Transient Temperature Influences on Soil Behavior

ROBERT E. PAASWELL, State University of New York at Buffalo

In cases where changes in temperature affect soil responses such as strength of compacted subgrades and embankments, changes in compaction properties during stabilization due to moisture migration, and changes in deformation of stressed clay soil layers, the ability to predict temperature profiles within the affected areas becomes highly desirable. This paper analyzes the temperature profile that is developed in soil samples for conditions of transient temperature changes at one boundary surface. The study is divided into two parts. The first is concerned with temperature distribution as affected by local inhomogeneities for nonsaturated soils. Close study is also made on moisture changes as affected by this surface temperature change. The second part is concerned with the pattern of temperature increase and decrease imposed on a stressed, saturated soil. For this case the theory of heat conduction was used to predict the temperature distribution within the soil sample.

For the study of the first part, a parabolic form of temperature increase was applied at the upper boundary surface of an insulated cylinder to an unsaturated sample of a clay soil compacted at varying moisture contents and density states. The resultant temperature-time-space distribution curves were analyzed and the resultant moisture content distribution curves were compared with the initial moisture content distribution curves. In the constant stress tests, a thermal pulse was applied at the upper surface of a saturated consolidated soil, and the resultant temperature-time-space distribution was determined.

The tests indicate that establishment of temperature profiles enables correlation to be made between surface temperature changes and moisture changes on a transient basis. Where surface boundary conditions can be accurately modeled, heat conduction theory, assuming a sample homogeneity, can be used to predict these temperature profiles.

•IN RECENT YEARS there has been a growing body of research on the physical and mechanical properties of soils and their dependence on or independence of temperature. The use and interpretation of laboratory models is the first step in the interpretation of field performance. The use of uniform thermal coefficients, for example, and the analysis of the influence of sample inhomogeneities on temperature changes within the sample caused by boundary changes can be closely analyzed in the lab and the findings transferred to the field. This would be useful in the analysis of field performance of compacted soils, where temperature changes might cause moisture migration and strength changes, altering the response of the soil to imposed stresses. Another occurrence could be the case of a confined soil layer subject to temperature increases that result in pore pressure fluctuations and subsequent deformation. Through the use of mathematical models and physical models, close study can be made of the influences of temperature changes on soil behavior.

In the study of the effects of temperature changes on soil behavior, much research has been made on various steady-state conditions, i. e., the properties of soil at one temperature vs the properties of the same soil at another higher, or lower, constant temperature. Field conditions, however, are far from steady-state, and it becomes

important to establish how soil characteristics change during temperature changes, i. e., the transient case.

The familiar and frequently used theory of heat conduction, when applied to soils, often includes the assumption that the soil is homogeneous. For soil of a consistent moisture density pattern throughout, this assumption may be used, by considering that this three-phase material has uniform thermal characteristics such as conductivity and diffusivity (1). Soil compacted to a given density at a specific site during the process of stabilization, or soil that exists naturally above the water table, is generally not uniformly homogeneous, and the nature of the actual heat transfer as measured by temperature patterns influenced by localized inhomogeneities represents a portion of the work studied herein. The warming cycle during the day causes temperature increases in the upper surfaces of the soil, and these temperature increases result in some moisture changes in the affected layers. One purpose of the first series of tests is to investigate the manner in which the measurable surface temperature pattern (here, increasing only) influences the temperature distribution pattern within the sample and the moisture pattern within the sample, and the extent to which this pattern is influenced by local inhomogeneities in moisture content.

In a saturated soil, the utilization of heat conduction theory becomes simplified due to the more homogeneous state of the soil. Field temperature conditions, however, are not steady-state, so the use of heat conduction theory, with time-dependent temperature boundary conditions to predict field temperature patterns, becomes desirable. By being able to predict subgrade temperatures, given surface distributions, it should then be possible to estimate changes in other temperature-dependent soil properties, such as strength, for a given environmental situation. The sinusoidal temperature pattern is a simplified approximation of a naturally occurring temperature pattern at ground surface. In the second series of tests reported here, a comparison is made between the temperature pattern predicted by the theory of heat conduction and the pattern as modeled in the laboratory on a stressed-saturated clay sample. In addition, some attention is given to the deformation induced in the sample due to the changing surface temperature.

