The Physics and Chemistry of Frozen Soils

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> The engineering properties of frozen soils—strength, thermal conductivity, dielectric constant, and electrical conductivity are affected by the phase composition of water in soil i.e., the ratio of unfrozen water to ice. The main factors determining the amount of unfrozen water are temperature, surface area, and salt content; minor factors are mineralogical composition, structure, and pressure. The amount of unfrozen water in frozen soils as a function of temperature and pressure can be calculated, using a thermodynamic treatment, for clays from swelling pressure data, and for granular soils from moisture characteristic curves at room temperature. The calculated data show good agreement with experimental data.

> The unfrozen film conducts water and ions under the influence of temperature gradients, electrical gradients, and external pressure differences. The rate of migration depends on the surface area and the thickness of the film of unfrozen water. Since the thickness of the film decreases with decreasing temperature, the rate of migration of water through frozen soils rapidly increases as the freezing point is approached.

•THE ENGINEERING behavior of soils is strongly influenced by the interaction of mineral material with water, and the effect of naturally encountered temperatures must invariably be assigned to changes caused by temperature in the interaction of water with the mineral surface or to changes in the properties of water itself. In unfrozen soils the force systems that act on the water are capillarity and adsorption. The forces of capillarity arise from the surface tension at air-water interfaces, and the temperature effect will be proportional to the change in surface tension. For water, the surface tension changes from 75.6 dynes/cm at 0 C to 71.97 dynes/cm at 25 C. The effect of temperature on adsorption forces is more difficult to analyze. One model for physical adsorption on negatively charged surfaces that has been proposed is the theory of the diffuse electric double layer. Differentiating expressions resulting from this model with respect to temperature would be a refinement not warranted by the present accuracy of the model. In general, the effect of temperature on the value of engineering properties of unfrozen soils is neglected in practice.

In frozen ground the effect of temperature, particularly in the range of naturally encountered temperatures, is very large. The reason is that temperature changes not only the interaction of water with soil, but also the ratio of the amount of liquid water to ice. The effect of temperature in frozen ground is similar to the combined effect of change in water content and temperature in unfrozen ground. The paper reviews recent work on the phase composition of water in frozen ground and its importance to engineering properties.

PHASE COMPOSITION OF FROZEN SOILS

In a frozen soil, unfrozen adsorbed water can exist in equilibrium with ice over a large temperature range below freezing. The adsorbed water freezes gradually as the temperature is lowered, decreasing the thickness of the unfrozen film. The ice phase at each temperature is in equilibrium with an unfrozen water layer whose properties are constantly changing as the temperature is lowered. The reason for the large

- 78

freezing point depression of the unfrozen water is the presence of exchangeable ions, which are concentrated in thin films upon freezing, and specific adsorption forces emanating from a charged surface.

Figure 1 shows how the liquid water content might change in a soil column in which the temperature changes with distance. At temperatures above freezing all water is in the form of liquid water. At some temperature just below 0 C ice forms and only a certain amount of unfrozen water remains in the liquid form. By lowering the temperature further the amount of unfrozen water is reduced as more water freezes to ice.

Since the unfrozen liquid water is in equilibrium with the ice at any particular temperature and pressure, equilibrium thermodynamics is particularly well-suited to relating physical measurements on soils at above-freezing temperature, such as swelling pressure and soil water tension, to the amount of unfrozen water. Thermodynamic relations only apply to the particular soil under consideration, and cannot be used to predict changes between different soil systems. The important soil parameters are surface area and temperature. Minor variations are caused by such parameters as mineralogical composition and kind of exchangeable ions. In Figure 2 the unfrozen water content is given as a function of temperature for several soils.

The thermodynamic relationships for soils at or below the freezing point are well understood (13, 14). The physical argument is as follows: The unfrozen water exists in the presence of ice, and the two phases of water are in a stable equilibrium. The partial molar free energy of ice, \overline{F}_i , is equal to that of the unfrozen, \overline{F}_{1w} , Hence,

$$\overline{\mathbf{F}}_{\mathbf{i}} - \mathbf{F}^{\circ} = \overline{\mathbf{F}}_{\mathbf{l}\mathbf{W}} - \mathbf{F}^{\circ}$$
(1)

where F° is the free energy of the standard state, which is chosen to be bulk supercooled water at the same temperature as the sample. The ice that forms in frozen soil has been found to be ordinary ice (3), apparently not influenced by the presence of the soil surface. Thus the vapor pressure of ice in soil is the same as ice in bulk. Ice in frozen ground differs in this respect from water in unfrozen ground, where the properties and vapor pressure of water are strongly influenced by a soil surface.

