1.7	5.1	3.4	3.8
	Bin discharge [b]	39.0	0.21	2.90	245.8	5.9	239.9	0.5	1.7	15.7	2.4	15.5
No. 4	Stockpile	7.3	0.31	0.64	10.6	10.7	0.0	0.6	0.8	3.2	3.3	0.0
	Belt	9.5	0.31	0.64	3.9	2.0	1.9	0.6	0.8	2.0	1.4	1.4
	Bin discharge [b]	10.6	0.31	0.64	32.8	1.6	31.2	0.6	0.8	5.7	1.3	5.6
No. 8	Stockpile	2.7	0.20	0.25	2.0	0.9	3.8	0.5	0.5	1.4	1.4	0.0
	Belt	3.3	0.20	0.25	2.1	0.2	0.4	0.5	0.5	0.9	0.4	0.8
	Bin discharge [b]	4.3	0.20	0.25	0.0	0.7	3.4	0.5	0.5	1.9	0.6	1.8
$\overline{A}$	Stockpile	2.314	0.0007	—	0.0412	0.0387	0.0025	0.03	—	0.203	0.197	0.050
	Belt	2.432	0.0007	—	0.0156	0.0059	0.0097	0.03	—	0.125	0.077	0.098
	Bin discharge [b]	2.472	0.0007	—	0.1258	0.0038	0.1220	0.03	—	0.354	0.062	0.350

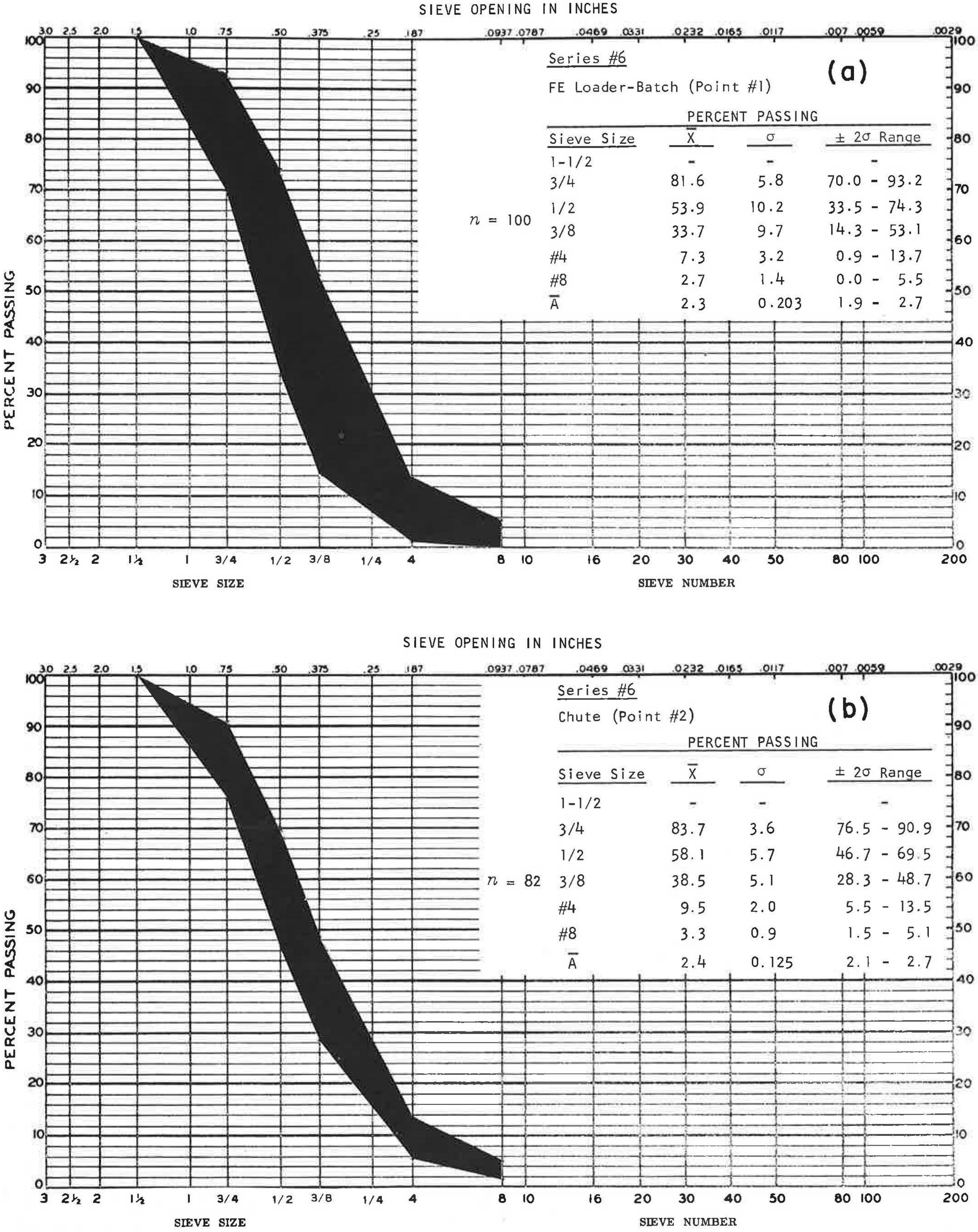
[a] See Table 13 for description of aggregate. [b] To weigh hopper.

TABLE 37

SUMMARY OF GRADATION AND STATISTICAL PARAMETERS, SERIES 6 AGGREGATE [a]

SAMPLING POINT NO.	LOCATION	AVERAGE SIZE, $\overline{X}$ % PASS. ¾ IN.	% PASS. ⅜ IN.	HUDSON $\overline{A}$	50% LEVEL STD. DEVIATION OVERALL, σ_{o-50}	WITHIN-BATCH, σ_{b-50}	BATCH-TO-BATCH, σ_{l-50}	SLOPE	HUDSON $\overline{A}$ STD. DEVIATION OVERALL, $\sigma_{o-\overline{A}}$	WITHIN-BATCH, $\sigma_{b-\overline{A}}$	BATCH-TO-BATCH, $\sigma_{l-\overline{A}}$	SEGREGATION INDEX BASED ON $\overline{A}$, $S_{\overline{A}}$	BASED ON 50% P, S_{50}	DEG. OF OVERALL VARIABILITY, DOV	DEG. OF SEGREGATION, D OF S
1	Stockpile	81.6	33.7	2.314	10.3	9.8	3.2	1.73	0.203	0.197	0.050	1.1	1.1	20.6	6.4
2	Belt	83.7	38.5	2.432	5.9	3.6	4.6	1.73	0.125	0.077	0.098	2.6	2.7	11.8	9.2
3	Bin discharge [b]	83.8	39.0	2.472	17.4	2.6	17.2	1.73	0.354	0.062	0.350	33.1	44.5	34.8	34.4

[a] See Table 13 for description of aggregate. [b] To weigh hopper.



Series #6

FE Loader-Batch (Point #1)

Sieve Size	PERCENT PASSING $\bar{X}$	σ	± 2σ Range
1-1/2	-	-	-
3/4	81.6	5.8	70.0 - 93.2
1/2	53.9	10.2	33.5 - 74.3
3/8	33.7	9.7	14.3 - 53.1
#4	7.3	3.2	0.9 - 13.7
#8	2.7	1.4	0.0 - 5.5
$\bar{A}$	2.3	0.203	1.9 - 2.7

Series #6

Chute (Point #2)

Sieve Size	PERCENT PASSING $\bar{X}$	σ	± 2σ Range
1-1/2	-	-	-
3/4	83.7	3.6	76.5 - 90.9
1/2	58.1	5.7	46.7 - 69.5
3/8	38.5	5.1	28.3 - 48.7
#4	9.5	2.0	5.5 - 13.5
#8	3.3	0.9	1.5 - 5.1
$\bar{A}$	2.4	0.125	2.1 - 2.7

Figure 40. Aggregate gradation charts for Series No. 6 samples taken at (a) stockpile, (b) chute, and (c) bin discharge to weigh hopper.

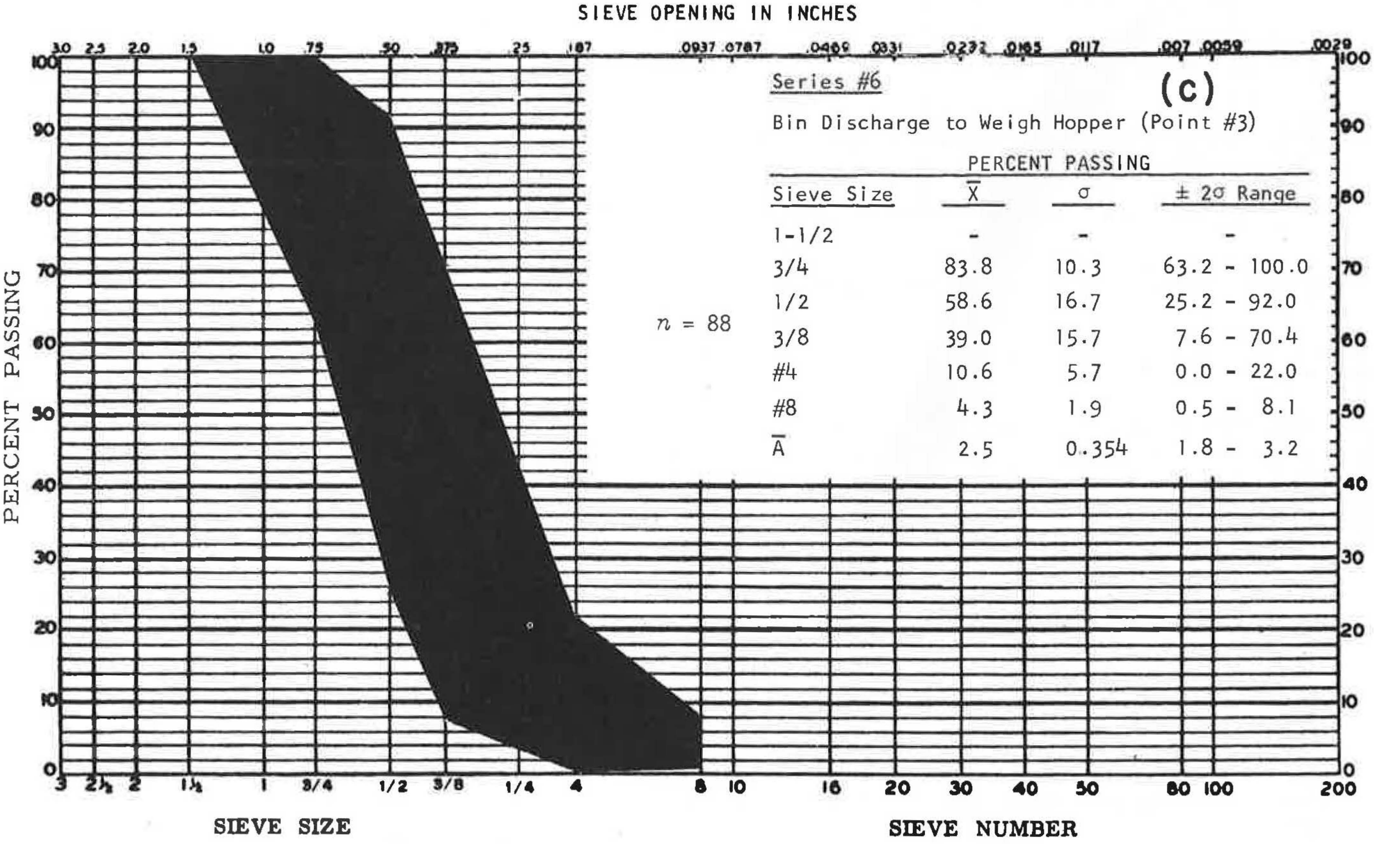
for the next 16 samples, the gradation reverted to the 60 to 75 percent minus ½-in. level and was again reasonably uniform until there was a minor surge of 40 percent passing ½-in. material. Note that, again, this was a pair of samples. The level then returned to about 68 percent passing and the pattern was carried down to the average of four successive test portions showing less than 20 percent minus ½-in. slag. It is beyond the scope of this initial part of the project to explore the reasons for this surging, but it is apparent that the large overall variability measured is a direct result of segregation within the storage bin. For extended periods the flow of aggregate from this bin was finer than average, but reasonably uniform, then the segregated coarser particles were released periodically as slugs or surges.

This whole situation leads to some rather interesting observations and questions from the construction control viewpoint. The situation, in brief, is as follows:

1. The specifications call for 25 to 60 percent passing the ½-in. sieve. These are the broad band limits, and it is normally anticipated that the construction controls on any one plant using aggregate from a single source on a steady production flow basis would result in much narrower operating limits within this broad band.
2. The material fed to the overhead storage bins averaged 58.1 percent passing the ½-in. sieve, which is too close to the upper specification limit. The important point, however, is that this aggregate, as fed to the storage bin, was reasonably uniform and the individual test results seemed to be randomly distributed around the average. The overall standard deviation, $\sigma_o = 5.7$, and the $\pm 2\sigma$ limits are 46.7 to 69.5. This is the feed to the overhead storage bins.
3. Due to segregation on the way into or within these 50-ton storage bins, however, the aggregate that drops into the weigh hopper to make up the individual batches of concrete is something else again. Of the 44 batches sampled during this day's production, 31 (about 70 percent) were made with aggregate that was too fine, ranging from about 58 percent to nearly 80 percent passing the ½-in. sieve, with most of the points distributed quite uniformly in the 66 to 74 percent range. The other 13 batches, representing about 30 percent of the day's production, were made with aggregate having less than 50 percent passing the ½-in. sieve; with four batches, or about 9 percent of the production, falling right at or below the lower limit (25 percent) of the broad band of the specifications.

The random test results obtained and the analysis of these data lead to some pertinent observations and questions of obvious concern from the construction control viewpoint. Some of these are as follows:

1. Inasmuch as the concrete from this plant has ap-



Series #6

(c)

Bin Discharge to Weigh Hopper (Point #3)

$n = 88$

Sieve Size	PERCENT PASSING $\overline{X}$	σ	$\pm 2\sigma$ Range
1-1/2	-	-	-
3/4	83.8	10.3	63.2 - 100.0
1/2	58.6	16.7	25.2 - 92.0
3/8	39.0	15.7	7.6 - 70.4
#4	10.6	5.7	0.0 - 22.0
#8	4.3	1.9	0.5 - 8.1
$\overline{A}$	2.5	0.354	1.8 - 3.2

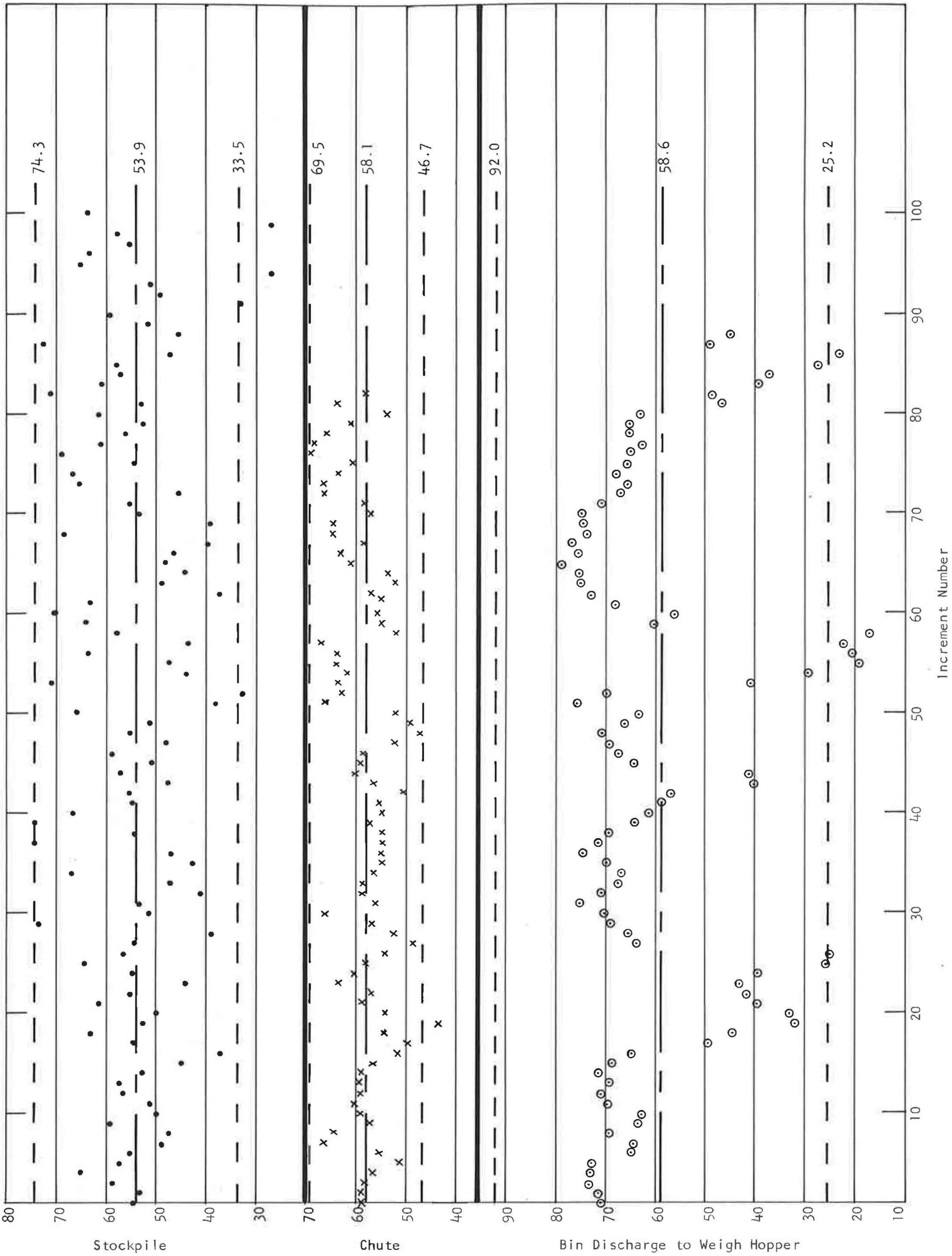


Figure 41. Individual test values, percent passing ½-in. sieve, Series No. 6.

parently been giving satisfactory service, what is the importance of coarse aggregate gradation control in portland cement concrete? Can the road or structure tell the difference between the 70 percent of the output of this plant using fine slag, most of which was outside of the upper broad band specification limit, and the 30 percent of the output produced during the same day with coarse slag with about 9 percent of the day's production at or below the lower limit of the broad specification band? Is there a real difference between 70 percent passing the ½-in. sieve and 25 percent passing, insofar as the quality of the finished product is concerned?

2. What is there about this storage bin that would cause the high degree of segregation noted? Why should the feed to the bin be reasonably uniform, but the discharge come out 70 percent of the time on the fine side and 30 percent of the time with periodic surges of coarse aggregate? As stated, it is beyond the scope of this first part of the project to delve further into theories of hopper design. This work does, however, pinpoint a definite need for further study and a potentially fruitful area for additional research.

3. There are many obvious questions from the construction control viewpoint, most of which are related to just where and when and how the aggregate feeding this plant should be sampled. An inspector can get totally misleading test results if he happens to sample the feed to the weigh hopper at the wrong time.

4. These data rather dramatically emphasize the need to take a look at the whole picture. To be guided by any one of the parameters would lead to a poor and possibly erroneous understanding of the construction control problems associated with this plant. For instance, the standard deviations, σ_o, σ_b, σ_t, or the various other ratios or parameters, give no hint of the pattern of surging at the bin discharge shown in Figure 41. This is a fine example of the advantages of maintaining a process control chart that will delineate these trends and show what is actually going on at the plant.

Series No. 7—Uncrushed Gravel (1 in.-No. 4) Aggregate for PC Concrete

PLANT DESCRIPTION AND PROCESS FLOW

This sampling was from the process stream (barge, conveyor belt to storage bin, feed to weigh hopper) of the aggregate supply to a large central-mix concrete plant in Philadelphia, Pa. The resulting concrete is used for State highway, municipal, and private work in the area. The concrete is tested daily by a private laboratory, which certifies as to the quality with respect to specification strength requirements.

The uncrushed gravel aggregate is dredged from the Delaware River near Philadelphia. It is produced to meet Pennsylvania 2-B gradation specifications (Equivalent to Simplified Practice Recommendation Grading No. 57). As the aggregate is dredged, it is screened and loaded into a general purpose barge with a capacity of 900 to 1,000 tons (sampling point No. 1). Upon arrival at a wharf adjacent to the concrete plant, the barges are unloaded with a clam-bucket into a silo-type surge bin. From this bin, a belt conveyor (sampling point No. 2) elevates the aggregate to overhead storage bins (sampling point No. 3), from which it is gravity fed into the weigh hopper.

VARIATION IN GRADATION AT POINT OF USE

Sample increments of aggregate were taken at sampling point No. 3 by sliding specially designed sampling tools (Fig. 17) into the stream of gravel flowing into the weigh hopper. Two increments, having an average weight of 25 lb each, were taken from each 9-cu yd batch selected by the use of a table of random numbers. The entire increment was used as a test portion and was sieved to refusal on a Weston rotary sieve.

The specification requirements for the gradation of the gravel, together with the overall variation and batch-to-batch variations, are given in Table 38. The overall variation shows the limits within which 95 percent of results of tests on single increments, selected at random, could be expected to fall. The batch-to-batch variation shows the probable limits within which the gradation of the coarse aggregate in 95 percent of the batches would fall, if the entire batch was put through the sieves.

VARIATION IN GRADATION AT OTHER SAMPLING POINTS

To compare the specified gradation with the gradation at the source and at intermediate sampling points, random sample increments were taken in duplicate from the barges (sampling point No. 1) and from the belt (sampling point No. 2) carrying the aggregate to the plant bins. Samplings were timed so that essentially the same LOT of aggregate was sampled at all points. The overall variations found at these points are compared with the specification limits in Table 39.

SERIES 7 SUMMARY CHARTS

The relative variances and standard deviations of the percentages passing the different sieves are summarized in Table 40. The relative variance of the Hudson $\overline{A}$ values is shown in Figure 42. The aggregate grading charts showing the $\pm 2\sigma$ envelopes at each of the three sampling points are presented in Figure 43. The individual test result values of percent passing the ⅜-in. sieve are plotted in the order of sampling from the barge, from the belt, and at the feed to the weigh hopper in Figure 44. The gradation and statistical summary parameters for this series are presented in Table 41.

