Influence of Seepage Stream on the Joining of Frozen Soil Zones in Artificial Soil Freezing

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In artificial soil freezing, the presence of a seepage stream disturbs the development and joining of frozen soil zones around the freezing pipes due to heat gain from the seepage stream. In order to obtain the mutual quantitative relationships among velocity of seepage stream, representative length of freezing zone, permeability coefficient of soil, temperature of seepage stream and coolant, and distance between freezing pipes and radius of the pipe, a theoretical study has been performed. The study consists of two parts. In the first part, the author has investigated by hydraulics theory and Darcy's law how to estimate the amount of dam-up in front of the frozen zones as freezing of the soil progresses. In the second part, the critical dam-up head, below which it is possible to achieve the joining of frozen zones under the presence of seepage stream as intensified by dam-up head, is solved mathematically.

•AT PRESENT, in the application of artificial freezing of soils, the ice-soil curtain can be achieved by cooling the freezing pipes laid under the ground. In the absence of seepage, it is obvious that the frozen-soil cylinders around the freezing pipes grow until they join, and the planned frozen-soil curtain is achieved. In the presence of seepage, however, the heat carried by it may often cause difficulties in joining of the ice-soil cylinders (1). These phenomena are frequently experienced with the spread of the freezing region (2). For instance, Khakimov (3) noted that when the method of artificial freezing of soils was carried out on the Irtysh riverside in the Soviet Union in 1952, the seepage flow caused difficulties in the joining of the frozen-soil cylinders and the freezing time was delayed about four months.

This paper tries to establish a criterion for determining whether the joining of frozen-soil cylinders can be achieved in spite of the above-mentioned influence from seepage flow. This phenomenon is the consequence of complicated hydrodynamics and heat transfer, and so the result depends on several assumptions. However, the result itself has a definite form and is considered to represent the process of the phenomenon sufficiently as the first approximation, and to be useful in practice.

In this paper the author describes, when an obstacle is formed by soil freezing in a uniform seepage, how large a dam-up arises before and behind it, and then derives a criterion for determining whether the frozen-soil curtain can be achieved in the presence of the seepage flow caused by the hydraulic gradient accompanied by the dam-up.

CHARACTERISTICS OF THE PHENOMENON

First we consider the following case. As shown in Figure 1, a finite number of freezing pipes have been arranged in a row in order to form a frozen curtain, and the seepage flow, having a uniform velocity u_∞ (natural velocity), is perpendicular to the line in which the pipes are arranged. After freezing, frozen-soil areas grow around the freezing pipes, an obstacle against the seepage flow appears, and the velocity u_∞ varies.

In our experience in cases like this, after sufficient freezing time, almost frozen soil cylinders join to the ice-curtain, but the areas of large dam-up or high permeability coefficient often do not join. Here, dam-up is the greatest at the central part (the

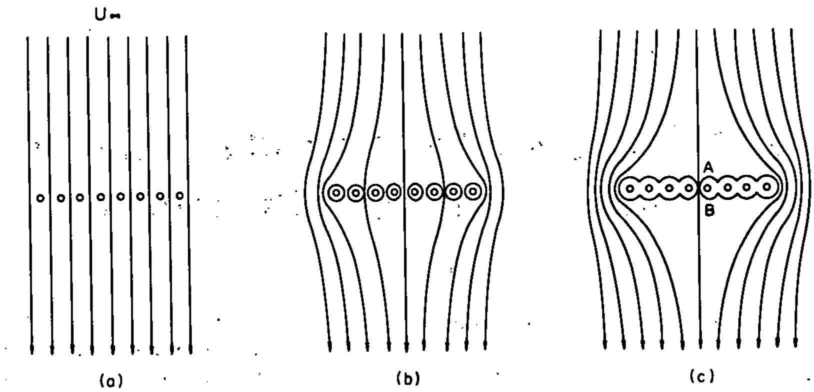


Figure 1.

point A, B), so we will calculate the quantity of dam-up at the point A, B and investigate the influence of the seepage flow caused by dam-up between A and B on the joining of the frozen-soil cylinders.

QUANTITY OF DAM-UP ACCOMPANYING THE FREEZING PROCESS

When groundwater moves among soil particles, the flow is laminar in the case of small hydraulic gradients, and it is well known that the following formula represents the velocity caused by the gradient according to Darcy's law:[1]

$$U_X = -\lambda \frac{dp}{dx} \tag{1}$$

where

U_X = the flow velocity in the direction of x, in m/h;
λ = the permeability coefficient, in m/h; and
$\frac{dp}{dx}$ = the hydraulic gradient in the direction of x, in m/m.