TRANSIENT TEMPERATURE PROPAGATION

Recompacted soil cylinders were subjected to a pattern of increasing temperature at the upper boundary surface, with the remaining surfaces insulated. In these tests, the original moisture content distribution was established to ascertain the degree of homogeneity attained by compaction and the influence of this pattern of homogeneity on resultant temperature and moisture distributions.

Table 1 lists 16 samples of a soil indigenous to the Buffalo area, PI = 30, LL = 54, specific gravity = 2. 73, that were compacted at nominal moisutre contents as given. These samples were compacted in insulated cylinders 10 cm in diameter and 15. 6 cm in height. An embedded copper foil heater was placed at the upper axial surface, controlled by a variable power source, which enabled the surface temperature pattern to have the desired time history. Temperature records were maintained by thermocouples throughout the sample connected to a recording potentiometer (Fig. 1).

In order to show the temperature distribution and moisture redistribution during these increasing temperature tests, samples 3, 8, 9, and 13 are compared showing temperature-time and moisture content sample height distributions in Figures 1 and 2. The temperature tests consisted of increasing the temperature at the upper surface while the other surfaces remained insulated.

The surface temperature increase, nonlinear in form, was from ambient temperature (26 C ±) to 80 C ± in 240 minutes. This value was recorded by thermocouple No. 1 (TC 1). The temperature propagation throughout the interior of the sample is shown by the temperature-time profile for thermocouple locations TC 2 through TC 5. It has been noted (1) that thermal diffusivity increases with increasing moisture content and decreasing density. This is shown clearly in Figure 1 where, for example, the propagation of the 50 C isothermal line from the surface to TC 2 takes 52 minutes for sample 3, 50 minutes for sample 8, 40 minutes for sample 9 and 42 minutes for

TABLE 1

SAMPLE PROPERTIES—TRANSIENT TEMPERATURE TESTS

Sample No.	Nominal Initial Moisture Content, w_i percent	Void Ratio, e	Density, γ_s, g/cc
1	19.2	1.16	1.24
2	19.3	1.24	1.21
3	19.8	1.39	1.14
4	20.6	1.03	1.34
5	30.0	0.83	1.49
6	30.6	0.80	1.51
7	31.0	0.80	1.51
8	31.7	0.82	1.50
9	39.4	1.09	1.31
10	40.5	1.12	1.28
11	41.1	1.14	1.27
12	41.5	1.12	1.28
13	50.2	1.36	1.15
14	50.3	1.38	1.15
15	51.5	1.39	1.14
16	54.4	1.50	1.09

sample 13. Sample 15 has a greater moisture content but a smaller density than sample 9. It is also noted that in sample 3, the surface temperature rises to 81 C before there is a 1 C temperature rise at TC 5, whereas for the other samples noted the surface temperature was less than 75 C (surface temperature increase, ΔT = 49 C) before a 1 C temperature increase occurred at TC 5.