The following relationship exists between $(F_1 - F^{\circ})$ and vapor pressure:

$$\mathbf{F}_{i} - \mathbf{F}^{\circ} = \overline{\mathbf{F}}_{iw} - \mathbf{F}^{\circ} = \mathbf{RT} \ln \frac{\mathbf{P}_{ice}}{\mathbf{p}_{o}}$$
(2)

where p_{ice} is the vapor pressure of ice and p_0 that of bulk supercooled water. The relative humidity of frozen soil, p_{ice}/p_0 , which in unfrozen ground is equal to the ratio of the vapor pressure of soil water, p_1 , to bulk water, p_{lw}/p_0 , is not appreciably affected by temperature at water contents of practical interest.





Figure 1. Liquid water content as a function of distance from the cold plate. In the frozen part ice is also present.

Figure 2. Unfrozen water content, U, as a function of temperature for three soils.

For a given soil the relationship between water content and soil water tension is called the moisture characteristic curve of the soil. The partial molar free energy of soil water, \overline{F}_{1W} - F°, is related to the soil water tension, ΔP , by

$$\overline{F}_{l} - F^{\circ} = -V_{l} \Delta P \tag{3}$$

where V_1 is the partial molar volume of liquid water. A moisture characteristic curve is usually determined at room temperature. For practical purposes the temperature effect on the moisture characteristic curve, related to the heat of wetting, can be neglected. A moisture characteristic curve can thus be considered as a relationship between partial molar free energy and water content over a fairly large temperature range.

Equations 2 and 3 are alternative expressions for the partial molar free energy of liquid water. Equation 2 is particularly useful in frozen soil when ice is also present. The unfrozen water content can be found from a moisture characteristic curve by first constructing a plot of $(\overline{F}_1 - F^\circ)$ vs water content by means of Eq. 3. This curve is valid over a large temperature range. Then a curve of $(\overline{F}_1 - F^\circ)$ vs temperature is constructed by means of Eq. 2, applicable strictly to frozen soil when ice and water are in equilibrium. In Figure 3 the amount of unfrozen water is plotted both as a function of $\overline{F}_1 - F^\circ$ and temperature for Lower London clay.

The same relationship that exists between tension and $(\overline{F}_1 - F^\circ)$ also exists between swelling pressure, π , and $(\overline{F}_{1w} - F^\circ)$. It is of the form

$$\overline{\mathbf{F}}_{1\mathbf{w}} - \mathbf{F}^{\circ} = -\overline{\mathbf{V}}_{1} \pi \tag{4}$$

Swelling pressures obtained at room temperature can thus also be used to determine amounts of unfrozen water.

For temperatures very close to freezing (0.2 C) the assumption that the free energy of ice in soil is equal to that of bulk ice may not be exactly true. If the ice consists of very small grains a correction may have to be applied for the surface energy (11). This correction becomes small below -0.5 C.

In discussing the strength of frozen ground it is important to know how the amount of unfrozen water changes with pressure (2, 13, 14, 9). Equation 1 can be used to calculate the pressure effect. Differentiating Eq. 1 with respect to pressure yields

$$\left(\frac{\delta \mathbf{F}_{i}}{\delta \mathbf{P}}\right)_{T} - \left(\frac{\delta \mathbf{F}^{\circ}}{\delta \mathbf{P}}\right)_{T} = \left(\frac{\delta \mathbf{\overline{F}}_{l}}{\delta \mathbf{P}}\right)_{T} - \left(\frac{\delta \mathbf{F}^{\circ}}{\delta \mathbf{P}}\right)_{T}$$
(5)



Figure 3. Liquid water content and partial molar free energy as a function of temperature for Lower London Tertiary clay.