In the case where soils are uniform and homogeneous, it is known from this equation that the velocity distribution around the frozen soil is equal to that of the incompressible perfect fluid by regarding λp as a velocity potential Φ, i.e.,

$$\Phi = \lambda p \tag{2}$$

Thus, if the velocity potential Φ of the incompressible perfect fluid is known, the pressure p in the groundwater can be calculated as

$$p = \frac{\Phi}{\lambda} \tag{3}$$

In the case of two dimensions, the velocity potential Φ around the several forms of obstacles in the incompressible perfect fluid is often solved by means of conformal representation. For example, where the flow is perpendicular to a flat plate, as shown in Figure 2, the velocity potentials at the point A, B are respectively (4)

[1] In this paper the author treats only the case in which seepage flow follows Darcy's law. Where the seepage flow is turbulent, no application of the method of artificial freezing of soils can be done.

$$\left.\begin{aligned}\Phi_A &= \frac{1}{2}\,\ell\,U_\infty \\ \Phi_B &= -\frac{1}{2}\,\ell\,U_\infty\end{aligned}\right\} \tag{4}$$

Therefore, the head of dam-up between A and B is

$$p_A - p_B = \frac{1}{\lambda}(\Phi_A - \Phi_B) = \frac{\ell}{\lambda}U_\infty \tag{5}$$

where

ℓ = the width of frozen curtain perpendicular to the direction of seepage flow, and
U_∞ = the velocity of seepage flow before freezing.

As an example, when we assume that, in a uniform soil layer with permeability coefficient $\lambda = 10^{-1}$ cm/sec = 3.6 m/hr, seepage rate U_∞ = 8 m/day = 0.33 m/hr flow, and the frozen-soil curtain with a width 100 meters perpendicular to the flow as shown in Figure 2, the quantity of dam-up head at the center of that curtain is

$$p_A - p_B = \frac{100}{3.6} \times 0.33 = 9.26\,\mathrm{m}$$

In order to examine the details, we study the section C-D in Figure 2(a). The head of groundwater of section C-D is as shown in Figure 2(b). The line CD is the groundwater level before formation of the obstacle. After that the level on the upstream side turns to CA and on the downstream side to BD and the water level between B and A becomes $(p_A - p_B)$. Then, if the ice-soil curtain is not attained at the area around A, the stream pours through that part, and in such a condition the ice-soil cylinders may remain open permanently.

Now, knowing the permeability coefficient λ and the natural velocity U_∞ or the hydraulic gradient dp/dx of the region where the method of artificial freezing of soils is to be applied, we can assume the maximum head of dam-up at the time the frozen-soil cylinders join to the ice-soil curtain is the above-mentioned relation. We thus try to establish a criterion for completing the ice-soil curtain in spite of the concentration of flow in the part of the curtain being frozen.

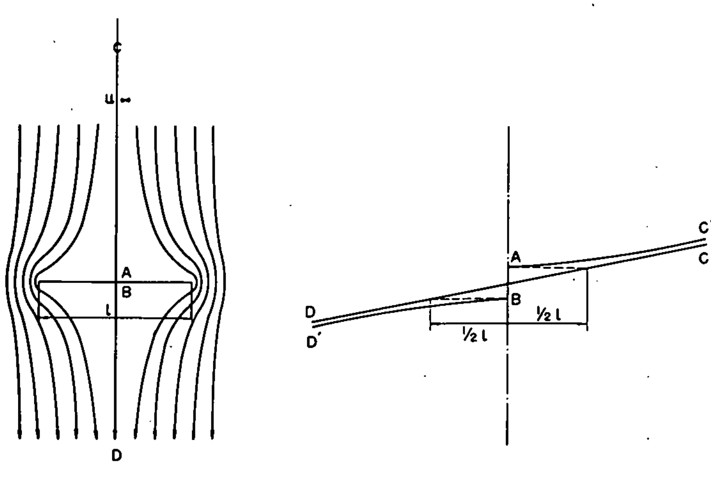


Figure 2.