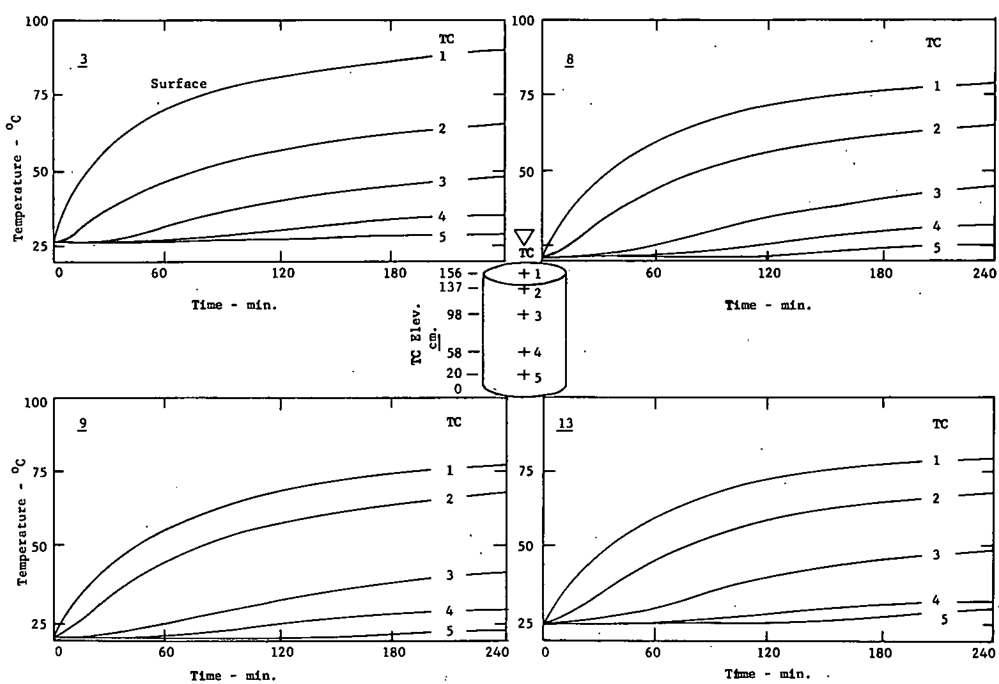

Figure 1. Temperature-time distribution in samples.

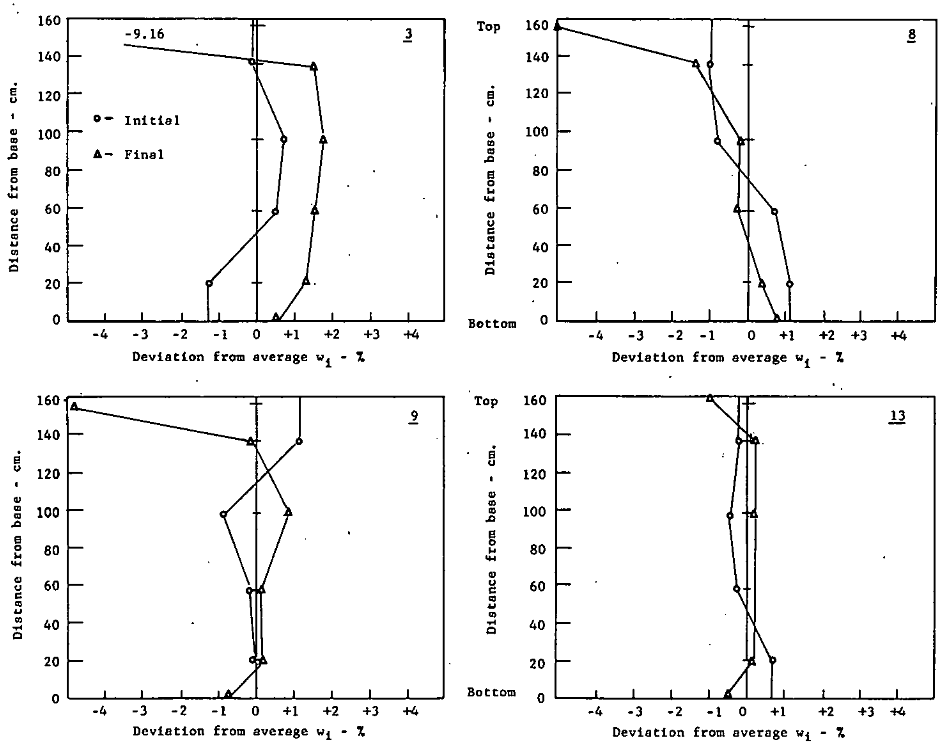

Figure 2. Initial and final moisture distribution in samples.

Of greater interest is the nature of the transmission of temperature through the soil as influenced by the moisture content. Figure 2 as noted shows the nominal initial moisture content w_i percent and the final moisture content w_f percent. The w_i percent is the average of the moisture contents taken as each of 4 layers was compacted into the container. There is a deviation of approximately 1 percent in no set pattern at the start of the test. It should be noted that, because there were initial moisture content gradients, however slight, throughout the sample, the natural process in the soil if no temperature tests were made would be some equilibrating moisture redistribution within the sample.

The most significant change in moisture content occurs near the heated surface, as noted in Table 2. The initial moisture content taken at the time of compaction is compared with the final moisture content near the surface taken immediately after completion of the test. It is seen that there is a substantial decrease in moisture content near the surface in every case. The moisture contents in Table 2 are also calculated as percent saturation for each sample in order to have a more uniform basis for comparison. The greatest decreases in percent saturation occur in samples 1 through 4. These also represent the greatest decrease in actual moisture content. Samples 1 through 4 were compacted at the smallest moisture contents (approximately 20 percent), and with the exception of sample 4, percent saturations less than 45 percent. The remaining samples were compacted at saturations near 100 percent.