using the relations

$$\left(\frac{\hbar \mathbf{F}_{i}}{\delta \mathbf{P}}\right)_{\mathbf{T}} = \mathbf{V}_{i} \text{ and } \left(\frac{\delta \mathbf{F}^{\circ}}{\delta \mathbf{P}}\right)_{\mathbf{T}} = \overline{\mathbf{V}}_{i}$$

where V_i and V° are the molar volumes of the solid and liquid state respectively. Equation 5 can be integrated between the limits of atmospheric pressure, P_a , and any other external pressure, P, which yield

$$\left(\overline{\mathbf{F}}_{1} - \mathbf{F}^{\circ}\right)_{\mathbf{P}} = \mathbf{P} - \left(\overline{\mathbf{F}}_{1} - \mathbf{F}^{\circ}\right)_{\mathbf{P}} = \mathbf{P}_{a} = \left(\mathbf{V}_{1} - \mathbf{V}^{\circ}\right) \left(\mathbf{P} - \mathbf{P}_{a}\right)$$
(6)

Equation 6 allows one to calculate the change in partial molar free energy with pressure at constant temperature, since both V_i (= 1.09 cm³/g) and V° (= 1.0 cm³/g) are known. Table 1 gives the change in the amount of unfrozen water caused by a pressure increase of 100 atm (1470 psi) at different temperatures for Lower London clay. The pressure effect rapidly decreases with temperature.

The phase composition of water in frozen soils thus depends first of all on temperature, and if it can be shown that the engineering properties of soils are determined by the amount of unfrozen water, one can expect the engineering properties of frozen ground to change dramatically over a relatively small temperature interval (0 to -10 C).

MASS TRANSPORT IN FROZEN SOIL

Water can be transported in frozen soils, and a study of the mechanism and rate at which this process occurs can be used to explain creep of frozen soils. Water movement can also alter the disposition of the ice phase, which often causes large changes in strength and conductance.

Frozen soils are essentially impermeable to laminar water flow and the dominant mechanism of mass transport is mainly by diffusion in the liquid phase. Diffusion can occur under the influence of temperature and external stress gradients. Mass transport occurs through the transition layer between the particle surface and the ice. Figure 4 illustrates the situation that might exist when the ice is placed in a temperature gradient field. The thickness of the film will be larger on the warm side than on the cold side, since the thickness of the transition layer is determined for a given substrate by temperature only. Thus, if there is a temperature gradient along a solid-ice interface, the thickness of the transition layer diminishes in the direction of decreasing temperature. The freezing point depression of the water in the film between a solid and an ice surface has a value equal to the negative temperature. To translate the freezing point depression into a driving force for water diffusion, the freezing point depression can perhaps be related to an equivalent osmotic pressure, π , by

$$\pi = \frac{h}{T\Delta V} \Delta T$$
(7)

TABLE 1	
INCREASE IN UNFROZEN WATER, AW, CAUSEI)
BY A PRESSURE INCREASE OF 100 ATM IN	
LOWER LONDON CLAY AT DIFFERENT	
TEMPERATURES	

I BMF BIGKI OILES			
Temperature, C	ΔW , g H ₂ O/g Clay		
-1.0	0.071		
-1.2	0.057		
-1.4	0.034		
-1.5	0.026		
-2.0	0.025		
-5.0	0.010		
-10.0	0.005		



Figure 4. Change in the phase composition of water with temperature.

The equation for diffusion in an ideal solution is

$$\gamma = \frac{\lambda (\mathbf{T})}{\mathbf{R}\mathbf{T}} \ \Delta \ \pi \tag{8}$$

where γ is mass flow rate, λ (T) is the coefficient for transfer of water, and R is the gas constant.

Substituting Eq. 7 in Eq. 8 yields

$$\gamma = \frac{\lambda (T) h}{RT^2 \Delta V} \Delta (\Delta T)$$
(3)

The coefficient of transfer, λ (T), is temperature-dependent. The coefficient can be expected to decrease rapidly with decreasing temperature below freezing. Also, since the water migrates over the surfaces of the soil particles, the flow rate at a certain temperature increases with increasing surface area, and clay-type soils have a higher coefficient of transfer than a silt or sand-type soil at the same temperature below freezing. Water migration under a temperature gradient in a cylindrical column of frozen soil was measured by the attenuation of gamma radiation. In Figure 5 the rate of flow of water in frozen Fairbanks silt is plotted as a function of temperature. The water migrates through the unfrozen water layer in the direction of decreasing film thickness. It should be observed that the rate of transfer is considerable in the temperature range from 0 to -5 C. A saturated unfrozen silt soil usually has a permeability on the order of 10^{-5} cm²/sec.

It was previously noted that an increase in pressure at constant temperature caused an increase in the amount of unfrozen water. Hence, stress gradients in soils also result in a change in unfrozen film thickness, and movement of water again will occur in the direction of decreasing film thickness. Under isothermal conditions the direction of flow is in the direction of decreasing stress.