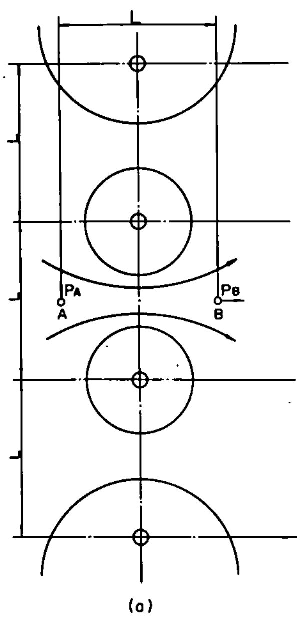
MODEL OF THE STREAM BETWEEN THE FREEZING PIPES CAUSED BY DAM-UP

As already mentioned, we experienced that the frozen-soil cylinders did not join at that area where the dam-up is large or the permeability coefficient is relatively large from the ground disturbance at the time of boring to set the freezing pipes. To simplify the problem, this section treats the simplest model of ice-soil cylinders left open and the assumptions for getting it. Figure 3(a) represents the incomplete part of the frozen-soil curtain. At this time, the other part is already finished, and the thickness of the curtain is almost equal to L (the distance between the freezing pipes). The flow pours in the part A, B and the difference of dam-up head, $p_A - p_B$, mentioned in the previous paragraphs, arises between the point A and B.

The flow between A and B is considered to be like Figure 3(a); however, it is very complicated for mathematical analysis. Therefore, the following assumptions are proposed:

1. The section of the ice-soil cylinder is assumed to be circular, as shown in Figure 3(b). Research by Yoshinobu (5) and field measurements by Khakimov (3) show that this assumption diverges only slightly from the real condition. Strictly the section is oval toward downstream, but it can be considered to be a circle.
2. The seepage flow between the freezing pipes flows fast around the frozen-soil cylinders and slow in the center, as indicated in Figure 3(a), but is assumed to have a uniform velocity profile as in Figure 3(b).
3. The head of dam-up is the value adopted when the ice-soil curtain is completed as shown previously, and is also assumed to occur at point A and B at a distance between the freezing pipes, in front of and behind the frozen-soil curtain, as shown in Figure 3(b).

In developing the theory on the basis of these assumptions, we must take care of some additional points. In the limiting case of the freezing radius R approaching $\frac{1}{2}$ L in the model, as in Figure 3(b), the induced velocity just before the achievement of the ice-soil curtain is infinite. This is considered to actually happen, but the quantity of the flow between freezing pipes is finite, and so the ice-soil cylinders grow together, forming the ice-soil curtain.



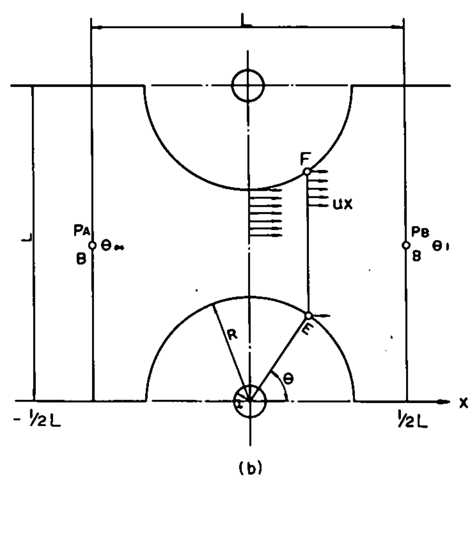


Figure 3.

THE QUANTITY OF INDUCED FLOW BETWEEN ICE-SOIL CYLINDERS

According to assumption 2, the quantity of water flowing between E and F in Fiugre 3(b) is, for $-R < x < R$,

$$V = \overline{EF} \times U_x = -(L - 2R \sin \theta)\, \lambda \frac{dp}{dx} \tag{6}$$

and for $-L/2 < x < -R$ and $R < x < L/2$,

$$V = L \times U_x = -L\lambda \frac{dp}{dx} \tag{7}$$

where

L = the distance between the freezing pipes,
R = the freezing radius, and
λ = the permeability coefficient.

Taking account that, for $-R < x < R$, $x = R \cos \theta$ and $dx = -R \sin \theta \cdot d\theta$, the transformation of Eqs. 6 and 7 is, for $-R < x < R$,

$$dp = \frac{V}{\lambda} \cdot \frac{R \sin \theta}{L - 2R \sin \theta} \cdot d\theta$$

and for $-L/2 < x < -R$ and $R < x < L/2$,

$$dp = -\frac{V}{\lambda} \frac{dx}{L}$$

Consequently, the resistance against flowing between A and B can be obtained by integration of that function about x from $-L/2$ to $L/2$, and its value must be equal to the dam-up head,

$$-\left(p_A - p_B\right) = \int_{x=-\frac{L}{2}}^{x=\frac{L}{2}} dp = \int_{x=-\frac{L}{2}}^{x=-R} dp + \int_{\theta=\pi}^{\theta=0} dp + \int_{x=R}^{x=\frac{L}{2}} dp$$