This would indicate that as moisture contents increased significantly, the moisture movement near the points of maximum thermal gradient decreased. The least moisture movement occurred at the highest moisture contents (samples 13-16). Because the surface temperature was maintained at less than 100 C, and because the upper surface was covered by the heating cap, moisture change due to evaporation at the surface was minimized.

TABLE 2

CHANGES IN MOISTURE CONTENT AND SATURATION DUE TO TEMPERATURE INCREASES—UPPER SURFACE

Sample No.	Initial Moisture Content, w_i Percent	Initial Percent Saturation, S_i Percent[1]	Final Moisture Content, w_f Percent	Final Percent Saturation, S_f Percent	Net Change Moisture Content, percent	Net Change Percent Saturation
1	18.7	44.0	7.9	18.5	8.8	25.5
2	18.8	41.0	12.4	27.0	6.4	14.0
3	19.7	39.0	10.6	21.4	9.0	17.6
4	22.3	59.0	12.9	33.0	9.3	26.0
5	31.1	103.0	28.4	94.0	2.7	9.0
6	31.8	108.0	29.0	99.0	2.4	9.0
7	31.1	105.0	29.1	99.0	2.0	6.0
8	30.7	106.0	26.8	89.0	3.9	17.0
9	40.5	98.0	34.5	86.0	6.0	12.0
10	40.6	99.5	38.8	94.5	1.8	5.0
11	41.3	101.0	38.3	93.0	3.0	8.0
12	41.5	101.0	39.8	97.0	1.7	4.0
13	50.0	100.0	49.3	98.5	0.7	1.5
14	50.1	99.0	48.1	95.0	2.0	4.0
15	50.0	102.0	49.5	100.0	0.7	2.0
16	54.3	99.0	52.6	96.0	1.7	3.0

[1]Initial percent saturation was computed based on original nominal moisture content. Thus inhomogeneities may cause some to appear greater than 100. These are given here to emphasize change in percent saturation, rather than to give individual values of percent saturation.

Figure 2 illustrates further the somewhat erratic pattern of initial and final moisture content distributions throughout the sample. Combined with the moisture decrease near the surface of the sample, there is a general moisture increase in the middle third of the sample. This would further amplify the moisture flow in the direction of the decreasing temperature gradient, especially at points where the temperature gradient is highest. The tests were always concluded when the surface temperature was still increasing. This meant that steady-state conditions were never realized so that water flows never reached an equilibrium state. In the sample with the highest density (i. e., the lowest permeability) the moisture increase was limited to the upper half of the sample, whereas in the other samples illustrated the moisture flow propagated further into the sample in the direction of decreasing temperature. It should be noted again that the moisture flow due to heat would be coupled with a general redistribution of moisture within the sample prior to the temperature increase. This too would cause the somewhat erratic nature of the temperature distribution. In all cases as represented by Figure 1 the temperature flow was uniform and continuous, that is, there were no sudden jumps due to inhomogeneities in the sample. The temperature pattern at successively deeper thermocouples was one of offset parallelism, as would be predicted by general heat conduction behavior. This point in fact is amplified in the latter part of this paper.

The uniform pattern of the temperature distribution suggests that the temperature propagation throughout a soil sample is not substantially affected by localized inhomogeneities but nonetheless has great influence on the degree of moisture movement within the sample. These results suggest that the soil-water system can be approached as a homogeneous unit in making preliminary predictions of temperature distribution due to transient temperature boundary conditions.