Besides mass transport of water, ions also migrate in frozen ground. For the purpose of evaluating frozen ground for storage or ultimate disposal of radioactive waste,

the self-diffusion of Na ions was measured in silts from northern Alaska (<u>16</u>). In Figure 6 the diffusion coefficient is given as a function of temperature. When the temperature is lowered and the film thickness decreases, the movement of the ions rapidly slows down. Nevertheless, in the temperature range from 0 to -5 C, ions have an appreciable diffusion coefficient, reduced only by a factor of 5 to 10 compared with unfrozen ground.



Figure 5. Rate of flow of water in frozen soil as a function of temperature below 0 C.



Figure 6. Apparent self-diffusion coefficient of silts from northern Alaska as a function of temperature (after Murrmann and Hoekstra, 16).

82

ENGINEERING PROPERTIES OF FROZEN SOILS

It has been shown that in general the engineering properties of frozen soils are very temperature-dependent in the temperature range where the phase composition of water changes appreciably. The important engineering properties for construction are strength and thermal properties.

Sections of frozen soils are sometimes used as load-bearing members of finite dimensions. The artificial freezing of a shell of ground for support during tunneling is an example. In such application both the short- and long-term strength must be considered; in particular the long-term strength is very temperature-dependent.

Serious failures occur when the permafrost degradates and frozen soil melts. The maintenance of temperature stability is, therefore, an important engineering objective. The thermal properties of soil enter into any meaningful analysis of thermal stability.

Electrical grounding of equipment and the design of broadcasting antennas require the electrical conductivity of the frozen soil as an input. The conductivity of a soil decreases rapidly when frozen. Yet, depending on the broadcasting frequency, frozen ground must be considered as a lossy dielectric. In the USSR, frozen soil sometimes is melted by passing an alternating current through a certain section.

All the engineering properties of frozen ground are influenced to a certain extent by the disposition of the ice. Different properties are encountered when ice is uniformly distributed or in lenses. These differences are not considered in this paper; here only an attempt is made to explain the temperature-dependence of the properties of frozen ground.

STRENGTH PROPERTIES OF FROZEN SOILS

In an unfrozen soil, shear stresses result from the friction between the particles in a soil skeleton. The value of the shear stress is normally expressed by Coulomb's equation:

$$\gamma = c + (\sigma - \mu) \tan \phi \tag{9}$$

where γ is the shear strength on a plane with a total normal stress σ and a pore pressure μ , and ϕ is the angle of internal friction. For wet cohesionless soil, c is zero and the soil has no unconfined compressive strength. For a clay soil, such as remolded Boston Blue clay with a water content of 29 percent, c is on the order of 3 kg/cm². The influence of temperature on the strength of unfrozen soil is considered small.

In frozen soil the ice forms a bond between the mineral particles, and the ice in the pores presumably can carry a stress. The shear stress of frozen soils is much higher than that of unfrozen soil, and frozen ground has been used for structural members of definite dimensions. The strength behavior of frozen ground is determined by many factors, among which are temperature, rate of stress applications, soil type, water content, and the disposition of the ice phase. The strength properties of frozen soils are highly temperature-dependent in the region of intensive phase transformations.

Figure 7 gives the ultimate unconfined compressive strength of a clay soil and sand soil at a strain rate of 0.2 in. per min. With faster rates of load application, higher ultimate strength values are observed. Many researchers found that for Ottawa sand the ultimate strength was related to the rate of stress application by

$$\sigma_{\max} = A (\dot{\sigma})^{\mathsf{D}} \tag{10}$$

where σ_{max} is the ultimate unconfined compressive strength and $\dot{\sigma}$ the time rate of load application.

The deformation of frozen soil under static loads has also been investigated in considerable detail. In Figure 8, the behavior of the strain with time at different levels of load application is given. In each curve several sections can be distinguished: Immediately after load application an instantaneous elastic strain arises. The strain then passes through a region of continually decreasing rate to a rate of steady creep, and finally the stage of progressive flow, characterized by an increasing rate of strain



Figure 7. Ultimate compressive strength of three soils as a function of temperature (after Sayles, 19).

leading to collapse. Vialov (21) determined strain as a function of time for several temperatures, and his data are given in Figure 9. In Figure 10 the strain as a function of time at a constant load and at a temperature of -0.3 C is given for different soil types.