$$= \int_{x=-\frac{L}{2}}^{x=-R} \frac{V}{\lambda} \cdot \frac{dp}{L} - \int_{\theta=0}^{\theta=\pi} \frac{V}{\lambda} \frac{R \sin \theta}{L - 2R \sin \theta}\, d\theta - \int_{x=R}^{x=\frac{L}{2}} \frac{V}{\lambda} \frac{dx}{L}$$

and thus

$$(p_A - p_B) \frac{\lambda}{V} = \int_{\theta=0}^{\theta=\pi} \frac{R \sin \theta}{L - 2R \sin \theta}\, d\theta + \frac{2}{L} \int_{x=R}^{x=\frac{L}{2}} dx$$

$$= 1 - b + \frac{1}{\sqrt{1-b^2}} \left\{\frac{\pi}{2} - \tan^{-1} \frac{b}{\sqrt{1-b^2}}\right\} - \frac{\pi}{2}$$

where

$$b = \frac{2R}{L} \tag{8}$$

Thus the quantity of the flow can be obtained in the expression,

$$\frac{V}{\lambda\,(p_A - p_B)} = \frac{1}{1 - b + \frac{1}{\sqrt{1 - b^2}} \left\{ \frac{\pi}{2} - \tan^{-1} \frac{b}{\sqrt{1 - b^2}} \right\} - \frac{\pi}{2}} = \frac{1}{f(b)} \tag{9}$$

The right-hand side of Eq. 9 is a function of b only, and variation of b from 0 to 1 shows the condition of varying flow between the freezing pipes from the beginning of the freezing to the completion of the ice-soil curtain.

The value of $1/f(b)$ is shown by curve 1 in Figure 4. According to Eq. 9, the quantity of the flow is

$$V = \frac{\lambda(p_A - p_B)}{f(b)}$$

and U_{max}, the fastest velocity in the center of the distance between freezing columns, is

$$\frac{V}{L - 2R} = \frac{\lambda(p_A - p_B)}{f(b)\, L(1 - b)} = U_{max}$$

Thus

$$\frac{U_{max} \times L}{\lambda(p_A - p_B)} = \frac{1}{f(b) \times (1 - b)} \tag{10}$$

Equation 10 represents a ratio in which the flow velocity increases in proportion to the freezing radius R during the process of freezing development. Taking b into the abscissa and $1/f(b)\,(1 - b)$ into the ordinate, curve 2 of Figure 4 is plotted, and it shows that, at $b \to 1$, i.e., just before completion of the ice-soil curtain, the velocity tends to infinity. But as shown in Figure 4, curve 1, the quantity V of the flow is finite. [Khakimov, (3) tried to analyze this problem, but his analysis does not satisfy a boundary condition of the flow around the freezing pipes, and so his result is far from the real state, especially when $b \to 1$, and the velocity is finite.]

Finally, the pressure distribution between A and B according to Figure 3(b) can be obtained as

$$p_A - p = \int_{x=-L}^{x} dp \tag{11}$$

Figure 5 shows how the hydraulic gradient between the freezing pipes changes with increasing freezing radius R. Notice the rapid increase of hydraulic gradient as R approaches L/2.

THE HEAT EQUILIBRIUM EQUATION

In the previous section we calculated the quantity of flow between frozen-soil cylinders in a stationary state on the basis of three assumptions. In this section we will

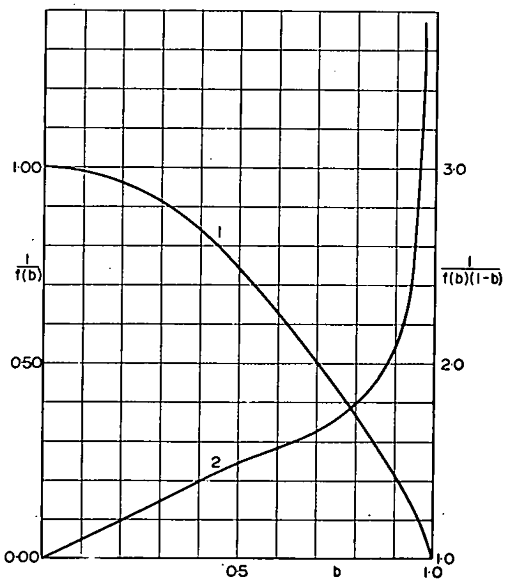


Figure 4.