These various figures and tables, as they apply to construction control for Series 7 alone, are discussed collectively in the following. Series 8 covers a very similar operation, which was purposely selected in an attempt to determine if a definite degree of variation, using the same type of aggregate but in a different plant, could be associated with a given handling process. Thus, observations common to both Series 7 and Series 8 are reserved for comment under Series 8.

It is apparent that the handling of the Series 7 rounded gravel resulted in mixing that greatly decreased the variability measured between the barge (sampling point No. 1) and the belt (sampling point No. 2). Part of this increased uniformity was then nullified by moderate within-bin segregation upon subsequent transfer to the overhead storage bins. The best picture of these effects is given in Figure 44 and by the parameters presented in Table 41. At the barge, the degree of overall variability, DOV, = 32.0, and the corresponding degree of segregation, D of S, = 25.0. The segregation index, $S_{\bar{A}} = 1.7$ and $S_{50} = 2.5$, is low because the within-batch variance on the barge is also relatively large.

The corresponding values for samples taken from the belt are DOV = 8.2, D of S = 6.8, $S_{\bar{A}} = 2.1$, and $S_{50} = 3.2$. Thus, the segregation index remains about the same, but the degree of overall variability and the degree of segregation are greatly reduced—decreased nearly four-fold.

The corresponding values on samples taken from the overhead storage bin discharge are DOV = 17.2, D of S = 14.6, $S_{\bar{A}} = 4.9$, and $S_{50} = 3.6$. Due to segregation in the overhead storage bin, the overall variability and the batch-to-batch segregation has about doubled, thus negating about half of the improved uniformity obtained in handling between the barge and the belt. The segregation index has also increased moderately, indicating some change in the pattern of segregation—a relative decrease in within-batch and an increase in batch-to-batch variability as the gravel was transferred from the barge through the overhead storage bins and discharged to the weigh hopper.

Figure 44, a plot of the individual test values of the feed to the weigh hopper (overhead storage bin discharge), indicates some tendency toward surging, but it is nowhere near as pronounced as that illustrated in Series 6. Between samples 30 and 40, and again between about 77 and 87, the gradation gets coarser (15 to 22 percent passing the ⅜-in. sieve), whereas between about sample 42 to 76 the grada-

TABLE 38

GRADATION OF SERIES 7 GRAVEL AT POINT OF USE

	PERCENT PASSING SIEVE		
SIEVE SIZE	SPEC. LIMITS	OVERALL VARIATION, $\pm 2\sigma_o$ LIMITS	BATCH-TO-BATCH VARIATION, $\pm 2\sigma_t$ LIMITS
1½ in.	100	100	100
1 in.	90-100	—	—
¾ in.	—	71-92	73-90
½ in.	25-60	—	—
⅜ in.	—	15-39	16-38
No. 4	0-10	2-9	3-8
No. 8	0-5	1-3	1-3

TABLE 39

GRADATION OF SERIES 7 GRAVEL AT OTHER SAMPLING POINTS

	PERCENT PASSING SIEVE, $\pm 2\sigma_o$ LIMITS			
SIEVE SIZE	SPEC. LIMITS	BARGE	TRANSFER BELT	WEIGH HOPPER
1½ in.	100	100	100	100
1 in.	90-100	—	—	—
¾ in.	—	61-93	73-85	71-92
½ in.	25-60	—	—	—
⅜ in.	—	0-40	18-27	15-39
No. 4	0-10	0-6	3-5	2-9
No. 8	0-5	0-2	1-2	1-3

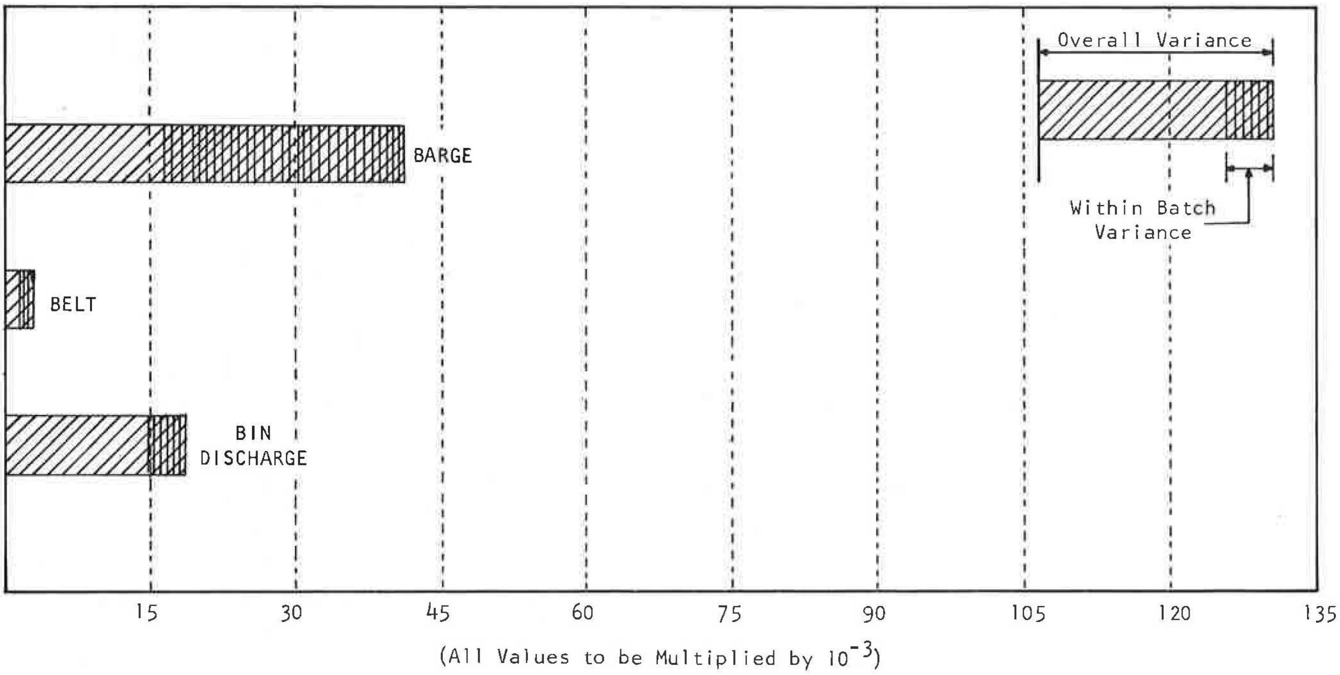


Figure 42. Relative variance of $\bar{A}$ *at different points in process stream, Series No. 7, uncrushed gravel (1 in.-No. 4), Pennsylvania (Plant No. 1).*

TABLE 40

SUMMARY OF GRADATION AVERAGES, VARIANCES, AND STANDARD DEVIATIONS, SERIES 7 AGGREGATE [a]

			VARIANCE					STANDARD DEVIATION				
SIEVE SIZE	SAMPLING POINT	AVG. % PASSING, $\overline{X}$	TESTING, σ^2_t	INHERENT, σ^2_a	OVERALL, σ^2_o	WITHIN-BATCH, σ^2_b	BATCH-TO-BATCH, σ^2_l	TESTING, σ_t	INHERENT, σ_a	OVERALL, σ_o	WITHIN-BATCH, σ_b	BATCH-TO-BATCH, σ_l
¾ in.	Barge	77.2	0.30	4.84	65.3	45.6	19.7	0.5	2.2	8.1	6.8	4.5
	Belt	79.1	0.30	4.84	9.7	8.4	1.3	0.5	2.2	3.1	2.9	1.1
	Bin discharge [b]	81.6	0.30	4.84	26.1	6.9	19.2	0.5	2.2	5.1	2.6	4.4
⅜ in.	Barge	19.1	0.18	2.25	112.4	65.2	47.2	0.4	1.5	10.6	8.1	6.9
	Belt	22.8	0.18	2.25	5.1	2.2	2.9	0.4	1.5	2.3	1.5	1.7
	Bin discharge [b]	26.7	0.18	2.25	37.5	8.2	29.3	0.4	1.5	6.1	2.9	5.4
No. 4	Barge	2.5	0.13	0.36	3.6	1.7	1.9	0.4	0.6	1.9	1.3	1.4
	Belt	3.6	0.13	0.36	0.3	0.1	0.2	0.4	0.6	0.5	0.3	0.4
	Bin discharge [b]	5.4	0.13	0.36	2.7	1.1	1.6	0.4	0.6	1.6	1.0	1.3
No. 8	Barge	0.8	0.01	0.16	0.6	0.1	0.5	0.1	0.4	0.8	0.2	0.7
	Belt	1.1	0.01	0.16	0.0	0.0	0.0	0.1	0.4	0.0	0.0	0.0
	Bin discharge [b]	2.2	0.01	0.16	0.4	0.2	0.2	0.1	0.4	0.6	0.4	0.4
$\overline{A}$	Barge	2.013	0.0003	—	0.0416	0.0251	0.0165	0.02	—	0.204	0.159	0.129
	Belt	2.095	0.0003	—	0.0029	0.0014	0.0015	0.02	—	0.054	0.037	0.039
	Bin discharge [b]	2.214	0.0003	—	0.0187	0.0038	0.0149	0.02	—	0.137	0.062	0.122

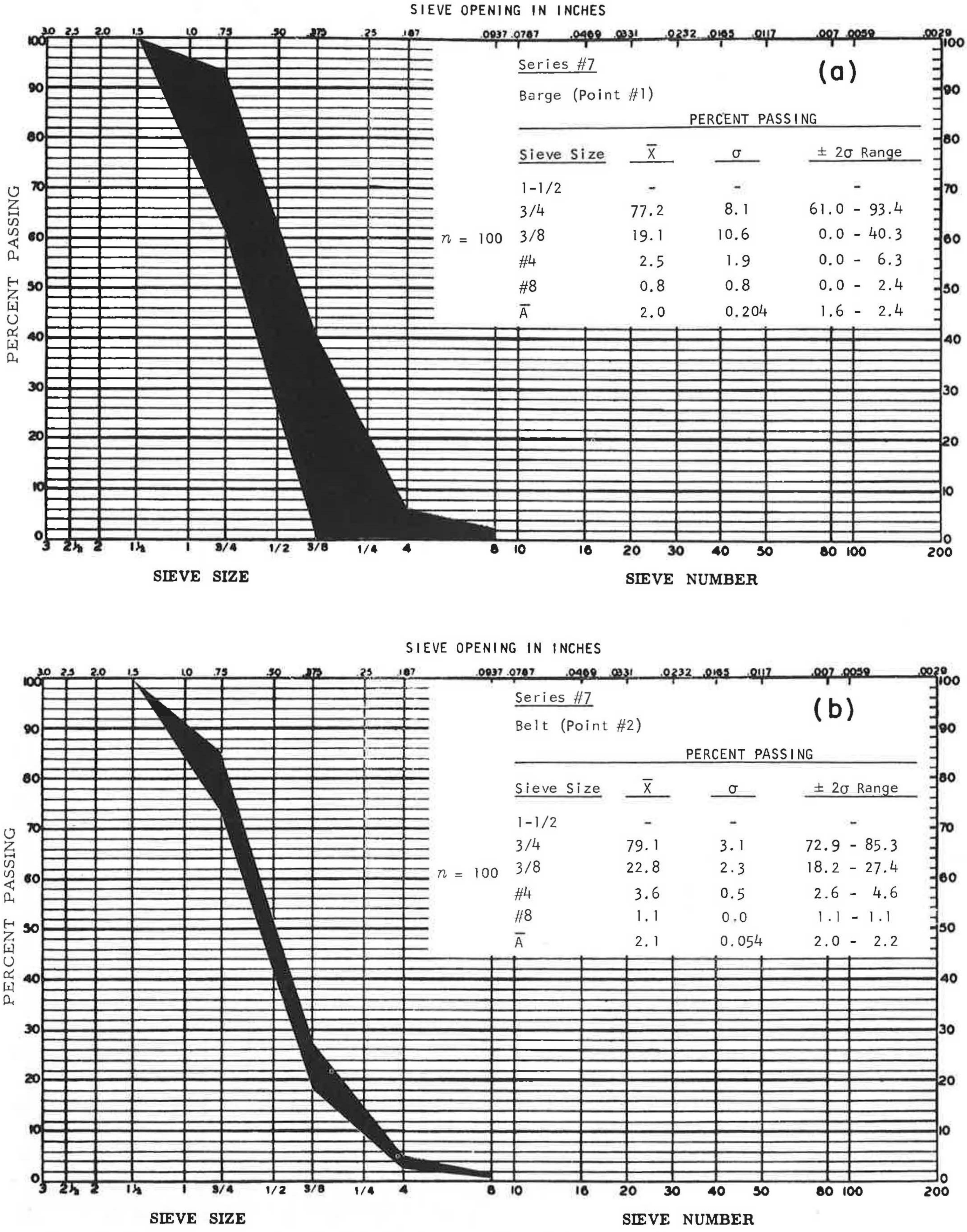
[a] See Table 13 for description of aggregate. [b] To weigh hopper.

TABLE 41

SUMMARY OF GRADATION AND STATISTICAL PARAMETERS, SERIES 7 AGGREGATE [a]

SAMPLING POINT		AVERAGE SIZE, $\overline{X}$			50% LEVEL STD. DEVIATION				HUDSON $\overline{A}$ STD. DEVIATION			SEGREGATION INDEX		DEG. OF OVERALL VARIABILITY, DOV	DEG. OF SEGREGATION, D OF S
NO.	LOCATION	% PASS. ¾ IN.	% PASS. ⅜ IN.	HUDSON $\overline{A}$	OVERALL, σ_{o-50}	WITHIN-BATCH, σ_{b-50}	BATCH-TO-BATCH, σ_{l-50}	SLOPE	OVERALL, $\sigma_{o-\overline{A}}$	WITHIN-BATCH, $\sigma_{b-\overline{A}}$	BATCH-TO-BATCH, $\sigma_{l-\overline{A}}$	BASED ON $\overline{A}$, $S_{\overline{A}}$	BASED ON 50% P, S_{50}		
1	Barge	77.2	19.1	2.013	16.0	10.0	12.5	2.14	0.204	0.159	0.129	1.7	2.5	32.0	25.0
2	Belt	79.1	22.8	2.095	4.1	2.3	3.4	2.14	0.054	0.037	0.039	2.1	3.2	8.2	6.8
3	Bin discharge [b]	81.6	26.7	2.214	8.6	4.5	7.3	2.14	0.137	0.062	0.122	4.9	3.6	17.2	14.6

[a] See Table 13 for description of aggregate. [b] To weigh hopper.



Series #7

Barge (Point #1)

n = 100

Sieve Size	$\overline{X}$	σ	± 2σ Range
	PERCENT PASSING		
1-1/2	-	-	-
3/4	77.2	8.1	61.0 - 93.4
3/8	19.1	10.6	0.0 - 40.3
#4	2.5	1.9	0.0 - 6.3
#8	0.8	0.8	0.0 - 2.4
$\overline{A}$	2.0	0.204	1.6 - 2.4

Series #7

Belt (Point #2)

n = 100

Sieve Size	$\overline{X}$	σ	± 2σ Range
	PERCENT PASSING		
1-1/2	-	-	-
3/4	79.1	3.1	72.9 - 85.3
3/8	22.8	2.3	18.2 - 27.4
#4	3.6	0.5	2.6 - 4.6
#8	1.1	0.0	1.1 - 1.1
$\overline{A}$	2.1	0.054	2.0 - 2.2

Figure 43. Aggregate gradation charts for Series No. 7 samples taken at (a) barge, (b) belt, and (c) bin discharge to weigh hopper.

tion of the aggregate feed to the weigh hopper is uniformly finer (at about the 32 percent minus ⅜-in. level).

Series No. 8—Uncrushed Gravel (1 in.-No. 4) Aggregate for PC Concrete

The plant at which this sampling was made is essentially the same, in all respects, as that used in the Series No. 7 sampling, although operated by a different company.

Essentially the same aggregate, and the same aggregate source, is involved as in Series 7. In this case the aggregate is dredged from a small bay off the Delaware River (called Van Scriver Lake) and screened at the production site. It is loaded directly into a barge (sampling point No. 1). The method of unloading and handling the aggregate is essentially the same, with the exception that the aggregate is dropped by clambucket into a relatively small feeder hopper, instead of a surge bin as was the case for Series 7. A belt conveyor (sampling point No. 2) carries the aggregate from the feeder hopper to the overhead storage bins (sampling point No. 3), from which it is dropped into the weigh hopper.

VARIATIONS IN GRADATION AT POINT OF USE

Due to the similarity of the plants, the same sampling procedure and sampling tools were used as for Series 7. Two increments, having an average weight of 25 lb each, were taken from each 9-cu yd batch selected by the use of a table of random numbers.

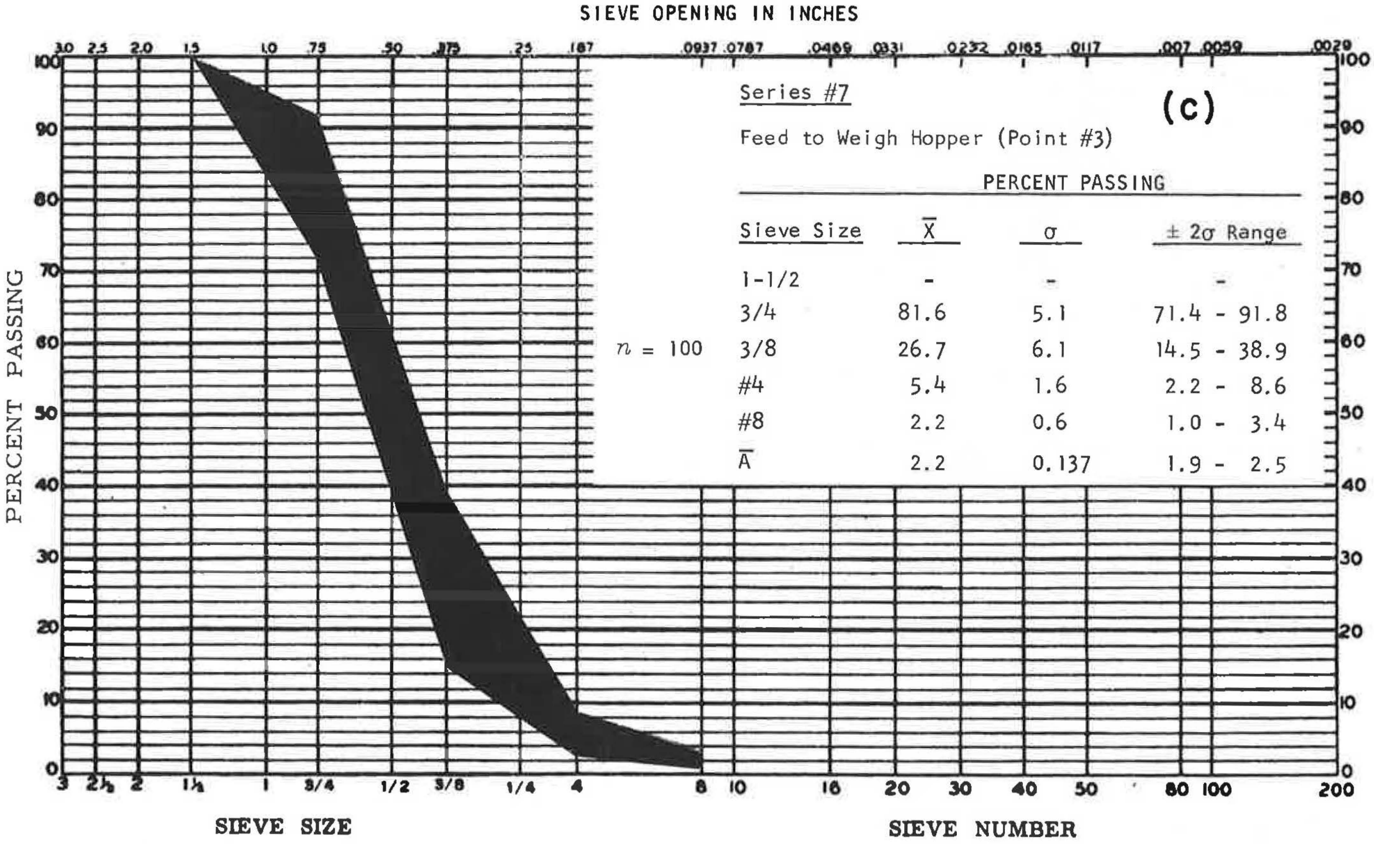
The specification requirements for the gradation of the gravel, together with the overall variation and batch-to-batch variations, are given in Table 42. The overall variation shows the limits within which 95 percent of results of tests on single increments, selected at random, could be expected to fall. The batch-to-batch variations shows the probable limits within which the gradation of the coarse aggregate in 95 percent of the batches would fall, if the entire batch was put through the sieves.

VARIATION IN GRADATION AT OTHER SAMPLING POINTS

The sampling points for this series were at the same relative locations as for Series 7. The overall variations found at these points are given in Table 43.