SINE PULSE PROPAGATION

To compare temperature distribution patterns as predicted by heat conduction theory with patterns as established in soil, tests were performed on recompacted saturated

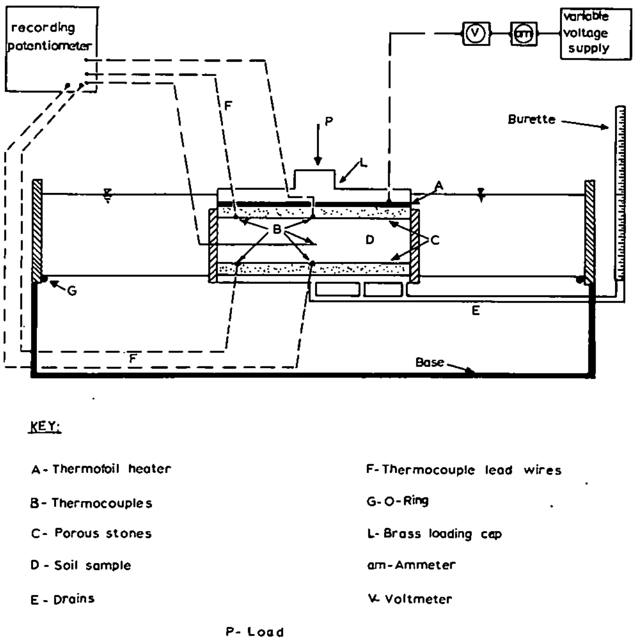

Figure 3. Sample test setup for sine pulse tests.

cylinders of the same soil. For these tests the temperature distribution is assumed sinusoidal at the upper boundary surface, with other surfaces remaining constant. In addition, these tests were used as preliminary tests in evaluating the influence of such a transient temperature pattern on deformation of a stressed soil.

The equipment used in these tests was described in some detail in a previous paper (3), but shall be described here briefly for clarity. A fixed-ring consolidometer, 6.34 cm in diameter, was adapted so that temperature increases could be made at the upper axial surface and temperature measurements made at the upper and lower surfaces and in the center of the sample (Fig. 3). The clay soil, premixed with distilled water at a moisture content greater than optimum, was compacted in the retaining ring to the desired density (Table 3). The soil was then saturated in distilled water under a seating load of 0.31 kg/cm^2. Drainage and saturation took place through the upper and lower surfaces. The heating element was contained within the loading cap and consisted of an etched upper foil heater surrounded by Kapton film and secured by an adhesive to the cap. The heating element was the same diameter as the ring, 6.24 cm, and was 0.0025 cm thick. Power was supplied through a variable voltage control. The loading cap also contained the upper

TABLE 3

PROPERTIES OF SAMPLES IN SINE PULSE TESTS

Sample	Dry Density, g/cc	Moisture Content at Compaction, percent	Max. Surface Temp., deg C	Max. Temp. Increase Above Ambient, deg C	Time to Attain T Max., minutes	Average Rate of Increase, deg C/minute	Max. Lower Surface Temp. Increase, ΔT_B, deg C	Time to Attain ΔT_B, minutes	Time Lag
1	1.39	36	71	46	16	2.9	11	18	2
2	1.36	38	71	46	16	2.9	14	16	0+
3	1.35	40	74	47	8	5.9	5	16	8
4	1.34	42	72	47	8	5.9	4	16	8
5	1.42	36	44	20	6	3.3	6	6	0+
6	1.47	32	72	49	10	4.9	7	16	6
7	1.47	32	73	40	6	6.7	1	6*	0+
8	1.47	32	43	20	6	3.3	1	8	2
9	1.47	32	45	19	6	3.2	1	6*	0+
10	1.47	36	42	19	4	4.7	1	12	8
11	1.47	33	42	15	4	3.8	1	6*	2
12	1.47	33	70	43	10	4.3	5	14	4
13	1.47	33	70	38	6	6.3	3	8	2