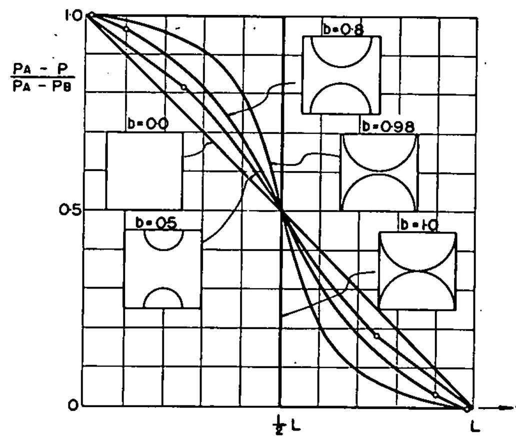


Figure 5.

calculate the temperature of the groundwater flowing downstream as the result of heat exchange.

If θ_∞ is the temperature of water from upstream and θ_1 is that of the water passed between the freezing columns to the downstream in Figure 3(b), the seepage is cooled from θ_∞ to θ_1 by passing between the freezing columns. Therefore the quantity of seepage flow in unit length perpendicular to the face of the paper in Figure 3(b) is

$$V = \frac{\lambda(p_A - p_B)}{f(b)}$$

and the heat quantity that cooled the flow is

$$\begin{aligned} q_1 &= V\gamma c\,(\theta_\infty - \theta_1) \\ &= \frac{\lambda\gamma c\,(p_A - p_B)\,(\theta_\infty - \theta_1)}{f(b)} \end{aligned} \tag{12}$$

where

γ = the specific gravity of the groundwater in kg/m^3, and
c = the specific heat of the groundwater in kcal/kg deg C.

Then, as shown in Figure 3(b), around the freezing pipes the frozen-soil cylinders grow to radius R, and the temperature of the face of the freezing pipes is kept at θ_c and of the freezing boundary at θ_f, and so the heat quantity transferred to the freezing pipes is

$$q_2 = 2\pi k_1\,(\theta_f - \theta_c)\,\frac{1}{\ln\frac{R}{a}} = 2\pi k_1\,\frac{\theta_f - \theta_c}{\ln\frac{L}{2a} - \ln\frac{L}{2R}} \tag{13}$$

where

k_1 = the thermal conductivity of the frozen-soil in kcal/m.h. deg C;
θ_f = the freezing temperature of the groundwater, usually = 0 C; and
a = the outside radius of the freezing pipe in m.

Because there is no heat exchange except the two terms mentioned, the Eqs. 12 and 13 are equal together, and the following formula is thus obtained:

$$\theta_1 = \theta_\infty - \frac{2\pi k_1 (\theta_f - \theta_c)}{\lambda\gamma c\,(p_A - p_B)} \times \frac{f(b)}{\ell n \frac{L}{2a} - \ell n \frac{L}{2R}} \tag{14}$$

Here, putting

$$F\left(\frac{2a}{L}, b\right) = \frac{f(b)}{\ell n \frac{L}{2a} - \ell n \frac{L}{2R}}$$

$$= \frac{1 - b + \frac{1}{\sqrt{1-b^2}}\left\{\frac{\pi}{2} - \tan^{-1}\frac{b}{\sqrt{1-b^2}}\right\} - \frac{\pi}{2}}{\ell n \frac{L}{2a} - \ell n \frac{L}{2R}} \tag{15}$$

then

$$\theta_1 = \theta_\infty - \frac{2\pi k_1 (\theta_f - \theta_c)}{\lambda\gamma c\,(p_A - p_B)} \times F\left(\frac{2a}{L}, b\right) \tag{16}$$

THE CRITICAL DAM-UP HEAD

The physical meaning of Eq. 16 is that the temperature of seepage passed through the unfrozen part is θ_1, when the frozen-soil curtain except for some unfrozen parts is achieved and a stationary state with freezing radius R around the freezing pipe is kept in the unfrozen part after a long lapse of freezing time in the situation in Figure 1. And it does not show when such a stationary state appears in the development of R. The radius R can be obtained only from the limit of the unsteady-state solution.