Frozen soil is a system of mineral particles and ice separated by films of unfrozen water. A film, even though very thin, exists at the ice-mineral particle interface down to very low temperatures. The increase in the strength of frozen soil with temperature reflects more the increased cohesive strength of the film than the temperature-dependence of the strength of ice.

The properties of the unfrozen film are also evident in the creep of frozen soil. The amount of unfrozen water was found to increase with pressure and flow of water will occur from levels of high stress to low stress. Creep is most likely the result of transport of water under stress gradients. Tsytovich postulated (22) that ice lenses can be transplaced by this mechanism. Ice will melt to replenish the films at the high-pressure end, and ice will accumulate at the low-pressure end. The transport of water under stress gradients is probably an important factor in stress relaxation in



Figure 8. Behavior of strain as a function of temperature at different load levels (after Vialov, 2<u>1</u>).

frozen ground.

The creep rate is apparently closely related to the amount of unfrozen water. The creep rate increases rapidly with increasing temperature, as seen in Figure 9. Also, at constant temperature the creep rate increases when the soil type contains more unfrozen water (Fig. 10). This observation is a strong argument against attributing the creep to the ice phase; soils of similar ice content may have widely varying creep rates depending on soil type.

Since the phase composition of the water in frozen soils constantly changes when the temperature is lowered, it is difficult to apply the concept of activation energy to this system (4). The theory of rate processes can be applied when



Figure 9. Strain as a function of time at constant load for a heavy sandy loam at several temperatures (after Vialov, 21).



Figure 10. Strain as a function of time at constant load for several soils at a temperature of -0.3 C (after Vialov, 21).

only the thermal energy of the moving molecules changes with temperature. In frozen soil, in addition to the change in thermal energy a gradual phase change also occurs.

As a result, when the theory of rate processes is applied to creep of frozen soils, activation energies are obtained as high as as 93.4 kcal/mole, which value is clearly incompatible with either movement in ice or water. A value of 93.4 kcal/mole would be equivalent to breaking approximately 9 hydrogen bonds. In ice, activation energies for creep are on the order of 13 kcal/mole (17). Finally, Vialov (21) stresses the importance of the structure of the frozen ground on the strength of the soil. The disposition of the ice phase, e.g., in lenses or uniformly distributed, strongly affects the strength of frozen ground. The Russian literature offers many detailed accounts on this subject.

THERMAL PROPERTIES OF FROZEN SOILS

Heat is transported in frozen soils mainly by conduction, that is, the transfer of thermal energy

on a molecular scale. Two independent thermal properties enter into a quantitative description of heat transfer by conduction, namely, the thermal conductivity, λ , and the heat capacity per unit weight, C. The quotient of these quantities often appears in heat flow equations; it is called the thermal diffusivity and denoted here by the symbol α .

The heat capacity per gram of unfrozen soil can be found by adding the heat capacity of the different soil constituents in one gram of soil. Thus, if X_S , X_W , and X_α denote the weight fractions of solid material, water, and air, respectively, the heat capacity is given by

$$C = X_S C_S + X_W C_W + X_{\alpha} C_{\alpha}$$
(11)

The third term can usually be neglected. The value of C_w is 1.00 cal/g C for water and C_s varies between 0.22 and 0.15 cal/g C depending on the mineral type.

The heat capacity of a frozen soil is somewhat more complicated (1, 2, 13, 24). The heat capacity is given by the sum of four terms, one for each phase—mineral particles, ice, and unfrozen water—plus a term that takes into account the latent heat involved when the phase composition changes with temperature. Thus, the heat required Q, to change the temperature of 1 g of frozen soil over a temperature interval ΔT , is given by (1)

$$Q = (C_{S} X_{S} + C_{I} X_{I} + C_{u} X_{u}) \Delta T + \int_{T}^{T+\Delta T} \Delta \overline{H}_{f} \left(\frac{\delta X_{u}}{\delta T}\right)$$
(12)

where C_s , C_i , C_u are the heat capacities of the mineral particles, the ice, and the unfrozen water respectively and likewise X_s , X_i , X_u are the weight fractions of the constituents; $\Delta \overline{H}_f$ represents the latent heat of phase change of unfrozen water in cal/g. The total weight of water, X_w , is the sum of the amount of ice and water:

$$\mathbf{X}_{\mathbf{W}} = \mathbf{X}_{\mathbf{i}} + \mathbf{X}_{\mathbf{u}} \tag{13}$$