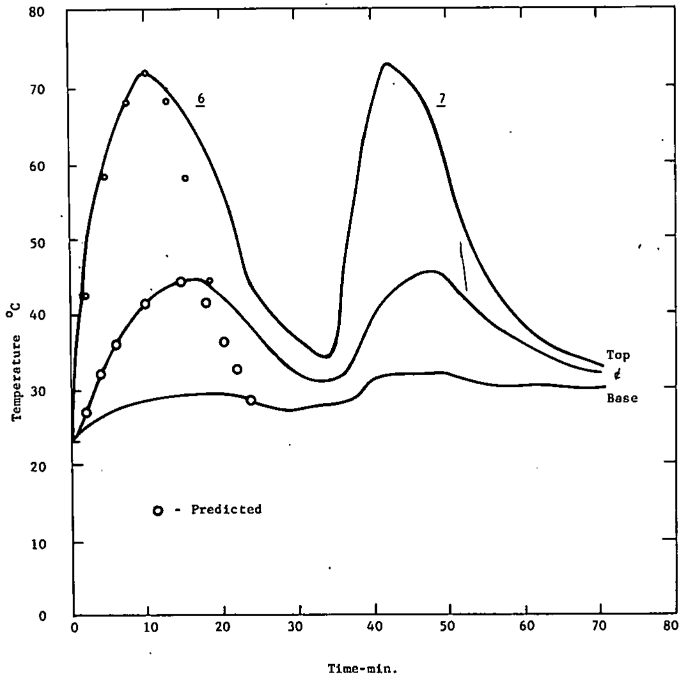

Figure 4. Temperature-time profiles, sine pulse tests, samples 6 and 7.

porous stone; thus there was a distance of 0.63 cm separating the heater and the upper sample surface. Fine wire copper-constantan thermocouples were placed at the surfaces of the upper and lower porous stones and in the center of the sample, permitting a temperature profile to be established across the thickness of the sample and in addition serving as a control on the temperature at the upper surface. To assume that no phase changes would take place within the sample, the maximum upper surface temperature was always less than 100 C. The shape of the upper surface temperature input was attained through control of the variable voltage supply, which was done manually for these tests. The sample was surrounded by a water bath at ambient temperature

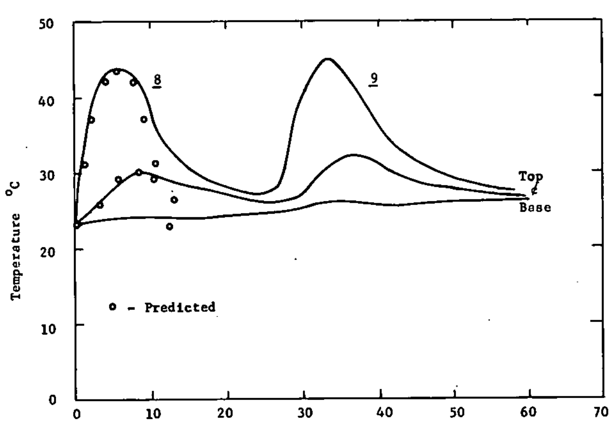

Figure 5. Temperature-time profiles, sine pulse tests, samples 8 and 9.

(25-30 C). The relative size of the water bath (15. 24 cm diameter) and the small source of the temperature input were believed to be adequate to approximate the desired boundary conditions of constant temperature at all points except the upper surface. The relatively short time of the temperature increases was also believed to add to the stability of temperature at the boundary.

The temperature tests were begun 24 hours after the samples had been placed in the water bath. The tests are summarized in Table 3. Figures 4 and 5 show the temperature-time history for samples 6, 7, 8, and 9.

To provide an estimate of the rate of temperature flow through the sample the following boundary value problem (3) was solved for a cylinder of radius a and height L:

$$\nabla^2 T = \frac{1}{k}\frac{\partial T}{\partial t} \qquad (1)$$

$$\begin{aligned} T\,(r,\ z = 0,\ t) &= T^* f(t) \qquad (2)\\ T\,(r,\ z = h,\ t) &= 0\\ T\,(r = a,\ z,\ t) &= 0\\ T\,(r,\ z,\ t = 0) &= 0 \end{aligned}$$

where

$$f(t) = \sin \pi t/t^* \qquad 0 \leqq t \leqq t^*$$

and

$$f(t) = 0 \qquad t^* < t \qquad (3)$$

Note that the zero temperature reference can be changed to an ambient, To, temperature reference by the addition of To. The solution to this problem was obtained as

$$\frac{T}{T^*} \quad \sin\frac{\pi t}{t^*} \sum_{n=1}^{\infty} \frac{2}{\alpha n} \frac{J_o\left(\alpha_n \frac{r}{a}\right) \sin h\left[\alpha_n \left(\frac{L-z}{a}\right)\right]}{J_1\,(\alpha_n)\ \sin h\left(\alpha_n \frac{L}{a}\right)}$$

$$-4 \sum_{n=1}^{\infty} \sum_{n=1}^{\infty} \frac{(m\pi)\ J_o\left(\alpha_n \frac{r}{a}\right) \sin\left(\frac{m\pi z}{L}\right)}{\left[m^2\pi^2 + \alpha_n^2 \left(\frac{L}{a}\right)^2\right] J_1\ (\alpha)_n\ \alpha_n} \cdot B \qquad (4)$$