However, because we do not aim at computing R or θ_1 in a stationary state, but at deciding whether the ice-soil curtain is formed or not, we must investigate the character of Eq. 16. On the right-hand side of Eq. 16 the second term

$$\frac{2\pi k_1 (\theta_f - \theta_c)}{\lambda\gamma c\,(p_A - p_B)}$$

is a positive constant number given by the dam-up head $p_A - p_B$ and the cooling temperture θ_c. Accordingly, θ_1 can be obtained from inspection of the character during the variation of b from 0 to 1 in Eq. 15. Both the numerator and the denominator of Eq. 15 are always positive, as shown in Figure 4, and thus

$$F\left(\frac{2a}{L}, b\right) > 0, \text{ for } \frac{2a}{L} < b < 1 \tag{17}$$

When b tends to $2a/L$, the denominator tends to zero and the numerator to one:

$$F\left(\frac{2a}{L}, b\right) \to +\infty, \text{ for } b \to +\frac{2a}{L} \tag{18}$$

Moreover, when b tends to 1, the numerator tends to infinity:

$$F\left(\frac{2a}{L}, b\right) \to +\infty, \text{ for } b \to 1 \tag{19}$$

and thus, for $2a/L < b < 1$, $F(2a/L, b)$ must have a minimum value at least. Practically, we see by the numerical calculation that it is a function with a minimum value as in Figure 6.

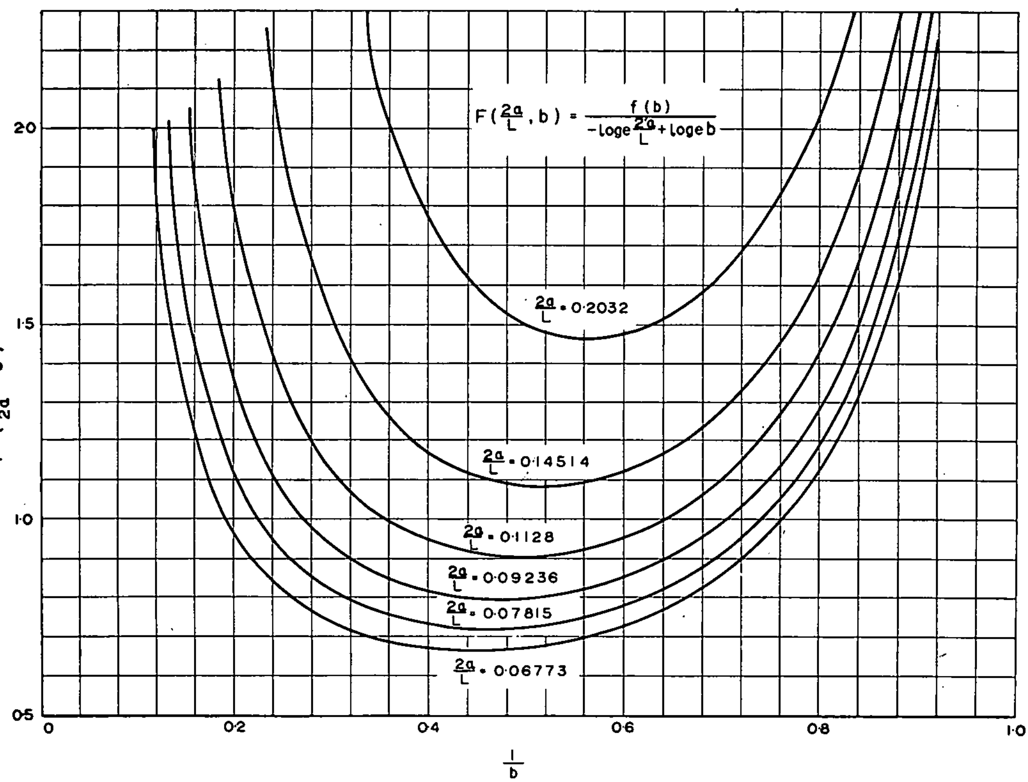


Figure 6.

This means that, for $2a/L < b < 1$, θ_1 has the only maximum value.

By supposing that b_{crit} represents b and that θ_1 is maximum, the following equation can be obtained:

$$\theta_{1\,max} = \theta_\infty - \frac{2\pi k_1 (\theta_f - \theta_c)}{\lambda \gamma c\,(p_A - p_B)} \cdot F\left(\frac{2a}{L}, b_{crit}\right) \tag{20}$$

Because the groundwater flows, $\theta_{1\,max}$ is larger than the freezing temperature θ_f, i.e., $\theta_{1\,max} > \theta_f$. And when the dam-up head $p_A - p_B$ properly becomes small, $\theta_{1\,max}$ approaches and finally equals θ_f. Hence, we call such a dam-up head a critical dam-up head, $(p_A - p_B)_{crit}$. The value $(p_A - p_B)_{crit}$ is obtained by transformation of Eq. 20 by substituting θ_f for θ_1:

$$(p_A - p_B)_{crit} = \frac{2\pi k_1}{\lambda \gamma c} \, \frac{\theta_f - \theta_c}{\theta_\infty - \theta_f} \times F\left(\frac{2a}{L}, b_{crit}\right) \tag{21}$$

This is the relationship we ask for, i.e., when the dam-up head $(p_A - p_B)$ expected to arise ahead of and behind the frozen-soil curtain is smaller than the critical dam-up head $(p_A - p_B)_{crit}$ given by Eq. 21, the following condition must be satisfied in order to complete the ice-soil curtain, namely, the temperature of the seepage across the unfrozen window cannot be higher than the freezing point θ_f. In other words, Eq. 21 is the sufficient condition for achievement of the ice-soil curtain.