SERIES 8 SUMMARY CHARTS

The relative variances and standard deviations of the percentage passing the different sieves are summarized in Table 44. The relative variance of the Hudson $\overline{A}$ values is shown in Figure 45. The aggregate grading charts showing $\pm 2\sigma$ envelopes at each of the three sampling points are



Series #7

Feed to Weigh Hopper (Point #3)

n = 100

	PERCENT PASSING		
Sieve Size	$\overline{X}$	σ	$\pm 2\sigma$ Range
1-1/2	-	-	-
3/4	81.6	5.1	71.4 - 91.8
3/8	26.7	6.1	14.5 - 38.9
#4	5.4	1.6	2.2 - 8.6
#8	2.2	0.6	1.0 - 3.4
$\overline{A}$	2.2	0.137	1.9 - 2.5

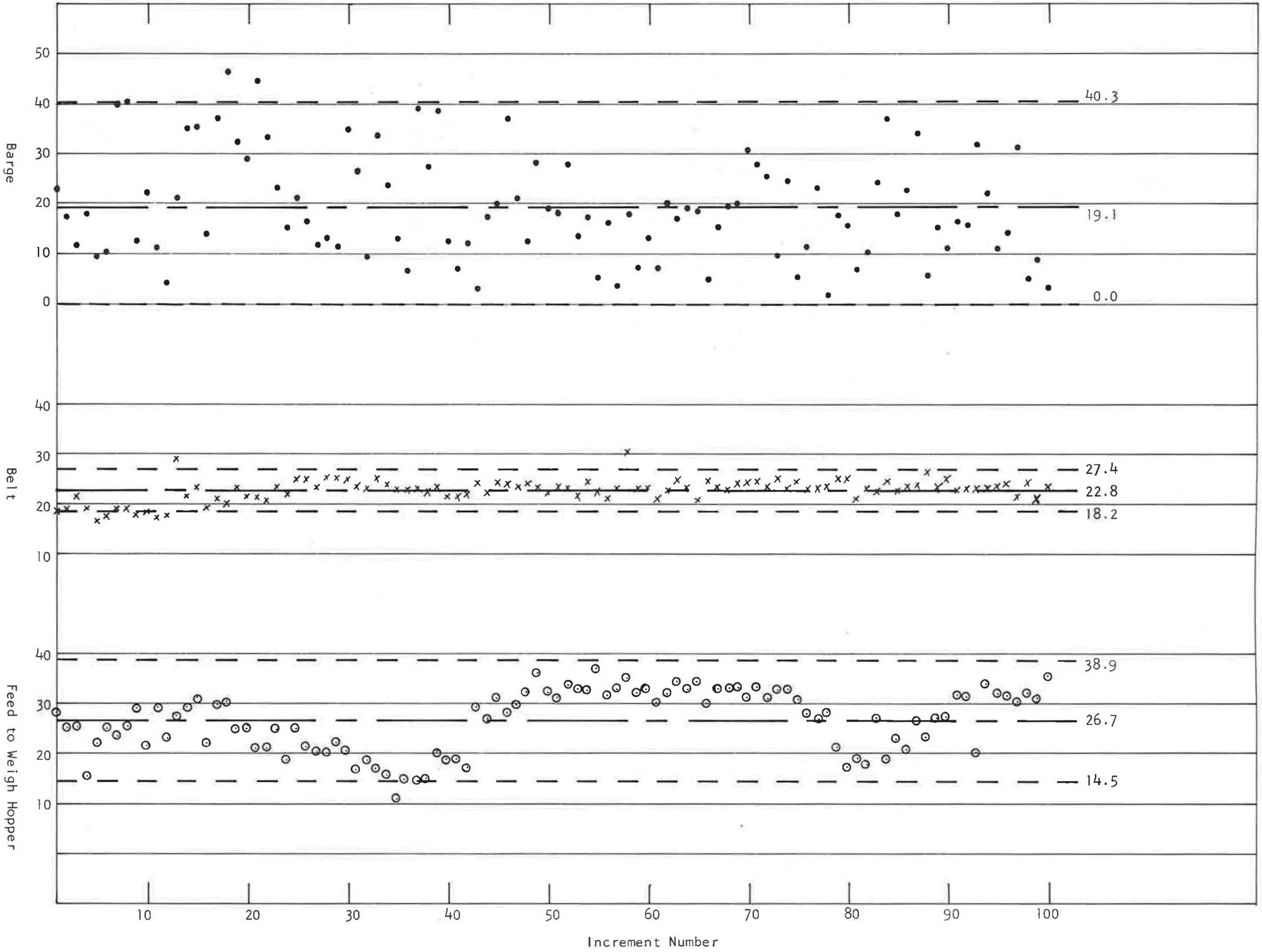


Figure 44. Individual test values, percent passing ⅜-in. sieve, Series No. 7.

TABLE 42

GRADATION OF SERIES 8 GRAVEL AT POINT OF USE

	PERCENT PASSING SIEVE		
SIEVE SIZE	SPEC. LIMITS	OVERALL VARIATION, $\pm 2\sigma_o$ LIMITS	BATCH-TO-BATCH VARIATION, $\pm 2\sigma_l$ LIMITS
1½ in.	100	100	100
1 in.	90-100	—	—
¾ in.	—	74-97	78-93
½ in.	25-60	—	—
⅜ in.	—	15-54	24-44
No. 4	0-10	1-15	3-12
No. 8	0-5	0-8	2-6

TABLE 43

GRADATION OF SERIES 8 GRAVEL AT OTHER SAMPLING POINTS

	PERCENT PASSING SIEVE, $\pm 2\sigma_o$ LIMITS			
SIEVE SIZE	SPEC. LIMITS	BARGE	TRANSFER BELT	WEIGH HOPPER
1½ in.	100	100	100	100
1 in.	90-100	—	—	—
¾ in.	—	63-100	72-92	74-97
½ in.	25-60	—	—	—
⅜ in.	—	4-60	20-41	15-54
No. 4	0-10	0-7	2-7	1-15
No. 8	0-5	0-3	1-4	0-8

presented in Figure 46. The individual test results of percent passing the ⅜-in. sieve are plotted in the order of sampling from the barge, from the belt, and at feed to the weigh hopper in Figure 47. The gradation and statistical summary parameters for this series are presented in Table 45.

The same general trend may be noted with respect to the decreased variability due to handling between the barge and the belt, part of which is again nullified due to subsequent segregation in the overhead storage bin. This time, however, the degree of improvement is not as great as that obtained in Series 7. The degree of overall variability at the barge, DOV, = 36.0, and the degree of segregation, D of S, = 30.6. The corresponding values at the belt (sampling point No. 2) are DOV = 14.8 and D of S = 11.0, less than a three-fold improvement. At the overhead storage bin discharge, DOV = 22.4 and D of S = 15.0

In this case, the segregation index at the barge, $S_{\bar{A}} = 3.8$ and $S_{50} = 3.6$, was higher than that obtained later at either the belt, $S_{\bar{A}} = 2.5$ and $S_{50} = 2.2$, or at the bin discharge, $S_{\bar{A}} = 2.1$ and $S_{50} = 1.8$. These values are not greatly different and indicate that the within-batch variability was relatively large at all three sampling points.

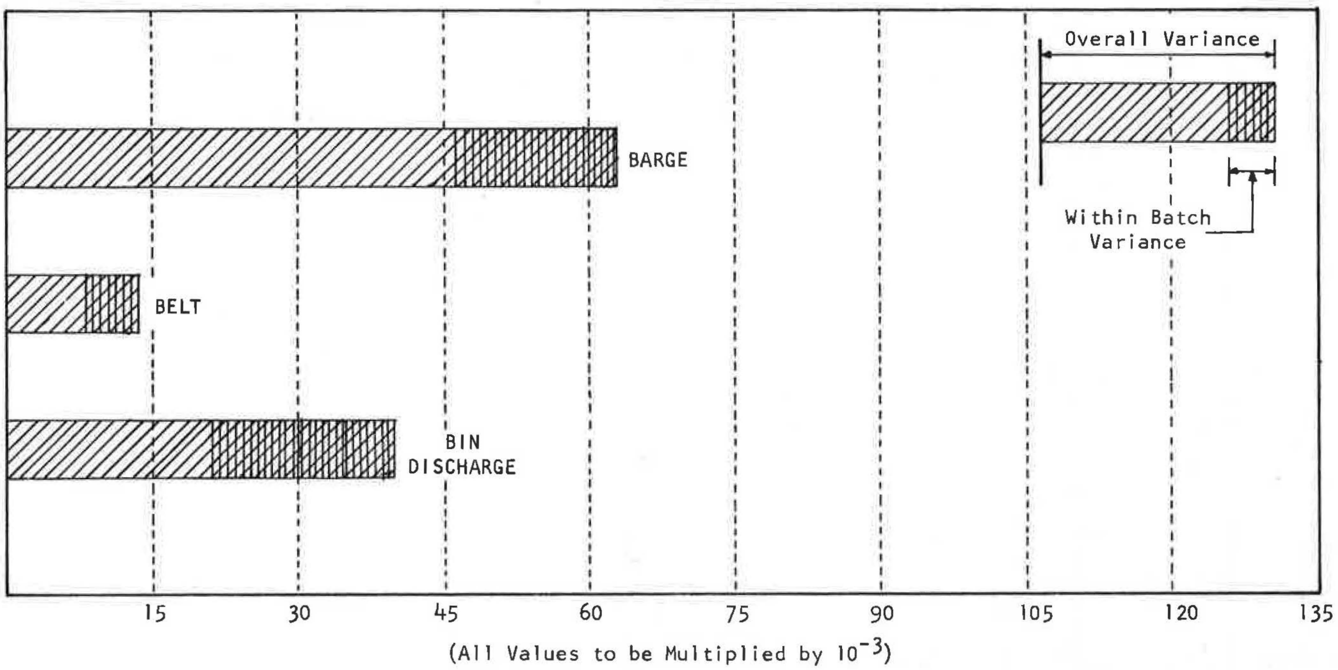


Figure 45. Relative variance of $\bar{A}$ at different points in process stream, Series No. 8, uncrushed gravel (1 in.-No. 4), Pennsylvania (Plant No. 2).

TABLE 44

SUMMARY OF GRADATION AVERAGES, VARIANCES, AND STANDARD DEVIATIONS, SERIES 8 AGGREGATE [a]

SIEVE SIZE	SAMPLING POINT	AVG. % PASSING, $\overline{X}$	VARIANCE: TESTING, σ^2_t	VARIANCE: INHERENT σ^2_a	VARIANCE: OVERALL, σ^2_o	VARIANCE: WITHIN-BATCH, σ^2_b	VARIANCE: BATCH-TO-BATCH, σ^2_l	STANDARD DEVIATION: TESTING, σ_t	STANDARD DEVIATION: INHERENT σ_a	STANDARD DEVIATION: OVERALL, σ_o	STANDARD DEVIATION: WITHIN-BATCH, σ_b	STANDARD DEVIATION: BATCH-TO-BATCH, σ_l
¾ in.	Barge	84.3	0.01	4.00	112.2	39.9	72.3	0.1	2.0	10.6	6.3	8.5
	Belt	82.2	0.01	4.00	24.3	9.2	15.1	0.1	2.0	4.9	3.0	3.9
	Bin discharge [b]	85.7	0.01	4.00	31.4	17.3	14.1	0.1	2.0	5.6	4.2	3.8
⅜ in.	Barge	32.0	0.02	2.00	193.7	53.1	140.6	0.1	1.4	13.9	7.3	11.9
	Belt	30.4	0.02	2.00	29.1	11.9	17.2	0.1	1.4	5.4	3.4	4.1
	Bin discharge [b]	34.4	0.02	2.00	88.9	64.2	24.7	0.1	1.4	9.4	8.0	5.0
No. 4	Barge	3.1	0.04	0.36	3.3	0.7	2.6	0.2	0.6	1.8	0.8	1.6
	Belt	4.8	0.04	0.36	1.6	0.7	0.9	0.2	0.6	1.3	0.8	0.9
	Bin discharge [b]	7.7	0.04	0.36	11.6	7.3	4.3	0.2	0.6	3.4	2.7	2.1
No. 8	Barge	1.2	0.03	0.16	0.5	0.2	0.3	0.2	0.4	0.7	0.4	0.5
	Belt	2.3	0.03	0.16	0.5	0.3	0.2	0.2	0.4	0.7	0.5	0.4
	Bin discharge [b]	4.1	0.03	0.16	4.4	3.6	0.8	0.2	0.4	2.1	1.9	0.9
$\overline{A}$	Barge	2.224	0.0001	—	0.0627	0.0166	0.0461	0.01	—	0.250	0.129	0.215
	Belt	2.233	0.0001	—	0.0135	0.0053	0.0082	0.01	—	0.116	0.073	0.090
	Bin discharge [b]	2.381	0.0001	—	0.0396	0.0184	0.0212	0.01	—	0.198	0.136	0.146

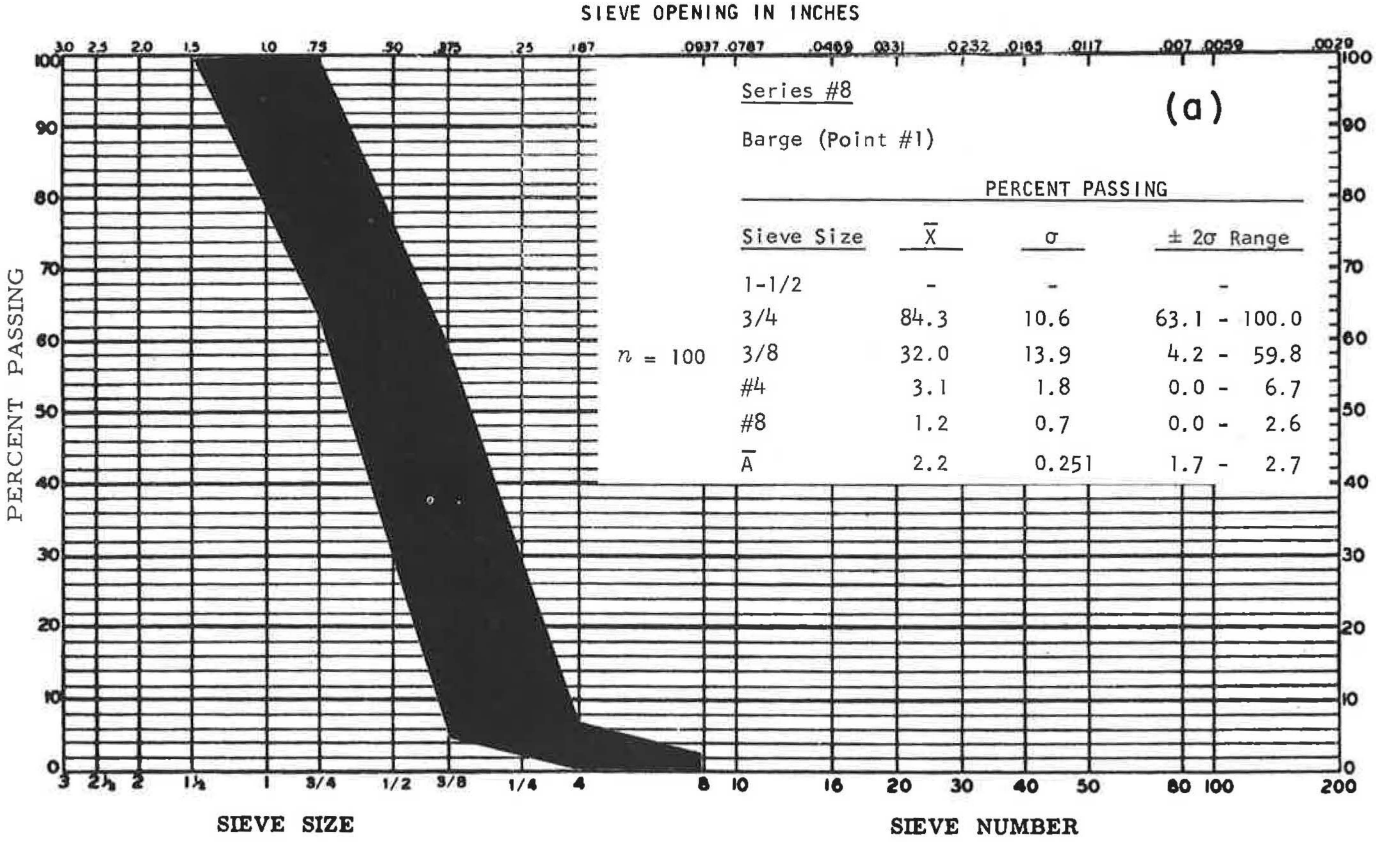
[a] See Table 13 for description of aggregate. [b] To weigh hopper.

TABLE 45

SUMMARY OF GRADATION AND STATISTICAL PARAMETERS, SERIES 8 AGGREGATE [a]

SAMPLING POINT: NO.	SAMPLING POINT: LOCATION	AVERAGE SIZE, $\overline{X}$: % PASS. ¾ IN.	AVERAGE SIZE, $\overline{X}$: % PASS. ⅜ IN.	AVERAGE SIZE, $\overline{X}$: HUDSON $\overline{A}$	50% LEVEL STD. DEVIATION: OVERALL, σ_{o-50}	50% LEVEL STD. DEVIATION: WITHIN-BATCH, σ_{b-50}	50% LEVEL STD. DEVIATION: BATCH-TO-BATCH, σ_{l-50}	SLOPE	HUDSON $\overline{A}$ STD. DEVIATION: OVERALL, $\sigma_{o-\overline{A}}$	HUDSON $\overline{A}$ STD. DEVIATION: WITHIN-BATCH, $\sigma_{b-\overline{A}}$	HUDSON $\overline{A}$ STD. DEVIATION: BATCH-TO-BATCH, $\sigma_{l-\overline{A}}$	SEGREGATION INDEX: BASED ON $\overline{A}$, $S_{\overline{A}}$	SEGREGATION INDEX: BASED ON 50% P, S_{50}	DEG. OF OVERALL VARIABILITY, DOV	DEG. OF SEGREGATION, D OF S
1	Barge	84.3	32.0	2.224	18.0	9.5	15.3	2.14	0.250	0.129	0.215	3.8	3.6	36.0	30.6
2	Belt	82.2	30.4	2.233	7.4	5.0	5.5	2.14	0.116	0.073	0.090	2.5	2.2	14.8	11.0
3	Bin discharge [b]	85.7	34.4	2.381	11.2	8.3	7.5	2.14	0.198	0.136	0.146	2.1	1.8	22.4	15.0

[a] See Table 13 for description of aggregate. [b] To weigh hopper.

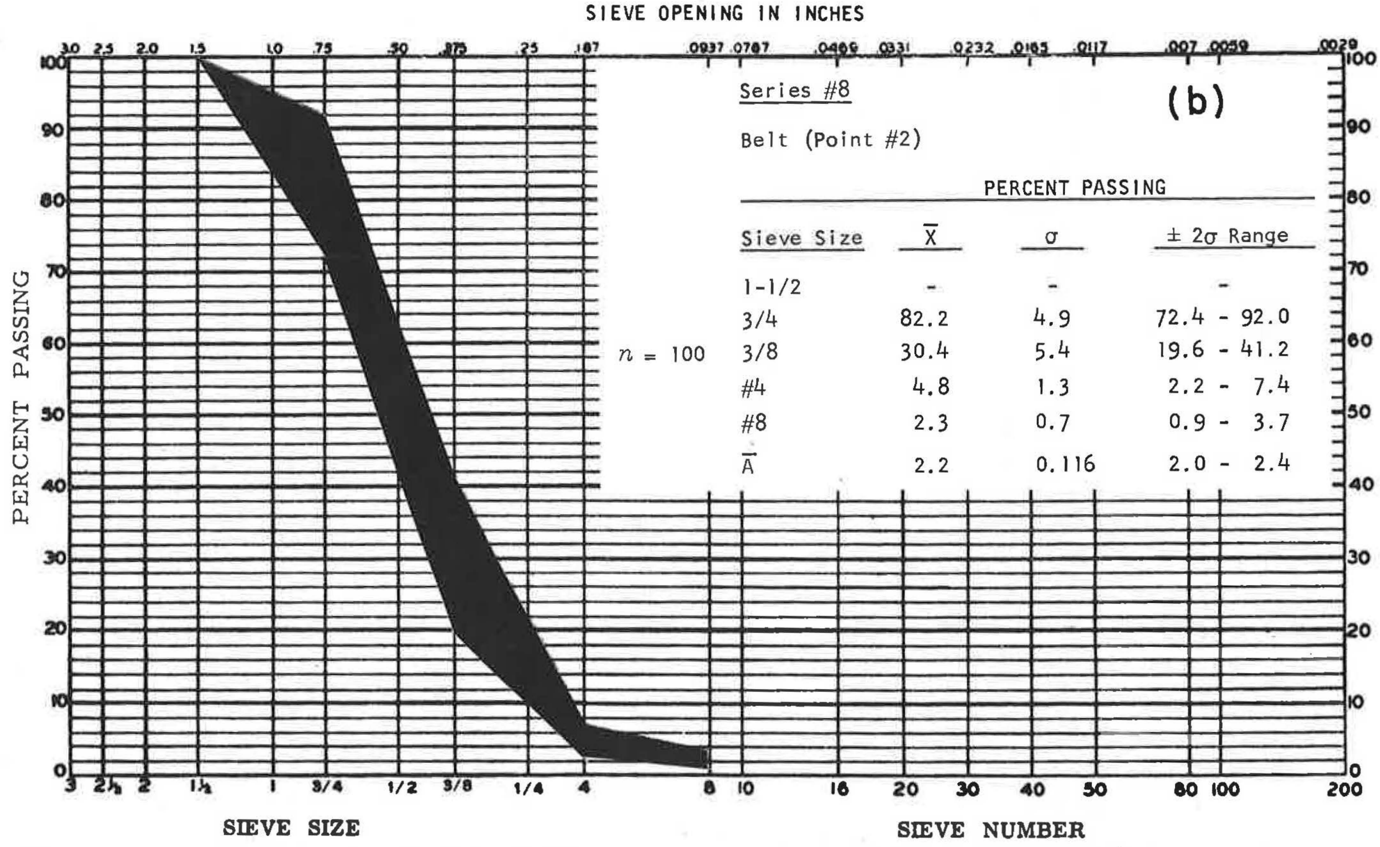


Series #8

Barge (Point #1)

n = 100

Sieve Size	$\overline{X}$	σ	± 2σ Range
	PERCENT PASSING		
1-1/2	-	-	-
3/4	84.3	10.6	63.1 - 100.0
3/8	32.0	13.9	4.2 - 59.8
#4	3.1	1.8	0.0 - 6.7
#8	1.2	0.7	0.0 - 2.6
$\overline{A}$	2.2	0.251	1.7 - 2.7



Series #8

Belt (Point #2)

n = 100

Sieve Size	$\overline{X}$	σ	± 2σ Range
	PERCENT PASSING		
1-1/2	-	-	-
3/4	82.2	4.9	72.4 - 92.0
3/8	30.4	5.4	19.6 - 41.2
#4	4.8	1.3	2.2 - 7.4
#8	2.3	0.7	0.9 - 3.7
$\overline{A}$	2.2	0.116	2.0 - 2.4

Figure 46. Aggregate gradation charts for Series No. 8 samples taken at (a) barge, (b) belt, and (c) bin discharge to weigh hopper.