Substituting Eq. 13 in Eq. 12 and dividing by ΔT yields

$$\frac{Q}{\Delta T} = C = C_{i} X_{w} + C_{s} X_{\Delta} + X_{u} (C_{u} - C_{i}) + \frac{1}{\Delta T} \int_{T}^{T+\Delta T} \Delta \tilde{H}_{f} \left(\frac{\delta X_{u}}{\delta T}\right) dT$$
(14)

In Figure 11 the heat capacity of Na montmorillonite (2, 13) and Leda clay (24) is given. The heat capacity can be seen to change appreciably between 0 C and -5 C where the phase composition undergoes large changes. The amplitude of the variation in C is related to the amount of unfrozen water and will thus increase the finer the soil.

The problem of expressing the thermal conductivity of a soil as a function of the conductivities and volume fractions of the soil constituents is a very complex task. McGaw (15) has made a critical analysis of heat conduction equations for granular materials. At low water content the thermal transports across solid-liquid interfaces to a large extent determine the heat transfer from grain to grain.

The change in thermal conductivity between frozen and unfrozen soils at low water content illustrates well the importance of interfaces in heat conduction. The thermal conductivity of water at 0 C is 1.4 mcal/deg C cm sec, and the thermal conductivity of ice at 0 C is 5.0 mcal/deg C cm sec. The conductivity of ice increases with decreasing temperature and is 5.5 mcal/deg C cm sec at -9 C. Upon freezing, both parallel or series flow models would predict an increase in thermal conductivity, since liquid water is replaced by the higher conductivity material, ice. McGaw (15) measured the thermal conductivities in unsaturated frozen Ottawa sand. His data are plotted in Figure 12. One data point at -4 C was obtained from Kersten (10). Although part of the



Figure 11. Apparent heat capacity of two clays as a function of temperature (from Anderson, <u>1</u>, and Williams, 24).

water freezes to ice with a higher conductivity, the transfer from grain to grain still has to occur through the films of liquid water. With decreasing temperature the film thickness decreases and the motion of the molecules is restricted, which apparently impedes the thermal transfer.

For soils at higher water content, the thermal conductivity no longer decreases upon freezing. Perhaps the ice matrix is continuous and some heat flow paths can bypass the liquid interface. In Figure 13 the ratio of the thermal conductivity at -4 C to the thermal conductivity at +4 C according to Kersten is shown. Thermal conductivity is clearly an area in which more data are required.

When the equation for one-dimensional heat conduction,

$$q = \lambda (T) \frac{dT}{dx}$$
(15)

where q is heat flow in cal/cm² sec and λ (T) is the thermal conductivity in cal/deg C cm² sec, is combined with the equation describing the conservation of energy,

$$\frac{d}{dx} (q) = \rho C \frac{\delta T}{\delta t}$$
(16)



Figure 12. Thermal conductivity of unsaturated frozen Ottawa sand as a function of temperature (from McGaw, <u>15</u>, and Kersten, <u>10</u>).

where ρ is density of the medium in gr/cm³ and C(T) is the heat capacity in gr/deg C, a one-dimensional heat transport equation results:

$$\rho C (T) \frac{\delta T}{\delta t} = \frac{S}{Sx} \left[\lambda (T) \frac{\delta T}{\delta x} \right] \quad (17)$$

1.2 Fairbanks SILT 1.1 Healy λ(-4°C) 1.0 (+4 °C) Sandy 0.9 LOAM 0.8 0.7 10 15 20 MOISTURE CONTENT,

1.3

Figure 13. Ratio of thermal conductivity at -4 C to +4 C as a function of moisture content for several soils (after Kersten, 10).

In most heat flow problems both λ and C are considered independent of temperature and Eq. 17 then reduces to

$$\frac{\delta \mathbf{T}}{\delta \mathbf{t}} = \frac{\lambda}{\rho C} = \frac{\delta^2 \mathbf{T}}{\delta \mathbf{x}^2}$$
(18)

and $\lambda/\rho C$ is called the diffusivity. Figure 12 and 13 illustrate that both the heat capacity, C, and the thermal conductivity, λ , are very temperature-dependent and can by no stretch of the imagination be considered as constants. Le Fur et al (12) studied the freezing of kaolinite suspensions and, by placing Eq. 17 in a finite difference form, valuated the apparent diffusivity of the medium. These data are plotted in Figure 14.