where

$$B = \frac{\pi\left[\lambda_{mn} t^* \cos\left(\frac{\pi t}{t^*}\right) + \pi \sin\left(\frac{\pi t}{t^*}\right)\right]}{(\lambda_{mn}\ t^*)^2 + \pi^2}$$

$$- \frac{\pi\ \lambda_{mn} t^* \exp\,(-\lambda_{mn} t)}{(\lambda_{mn} t^*)^2 + \pi^2} \quad 0 \leq t \leq t^* \qquad (5a)$$

or

$$B = \frac{-\pi\ \lambda mnt^* \exp\,(-\ \lambda mnt)\ [1 + \exp\,(\lambda mnt^*)]}{(\lambda mnt^*)^2 + \pi^2} \quad t^* < t \qquad (5b)$$

Values of thermal diffusivity based on density and saturated moisture content were computed using formulas given by Kersten. These values ranged from 0.21 to 0.23 cm^2/min. The theoretical solution for the temperature-time profile at the sample center and upper surface is shown as circular points in Figures 4 and 5. Where the actual time surface input resembles the theoretical sine input, the predicted results are quite accurate. However, the time input shows a temperature lag on the cooling part of the cycle that is due both to a slight heating of the water in the porous stone and a heating of the cap, which does not permit rapid cooling to the original ambient. This lag is reflected in the temperature profile throughout the sample and becomes more pronounced with depth in the sample, as can be seen by the figures.

As would be expected, the greater the period of the pulse, the greater the heat transferred into the sample, as manifested by greater temperature increases throughout. In samples 1 and 2, the time for the surface temperature to reach its maximum value was 16 minutes and the corresponding lower surface was between 11 and 14 C. From Table 3 it is seen that the maximum lower surface increase in all other cases was no greater than 7 C. It is also noted from repetitive cases that when a temperature decrease lag occurs, i. e., when there is a decay rather than a rapid drop-off followed by a rapid increase, there is a tendency for slight temperature increase throughout, as illustrated by the center and lower surface temperature profiles in Figures 4 and 5. Whereas the period of the pulse shows the relative insulating properties of the soil, the repetitive type pulse shows the net temperature buildup in the soil, which in actual field situations can be quite significant. The good agreement of the loading portion of the sine curve with predicted temperature distributions also suggests that the heat conduction equation can be used with some success in approaching the temperature distribution in the two-phase saturated soil by assuming that the material responds as a homogeneous solid. This assumption is made more valid by noting from the first test series that the moisture propagation decreases as the saturated moisture content increases.

In tests 1 through 4 deformation responses were noted together with the temperature profile. In all cases it was observed that there was a slight swelling initially coincident with the temperature increase, followed by a contraction with temperature decrease. To separate out the natural thermal expansion of the loading system due to temperature increases, calibrations were performed for the same pulses applied to the soil. These yielded, on the basis of a 2.54 cm height comparative brass sample, strains of 0.41 percent increase for a 20 C temperature increase above ambient and 0.82 percent for a 45 C temperature increase. These strains were then subtracted from the total expansion of the soil to determine the net strain, expansion or contraction of the soil system. These samples were subjected to a net stress of 0.31 kg/cm^2 applied at ambient temperature and the temperature tests were not commenced until consolidation under this stress had ceased (a value chosen as < 0.002 cm over a 4-hour period).

For tests 1 and 2, the temperature increase was applied over a 16-minute period, and the net change was 0.35 percent decrease in volume at peak strain. For tests 3 and 4, the maximum temperature increase was applied over half the time period in tests 1 and 2, and the net change at peak surface temperature ranged from 0.7 to 1.5 percent expansion. These strains were not completely recovered at the conclusion of the tests when the surface temperature was still somewhat above the original ambient temperature.

In a recent paper (4) Campanella and Mitchell have made a study of causes of this change in volume of soils due to various temperature increases. In their paper and in a previous paper by the author (5), it is noted that increases in temperature result in increases in pore pressure. Campanella and Mitchell further note that such increases lead, therefore, to a decrease in effective stress, which for undrained samples could result in natural swelling (unloading effect).