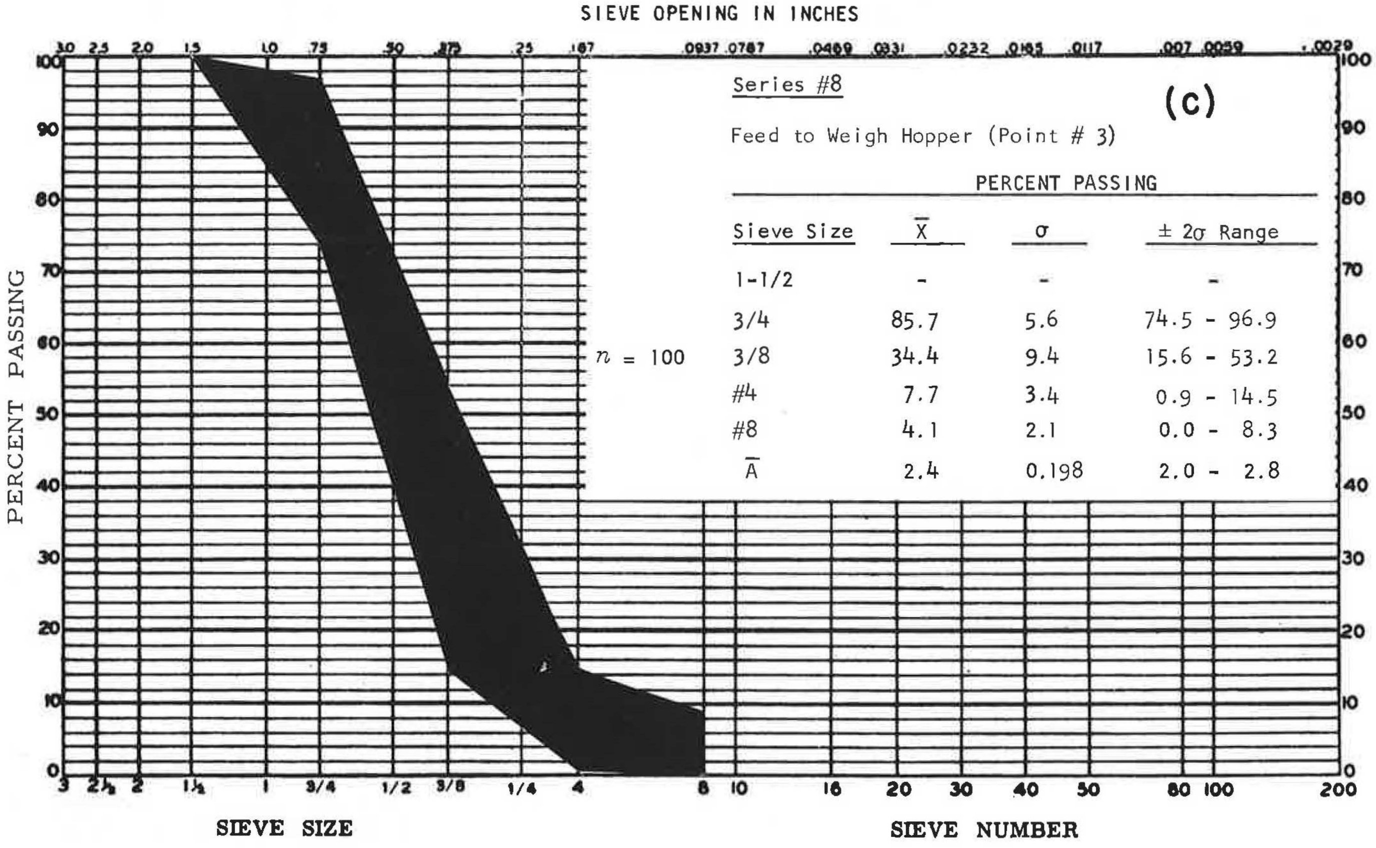
An assignable cause for high within-batch variability was noted at the overhead storage bin discharge. In this case, gravel was fed through two gates to the weigh hopper and because the flow time was quite short it was decided to take a test portion from under each gate at the same time (Fig. 17) for purposes of measuring within-batch variance. Some difference in the gradation of the gravel flowing from these two gates was observed at the time of sampling; however, the significance of this difference did not really become apparent until the data were analyzed. In Figure 47 the individual test values for percent passing the ⅜-in. sieve obtained on samples taken under the fine gate are shown as circles, whereas the corresponding values for the companion samples taken under the coarse gate are shown as solid squares. Average level, standard deviation, and 2σ limits are summarized in Table 46. Because of the assignable cause of difference in gradation at these two gates, the within-batch variance, σ^2_b, was estimated by a different statistical method for this sampling point.

It is apparent that there was significant segregation taking place in the overhead storage bin, and that the feed from one gate has different characteristics from the other, in both level, $\bar{X}$, and variability, σ. From the construction control viewpoint, the same basic questions are again germane: "Where should the inspector take his sample?" and "Can the road tell the difference?"

TABLE 46

COMPARISON OF RESULTS FROM BIN DISCHARGE GATES, SERIES 8

GATE	AVG. % PASS. ⅜-IN. SIEVE, $\bar{X}$	STD. DEV., σ	$\pm 2\sigma$ LIMITS
Fine	27.8	8.7	10-45
Coarse	40.9	10.0	21-61
Combined	34.4	11.4	12-57



Series #8

Feed to Weigh Hopper (Point # 3)

PERCENT PASSING

Sieve Size	$\bar{X}$	σ	$\pm 2\sigma$ Range
1-1/2	-	-	-
3/4	85.7	5.6	74.5 - 96.9
3/8	34.4	9.4	15.6 - 53.2
#4	7.7	3.4	0.9 - 14.5
#8	4.1	2.1	0.0 - 8.3
$\bar{A}$	2.4	0.198	2.0 - 2.8

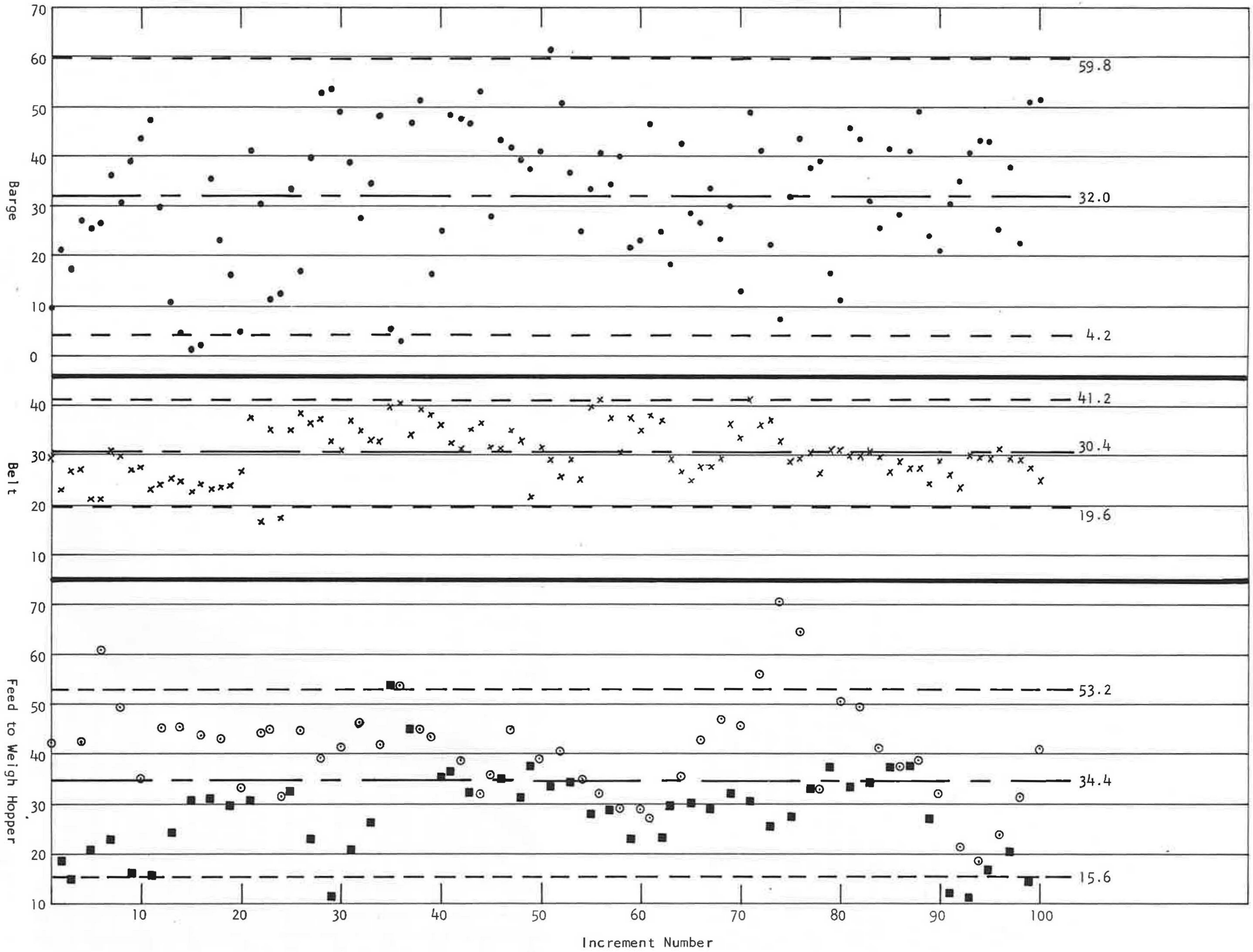


Figure 47. Individual test values, percent passing $\frac{3}{8}$-in. sieve, Series No. 8.

CHAPTER FIVE

ANALYSIS AND INTERPRETATION OF DATA

In the previous chapters the test data and findings are presented with discussion or interpretation largely limited to a particular experiment or series. In this chapter an attempt is made to analyze and interpret the overall data and findings as a whole. Various comparisons are made to explore relationships among results obtained at different locations using similar processes, sampling methods, etc., both with the same and with different types and gradation of aggregates. Some of these were planned, such as the comparison between Series 7 and 8, whereas other comparisons are possible because of similarities that just happened to occur among the eight locations selected for the main field investigation.

This chapter is divided into two main sections. The first is devoted to the primary objectives of this study as outlined in the project statement. In general, comments in this section are limited to points or considerations germane to the project title, "Evaluation of Construction Control Procedures."

As mentioned previously, however, there are some secondary benefits to be derived from this research in the way of supporting data on the relative degree of segregation associated with the stockpiling and handling procedures that happened to be in use in the eight commercial plants investigated. It should be emphasized again, however, that the primary objectives of this study were not to investigate the relative efficacy or merits of any particular transportation or aggregate handling technique, nor is any attempt made to completely analyze or "milk" the data made available in this study for objectives other than those related to construction control considerations. Nevertheless, where applicable, various comparisons are made among results obtained in this study and, in some cases, incorporating data from related projects 10-3 and 10-3(1). The second section, therefore, presents correlations and interpretations of some data which may, strictly speaking, be beyond the scope of the current project assignment, but which, nevertheless, are considered to be of enough importance in the overall aggregate handling problem to be included in this report.

CONSTRUCTION CONTROL CONSIDERATIONS

As stated in Chapter One, the basic problem in construction control of aggregate gradations is to estimate the average percent of an aggregate that will pass a given sieve, and the variations from that average, on the basis of measurements made on a sample taken from the LOT of aggregate. The engineers responsible for the design and control of aggregate mixtures must estimate the average percent passing the critical sieves so that the proportions of fine and coarse aggregate in the designed mixture will result in optimum workability and serviceability. They also must make sure that variations from this average are not so great as to require frequent adjustments in the mix proportions, or, if adjustments are not made, to result in batches or units of product that will have unacceptable workability or serviceability. In addition to meeting the engineering requirements, the engineer must not only satisfy himself, but also must be able to document, that the aggregate actually used in the construction was in substantial compliance with the specification requirements.

Obviously, the specification limits should be compatible with the engineering requirements and, insofar as possible, reflect that optimum balance between serviceability and construction cost that will result in maximum use value; i.e., minimum overall construction and maintenance costs over the service life of the facility. Both the average level and the variations from that average are needed to properly design and establish optimum construction control procedures to first define and then to arrive at the best practical compromise between engineering requirements and specification limits. Once established, the average and the variability are again the important parameters needed to put a procedure to work for control of a given material or element of construction and then to document the results.

There are three basic questions to be considered, as follows:

1. What are the engineering requirements?
2. What are realistic specification tolerances under normal, practical operating conditions?
3. What construction control procedures should be used to measure and document substantial compliance with the specification requirements?

Although a complete discussion is not required for this interim report, it is important to the interpretation of the findings of this study that certain factors common to these three questions be briefly reviewed. The reader is referred to a more complete analysis and discussion of these considerations contained in *NCHRP Report 17,* "Development of Guidelines for Practical and Realistic Construction Specifications."

There are two approaches to question No. 1, "What are the engineering requirements?" One is to base the average level and acceptable variability strictly on known or assumed technical considerations. In some cases enough is known, or is thought to be known, about engineering requirements to theorize or demonstrate in the laboratory that a change in aggregate gradation beyond a certain point will become intolerable from the quality or serviceability viewpoint. A limited discussion of some of these factors for portland cement concrete and for asphaltic concrete mixes is given in the following. The other approach is to measure the average level and the variability of known acceptable construction and to use these results for defining engineering requirements on the basis that they represent satisfactory serviceability experience. Requirements so defined may not reflect the ultimate in good engineering, but they are supposedly sound because they are based on criteria known by past experience to represent acceptable performance.

The pertinent factors in answering the second question, with relation to realistic specification limits and tolerances, can be derived from the same measurements of known acceptable construction just mentioned. The normal variability in gradation encountered with a number of different coarse aggregates in different plants and in different geographic locations provides a sound basis for establishing realistic limits because they were made under practical operating conditions using typical and customary construction control procedures. The acceptance limits for any specification must be practical and realistic, otherwise difficulties in enforcement or increased costs are inevitable. Realistic limits should be wide enough to accommodate normal variations in gradation, unless there are definite engineering requirements that justify the added expense of more rigid controls and measurement techniques.

Answers to the third question, regarding the appropriate construction control procedures to measure and document substantial compliance, involve a number of considerations. Some of these are discussed in greater detail in Chapter Six, which presents a recommended means for incorporating the results of this study into a complete aggregate specification. The point of sampling is an obvious consideration, as is the relative degree of variability associated with the sampling and testing; i.e., the within-batch variance as distinguished from batch-to-batch variability. Less obvious are the factors having to do with the degree of assurance that the engineer (or the State, or the Bureau of Public Roads) may believe to be warranted in documenting substantial compliance for specific specification requirements. Some requirements are more important than others and also some items of construction are more critical than others, even though governed by the same specification. So the number of test portions required in a given aggregate control situation depends not only on the magnitude of the variations, but also on the importance of the product under control and the degree of accuracy desired or deemed to be warranted by the people who are actually running the job or who are interpreting the findings for a particular control problem.

Regardless of the construction control procedure or other basis of engineering judgment, there is always a risk of accepting poor material and of rejecting good material. As shown by the variations noted in the eight commercial plants studied under this project, these risks are extremely large when the decision to accept or reject is based on a test result from a single test portion. Inasmuch as these were typical plants under normal control, it may be assumed that the magnitudes of the variations measured are also typical and normal. Further, it is suspected, but not known, that the actual construction control in many State highway departments, as practiced by "typical" inspectors, consists of taking single test portions, or at best "selecting" relatively few portions, rather than following a random sampling plan with a sufficient number of tests to materially reduce the sampling and testing errors. Therefore, the risks associated with coarse aggregate construction control are probably larger than most of the industry might have supposed.

It is apparent that all three of the basic questions outlined are interrelated and depend on a number of factors that cannot be independently and simply defined as engineering requirements, or realistic tolerances, or documentation requirements. One influences the other and they must be considered together. What constitutes a good construction control procedure for one product may incur unwarranted inspection and testing costs for another, or involve unacceptable risk for still a third.

There is another important factor, however, in connection with these construction control considerations that must not be overlooked; namely, that these studies revealed a number of assignable causes for variability, some of which obviously warrant corrective action quite independent of the basic quality versus cost question. The surging resulting from within-bin segregation is a good example. Some corrective action, in the form of improved hopper design or in the method of feeding bins or controlling the discharge from them, is obviously in order. Figure 41 graphically illustrates the extreme surging obtained in the feed to the weigh hopper in Series 6. There are other assignable causes for variability that should be corrected as a matter of good construction control procedure quite independent of specification considerations. The important point is that this study provides the engineer with techniques and tools whereby these assignable causes can be separated and quantified.

Conformance With Specification Limits

A rather startling result from analysis of the data derived from the main field investigation is the lack of general conformity to the specification limits governing the production of these eight commercial plants. This is a very touchy subject, however, because assurances were given that this was a research study and not an investigation, and that the owners would not be placed in any jeopardy due to lack of specification conformance. This is why the several series are identified merely by number and State. Although the 2σ comparisons are made against the specifications as a means of presenting the findings for the individual series, it so happens that conformance implications are not readily apparent because of differences in sieve sizes. The policy of obscuring identification with any individual plant is continued in the following by discussing the conformance results as they apply to all eight of the series as a whole, or to groups of similar-type aggregates.

The method of analysis has been to select one sieve size for each series that is reasonably close to the central range (i.e., 30 percent or more passing range), then to calculate the number of tests that would be outside of the specifications for that particular sieve size based on the overall standard deviation, σ_o, at the sampling point closest to incorporation of the aggregate into the finished product (usually feed to weigh hopper). For the six portland cement concrete aggregates conformance comparisons were made on percent passing the ⅜-in. or ½-in. sieves; for the asphaltic concrete aggregate, series 3, on the ¾-in. size; and for the well-graded base course aggregate, series 5, on the No. 4 sieve. The results reported are based on calculations of the percentage of single test results that would be outside of the specification limits assuming normal distribution. It should be noted that this analysis does not take cognizance of the influence of the assignable causes discussed.

On this basis, an average of 35.8 percent of all test results for all eight series may be expected to fall outside of the specification limits. The corresponding average for the group of six portland cement concrete aggregates is 25.5 percent. In other words, using normal construction controls and sampling and testing procedures, from one-fourth to more than one-third of the test results obtained at these commercial plants would be outside of the specification limits on the one sieve size used for this conformance analysis. This was a rather surprising finding and the implications are of obvious importance from the construction control viewpoint.

Only one of the plants was operating in what might be considered a reasonably satisfactory manner with respect to specification conformance. This single low result was 4.4 percent out. Next was a group of three series ranging from 13.0 to 21.9 percent out. Then three series in the range of 48.9 to 59.1 percent out, and the high value was 74.2 percent out. By "percent out" in each case is meant the percentage of the test results that would be outside of the specification limits for that particular sieve size, using the $\overline{X}$ and σ_0 values actually measured under the conditions cited. The reason for non-conformance is about equally split between displaced average level and high variability. In five out of the eight series, the average percent passing, $\overline{X}$, was either close to or, in one case, actually outside of the broad band limits specified. For two of these five series the variability was also high. In two cases $\overline{X}$ was reasonably close to the center of the band but σ_0 was high. In summary, one series was good for both $\overline{X}$ and σ_0; two series were out because of high σ_0; three series were out because of displaced level; and two series were out because both $\overline{X}$ was displaced and σ_0 was high.

A further analysis was made on the assumption that the average percent passing, $\overline{X}$, was at the midpoint of the specifications. In other words, if the plant was operating so that the average gradation was right in the middle of the band, then how many test results would fall outside of the specification limits if the distribution were normal and the standard deviation was that measured at the final sampling point for that series? Under these conditions the average percent out for all eight series is 18.4, with a range of 0 to 55.5 percent out. The average percent out for the six portland cement aggregates is 12.2, with a range of 0 to 29.4 percent out. Even under these very favorable conditions of $\overline{X}$ being right in the center of the band, only two of the plants were operating in what would normally be thought of as being under really good control. The percent out for these two series was essentially 0 and 2.9 (the 2.9 is the series that was 4.4 percent out when using the actual $\overline{X}$). Two series were at the 8.5 to 11.2 level; three in the 18.7 to 29.4 range; and then the one high value of 55.5 percent out.

Still a third analysis was made to calculate the percentage of individual test results that would be out of a broad band range of 30 to 70 percent passing if $\overline{X}$ were exactly 50 percent passing and the variability was the hypothetical standard deviation at the 50 percent passing level, σ_{o50}. Under these conditions an average of 9.3 percent of the test results for all eight series (8.0 percent for the six portland cement aggregates) would be outside of even these very broad limits of ± 20 percent with $\overline{X}$ exactly at the 50 percent passing level. In this case, however, only three of the series were high (12.4, 16.8, and 25.0) and all are probably subject to assignable cause influences. There was one plant at the 7.3 percent out level; the remaining four series are 2.0 or less (0, 0.2, 1.2, 2.0). It is interesting to note that at the 50 percent passing level, and with $\overline{X}$ exactly in the middle of the band, at only one of the eight plants could all of the test results be expected to fall within the limits of 30 to 70 percent passing.

Regardless of how the data are analyzed, one positive conclusion is that these studies have demonstrated without question that actual variations in gradation of eight commercial coarse aggregates exceed the limits of current specifications. The variations measured are considerably greater than was anticipated. There are a number of obvious interpretations that could be placed on these findings. There also are a number of implications that could be highly significant, not only from the viewpoint of construction control *per se,* but also that these findings create doubts as to the validity or importance of some engineering concepts of gradation criteria that have become generally accepted over the years. In a number of cases engineering judgment would seem to place many of the variations measured in a clearly unreasonable category for acceptable construction. Still, how is one to know?

The question has been raised repeatedly throughout this study as to whether the specification limits can be justified in light of the historical implication that the finished products produced at these plants have been performing satisfactorily. Although eight plants admittedly comprise only a small sample, there is no reason to believe that the six portland cement concrete plants are not more or less typical. Series 3 and series 5 are probably exceptional in that the specification limits are quite narrow and the average gradation level at the time of sampling was just not where it should have been. Discounting these two series entirely, however, still leaves the six portland cement concrete plants representing operations in five States with three uncrushed gravels (series 4, 7, and 8) one crushed gravel (series 2), one slag (series 6), and one crushed limestone (series 1). The gradation of these six aggregates is similar enough so that the results lend themselves to consideration as a group and so far as is known there was nothing atypical or unusual about any of these plants, or in the specifications under which they were operating. Still, on the average, 25.5 percent of the test results of the coarse aggregate feed to the weigh hoppers in these plants can be expected to be outside of the specification limits, assuming normal distribution at the $\overline{X}$ and σ_0 levels actually measured.