In many practical calculations one often assumes that all water freezes at the initial freezing temperature of the soil. The error of this approximation increases the finer mechanical composition of the soil.



Figure 14. Thermal diffusivity of a kaolinite suspension as a function of temperature (after LeFur et al, 12).

ELECTRICAL PROPERTIES OF FROZEN GROUND

Current flow in response to an electrical gradient in a frozen soil occurs almost entirely through the unfrozen water films. Conductance measurements on a frozen soil can, therefore, be used to investigate the disposition of the unfrozen water, and are particularly useful as an indication of changes in the liquid-like films caused by variation of an external factor such as temperature or pressure.

Soil particles usually carry a negative charge, and to remain electro-neutral this charge is balanced by an excess of positive ions in the water layers close to the surface. When a soil is frozen, relatively pure ice freezes out, the ions are expelled from the ice phase, and crowded in the thin films of unfrozen water. In these thin films the ions are relatively free to move along the surface, but the mobility decreases as the thickness of the film is reduced. Self-diffusion experiments of Na ions provided further proof that at least at temperatures within 10 C from zero, the ions are the most important charge carriers. In Figure 15, the conductance of a frozen silt is shown as a function of temperature. The conductance rapidly decreases in the temperature range from 0 C to -5 C and continues to decrease at a slower rate at temperatures below -5 C.

Previously it was noted that pressure increases the amount of unfrozen water. In Figure 16, the ratio of conductance at atmospheric pressure is plotted vs the conductance at a pressure of 1500 psi. The enhanced conductance with pressure is due to an increase in the amount of unfrozen water.

In summary, one can conclude that frozen soil is not to be regarded as a good insulator. Although ice and mineral particles are relative poor conductors, the transition layer between them causes frozen soil to have a conductance factor 5 to 10 less than the same soil unfrozen.

In order to satisfy the condition of electro-neutrality, the liquid adjacent to a negatively charged surface has an excess of positively charged ions. Thus, if an electric field, E, is applied along the surface, a net force, F, is exerted on a unit volume of liquid. If the excess charge density is ρ coulombs/cm³, and the electric field, E, is in volts/cm, then the force per unit volume of liquid in joules/cm³ is given by

A requirement for electro-osmosis is an excess of charge in the liquid layers adjacent to a charged surface. Frozen soil clearly meets these requirements. The exchangeable ions are not incorporated in the ice lattice, but are crowded in the liquid films on the particle surfaces.



Figure 15. Conductance of a frozen silt as a function of temperature.



Figure 16. Relative resistance at 1500 psi for Na montmorillonite samples as a function of temperature.



1	ΓA	BI	ЬE	2	

Soil	Temperature C	Initial Water Content	Final Water Content	
			Within 1 cm From Anode	Within 1 cm From Cathode
Wyoming bentonite	-2.0	341	275	456
	-1.5	265	134	310
New Hampshire silt	-1.5	30	27	32
	-1.0	28	21	40

INITIAL AND FINAL WATER CONTENTS AFTER EXPOSING FROZEN SAMPLE TO ELECTRICAL GRADIENT OF 1 VOLT/CM FOR 24 HOURS (Weight of Water per Weight of Clay × 100≸)

Furthermore, if these films are continuous through the frozen soil, water movement could occur by this mechanism. Verschinin et al (22) and Hoekstra and Chamberlain (6) measured electro-osmoses in frozen clays and silt. Table 2 gives the initial and final water contents after the frozen sample was exposed to an electrical gradient of 1 volt/cm for 24 hours. Water was transported in the frozen soil toward the cathode. Initially the anode area contained ice and unfrozen water, but in the final state all ice was removed from the anode region and large bodies of ice formed in the vicinity of the cathode. Shrinkage cracks might form at the anode.

The mechanism by which the ice is transported needs further discussion. The phase composition of frozen ground was found to depend on temperature and external pressure. An electrical field is not likely to influence the phase composition of frozen ground. When the film water is depleted at a certain location, some ice will melt to replenish the films and, likewise, water will freeze where the amount of liquid water exceeds the equilibrium value.

SUMMARY

The engineering properties of frozen soils are very dependent on temperature in the range from 0 C to -10 C. The temperature-dependence is related to changes in the amount of unfrozen water. The effect of temperature is most pronounced in soil types that contain large quantities of unfrozen water. The amount of unfrozen water increases in the direction sand > silt > clay.

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90