In the sine pulse tests, the temperature is applied rapidly to one surface of the sample. There is a significant difference in the resultant temperature propagation between this type of heating and a uniform heating of the entire soil sample. In the

latter, any increase in pore pressure would be reflected uniformly throughout the whole system, whereas in the former, pore pressure increases that are due to both thermal expansion of the saturating pore fluid and decrease of effective stress (total stress applied, external stress remaining constant) would increase at a decreasing rate in the direction of the decreasing temperature gradient. There are then two forces at work, somewhat opposing each other. As the sample swells due to heating of the pore fluid, the pore pressure increases, and the pressure can be dissipated by flow from the nearest drainage face. In this case the nearest drainage face is also the source of increasing temperature and the fluid would normally flow in the direction of decreasing temperature. Further, the rate of dissipation of pore pressure is controlled by the permeability of the system. If flow to the drainage face is limited while simultaneously the temperature is increasing, the result would appear temporarily as a net volume increase, as noted in samples 3 and 4. It is evident that further testing along these lines is necessary to confirm this.

The sine pulse tests, the tests by Campanella and Mitchell, and previous tests by the author (6) all show the importance of clearly delineating the boundary conditions in temperature tests in order to gain full understanding of the nature of temperature propagation and the resultant changes in deformation that may occur in a stressed sample.

CONCLUSIONS

The first test series on nonsaturated soils indicates that uniform temperature profiles with no discontinuities will be established in a recompacted soil when the initial moisture content variation throughout the sample depth varies as much as 2 percent in an erratic pattern. The soil can, then, be considered homogeneous with regard to its gross thermal properties. The increasing surface temperature causes moisture migration, as expected, with quantities of water (as evidenced by percent saturation) decreasing with increasing percent saturation.

The second test series indicated that, where boundary conditions can be modeled, heat conduction theory will predict temperature distributions within the sample. In addition, the sine temperature pulse applied to one surface of the stressed sample caused volume changes that result from a buildup and possible subsequent relief of pore pressure. These tests indicate that more research must be made into the interrelationship of the temperature boundary conditions, in the field and as modeled in the laboratory, and resulting subsequent deformation before more definite conclusions can be made.

It is known that the strength and density of compacted soil are dependent upon temperature. Because temperature changes in the field are continuous, establishment of continuous temperature profiles beneath the surface becomes desirable. By assuming homogeneity the ability to predict changes becomes simplified. Establishment of temperature profiles permits correlation to be made between surface temperature and moisture migration in a somewhat continuous, rather than steady-state, basis. Where recompacted soils are subject to stress, changing temperature patterns can create varying deformation patterns, depending on the boundary conditions (both flow and temperature) of the soil. While heat conduction theory can be used with some certainty, it becomes imperative to model the field conditions in the laboratory by proper choice of boundary conditions. Then, the use of heat conduction theory with proper laboratory testing should prove of great benefit in analyzing and predicting field soil performance under transient temperature conditions.

ACKNOWLEDGMENT

The research on the sine pulse was partially supported by a grant from the Research Foundation of the State University of New York. R. Eckerson carried out the experiments on the large cylinder tests.

REFERENCES

1. Kersten, M. Final Report, Laboratory Research for the Determination of Thermal Properties of Soils. Engineering Experiment Station, University of Minnesota, 1949.
2. Reno, W., and Winterkorn, H. Thermal Conductivity of Kaolinite Clay as a Function of Type of Exchange Ion, Density and Moisture Content. Highway Research Record 209, p. 79-85, 1968.
3. Paaswell, R., and Reismann, H. Transmission of a Thermal Pulse Through Soil. Paper T68 presented at American Geophysical Union, Washington, 1968.
4. Campanella, R., and Mitchell, J. Influence of Temperature Variations on Soil Behavior. Jour. Soil Mech. and Found. Div., ASCE, Vol. 94, No. SM3, Proc. Paper 5958, May 1968.
5. Paaswell, R. E. Thermal Influence on Flow From a Compressible Porous Medium. Water Resources Research, Vol. 3, No. 1, 1967.
6. Paaswell, R. E. Temperature Effects on Clay Soil Consolidation. Jour. Soil Mech. and Found. Div., ASCE, Vol. 93, No. SM3, Proc. Paper 5225, May 1967.