This finding is disturbing from the viewpoint of all three of the basic questions posed earlier in this section. If one accepts the general finding that large variations in the coarse aggregate for portland cement concrete do occur without apparent serious consequences, there is need to re-examine the following:

1. How much variation from the gradation used in design can occur without requiring adjustments in mix proportions, which, if not made, would detrimentally effect the placeability and/or serviceability of the mixture? Can more positive control be justified?
2. What are realistic specification tolerances under normal practical operating conditions? Should the specification limits be widened?
3. What constitutes substantial compliance and how many test portions are needed to attain the desired degree of confidence in the acceptance/rejection decision?

Construction Control of PCC Aggregates

This portion of the discussion is limited to the six portland cement concrete plants (series 1, 2, 4, 6, 7, 8). This is an appropriate group for special consideration inasmuch as the problem of construction control of coarse aggregates for portland cement concrete is of particular concern because these aggregates are generally not re-screened before proportioning into the concrete mixture. The most important coarse aggregate size with respect to both workability and mixing water is probably the ¾-in. to No. 4 fraction. Accordingly, the following discussion is confined to the percentages passing the ⅜-in. sieve at the six concrete proportioning plants investigated under this project.

Table 47 presents a summary of the pertinent parameters, all based on the percent passing the ⅜-in. sieve. In each case these parameters apply to test portions taken at the point where the aggregate was being fed to the weigh hopper.

The average level, $\overline{X}$, for the six series was 33 percent passing the ⅜-in. sieve, with a low of 27 (series 7) and a high of 39 (series 6). When one considers that these are widely scattered plants geographically and that four different types of aggregate are represented, the average gradation is quite consistent. On the surface this would seem to indicate that 33 ± 6 percent passing the ⅜-in. sieve is about right, if customary practice is taken as a reflection of optimum experience. This may or may not be the case and it is known, of course, that satisfactory concrete mixtures can be designed using other gradations of coarse aggregate. The average level is more likely to be a reflection of optimum gradation from the viewpoint of minimum waste; i.e., that 33 percent minus ⅜ in. is about the level at which most nominal 1½-in. maximum size aggregate normally runs, whether from a crusher or as naturally occurring uncrushed gravel.

Although the average level at these six PCC plants was reasonably constant, the gradation of individual test portions as the coarse aggregate was being fed to the weigh hopper varied widely, as shown by both the range, R, and the standard deviations. Individual batches of concrete were produced using coarse aggregate from a low of about 10 percent passing the ⅜-in. sieve to a high of about 70 percent. The extreme deviation from the average, $\overline{X}$, was -26 for one test portion in series 1, and $+36$ for one test portion in series 8. These are extreme values and include the full within-batch measurement errors and variations. It is probable that the actual outer limit variability was more in the order of ± 20 percent, or from about 15 to 55 percent passing the ⅜-in. sieve.

As shown, the actual batch-to-batch standard deviation of the percent passing the ⅜-in. sieve at the point of use ranged from a low of 3.9 (series 2) to a high of 15.5 (series 6). Excluding series 6, where the high batch-to-batch variation can be assumed to be due to an assignable cause which could be corrected, the normal batch-to-batch standard deviation appears to be about 6.0. This indicates that, if a sufficient number of increments are taken from each batch to average out most of the within-batch variation, a practical specification range would be $\pm 2\sigma_l$, or about ± 12 percent from the desired average passing the ⅜-in. sieve. In most cases of normal operation this range should include about 95 percent of the batches, and the statistical acceptance plan should be designed accordingly.

Whether one considers the extreme limits of 15 to 55 percent passing as estimated from the range values or the

TABLE 47

SUMMARY OF PARAMETERS FOR PCC AGGREGATES

SERIES	NO. OF TEST PORTIONS, n	% PASS. ⅜-IN. SIEVE[a] AVERAGE, $\overline{X}$	% PASS. ⅜-IN. SIEVE[a] RANGE, R	STANDARD DEVIATION OVERALL, σ_o	STANDARD DEVIATION WITHIN-BATCH, σ_b	STANDARD DEVIATION BATCH-TO-BATCH, σ_l
1	100	37	11-61	12.0	2.7	11.6
2	100	33	21-41	4.0	0.7	3.9
4	100	35	18-43	5.8	2.5	5.3
6	88	39	9-61	15.7	2.4	15.5
7	100	27	11-36	6.1	2.9	5.4
8	100	34	11-70	9.4	8.0	5.9

[a] At feed to weigh hopper.

adjusted batch-to-batch standard deviation, $\sigma_l = 6$, the question arises as to the effect of such large variations on the workability and serviceability of the concrete in which these batches of aggregate were used. One line of investigation is to estimate the change in voidage of the plus No. 4 aggregate, inasmuch as this is one of the considerations in some methods of design of concrete mixtures. In theory, the minimum voids would be obtained by grading the coarse aggregate in accordance with the maximum density curve. According to this theory, the ideal gradation would be 100 percent passing the 1½-in. sieve, 74 percent passing the ¾-in. and 55 percent passing the ⅜-in. Assuming 45 percent voids in 100 percent ⅜-in. to No. 4 aggregate, the foregoing gradation would theoretically reduce the voidage of the mixture to 38 percent, or a maximum range in voids between the best possible and worst possible conditions of about 7 percent. Because most practical concrete mixtures are usually slightly oversanded, it is doubtful if variations in voidage within this range would seriously effect the workability of the concrete.

Another line of investigation is to estimate the effect of the variations in gradation on the gallons of water per cubic yard required for a certain slump, say 2 to 4 in. If the gradation of the total solids, including the cement, is such that the $\bar{A}$ value is about 4.5, the estimated water requirement would be about 34 gal per cubic yard of mixture for a 2- to 4-in. slump concrete. The variation in $\bar{A}$ corresponding to a variation in percent passing the ⅜-in. sieve of from 15 to 55 percent is about 0.9, and this variation is estimated to change the water requirements for a 2- to 4-in. slump by about 4 gal per cubic yard. This is enough to require adjustment. It is further estimated that with the plant set for a 3-in. slump, the variation from 15 to 55 percent passing the ⅜-in. sieve could cause the slump to go as low as 1 in. or as high as 6 in., if the water content was not adjusted. Insofar as the effects on water requirements are concerned, the variations observed in percent passing the ⅜-in. sieve are probably significant from the viewpoint of both consistency and compressive strength.

The foregoing estimates are admittedly approximate and cannot be substantiated without further experimental work. However, they do indicate that although the extreme variations in gradation of coarse aggregate indicated by sampling actual batches may not affect workability, they probably would require adjustment in the quantity of mixing water in order to maintain a constant slump which, in turn, would result in significant variations in compressive strength. Further research is needed to quantitatively evaluate these effects and determine what engineering limits should be applied for coarse aggregate gradation. In general, it appears that a realistic specification for coarse aggregate could have wider limits for actual variation than some current specifications, but that these limits would not necessarily be wide enough to include the variations resulting from some current methods of handling and storing aggregates.

The accuracy obtained in estimating the average percent of a LOT passing a given sieve will depend on the number of increments in a sample and on the total weight of the sample. The required number of increments can be determined from Eq. 9 or from the nomograph (Fig. 8). Use of either requires that an advance estimate of the standard deviation be available and that the desired degree of accuracy be stated. The total sample size can be found by use of Eq. 2.

If the desired degree of accuracy is ± 3 percent passing the ⅜-in. sieve and a 95 percent confidence level is acceptable, the data indicate that sampling the feed to the weigh hopper would require 10 to 80 increments, depending on the series and the uniformity of the aggregate at that sampling point. This is based on the overall standard deviation, making no allowance for surging or other assignable causes of variation. In other words, if one wants to know the average level of the percent passing the ⅜-in. sieve at the point of use within ± 3 percent and is willing to accept a wrong answer 5 percent of the time (1 out of 20), the average must include from 10 to as many as 80 test increments taken from the feed to the weigh hopper at the six concrete plants studied in this investigation.

The number of increments taken from each batch should be sufficient to average out within-batch variation, which will disappear when the concrete is mixed. This study indicated that within-batch variation was a significant part of the overall variation, and that in most cases a number of increments should be taken from each sampled batch. Except for one very low value (series 2) and one high value (series 8), both due to unusual circumstances, the within-batch standard deviation had a range of from 2.4 to 2.9. This indicates that when a LOT is to be accepted or rejected on the basis of a small number of sampled batches at the point of use, the test portion of coarse aggregate from each batch should be made up of about 5 or 6 increments if the batch average of the percent passing the ⅜-in. sieve is to be determined with an accuracy of about ± 3 percent.

Study of the data shows that the main component of within-batch variation, in most cases, is local segregation of aggregate sizes in different parts of the batch, and that inherent variance is significant only in the case of gradations containing large (plus ⅜ in.) particles. The least significant component of either within-batch or overall variance appears to be testing error, even when gradation tests are made in a routine manner. This indicates that the risks associated with acceptance sampling and testing cannot be significantly reduced by efforts to increase the accuracy of the test method. The reliability of results of gradation tests on a sample of aggregate depends primarily on the number of random increments in the sample, and on the total weight of the sample.

The total sample size required to attain a given degree of accuracy of average percent passing a given sieve size will depend on the gradation and the average particle size weight of the material retained on that sieve. Assuming 100 percent passing the 1½-in., 85 percent passing the ¾-in., and 30 percent passing the ⅜-in. sieve, the total sample weight should be about 20 lb for an accuracy of ± 3 percent. If an accuracy of ± 1 percent is desired, a total sample weight of about 200 lb would be required. The manner in which this total sample weight is divided among different batches or the number of increments taken per batch will depend on the relative magnitude of the within-batch variance, the batch-to-batch variance, and the objectives of the particular construction control or sampling plan.

In cases where a large number of increments must be taken, the total sample weight could exceed the minimum requirement, but could be reduced by coneing and quartering or with a sample splitter. In cases where only few increments are needed to average out the effects of segregation, either the size or the number of increments may be increased so that the required total sample weight can be obtained. In theory, the size of the increments depends only on the number of increments and the required total sample weight, but obviously the size must be sufficient to minimize the risk of excluding any of the larger particles. This minimum size is not definitely known, but a weight of 25 times the weight of the largest particles in the gradation is suggested, resulting in a minimum increment weight of about 2 lb for 1½-in. maximum size aggregate. The size of the test portion is a function of the particular equipment (sieves and shaker) to be used. The test portion may consist of a number of increments.

For any given construction control situation, the relationship between the desired accuracy and the balance of the number of batches to be sampled, the size and number of increments per batch, the average particle weight, and the total sample weight is presented and discussed in Chapter Two (Eq. 8).

Construction Control of AC Aggregates

For asphaltic concrete or, in fact, for any of the bituminous paving mixtures, aggregate gradation requirements for both average level and variability are quite critical, inasmuch as aggregate gradation, aggregate voids, and optimum asphalt content are closely related. It is common practice to design a specific mix for a given job and to apply job-mix tolerances to both the aggregate gradation and the asphalt content. In addition, some States and agencies specify stability, flow, voids filled, etc., for various classes of paving mixtures. Because the asphalt content is usually held constant once the job-mix formula is set, it is necessary to control the aggregate gradation within reasonably close limits if the proper asphalt/void ratio is to be maintained. This necessity for close control of the gradation justifies the added expense of the control exercised by the screens over the contents of the hot bins for most high-type paving mixtures. Unfortunately, it is not practical to separate and recombine the finer aggregate (minus No. 10). Therefore, the gradation of the important No. 1 bin must be controlled at the cold feed. Although this initial study is concerned almost entirely with coarse aggregate, the construction control of the fine aggregate for bituminous mixtures is an important consideration that might profitably be made a part of the follow-up work under this project.

Although rescreening of the coarse aggregates into two or three hot-bin sizes removes some of the potential consequences of large variability at the cold feed, it does not remove all of them by any means. There are two possibilities. If there are surges of the finer sizes of coarse aggregate there is a tendency for this material to carry over into the bins intended to contain only larger sizes. This "overrun" results in a change in the actual gradation of the aggregate in the mixture and variations in the voidage and texture of the compacted pavement. On the other hand, long-term variations from coarse to fine, as shown in Figure 32, result in unbalanced bins. Unless these variations are compensated for by adjustment of the cold feed, one of the bins is certain to either run empty or overflow. In the case of a manually operated plant this presents a great temptation to the man on the scales to "pull heavy" or "pull light" on a bin; again, the result is a variation in the characteristics of the pavement. In the case of a fully automated plant, a low bin would cause a delay in production. For these reasons, elimination of assignable causes of variations in coarse aggregate gradation is of importance for hot-plant operators as well as concrete producers.

Construction Control of Coarse Aggregate Base Courses

Recent studies (HR 10-3(1)) indicate that the gradation of the coarse aggregate in a mechanically stabilized base can have an influence on degradation. If the gradation is such as to approximate the maximum density curve the weaker aggregate particles are protected from crushing by the support of the finer particles that fill the voids. An extreme variation in coarse aggregate gradation from coarse to fine could increase the voids to the extent that this support would be decreased and there would be point contact between the coarse aggregate particles. Under these conditions the pressures developed by vibratory or steel-wheeled rolling could result in crushing or splitting of the particles. Because base mixtures are usually blended by combining aggregates direct from stockpiles, without rescreening, methods of control of handling and stockpiling that will result in reasonable uniformity of gradation appear worthy of further study.

A preliminary inference from the limited measurements made to date is that there is a lesser tendency for well-graded aggregates to segregate. As indicated in Table 33, the overall standard deviation at the 50 percent passing level, σ_{o50}, increased only a moderate amount as the material was handled from the plant to the truck to the roadway (corresponding σ_{o50} values were 4.1, 6.4, 6.6). The same general finding was obtained in the HR 10-3(1) study, in which two well-graded aggregates, one hard and one soft, were sampled at the belt feeding the pugmill, after pugmill mixing, and after transporting and placing on the roadway using different methods of spreading and compacting. The change in standard deviation, σ_o, of the percent passing the ⅜-in. sieve was from about 3.5 to a high of about 6.2 for both the hard and soft aggregate. In general, these limited findings indicate that the variability likely to occur in the normal handling of well-graded base course aggregates that have been premixed and brought to optimum moisture content is not great.

Effect of Sampling Point on Construction Controls

One objective of the main field investigation was to measure just how important the point of sampling might be for coarse aggregate in typical commercial plants under normal construction control. It is apparent from these studies that the point of sampling can have a great deal to do with both the average level and the variability of the test results obtained. Some such effects were, of course, known to exist, but the relative magnitude of the differences under practical operating conditions needed to be defined. As might be expected, the effects differ widely, depending on the aggregate, the process, and the sampling procedure.

Taking an overall look first, the number of test increments necessary to attain a ± 3 percent degree of accuracy at the 95 percent confidence level may be used to show relative differences. It has already been noted that sampling the feed to the weigh hopper for the six PCC aggregates would require taking 10 to 80 increments under these conditions; i.e., to measure the average level of the percent passing the ⅜-in. sieve within ± 3 percent and be right 19 out of 20 times. On the same basis, sampling from a truck, stockpile, or barge would require 25 to 120 increments, depending on the series and on how badly the aggregate was segregated. As noted in Chapter Three, truck sampling gave test results which were about 10 percentage points on the fine side of the average in each of two cases (series 2 and series 5). Sampling at the source (crusher or screening plant) would require only 10 to 15 increments, whereas sampling from the belt (generally the belt feeding the overhead storage bins) would require from 5 to 27 increments to determine the average percent passing the ⅜-in. sieve with an accuarcy of ± 3 percent.

In general, these findings show that the more a coarse aggregate is handled the greater is the chance of inducing variability of significance from the construction control viewpoint. There is more to be learned on this subject, however, when one gets away from the overall generalizations to a closer look at the individual series or to groups of related operations.

Series 2 and series 4 illustrate that coarse aggregate can be satisfactorily taken from the crusher or from the screening plant, transported, stockpiled, and then transferred to a belt feeding the weigh hopper or plant bins. In each of these series the variability at point of use in the plant is slightly less than it was at the crusher or the screening plant. In both series some segregation occurred at intermediate points and an inspector would likely obtain a false reading, using customary sampling techniques at these intermediate points. Series 5 is quite similar in that the variability of the aggregate at the point of use on the roadway is reasonably close to that at the plant. In series 7 there is both within-batch and batch-to-batch segregation evident in sampling river gravel from the barge. However, this segregation is materially reduced in handling across a transfer belt to the overhead storage bins feeding the weigh hopper. The degree of overall variability, DOV, for series 2, 4, 5, and 7 is 8.6, 16.0, 13.2, and 17.2, respectively. Inasmuch as DOV is the same as the $2\sigma_0$ limits at the 50 percent passing level, these values represent a maximum range (widest part of the broad gradation band) within which 95 percent of the individual test results should fall. The corresponding degree of segregation, D of S, values ($2\sigma_{l50}$) are 8.2, 14.6, 10.8, and 14.6, respectively, indicating that batch-to-batch variability, the more important from a quality viewpoint, is reasonable. Thus, these four series illustrate that it can be done.

In the other four series there are assignable causes for decreased uniformity as the aggregate is handled through the process stream. In series 3 the assignable cause was severe segregation in the stockpile which carried through to the cold feed bin. In series 6, 1, and 8 the assignable cause is associated with segregation in the storage bin feeding the weigh hopper. Correction of these assignable causes by improved hopper design or method of charging the bin could well result in acceptable uniformity at each of these points. At the final sampling point for series 1, 3, 6, and 8 the degree of overall variability, DOV, values are 26.0, 29.0, 34.8, and 22.4, respectively. The corresponding degree of segregation, D of S, values are 24.6, 27.2, 34.4, and 15.0, respectively. This significantly greater level of variability is sufficient to cause difficulties in construction control and probably to affect the quality of the finished product.

In summary, four of the series (Nos. 2, 4, 5, 7) end up operating in a reasonably satisfactory manner insofar as variability at the point of use is concerned. Of the remaining four series (Nos. 1, 3, 6, 8), there were assignable causes in each case for the lack of uniformity measured—assignable causes which supposedly could be alleviated.

There is no way, however, in which an inspector can appraise the construction control situation in a strange plant or make a meaningful and valid decision regarding conformance to the specifications by taking the customary too few test portions from any one of the normal sampling points. The findings of this study make it apparent that some rather drastic changes in customary practice are in order if effective construction controls are to be exercised and the results documented for State and BPR records. Three main steps are indicated, as follows:

1. To ferret out assignable causes, such as the surging noted in the overhead storage bins feeding the weigh hoppers, process control charts plotting successive test results obtained over an extended operating period should be prepared. Whether these charts should be the responsibility of the plant operator or of the inspector is another question, but the usefulness of the simple line charts shown for each series in Chapter Four is obvious. The condition shown in Figure 41, or in Figure 32, could lead to totally erroneous construction control decisions or, at best, be very confusing to the average inspector unless the situation is clarified by plotting at least a day's run, and preferably a week or more. Actually, all concerned, but particularly the producer or plant operator, have much to gain by keeping these or similar process control charts on one or more key gradation points on a continuing basis. They help the operator predict trends or real changes requiring corrective action; conversely, they provide a valuable deterent to unwarranted action or rejection based on one or two test results. There are other charting techniques which incorporate additional advantages from the viewpoint of both operation and documentation. Regardless of the degree of sophistication ultimately selected, however, some method of process control charting is a strongly recommended first step.

2. Once the significant assignable causes have been defined and minimized or eliminated where practical, selection of the sampling point for routine quality control becomes a matter of judgment and convenience balance. Hard and fast rules are not applicable because the situation will obviously vary from plant to plant. For series 1, 6, and 8 in particular, sampling from the belt, or from any prior point for that matter, would yield test results bearing no resemblance to those on the aggregate actually fed to the weigh hopper. On the other hand, there is no reason why a more convenient earlier sampling point should not be used for the routine checking of construction control *if* there is an established relationship with the level and batch-to-batch variability of the aggregate that actually goes into the finished mix. Such a relationship must be constant as shown by periodic spot-checking. In other words, the inspector does not necessarily have to sample from an inconvenient point merely because it is closer to the finished product if he can prove to himself, and periodically check, that the test results obtained at another point in the process stream do, in fact, reflect the gradation of the aggregate that is incorporated into the finished product.

This can be made a bit more specific for typical PC concrete coarse aggregate process flow on the basis of the six plants investigated in this study. Sampling from the belt feeding the overhead storage bins could have considerable advantages from the viewpoint of both convenience and ease of randomization, depending on the plant setup. Once any intolerable variability occurring in the transfer to, or within the bin has been corrected, sampling from the belt could be a satisfactory construction control procedure and might well be preferred from the viewpoint of safety and ability to avoid local segregation or sampling error. As discussed in Chapter Three, the belt sampling procedure shown in Figure 13 is a preferred method where practical.

3. Lastly, the required number of test portions must be taken in strict conformance with the assigned random sampling plan. Deviations in either the number or the size of the increments taken or the introduction of bias in sample selection may completely upset the validity of the statistical approach to construction control by changing both the buyer's and the seller's risk and the confidence levels designed into the plan and supposedly warranted by the engineering requirements. The importance of avoiding bias and of sticking to the specified random sampling plan may take some special training or indoctrination, and possibly a new look at administrative controls, because a somewhat different view of the inspector's role and his responsibility is involved.

All of the foregoing is predicated on the design of a random sampling plan which will give the purchaser (the State) reasonable assurance of obtaining a satisfactory product and which is also realistic from the viewpoint of both execution by the inspector and ability to conform for an acceptable cost on the part of the contractor or producer. This sounds like a big order, requiring a knowledge of statistics and other skills beyond the capacity of many materials engineers. Such is the case only for the basic analyses, development of guidelines, etc.—work which has been done and is reported partly herein, but in detail in *MW Technical Report 201,* entitled "A Plan for Expediting the Use of Statistical Concepts in Highway Acceptance Specifications," prepared for the Bureau of Public Roads in 1963; and *NCHRP Report 17* (HR 10-1), entitled "Development of Guidelines for Practical and Realistic Construction Specifications." Some of the principles as they apply to coarse aggregates are presented herein in Chapter Two and discussed earlier in this chapter. These principles are illustrated and incorporated into the design of a complete specification in Chapter Six. Practical application of these principles in the field becomes a matter of applying routine disciplines requiring only a high school knowledge of simple mathematics and the ability to follow instructions.