EXAMINATION OF THE SOLUTION

The expression for the critical dam-up head consists of the product of three factors, as seen in Eq. 21. The first, $(2\pi k)/(\lambda\gamma c)$, is a factor determined from the property of the soils and the groundwater; the second, $(\theta_f - \theta_c)/\theta_\infty - \theta_f)$, is determined from the temperature of the ground before freezing and the surface of the freezing pipes; the third $F(2a/L, b_{crit})$ is determined from the radius of the freezing pipe and the distance between the pipes.

In this section we will examine the influence of these factors on the formation of the ice-soil curtain.

Influence of Properties of Soils

We will try the order estimation of Eq. 21. The order of $(\theta_f - \theta_c)/(\theta_\infty - \theta_f)$ and $F(2a/L, b_{crit})$ are 1, as indicated later. In the first factor, k is the thermal conductivity of the frozen soil $\doteqdot$ 2 kcal/m.h. deg C; γ is the specific gravity of the groundwater $\doteqdot$ 1000 kg/m^3; and c is the specific heat of the seepage $\doteqdot$ 1.0 kcal/kg deg C.

In urban civil engineering, dam-up head caused by the formation of a frozen curtain is not over 10 meters. If the dam-up head is over 10 meters, groundwater will overflow on the earth's surface. Accordingly, a frozen-soil curtain is achieved when the following condition of the permeability coefficient λ is satisfied:

$$\lambda < \frac{2\pi k}{\gamma c\,(p_A - p_B)_{crit}} \cdot \frac{\theta_f - \theta_c}{\theta_\infty - \theta_f} \cdot F\left(\frac{2a}{L}, b_{crit}\right)$$

$$\doteqdot \frac{10}{1000 \times 1 \times 10} \times 1 \times 1 = 10^{-3}\ \text{m/h} \doteqdot 10^{-4}\ \text{cm/sec} \qquad (22)$$

In other words, neglecting seepage has no noticeable influence when applying the method of artificial freezing of soil in uniform silt and clay in urban civil engineering.

However, when applying the method in a tunnel or deep pit in an impervious stratum where the dam-up head can be very large, the influence of seepage must be taken into account even in the case of permeability coefficients less than 10^{-4} cm/sec.

By the foregoing analysis we know that, when soil property improvement is necessary for lowering the permeability coefficient, it is efficient to use freezing pipes. The soil properties between the freezing pipes are, however, liable to be disturbed by boring and other factors.

Influence of Temperature of Ground and Coolant

The freezing point of groundwater θ_f in the second factor on the right-hand side of Eq. 21 is 0 C, if salts are not present. Thus, substituting $\theta_f = 0$, the following is obtained:

$$\frac{\theta_f - \theta_c}{\theta_\infty - \theta_f} = - \frac{\theta_c}{\theta_\infty} \qquad (23)$$

Consequently, we see that the critical dam-up head is inversely proportional to the initial temperature of the ground θ_∞. The nearer θ_∞ is to 0 C, the larger is the critical dam-up head, and at 0 C the ice-soil curtain is achieved however large the seepage. Jumikis (7) points out the difficulty of formation of a frozen-soil curtain in the case where the velocity of the seepage flow is faster than 2 m/day as the weak point of the artificial freezing of soils, but he does not indicate the condition of the dam-up and the temperature of the ground in that case. His suggestion is thereby limited in special cases. We also see that the critical dam-up head is in proportion to the temperature of the surface of the freezing pipes θ_c, and thus the freezing method by liquified nitrogen refrigerant ($\theta_c \doteqdot$ - 180 C) is nine times more effective than the usual brine (θ_c = -20 C).