ADDITIONAL CORRELATIONS AND RELATED FINDINGS

Although complete coverage is beyond the scope of this interim report, the 50 percent passing level parameters provide a means of comparing related groups of data on aggregates of different gradations. These parameters, based on the variability projected for the 50 percent passing level (the widest part of the gradation band), are discussed in Chapter Two and summarized in the following for the convenience of the reader.

The standard deviation at the 50 percent passing level is estimated by plotting the standard deviation, σ, versus $\sqrt{P(100 - P)}$ on log-log paper, then reading off the σ which corresponds to the $P = 50$ percent passing level. The values thus determined corresponding to σ_{o50}, σ_{b50}, and σ_{l50}, together with the slope of the line and the corresponding parameters for each sampling point, are presented in tabular form for each individual series in Chapter Four.

Segregation index, S_{50}, is the ratio of the overall variance to the corresponding within-batch variance at the 50 percent passing level; that is

$$S_{50} = \sigma^2_{o50}/\sigma^2_{b50} \tag{12}$$

Degree of overall variability,

$$\text{DOV} = \frac{\sigma_{o50}}{\sqrt{P(100 - P)}} \times 100 \tag{13a}$$

Because P, by definition, equals 50 percent in this case, the equation reduces to

$$\text{DOV} = 2\sigma_{o50} \tag{13b}$$

Degree of segregation, D of S, is a companion parameter based on the batch-to-batch standard deviation, σ_{l50}, at the 50 percent passing level.

Inasmuch as $P(100 - P)$ is the variance hypothetically associated with maximum variability on the basis of the binomial theorem, $\sqrt{P(100 - P)}$ becomes the standard deviation theoretically associated with complete segregation. The parameters DOV and D of S may therefore be thought of as a percentage of total or complete separation of a test portion of aggregate into two equal weights, one with all particles smaller, and the other with all particles larger, than the openings in a hypothetical sieve, sized to yield exactly this 50 percent passing level.

The first comparison is presented in Table 48, which gives the relative difference in these parameters as a function of the point of sampling. There are four principle sampling points involved. First is the point closest to the crusher or screening unit, average DOV = 11.6. Next is the stockpile or barge, average DOV = 29.1. Third is the transfer to overhead storage bins, or, in one case, directly to the feed hopper, average DOV = 12.0. The final sampling point is the bin discharge to the weigh hopper or cold feed, average DOV = 24.2. Four of the 27 sampling points included in this main field investigation do not lend themselves to the foregoing groupings, but are presented separately in Table 48 for completeness.

Another comparison is shown in Table 49, which gives

TABLE 48

COMPARISONS BASED ON SAMPLING POINT

SERIES NO.	SAMPLING POINT	AGGREGATE		SEG. INDEX, S_{50}	*DOV*, $2\sigma_{o50}$	D OF S, $2\sigma_{i50}$	SLOPE
		TYPE	SIZE RANGE				
(*a*) Crusher, Screening Unit or Blending Plant							
1	1	Crushed stone	1 in.-No. 4	4.3	9.6	8.6	1.7
2	1	Crushed gravel	¾ in.-No. 4	3.5	9.0	7.6	1.7
3	1	Crushed stone	1½ in.-⅜ in.	1.4	14.1	8.0	1.7
4	1	Rounded gravel	1 in.-No. 4	2.9	17.0	13.8	2.1
5	1	Crushed stone	1 in. No. 200	4.6	8.2	7.2	3.1
Avg.				3.3	11.6	9.0	
(*b*) Stockpile or Barge							
1	2	Crushed stone	1 in.-No. 4	9.5	27.2	25.8	1.7
2	3	Crushed gravel	¾ in.-No. 4	1.8	15.6	10.2	1.7
3	2	Crushed stone	1½ in.-⅜ in.	2.1	45.0	32.8	2.1
4	2	Rounded gravel	1 in.-No. 4	5.8	29.0	26.4	2.1
4	3	Rounded gravel	1 in.-No. 4	3.5	18.8	16.0	2.1
7	1	Rounded gravel	1 in.-No. 4	2.5	32.0	25.0	2.1
8	1	Rounded gravel	1 in.-No. 4	3.6	36.0	30.6	2.1
Avg.				4.1	29.1	23.8	
(*c*) Transfer Belt to Storage Bin or Feed Hopper							
1	3	Crushed stone	1 in.-No. 4	27.5	16.6	16.2	1.7
2	4	Crushed gravel	¾ in.-No. 4	10.9	8.6	8.2	1.7
6	2	Slag	1 in.-No. 8	2.7	11.8	9.2	1.7
7	2	Rounded gravel	1 in.-No. 4	3.2	8.2	6.8	2.1
8	2	Rounded gravel	1 in.-No. 4	2.2	14.8	11.0	2.1
Avg.				9.3	12.0	10.3	
(*d*) Bin Discharge to Weigh Hopper or Cold Feed							
1	4	Crushed stone	1 in.-No. 4	20.1	26.0	24.6	1.7
3	3	Crushed stone	1½ in.-⅜ in.	8.4	29.0	27.2	1.7
4	4	Rounded gravel	1 in.-No. 4	6.4	16.0	14.6	2.1
6	3	Slag	1 in.-No. 8	44.5	34.8	34.4	2.1
7	3	Rounded gravel	1 in.-No. 4	3.6	17.2	14.6	2.1
8	3	Rounded gravel	1 in.-No. 4	1.8	22.4	15.0	2.1
Avg.				14.1	24.2	21.7	
(*e*) Trucks							
2	2	Crushed gravel	¾ in.-No. 4	6.3	20.0	18.6	1.7
5	2	Crushed stone	1 in.-No. 200	2.0	12.8	8.8	3.1
Avg.				4.2	16.4	13.7	
(*f*) Roadway							
5	3	Crushed stone	1 in.-No. 200	3.2	13.2	10.8	3.1
(*g*) Front-End Loader Batches							
6	1	Slag	1 in.-No. 8	1.1	20.6	6.4	1.7

the relative magnitudes of the 50 percent passing level parameters at the point of minimum variability and at the point of maximum variability for each series, grouped according to aggregate type. Of the eight aggregates in the main field investigation, three are crushed stone, three rounded gravel, one crushed gravel, and one slag. Although there are obviously insufficient data to draw valid conclusions, there is some inference that rounded gravel is slightly more prone to segregate than angular or rough-textured particles of the same size. A further inference is that crushed stone of 1½-in. maximum size has a greater tendency to segregate than does 1-in. maximum size crushed aggregate. The final inference is that the base course aggregate, having an extended gradation from 1 in. down through minus 200, has less tendency to segregate, but it should be noted that this material was premixed to optimum moisture content and was not stockpiled. All three of these inferences are substantiated by the findings of HR 10-3(1); or, conversely, the indications that might be garnered from these limited data on eight widely scattered commercial plants lend some support to the HR 10-3(1) general conclusions.

The 50 percent passing level parameters for stockpiles built under projects 10-3 and 10-3(1) are given in Table 50. At the time these projects were reported, the concept of these parameters based on the 50 percent passing level projections had not been developed. They are presented herein largely to get them "on the record" as a matter of general interest and for whatever value they might have as guidelines for enhancing interpretation of the variations measured in this study. It should be noted, however, that the method of stockpile sampling was different, therefore direct comparisons should be approached with caution. In projects 10-3 and 10-3(1), the entire stockpile was taken down, usually with a front-end loader, and the aggregate put across a belt for the taking of test portions, as shown in Figure 13. Two of the stockpiles (series 1 and 2) and the two barges (series 7 and 8) were sampled, using both methods. As noted in Chapter Three, the variability, as measured by overall standard deviation, σ_o, using the hand shovel method, is about double that obtained using the belt method. Nevertheless, the parameters presented in Table 50 are of value because of association with known good and known questionable or poor construction practice, insofar as the building of stockpiles is concerned.

The pattern shown by the values of the slope of the log σ versus log percent passing line is also of interest. The crushed aggregates consistently plot to a slope of 1.7, except when they are highly segregated. The rounded gravel plots to a slope of 2.1, independent of the degree of segregation. The extended gradation, 1-in. to No. 200 aggregate, series 5, does not plot to the nice straight line normally obtained with the coarse aggregates, but tends to curve in the direction of increased variability as the gradation approaches 50 percent from either direction. The value given of 3.1 is an approximation as a reasonable "eyeball" compromise of the curvature. This same curvature and the same approximation, 3.1, was also observed in corresponding

TABLE 49
COMPARISONS BASED ON TYPE OF AGGREGATE

SERIES NO.	SAMPLING POINT		AGGREGATE		SEG. INDEX, S_{50}	DOV, $2\sigma_{o50}$	D OF S, $2\sigma_{l50}$	SLOPE
	NO.	LOCATION	TYPE	SIZE RANGE				
			(*a*) AT POINT OF MINIMUM VARIABILITY					
1	1	Crusher	Crushed stone	1 in.-No 4	4.3	9.6	8.6	1.7
3	1	Crusher	Crushed stone	1½ in.-⅜ in.	1.4	14.1	8.0	1.7
5	1	Plant	Crushed stone	1 in.-No. 200	4.6	8.2	7.2	3.1
2	1	Crusher	Crushed gravel	¾ in.-No. 4	3.5	9.0	7.6	1.7
4	1	Plant	Rounded gravel	1 in.-No. 4	2.9	17.0	13.8	2.1
7	2	Belt	Rounded gravel	1 in.-No. 4	3.2	8.2	6.8	2.1
8	2	Belt	Rounded gravel	1 in.-No. 4	2.2	14.8	11.0	2.1
6	2	Belt	Slag	1 in.-No. 8	2.7	11.8	9.2	1.7
			(*b*) AT POINT OF MAXIMUM VARIABILITY					
1	2	Stockpile	Crushed stone	1 in.-No. 4	9.5	27.2	25.8	1.7
3	2	Stockpile	Crushed stone	1½ in.-⅜ in.	2.1	45.0	32.8	2.1
5	3	Roadway	Crushed stone	1 in.-No. 200	3.2	13.2	10.8	3.1
2	2	Truck	Crushed gravel	¾ in.-No. 4	6.3	20.0	18.6	1.7
4	2	Stockpile	Rounded gravel	1 in.-No. 4	5.8	29.0	26.4	2.1
7	1	Barge	Rounded gravel	1 in.-No. 4	2.5	32.0	25.0	2.1
8	1	Barge	Rounded gravel	1 in.-No. 4	3.6	36.0	30.6	2.1
6	1	Front-end loader	Slag	1 in.-No. 8	1.1	20.6	6.4	1.7
6	3	Bin discharge	Slag	1 in.-No. 8	44.5[a]	34.8[a]	34.4[a]	1.7

[a] Abnormal distribution.

TABLE 50

50 PERCENT PASSING LEVEL PARAMETERS FOR PROJECT 10-3 AND 10-3(1)

NO.	TYPE OF PILE	SEG. INDEX, S_{50}	DOV, $2\sigma_{o50}$	D OF S, $2\sigma_{l50}$	SLOPE
		(*a*) PROJECT 10-3			
—	Parent pile	1.6	9.4	5.8	1.7
11	Flat-mixed	1.4	8.8	4.8	1.7
1	Cast and spread	1.7	9.6	6.2	1.7
10	Ramped	1.8	10.0	6.8	1.7
3	Flat-layered	2.6	12.0	9.4	1.7
9	Truck-dumped	3.0	12.8	10.4	1.7
6	Flat-layered	4.0	14.8	12.8	1.7
8	Tiered (bermed)	7.4	20.0	18.6	1.7
5	Coned, tent	9.9	23.2	22.0	2.1
2	Double cone	15.0	28.6	27.6	1.7
7	Single cone	18.8	32.0	31.2	2.1
4	Single cone	24.3	36.4	35.6	2.1
	(*b*) PROJECT 10-3(1), 1½-IN. TO ⅜-IN. CRUSHED STONE				
	Starting pile	4.0	9.6	8.4	—
	Coned	18.9	21.0	19.4	—
	Cast and spread	10.0	10.8	10.2	—
	Truck-dumped	10.0	15.2	14.4	—
	(*c*) PROJECT 10-3(1), 1-IN. TO NO. 4 ROUNDED GRAVEL				
	Starting pile	7.8	20.2	18.8	2.1
	Coned	16.3	42.0	40.6	2.1
	Cast and spread	3.8	24.0	19.2	2.1
	Truck-dumped	4.5	17.4	15.4	2.1

plots of log σ versus log P for the well-graded aggregates used for the degradation portion of the HR 10-3(1) project. It will be noted that no value was shown for the slope of the line for the 1½- to ⅜-in. crushed stone aggregate in Project HR 10-3(1). This aggregate was too clean, due to washing and scalping out of the fines. There just was not enough material passing the No. 4 and No. 8 sieves to provide a reasonable measure of variability in this finer sized part of the curve.

The significance of this slope or why the values 1.7 and 2.1 keep recurring has not been established as yet, or at least not well enough for presentation at this time. Some theorizing and some additional development of a mathematical model suitable for relating the pertinent variables and predicting their influence has been accomplished. Further work along these lines and development of a more sophisticated and statistically sound model is recommended as part of the continuation of this project.

CHAPTER SIX

DESIGN OF A COMPLETE SPECIFICATION

An essential part of "construction control" is the specification. This was recognized in the Project Statement, which gives as one of the objectives:

> Recommendation as to practical means for incorporating the results of this study into highway construction specifications and procedures to provide a basis for the acceptance or rejection of aggregates.

Accordingly, attention is now focused on this important subject. The pertinent statistical concepts and the analysis of variance results presented in Chapter Two, the findings of the various experiments and field investigations reported in Chapters Three and Four, and the interpretation of the data discussed in Chapter Five, are incorporated where appropriate.

Presented first is a brief review of the factors influencing allowable gradation tolerances and consideration of the existing variations in gradation found. Next is a listing of the essential parts of a complete aggregate specification. An illustrative model specification for a coarse aggregate for portland cement concrete is then presented; following which the basis for each point in the model is explained and related back to the discussion of the essential parts of a complete aggregate specification necessary for adequate and practical construction control.

APPLICATION OF FINDINGS TO SPECIFICATION CONSIDERATIONS

The acceptance limits for any specification must be practical and realistic; otherwise, difficulties in enforcement or increased costs are inevitable. Realistic limits should be wide enough to accommodate normal variations in gradation unless there are definite engineering requirements that justify the added expense of more rigid controls and measurement techniques. In general, the engineering requirements for highway construction aggregates are related largely to a surface area or aggregate voidage. Although it is beyond the scope of the present project to attempt answers to the basic question regarding the degree of uniformity that can be justified, limited discussions of some of the engineering aspects have been presented in Chapter Five for PCC aggregates, bituminous mixtures, and crushed aggregate base course.

Studies conducted over a wide geographical area during the work accomplished under this project indicate that actual variations in gradation exceed the limits of current specifications. Because there is no reason to suspect that the plants selected are abnormal, it is assumed that the results obtained are representative of typical commercial production, at least insofar as the six PCC aggregates are concerned. This leads to the question as to whether the limits specified can, in fact, be justified in light of the historical implication that the concrete produced at these plants has been performing satisfactorily.

Thus, the basic problem of defining realistic tolerances becomes a matter of judgment in choosing between traditional concepts of acceptable engineering criteria and the random variability found to be associated with typical plants operating under normal construction controls. The findings of this study help in three ways, as follows:

1. Techniques have been developed and suitable parameters defined to measure the important batch-to-batch variability, as distinguished from the sampling and testing errors or localized within-batch segregation, which is normally negated by a subsequent mixing operation.
2. The importance of the location of the sampling point and the need to distinguish and locate assignable causes of variability has been demonstrated. Obviously, variations due to assignable causes which are practical to correct should not be weighed in specification limit considerations, but it is highly desirable to have a method for pinpointing them, so that they can be minimized or eliminated when feasible.
3. Finally, the engineer, as the result of this study, now has some typical measurements of both the batch-to-batch segregation important to quality and the normal measurement errors typical to commercial plants using different types of coarse aggregates.

The important point is that there are now available the tools and sufficient data on typical commercial plants to define the problem and provide a sound basis for the use of engineering judgment in preparing more realistic and practical specifications. The further definition of just what constitutes an acceptable variability for different aggregates and uses remains to be evaluated, as does the actual quality improvement related to the increased costs involved in reducing that variability.

ESSENTIAL PARTS OF A COMPLETE AGGREGATE SPECIFICATION

A review of current specifications indicates that, in many cases, they are incomplete, are subject to more than one interpretation, do not distinguish between characteristics of different levels of criticality, and fail to give definite criteria for acceptance or rejection. Some do not specify a sampling plan, and others merely require a measurement on a single test portion, irrespective of tolerances or variability. In some cases, limits are set for some measurable property, but no method of test is specified. The net result is that many such specifications are not fully enforceable, and there is certain to be a lack of uniformity with respect to both interpretation and enforcement. To avoid these faults, a specification for construction aggregates should be clear with respect to the following details:

1. Point of Sampling.—These research efforts have emphasized the fact that no highway specification can be complete unless the point of sampling is clearly stated. As can be seen from the earlier presentations, significant fluctuations are present in aggregate gradations at various locations between the point of production and the point of use. Each time the aggregate is handled, loaded, or transported to a different site, some change in variability occurs. Indicated variations are of such magnitude and so closely related to point of sampling that the acceptance or rejection of a particular aggregate may depend entirely on the location from which the sample was obtained.

2. Acceptance Plan.—Another essential element of a complete specification is a definite, clearly stated acceptance plan. Such a plan will describe the basis for acceptance or rejection, the number and weight of test portions to be obtained, and the method of interpretation of test results. The absence of a detailed acceptance plan leaves these items to the discretion of the inspector or engineer, which can result in nonuniformity of enforcement.

A proper plan will require that all test portions be obtained by a statistically random procedure, so that each increment will have an equal or known chance for inclusion. It will also state the method of determining compliance with the specification requirement(s).

3. Number of Test Portions.—Few, if any, present-day specifications mention the number of tests to be used as a basis for an acceptance decision. As a result, some organizations use a single gradation test as a basis for acceptance of a stockpile of aggregate, or as a basis for design of a portland cement or bituminous paving mixture. This procedure often results in considerable difficulty in maintaining control once the project construction begins. The engineer will usually find it necessary to make adjustments in his materials proportions to compensate for the changes in gradation from that reported prior to the start of the operation.

The number of required test portions is primarily affected by (1) the desired confidence limits (i.e., the range that, with a given probability, will include the true average), and (2) the variability of the material. In addition to these factors, the number of tests specified must be within certain limits (based on cost per test), and must be capable of being completed within a reasonable time so that the results can be used effectively. For example, it would obviously be impracticable to require 50 gradation tests per day on a bituminous paving project, although a closer degree of assurance would result from this requirement. For this reason, the required number of tests must often be a compromise between practical and statistical considerations, and in some cases, wide confidence limits must be tolerated.

If the standard deviation is known or can be estimated with a reasonable degree of accuracy, a minimum number of test increments may be used to obtain given confidence limits. If, however, the pattern and amount of variability is not known, more test increments are necessary to obtain an equivalent accuracy.

4. Summary of Essential Parts of a Specification.—To provide firm ground for an acceptance decision, a complete aggregate gradation specification should include:

(a) The total number of test portions to be taken from each LOT.
(b) The requirement that the test portions shall be taken from units or batches located by means of a random sampling plan.
(c) The minimum weight of the test portion to be taken from each batch, the minimum number of increments taken to make up each test portion, and the definition of the batch.
(d) The point of sampling, and the sampling tool to be used.
(e) The standard test method to be used.
(f) The desired percentage passing each of stated sieve sizes.
(g) Tolerances on the desired percentages for the values computed from a stated number of tests.
(h) The method of determining compliance with the stated tolerance limits.
(i) Action to be taken in case of nonconformance.

ILLUSTRATIVE MODEL SPECIFICATION FOR CONCRETE COARSE AGGREGATE

The principles discussed in the foregoing are incorporated in the following model specification, and the basis for each point is explained in the ensuing paragraphs.

> Grading Requirements for Coarse Aggregate for Concrete
>
> Five(5) test portions shall be taken from each LOT as near as possible to the point of use in accordance with a definite random sampling plan. Each test portion shall weigh not less than 16 pounds and shall be composed of not less than three (3) increments taken at random from each unit (batch or mixer-truck charge) of aggregate at the time of proportioning by means of an approved sampling tool. The five (5) test portions shall be individually tested in accordance with the provisions of AASHO Method T 27. The percent passing each sieve size, as shown by the average of the five tests, shall be within the limits shown in the table below. The R value will be found by taking the difference between the smallest and largest percentage passing that sieve in the group of five acceptable* test results.

Percentage Passing Sieves, Limits for Computed Values from 5 Tests

Sieve Size	Coarse Limit (%)	Desired Average (%)	Fine Limit (%)
1½ in.	100	100	100
¾ in.	$70 + 0.3R$	84	$98 - 0.3R$
⅜ in.	$11 + 0.3R$	30	$50 - 0.3R$
No. 4	0	7	$15 - 0.3R$
No. 8	0	3	$8 - 0.3R$

Aggregate or concrete containing aggregate not meeting these provisions will be paid for at a reduced price. This reduction will be based on the cost of specification aggregate and the estimated percentage of the aggregate meeting specification requirements on each sieve, as shown in the following table.

$\frac{\overline{X}-70}{R}$ or $\frac{98-\overline{X}}{R}$	Reduction in Contract Price of Coarse Aggregate (%)
0.30 to 0.25	0.0 (Warning)
0.24 to 0.22	5.0
0.21 to 0.16	10.0
0.15 to 0.10	25.0
0.09 to 0.00	50.0
Negative value	100.0 (Reject)

---

* Very large or very small individual test values will be tested in accordance with ASTM Method E 178, Recommended Practice for Dealing with Outlying Observations. If a test value is discarded in accordance with this procedure, the results of a test on a reserve test portion (if available) will be substituted. If a reserve portion is not available, the acceptance decision will be based on the average and range of the remaining portions.

BASIS OF ILLUSTRATIVE SPECIFICATION

The method of designing the illustrative specification is discussed in the following for each step in the foregoing "Summary of Essential Parts of a Specification," and the sampling plan shown in Figure 48. The general method is described in detail in Chapter Two of *NCHRP Report No. 17,* HR Project 10-1, and references in the following discussion are to tables and figures in that report.

Number of Test Portions

The number of test portions to be taken from each LOT is necessarily a compromise between economic and statistical considerations. Time, manpower, and cost must be equated with such interrelated factors as the average test values for good (acceptable) material and the average test values for poor (nonacceptable) material, and the probability of rejecting poor material and the probability of rejecting good material. In the case of the example, this compromise resulted in using the average of five tests. This is the appropriate number of test portions for a material having a minor criticality classification and the probability of rejecting good material is about 0.4 percent, while the probability of rejecting poor material is 90 percent (Table 6, *NCHRP Report 17*).

If only the required number of test portions is taken at the time of sampling, there is always the possibility that a test result from one or two portions will be an extreme value, which should be discarded if classed as an outlier by ASTM Method E 178. If no reserve test portions are available, the acceptance decision must be based on the average and range of the remaining test results. This would upset the statistical basis on which the acceptance plan is based and would decrease the probability of rejecting poor material to about 85 percent if only four acceptable test results were available, and to about 76 percent if only three were available.

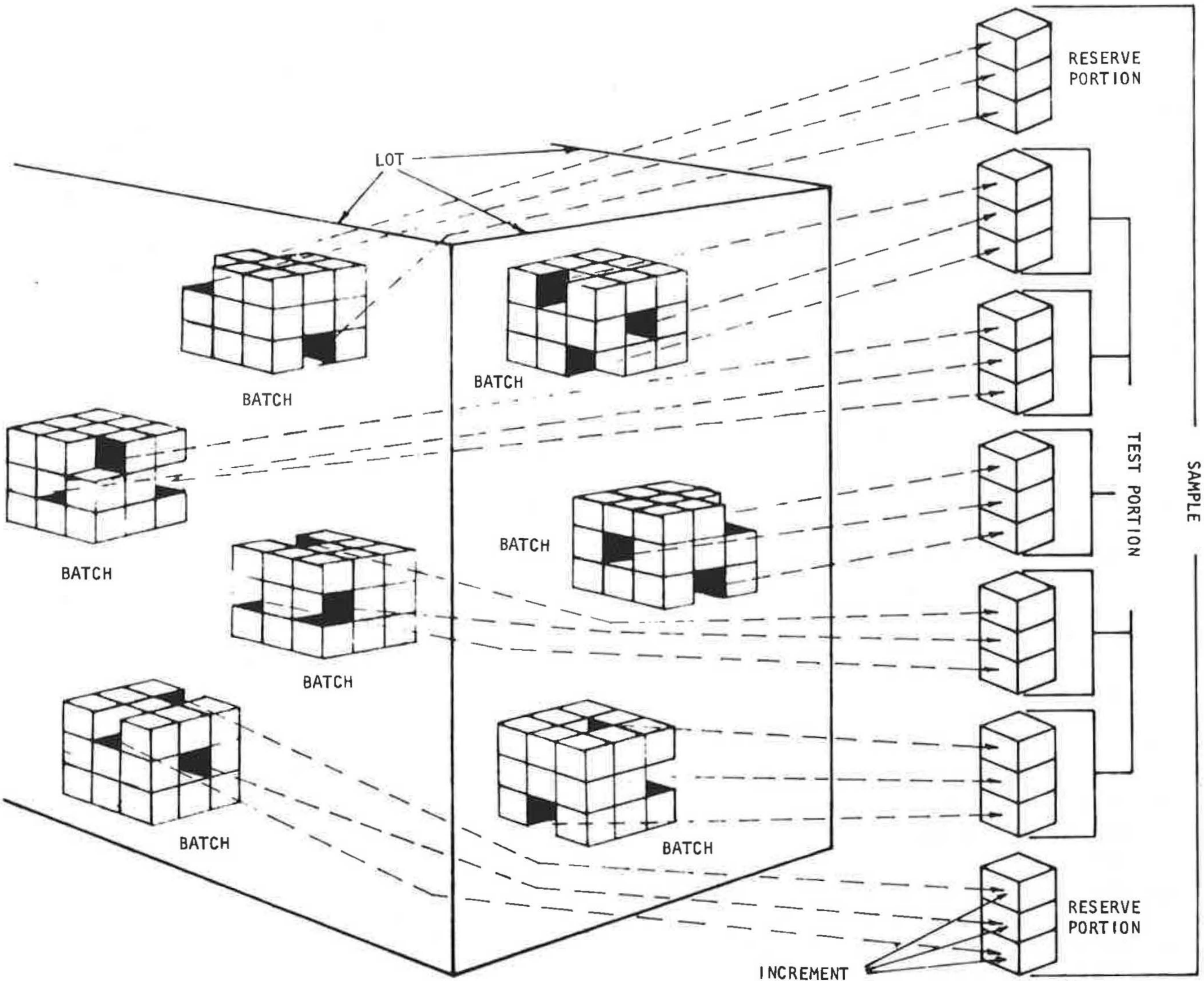


Figure 48. Sampling plan for model specification.

The frequency of test portions that obviously do not follow the normal random pattern of variation can be predicted only on the basis of experience. Obviously, the reason for the occurrence of any extreme values of test results on samples should be investigated. A very large or very small test result does not necessarily mean that the aggregate actually going into the batch is not substantially within specification limits, because such a result may be due to sampling error. In any case, whether due to improper process control or improper sampling technique, all possible corrective action should be taken to prevent future reoccurrence of biased samples. However, to insure that the planned probability of rejection of poor material can be applied equitably to all producers, it is recommended that two reserve test portions be taken initially at the time of sampling. This should not greatly increase the cost, and would largely eliminate the possibility of having to make an acceptance decision with increased risks of error. If continued use of the sampling plan shows that the reserve portions are infrequently needed, they can be discontinued.

Random Sampling Plan

If decisions are to be made based on known probabilities of rejection, the test portions comprising the sample must be taken absolutely without bias. This means that the location of the sampling points, in time or space, must be selected entirely by the use of a table of random numbers.

Minimum Weight of Test Portions

The minimum weight of the test portions is found by dividing the required total sample weight by the number of tests on which acceptance is based, five. The total weight of the sample depends on the desired accuracy of the result (see Fig. 7). In the case of the illustrative specification, where acceptance is based on five tests, the total sample weight of 80 lb was chosen to give an accuracy of about ±1.5 percent for the percentage passing the ⅜-in. sieve. Dividing 80 lb by 5 resulted in the minimum test portion weight of 16 lb for each of the five test portions to be taken.

At least three increments should be taken from different points in each batch of aggregate so as to help average out within-batch variation. The batch is defined as the quantity of aggregate that will receive further mixing as a unit. For example, there may be considerable segregation of fine and coarse particles in the weigh hopper when batches of aggregate are weighed into a mixer-truck, but this segregation will be removed during the mixing of the concrete.

Point of Sampling

The point of sampling was chosen at the weigh hopper so that tests on samples would represent the actual aggregate in the product, such as a mixer-truck load of concrete. Although it may be advantageous to the producer to sample aggregate at the source, or more expedient for the inspector to sample aggregate at some intermediate point in the process stream, the results of this investigation clearly show that there is very little relationship between the variation in gradations found at these points and the variations found at the point of use. For this reason, acceptance samples should preferably be taken close to the point of incorporation into the finished product, or item of construction. Routine process control samples may be taken at some earlier point if a definite relationship has been established with both level and variability at the point of use.

Standard Test Method

The standard test method to be used is stated in the specification. If modifications of the standard test are to be made, they should be complete and clearly described.

Desired Gradation

The specification writer should state the appropriate or desired gradation for the intended purpose. This gradation may be based on an engineering or theoretical basis, but preferably should be the average of gradations in actual use that have given satisfactory performance. In the case of the illustrative specification, the average gradation of two concrete plants (Series 7 and 8) was taken as the desired gradation. In practice, the average of all similar plants that produced satisfactory concrete would be used.

Tolerances on Desired Percentages

As in the case of the desired average gradation, tolerances on this average may be based on an engineering or theoretical basis, but preferably should be based on the range of the percentages found in gradations in actual use that have given satisfactory performance. This range should be determined by actual investigation on a statistical basis, such as described herein, instead of reference to historical data, which may have been subject to varying degrees of bias.

In the case of the illustrative specification, the average gradation ($\overline{X}_7$ and $\overline{X}_8$) and the standard deviations (σ_7 and σ_8) for the percentages passing each sieve were calculated from the data obtained from tests on samples taken at the plants designated as Series 7 and Series 8 in this report. The average standard deviation, $\bar{\sigma} = \sqrt{\sigma^2_7 + \sigma^2_8}$ was calculated for each sieve size and the specification limits were obtained by subtracting twice this value from $\overline{X}_7$ and adding twice this value to $\overline{X}_8$. The appropriate acceptance limits for material of minor criticality were taken from Table 6, *NCHRP Report 17*, and are $\overline{X}_p{}' \pm 0.295R$. This was rounded to $0.3R$.

Sieve Size	*Percent Passing* $\overline{X}_7$	$\overline{X}_8$	Avg. $\bar{\sigma}$	($\overline{X} \pm 2\bar{\sigma}$) *L*	*U*
¾ in.	82	86	5.8	70	98
⅜ in.	27	34	8.0	11	50
No. 4	5	8	3.4	0	15
No. 8	2	4	1.8	0	8

In practice, the appropriate standard deviation might be that found by averaging the variances (by statistical methods) measured at all plants producing a satisfactory product.

Method of Determining Compliance

The stated method of determining compliance with the tolerance limits for the average of the five tests is spelled out. Compliance is a function of both the average, $\bar{X}$, and the range, R, of the five test portions.

It should be noted that although the specification limits were derived by the use of a selected standard deviation this method does not require that the standard deviation of the sampled LOT of aggregate be known or that the sample standard deviation be computed. If the actual standard deviation is greater than that of the well-controlled process on which the specification was based, there will be increased probability of rejection; if the actual standard deviation is less, there will be increased probability of acceptance (see Fig. 3), thus providing a real incentive for improved uniformity.

Action to Be Taken

To insure uniformity of specification enforcement, the action to be taken if the results of the tests indicate noncompliance with the given limits should be clearly stated. Severe penalties, such as rejection of aggregate or product, are difficult to enforce under practical circumstances. In many cases, the LOT in question has been incorporated into the finished work before the test data become available and nonpayment for, or rejection of, a part of the finished work leads to contention and expensive after-examination. For these reasons, a graduated scale of penalties is included. These penalties can be based on the cost of the aggregate and the estimated percentage of the aggregate that was within the specification limits, as indicated by the value of the statistics computed from the sample, shown in the illustrative specifications. This type of penalty encourages the contractor or producer to exercise quality control and to resize, remix, or otherwise prepare the aggregate as necessary to meet requirements, prior to acceptance sampling.

The percent within tolerance (PWT) is found by the use of Table 5, *NCHRP Report 17,* and the adjusted contract price is determined accordingly for noncompliance on any sieve, as follows:

Percent Within Tolerance*	$\frac{\bar{X} - 70}{R}$ *or* $\frac{98 - \bar{X}}{R}$	*Reduction in Price (%)*
72 to 76	0.30 to 0.25	0.0 (Warning)
69 to 71	0.24 to 0.22	5.0
64 to 68	0.21 to 0.16	10.0
58 to 63	0.15 to 0.10	25.0
50 to 52	0.09 to 0.00	50.0
Less than 50	Negative value	100.0 (Reject)

* These are the single-limit values which apply to the lower or the upper tolerance limit (70 or 98). However, unless R is very large, they will closely approximate the percentage of individual test results falling within the 70 to 98 broad band limits.

For example, if the average, $\bar{X}$, of tests on five test portions is 72 percent passing the ¾-in. sieve, and the difference between the smallest and largest test values, R, is 10 percent, the lower limit is $70 + 0.3R = 70 + 3 = 73$, and the aggregate is out of specifications on that sieve. The penalty is computed by $(\bar{X} - 70)/R = (72\text{-}70)/10 = 0.20$. This indicates that only 67 percent of the individual test results obtained by sieving 16-lb test portions as specified will fall within the broad band of 70 to 98 percent passing (33 percent of the individual tests will show less than 70 percent passing). The suggested penalty in this case is a 10 percent reduction in unit price, or, in other words, that the producer should be paid only 90 percent of the contract price for that particular LOT.

CHAPTER SEVEN

PRECISION STATEMENT

OBJECTIVES

An important objective of this project was to develop a precision statement, if needed, for the aggregate gradation test. To accomplish this work, current ASTM, AASHO, and Federal procedures pertaining to the determination of aggregate gradation were reviewed to determine the presence or absence of a satisfactory precision statement. This was followed by a study of the classification of test methods with respect to the experimental and theoretical work accomplished in this and related projects. Finally, prototype precision statements for the sieve analysis of coarse aggregates were developed and expressed in both the ASTM D-2 and the ASTM D-4 (E-177) formats.

CURRENT STATUS

A review of aggregate test procedures conducted in accordance with the preceding objectives indicated that the only gradation test procedures that contained some form of precision statement were three ASTM methods of test. These were:

D 451-63 Test for Sieve Analysis of Granular Mineral Surfacing for Asphalt Roofing and Shingles;

D 452-63 Test for Sieve Analysis of Nongranular Mineral Surfacing for Asphalt Roofing and Shingles; and
D 546-55 Test for Sieve Analysis of Mineral Filler.

Of these three gradation test procedures, only D 546 is directly applicable to the objectives of this project. There is no precision statement for ASTM Standard Method of Test for Sieve or Screen Analysis of Fine and Coarse Aggregates C 136-63, or the similar AASHO Designation T 27-60, and Corps of Engineers CRD-C 103-60, and a precision statement is needed for this procedure.

DEVELOPMENT OF PRECISION STATEMENT FOR AGGREGATE GRADATION TEST

On January 25, 1965, the Task Force on Precision Statements, Subcommittee 1e, Committee D-4, ASTM, recommended that the following action be taken:

> (1) Discard the present format for precision statements in D-4 methods and express the precision of the method in terms of the estimated standard deviation of the measuring process, designated σ_p in E-177. Two "sigma's" shall be given. One shall be applicable when results within a single laboratory by a single operator are of concern. This shall be designated "within laboratory variability," and shall be single-laboratory-operator-apparatus-multi-day precision as defined in E-177. The second term shall be designated "between laboratory variability," and shall be multi-laboratory-operator-apparatus-day precision as defined in E-177.
>
> The general format of the precision statement shall be as follow:
>
> 0. *PRECISION.* The estimated standard deviation (σ_p') for within laboratory variability and between laboratory variability are as follows:
>
> 0.1—Within laboratory variability (single-laboratory-operator-apparatus-multi-day precision . . . ________
> (Express in units of test or percentage of mean value, whichever is applicable. Different values for different materials or levels of test values shall also be given when necessary; use tabular forms in such cases.)
>
> 0.2—Between laboratory variability (multi-laboratory-operator-apparatus-day precision . . . ________
> (Express in a manner as "within laboratory variability.")
>
> For definition and use of terms relating to precision, see ASTM Method E-177.
>
> (2) Prepare an informational memorandum for circulation to all members of Committee D-4 explaining the changes in concept between the use of the σ_p as the precision statement and the use of "repeatability" and "reproducibility" ($2\sqrt{2}\ \sigma_p'$). The minimum tests required to determine an adequate estimate of σ_p' and the method of its computation shall be included. The memorandum shall also discuss the various combinations of "sigma" of interest under different situations.
>
> (3) During an interim report to establish the relationship between the previous format and the present format, include in the precision statement a second footnote reading as follows:
>
> (2) For comparing results of different runs of the same material in the same laboratory or for comparing results obtained by different laboratories on the same material, the parameter $2\sqrt{2}\ \sigma_p'$, defined in E-177 as the difference two sigma limits (D2*S*), is often used. For within laboratory variability this parameter is usually designated as "repeatability" and for between laboratory variability, the term "reproducibility" has been used. The "difference two sigma limits" is the difference between two results that will be exceeded about 5 percent of the time. When this difference is exceeded, the possibility exists that the deviation may be caused by other than normal variations in the method and, therefore, one or more of the values are "considered suspect." Under these circumstances the operator should look for possible causes of the deviation, and, when possible, retests should be made. The "difference two sigma limits" (D2*S*) should not be confused with "two sigma limits" (2*S*). The latter defines the range in which 95 percent of the test results will fall for tests on the same LOT of material (average $\pm 2\sigma_p'$).

As is indicated by the different approaches described, there is still some question as to the final format for ASTM precision statements. In the precision statement for gradation test of aggregates in this report, two formats are included for purpose of comparison.

CLASSIFICATION OF TEST METHODS

Not all standard tests are adaptable to precision statements. In a recent report of Subcommittee 11a (Evaluation of Data) of ASTM Committee C-9, it was suggested that ASTM designations be divided into four classes or groups, as follows:

Group I. Methods of tests for which it appears possible to obtain a measure of repeatability and reproducibility having little or no sample variance, either by repeating the test on the same sample or by making synthetic samples.

Group II. Methods for which the measure of repeatability and reproducibility will necessarily include a component of variability introduced by sampling. That is, the method cannot be applied a number of times to the same sample and it does not appear that synthetic samples can be made.

Group III. Methods of test in which multiple specimens are required. These methods might possibly

CHAPTER EIGHT

CONCLUSIONS, RECOMMENDATIONS, AND SUGGESTED RESEARCH

This chapter presents conclusions and recommendations based on laboratory investigations and the results of samplings of five types of coarse aggregates at eight production plants.

CONCLUSIONS

The following conclusions are drawn from the results obtained:

1. With usual handling and bin storage procedures, gradation of coarse aggregate varies over a wide range at the point of use.
2. Variation of coarse aggregate gradation is usually at a minimum at the point of production (crusher or screening plant).
3. Variation of gradation of coarse aggregate at intermediate points in the process stream does not follow a uniform pattern, but tends to increase with each handling involved from point of production to point of use.
4. Segregation is the largest cause of variation in coarse aggregate gradation, but no one handling procedure as investigated herein can be isolated as the principal cause of segregation.
5. Due to different degrees of segregation of the coarse aggregate at various points in the process stream, compliance with specifications depends largely on where the sample is taken and on the weight and number of increments in the sample.
6. A large number of sample increments are required to estimate the true average gradation of a coarse aggregate with acceptable accuracy. The greater the batch-to-batch segregation, the more test portions are required, while the greater the within-batch segregation, the more increments must be taken for each test portion to reduce sampling error. Testing error appears to have a relatively insignificant effect on the accuracy of the estimate.
7. The weight of coarse aggregate test portions recommended in ASTM C 136-63 and AASHO T-27 does not appear to be sufficient to provide the desired precision or to accurately determine the true average gradation.
8. Real compliance with many current specifications would require that coarse aggregates be rescreened or remixed at the point of use. However, there was no apparent indication of inferior quality as a result of the relatively high levels of variability of the aggregates in the cases studied.
9. An analysis of results at the 50 percent passing level can be used as the basis for developing basic mathematical equations important in preparing construction controls and specifications for coarse-aggregate gradations.
10. Two new parameters, degree of overall variability (DOV), and degree of segregation (D of S), have been developed as further measures of the level of over-all and batch-to-batch segregation of an aggregate gradation. These two parameters are based on ratios comparing standard deviation values with complete (theoretical maximum) segregation at the 50 percent passing level. DOV values were found to range from 8.2 to 45.0, while D of S values ranged from 6.4 to 34.4.
11. Development of a precision statement for the gradation test of coarse aggregates which is applicable to all conditions must include consideration of the inherent variance, which is not constant for all gradations.

RECOMMENDATIONS

1. Evaluation of the effectiveness of control procedures and compliance of a coarse aggregate gradation with specifications should be based on a sample taken as near as possible to the point of use.
2. Recognition should be taken of the necessity of taking test portions and increments in a random pattern, the number of test portions required, and the weight of the sample required for the accuracy desired.
3. Revised specifications for coarse aggregate should state the sampling point, the sampling procedure, the test method on which compliance with realistic limits will be judged, the method of determining compliance, and the action to be taken in case of noncompliance.
4. When sampling of coarse aggregate at the point of use is impractical, test portions should preferably be taken from a belt or flowing stream of aggregate. Time or location of samplings should be based on the use of a table of random numbers to eliminate bias.
5. Sampling coarse aggregate from stockpiles is not recommended. When there is no alternative, the following technique should provide the most nearly unbiased sample practical under the unfavorable conditions.
    - (a) Calculate in advance the required number of test portions that are to be secured, and the minimum weight of each test portion.
    - (b) Use a random sampling scheme to determine the location from which each test portion is to be secured.
    - (c) Use an approved sampling tool to prevent loss of coarse aggregate particles.
    - (d) Clear away several inches of surface aggregate before inserting the shovel to remove an increment of a test portion. Use a barrier or shield on the upper side of the sampling area to prevent aggregate at a higher level from tumbling into the work area. Take three or more increments (scoopful) for each test portion.
    - (e) Pass all of each test portion of aggregate through the test sieves and sieve to refusal.

PROPOSED PLAN FOR CONTINUING RESEARCH STUDIES

The initial phase of this project established a pattern of variability of aggregate gradation at various points in the process stream of several operating plants. The magnitude of variation found at these locations was much greater than apparently has been normally assumed, as shown by a review of current gradation specifications (Chapter Five, *NCHRP Report 17*). It was not within the scope of this initial study, however, to evaluate the effects of gradation variations, but rather to measure their relative magnitude.

The recommendation for continuation studies suggests broadening the scope to include two possible courses of action—one involving a study of aggregate variation associated with the production of bituminous mixes (Plan 1), and the other involving a study of the effect of gradation variations in the strength of PC concrete (Plan 2). Plan 1 would be divided into three major areas of study: (1) a measure of variation found in asphalt plant hot-bin gradations, (2) determination of optimum increment size with respect to the maximum particle size of aggregate, and (3) development of a mathematical model to describe the mechanism of segregation. The two latter areas of study would be equally applicable to any type of aggregate gradation, regardless of the purpose for which the aggregate would be used. Plan 2 would provide for a study of the effect of variation in coarse aggregate gradations on the compressive strength of concrete, with the objective of better defining practical permissible variability.

In either case, a comprehensive literature review would be made, from which an interim report on current status would be developed. This literature search would involve the screening of available technical articles for appropriate information pertaining to the specific problem, and would be culminated with the interim status report.

It is believed that Plan 1 would provide the more significant data with respect to construction control, meanwhile making full use of previous research findings such that there would be no needless duplication of effort.

Plan 1

Although the initial studies under this project furnished some indication of variation among typical concrete plants, there are still several other areas requiring additional research before a complete system of construction control can be developed. At this time, it is not known whether the same relative magnitude of variation should be expected in other types of paving mixture operations as was found at the ready-mix concrete plants. Control limits developed on the basis of completed research may have restricted application until additional data are acquired.

It is therefore proposed that the continuation studies include an investigation of variations in bituminous plant hot-bin gradations with respect to significant factors of plant design and operation. A fundamental difference exists between the proportioning procedure for asphalt and PC concrete mixes in that aggregates for bituminous mixes are rescreened prior to introduction into the mixtures, whereas aggregates for concrete mixes are usually introduced without resizing. An investigation of the type proposed could furnish an answer as to whether the greater degree of control exercised in the production of bituminous mixes, at correspondingly higher costs, is fully justified. This study would include a variety of mixture types involving varying maximum size aggregates. It is, more or less, standard procedure to screen the aggregates into three or four size ranges when producing a bituminous mixture employing a maximum aggregate size in the order of 1½ in. Questions which might logically arise include: (a) Is such a degree of control really necessary? (b) What is the normal variation associated with current operational procedure? (c) Could a smaller number of hot bins be used and achieve the same end result?

An integral part of this study would include the determination of optimum increment size with respect to maximum particle size. In many cases, current guidelines appearing in AASHO and ASTM Standards do not provide sufficient test portion weight to furnish an acceptable degree of accuracy. There is considerable evidence of nonuniformity in the requirements for acceptance standards as related to increment size. It is suspected that these guidelines were arbitrarily established rather than being determined on the basis of a controlled experiment. The proposed research would provide the basis for the development of a reliable sampling guide.

An adjunct to these studies would be a study of basic mathematical principles relating to segregation of graded aggregates with respect to specification requirements. This particular aspect was given a cursory review during preparation of this report, but neither time nor available funds would permit the intensive study necessary to provide basic mathematical formulations for describing the segregation mechanisms. Such mathematical models would be invaluable in most applications wherein segregation or variation considerations may be involved.

The tying together of the three major items proposed in Plan 1 for additional study would insure a more absolute basis for the establishment of broad construction controls.

Plan 2

A laboratory designed experiment would be conducted using a fractional factorial design to determine the limits within which variations in grading do or do not affect quality (compressive strength) of concrete. This study would involve a large number of 6-in. × 12-in. concrete cylinders in which one brand of cement, two coarse aggregates (gravel and crushed stone), two cement contents or two water/cement ratios, one consistency (slump), and one fine aggregate would be used. Several gradations (from coarse to fine) of coarse aggregate with each of the foregoing variables would be studied. These gradations would be adjusted to reflect the degree of variation found in the previous work of this project so that the effects of gradation over a realistic range could be properly assessed. The initial design would be for 3,000-psi concrete, and three cylinders, representing each combination of variables, would be prepared for testing at 7, 14, and 28 days. After all test data are assembled, appropriate statistical treatment would be applied to extract necessary parameters for a proper statistical and engineering evaluation. From these data, the significance of gradation control as related to portland cement concrete could be determined. A suggested acceptance specification for concrete aggregate would be prepared on the basis of this experimental work.

APPENDIX A

GLOSSARY

This listing provides explanations of statistical, mathematical, and technical terms as used in this report. In individual items, significant associated terms explained elsewhere in the Glossary are capitalized.

ACCEPTANCE DECISION — A determination of acceptability of a material, product, or process based on statistical or mathematical principles.

ACCURACY — The agreement between a measured value and a true value.

ACCURATE — Refers to agreement between the true value and a measured value.

AGGREGATES (COARSE) — Certain specified gradations of mineral particles usually larger than ¼ in. in size.

ASSIGNABLE CAUSE — A relatively large factor, usually due to error or process change, which contributes to VARIATION and whose effects are of such importance as to justify time and money required for its identification.

AVERAGE ($\overline{X}$) — A measure of central value which usually refers to the arithmetic mean, obtained by dividing the sum of n values by n.

BATCH — A unit or a subdivision of a LOT, such as a mixer-truck load of concrete, or a square yard of subbase.

BIAS — A constant error, in one direction, which causes the AVERAGE of a number of measurements to be offset from the true value of the true measure of central tendency.

CONFIDENCE LIMITS — The maximum and minimum values which define the confidence interval.

CONTIGUOUS — Having contact on most of one side.

CORRELATION — A relationship which exists between two or more variables, and is often expressed as a ratio known as the correlation coefficient.

CRITICALITY — The classification of various factors of specifications as they affect safety, performance, or durability.

DATA — Measurements collected for a planned purpose and suitable for the inference of conclusions.

DEGREE OF ASSURANCE — The probability that a confidence interval has of including the true value. Also called confidence coefficient or confidence level.

DEGREES OF FREEDOM (d.f.) — The number of measurements (n) less the number of constants derived from them. When only one AVERAGE has been taken, d.f. = $(n - 1)$. When the VARIATION around the group averages is determined, the degrees of freedom are the total number of measurements in the groups less the number of groups.

DEGREE OF SEGREGATION — A measure of the principal source of VARIATIONS in the gradation of aggregate. It is computed by dividing the OVERALL VARIANCE by the maximum or parent VARIANCE.

EXPERIMENTAL ERROR — The difference between measurements on two identically treated units.

FINENESS MODULUS (FM) — An empirical factor obtained by adding the total percentages of a SAMPLE of the aggregate retained on each of the STANDARD SIEVES and dividing by 100. These sieves include the No. 100, No. 50, No. 30, No. 16, No. 8, No. 4, ⅜ in., ¾ in., 1½ in., and larger, increasing in the ratio of 2 to 1.

GROUP — Replicate test portions taken from a single batch.

HUDSON $\overline{A}$ — The term for a factor which expresses the relative coarseness of an aggregate gradation in a single number. It is found by summing the percentages passing the 1½ in., ¾ in., ⅜ in., No. 4, No. 8, No. 16, No. 30, No. 50, No. 100, and No. 200 sieves and dividing by 100.

INCREMENT — The smallest unit removed from a LOT during sampling.

INHERENT VARIATION — A VARIATION due to RANDOM or insignificant causes.

INHERENT VARIANCE (σ^2_a) — A VARIANCE due to RANDOM or insignificant causes.

ITERATION — A method of finding a required value by means of successive estimates.

LOT — An isolated quantity of material from a single source. A measured amount of construction assumed to be produced by the same process. When several true LOTS are combined, the result is a "grand lot."

NORMAL CURVE — A curve having a bell-shaped form which depends on values of $\overline{X}'$ and σ', and which shows the distribution of individual values of measured characteristics about their AVERAGE.

NORMAL DISTRIBUTION — A distribution represented by the NORMAL CURVE.

NORMAL DISTRIBUTION CURVE — See NORMAL CURVE.

OVERALL VARIANCE (σ^2_0) — The sum of all RANDOM ERRORS and ASSIGNABLE CAUSES, which may be expressed as the sum of several VARIANCES.

PARAMETER — A constant or coefficient that describes some characteristic of the distribution of a series of measurements.

PORTION — Any small part of a larger quantity.

PRECISION — The VARIANCE of repeated measurements of a characteristic.

PRECISION STATEMENT — A statement defining the limits of VARIANCE of repeated measurements of a characteristic.

PROBABILITY SAMPLING — A method of sampling in which every part of the material or product to be sampled has a known chance of inclusion.

PROCESS CONTROL — A method based on the application of STATISTICS used to regulate the uniformity of a material, product, or process.

RANDOM — Without aim or reason, depending entirely on chance. When a sampling process is said to be RANDOM, each item in the frame has an equal probability of being chosen.

RANDOM ERRORS — Differences from the true value, due to chance, which behave as though chosen at RANDOM from a probability distribution.

RANDOM NUMBER — A number selected from a table of RANDOM sampling numbers.

RANDOM SAMPLE — A SAMPLE is said to be RANDOM when each item in the frame has an equal probability of being chosen.

RANGE — The difference between the highest and lowest values in a group of measurements.

REPEATABILITY — The RANGE within which repeated measurements are made by the same operator on the same apparatus. Essentially, the PRECISION of the test.

REPRODUCIBILITY — The RANGE within which check measurements by different operators on different apparatus should agree under definitely stated conditions.

SAMPLE — A small part of a LOT which represents the whole. A sample may be made up of one or more INCREMENTS or TEST PORTIONS.

SEGMENT — An arbitrary division of a LOT, which may be either real or imaginary.

SIGMA (σ) — A term used in STATISTICS to indicate the value calculated from the differences between the individual measurements in a group and their AVERAGE. Also called STANDARD DEVIATION (SD).

SIGNIFICANT DIFFERENCE — A spread between two values too great to be due to chance alone, usually proved by a statistical test, as distinguished from a technically or economically meaningful difference.

SIGNIFICANT NUMBER — The smallest digit of a number that would have an effect on the ACCURACY of an answer determined by using that number.

STANDARD DEVIATION (σ) — A term, used in STATISTICS to indicate the value calculated from the difference between the individual measurements in a group and their AVERAGE.

STANDARD SIEVES — Those screens used in aggregate gradation analysis in which the size of the openings is successively halved as the sizes decrease. These sieves are as follows: 1½ in., ¾ in., ⅜ in., No. 4, No. 8, No. 16, No. 30, No. 50, No. 100, and No. 200.

STATISTIC — A summary value such as $\overline{X}$, σ, or R, computed from a group of measurements.

STATISTICS — The science which deals with the treatment and analysis of numerical DATA. Also, a collection of numerical DATA.

STATISTICAL ANALYSIS — A mathematical method of obtaining meaningful information from DATA.

TEST PORTION — The part of a SAMPLE actually tested. Usually obtained by reducing the SAMPLE or SAMPLE INCREMENTS by quartering, riffling, or taking an aliquot quantity.

TESTING ERROR (σ^2_t) — VARIATION caused by reducing a SAMPLE to a TEST PORTION and to the lack of REPEATABILITY of the test method.

UNIFORMITY COEFFICIENT (C_u) — The ratio of the diameter of the 60 percent finer point to that at the 10 percent finer point on the gradation curve.

VARIABILITY — A tendency to be variable.

VARIANCE — The square of the SIGMA of the SAMPLE (σ^2) or of the true value $(\sigma')^2$.

VARIANCE COMPONENT — Any one of the several sources of VARIANCE which is combined to produce the OVERALL VARIANCE. These components include σ^2_t, σ^2_s, σ^2_a, σ^2_l, and σ^2_b.

VARIATION — Differences, due to any cause, in measured values of a measurable characteristic.

WITHIN-BATCH VARIANCE (σ^2_b) — A VARIANCE having a value that depends on the amount of difference of the measurements on two INCREMENTS taken from the same BATCH.

WITHIN-LOT VARIANCE (σ^2_l) — A VARIANCE having a value that depends on the amount of difference among INCREMENTS taken from different parts of a LOT.

APPENDIX B

LABORATORY TEST OF VALIDITY OF BASIC THEORY OF INHERENT VARIANCE OF AGGREGATE GRADATION

As discussed in Chapter Two, the equation selected for the purpose of estimating the standard deviation, σ_a, of an aggregate gradation due only to inherent variance was

$$\sigma_a = \sqrt{\frac{P(100 - P)g}{454\,W}} \qquad (2)$$

in which

P = the percent by weight of the aggregate passing a designated sieve;

σ_a = the standard deviation of this percentage;

$\bar{g}$ = the average particle weight, in grams, of all particles larger than the openings in the designated sieve; and

W = the total weight, in pounds, of aggregate passed through the sieves.

To check the validity of this equation, a designated experiment was performed, using both crushed stone and uncrushed gravel aggregates.

The first aggregate selected for study was a crushed limestone produced from a quarry in Nebraska. This material has a bulk specific gravity of 2.66, an apparent specific gravity of 2.73, a Los Angeles abrasion loss of 25 percent, an absorption of 1 percent, and a gradation and particle weight as given in Table B-1a. With this material, samples of 20, 10, and 5 lb were run concurrently so that the effect of the test portion (weight) could be studied at the same time.

The second aggregate selected for this laboratory study was a rounded gravel from Wyoming, with a gradation and particle weight as given in Table B-1b.

To determine the actual inherent variance, σ'^2_a, it is necessary to determine the gradation of a large number of test portions taken from a perfectly-mixed LOT of aggregate without introducing segregation effects.

Mixing such a material with conventional equipment so that the arrangement of the particles is completely randomized, and maintaining complete control of segregation while removing and returning test portions, presents many difficulties. For this reason, the material was continuously mixed, and samples extracted, with a Gilson sample splitter.

This device consists of a frame supporting a hopper of about 90-lb capacity, 24 in. long and 18 in. wide. The cross section is that of a right triangle with apex at the bottom, so that the depth at the center is 8 in. The sides of the hopper can be moved by a lever so that a slotted opening is formed in the center for the length of the hopper. The aggregate in the hopper drops through this opening onto the splitting bars, which are 2 in. wide and separated by 2-in. slots. These bars are set at a 45° angle with a slot opposite each bar so that the aggregate is separated into two approximately equal parts, which are caught by two pans placed beneath the bars. There are six openings over each pan so that, in effect, the aggregate is separated into twelve parts with even-numbered parts going into one pan and odd-numbered parts into the other. The pan into which an aggregate particle falls is determined entirely by chance.

In the case of the limestone material, the aggregate was first separated into four size fractions by use of a Gilson sieve shaker. These fractions were then combined, by weight percentage, into five 40-lb batches, designated as A, B, C, D, and E. At this point, the particles could be considered to be in ordered arrangement, because each batch contained nearly identical percentages, or numbers, of each sized particle. To randomize the arrangement, the batches were mixed by adding batch A to batch B, splitting the mixture in half with the Gilson sample splitter, returning one-half of the mixture to batch B, and adding the other one-half to batch C, and so on, as outlined in Table B-2.

TABLE B-1

GRADATION AND PARTICLE WEIGHT OF EXPERIMENTAL AGGREGATES

SIEVE SIZE	PERCENT PASSING	PERCENT RETAINED	AVG. PARTICLE WEIGHT (GM)
(*a*) NEBRASKA CRUSHED LIMESTONE			
1½ in.	100	0	—
¾ in.	87	13	29.0
⅜ in.	35	52	4.3
No. 4	8	27	0.61
No. 8	0	8	0.09
(*b*) WYOMING ROUNDED GRAVEL			
1½ in.	100	0	—
¾ in.	84	16	16.2
⅜ in.	27	57	4.3
No. 4	3	24	0.81
No. 8	1	2	0.11
Pan	0	1	—

At this point, the mixture was considered to be essentially randomized and one-half of mixture (10) was split into 20-lb, 10-lb, and 5-lb test portions. These portions were tested for gradation using a Gilson sieve shaker, recombined with the untested part of the half, and returned to the batch. The other half was combined with the next mixture, and so on, the mixing being carried on continuously as test portions were acquired, tested, and returned to the mixtures. This mixing cycle is shown in Figure B-1.

The previously described mixing-testing cycle was used for the crushed limestone aggregate only. A similar pattern was followed for the rounded gravel, except that 50-lb units were mixed and then split into test portions of approximately 25 lb each. All test portions were sieved to refusal and the sieved fractions weighed with special care so that testing error would be negligible. The raw data acquired by means of these experiments are available on special request to HRB or the Research Agency.

A summary of computed versus the obtained experimental inherent standard deviation is given in Table 2. As shown by the 95 percent confidence limits in this table, agreement between theoretical and experimental values was quite good, providing test portions of approximately 20 lb or more were used. Accordingly, Eq. 2 was considered to be appropriate for use in estimating this important and heretofore poorly defined factor of "inherent variance."

The solution of Eq. 2 requires that the average particle weight, $\bar{g}$, be known. This value can only be determined accurately by counting, then weighing, a very large number of particles. However, a method based on theory, but

TABLE B-2

STEPS IN RANDOMIZING AGGREGATE PARTICLES IN EXPERIMENTAL BATCH MIXTURES

MIXTURE	PROCEDURE
(1)	$\frac{A+B}{2}$ (One-half to B, one-half to C)
(2)	$\frac{\frac{A+B}{2}+C}{2}=\frac{A+B+2C}{4}$
(3) D	$\frac{\frac{A+B+2C}{4}+D}{2}=\frac{A+B+2C+4D}{8}$
(4) E	$\frac{\frac{A+B+2C+4D}{8}+E}{2}=\frac{A+B+2C+4D+8E}{16}$
(5) B $\left(\frac{4}{2}+\frac{1}{2}\right)$	$\frac{\frac{A+B+2C+4D+8E}{16}+\frac{A+B}{2}}{2}=\frac{9A+9B+2C+4D+8E}{32}$
(6) C $\left(\frac{5}{2}+\frac{2}{2}\right)$	$\frac{\frac{9A+9B+2C+4D+8E}{32}+\frac{A+B+2C}{4}}{2}=\frac{17A+17B+18C+4D+8E}{64}$
(7) D $\left(\frac{6}{2}+\frac{3}{2}\right)$	$\frac{\frac{17A+17B+18C+4D+8E}{64}+\frac{A+B+2C+4D}{8}}{2}=\frac{25A+25B+34C+36D+8E}{128}$
(8) E $\left(\frac{7}{2}+\frac{4}{2}\right)$	$\frac{\frac{25A+25B+34C+36D+8E}{128}+\frac{A+B+2C+4D+8E}{16}}{2}=\frac{33A+33B+50C+68D+72E}{256}$
(9) B $\left(\frac{8}{2}+\frac{5}{2}\right)$	$\frac{105A+105B+66C+100D+136E}{512}$
(10) C $\left(\frac{9}{2}+\frac{6}{2}\right)$	$\frac{241A+241B+210C+132D+200E}{1024}$

slightly modified to conform to experimental results, was developed for estimating this value.

The first step was to estimate the average size, $\bar{d}$, of the particles passing a sieve having openings of size d_1 and retained on a sieve having openings of size d_2. The equation used for this purpose was

$$\bar{d}=\frac{0.4343(d_1-d_2)}{\log\left(\frac{d_1}{d_2}\right)} \tag{B-1a}$$

When standard sieves are used, $d_1/d_2=2$, and Eq. B-1a is simplified to

$$\bar{d}=1.443(d_1-d_2) \tag{B-1b}$$

The next step was to compute the average particle weight, $\bar{g}$. The equation, based on counting-weighing many thousands of particles of different aggregates, is

$$\bar{g}=0.003(\bar{d})^{2.8} \tag{B-2}$$

To illustrate the use of Eqs. B-1b and B-2 by example, suppose a base material had 99 percent passing a 2½-in. sieve, and 50 percent passing a 1-in. sieve. The sieve openings, in mm, are 2½-in. = 64.0, and 1-in. = 25.4.

Substituting in Eq. B-1b gives $\bar{d}=\frac{0.4343(64.0-25.4)}{\log(64.0/25.4)}$ $= 41.75$ mm.

Substituting in Eq. B-2 gives $\bar{g}=0.003\times 41.75^{2.8}=$ 103.5 gm.

It is believed that this equation is sufficiently accurate for estimating the value of $\bar{g}$ of either rounded or angular particles in the range of 2.60-2.70 *bulk* specific gravity. For very heavy or very light particles, the value of $\bar{g}$ should be adjusted by the use of the factor, Bsg/2.65.

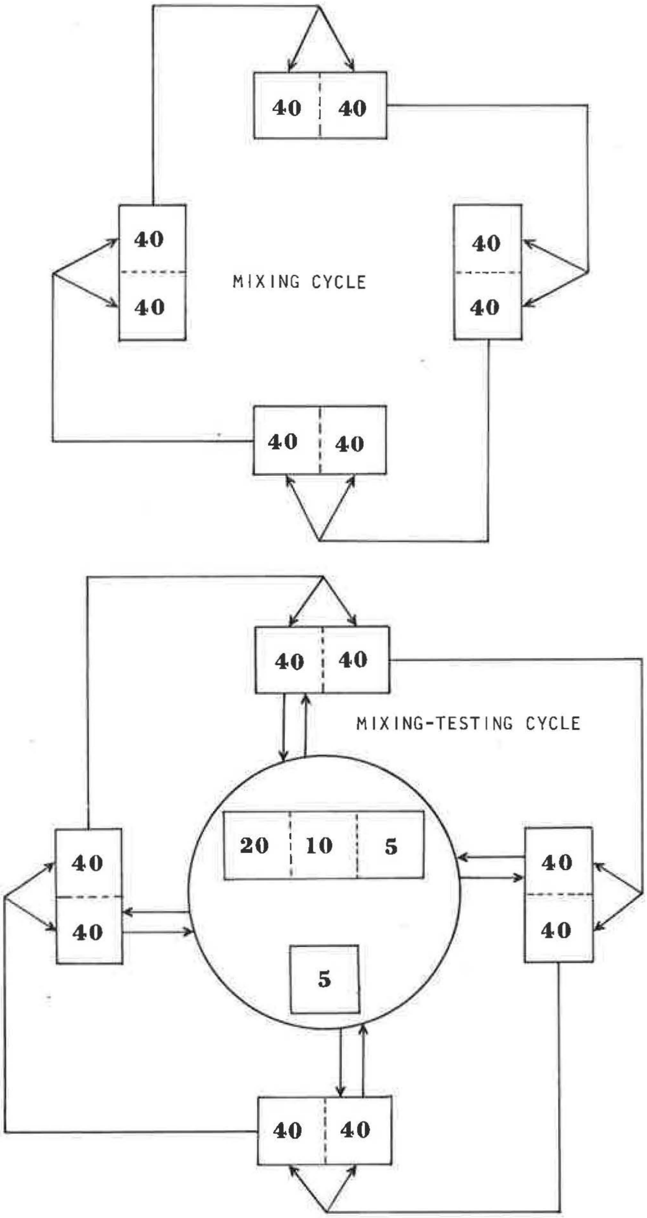


Figure B-1. Mixing-testing experiment to determine inherent variance of aggregate gradations.

APPENDIX C

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