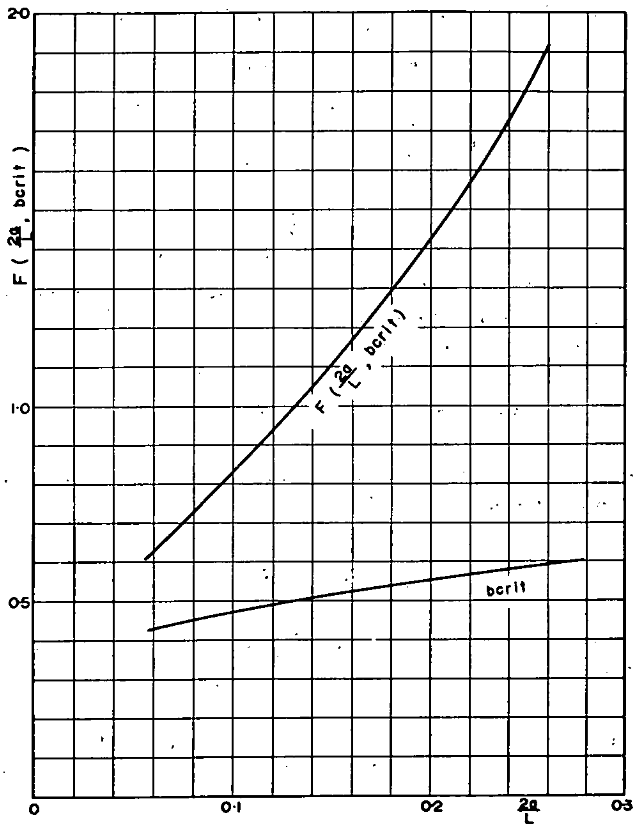


Figure 7.

Influence of Radius of Freezing Pipes and Distance Between Them

The third factor in the right-hand side of Eq. 21, $F(2a/L, b_{crit})$, is the minimum of Eq. 15. This factor has a parameter 2a/L, and enlarges with an increase of its value.

Figure 7, in which the abscissa is 2a/L and the ordinate $F(2a/L, b_{crit})$, shows that the relationship between them has a direct proportionality, and so at the same radius of pipes and half the interval, the critical dam-up head doubles, and at the same interval and half the radius, it becomes half.

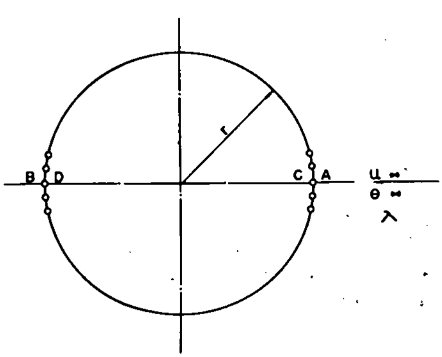


Figure 8.

EXAMPLES OF THE CALCULATION

In the case of comparatively simple forms of ice-soil curtain, we can calculate a dam-up head by the method of Eq. 5 when the value of seepage velocity U_∞ and permeability coefficient λ before freezing are known. In this section we make practical computations for two specific cases.

The Case of a Circular Shaft

In the case where the freezing pipes are set around a circle with the radius r as shown in Figure 8, if we write the velocity of the seepage flow before freezing U_∞ the difference of the velocity potential at points A and B is, as shown earlier,

$$\Phi_A - \Phi_B = 4rU_\infty \tag{24}$$

Note that if the seepage velocity U is measured directly by a dyestuff or method of electric conductivity, it must be corrected as follows:

$$U_\infty = \frac{A\nu}{A} U$$

where $A\nu$ is the effective section-area of the pore and A is the apparent flowing-section area. Although the porosity n is commonly used for $A\nu/A$, it should be measured accurately. Using Eq. 3, we obtain

$$p_A - p_B = \frac{1}{\lambda}(\Phi_A - \Phi_B) = \frac{4r}{\lambda} U_\infty \tag{25}$$

When the frozen curtain is almost completed, the dam-up head at point A, C is considered to be half that described:

$$p_A - p_C = \frac{2r}{\lambda} U_\infty$$

Now, when $(p_A - p_C)$ is equal to the critical dam-up head given by Eq. 21, the critical velocity $U_{\infty crit}$ appears; i.e.,

$$\frac{2r\, U_{\infty crit}}{\lambda} = (p_A - p_C)_{crit} = \frac{2\pi k}{\lambda\gamma c}\, \frac{\theta_f - \theta_c}{\theta_\infty - \theta_f}\, F\left(\frac{2a}{L}, b_{crit}\right)$$

$$\therefore\ U_{\infty crit} = \frac{\pi k}{r\gamma c}\, \frac{\theta_f - \theta_c}{\theta_\infty - \theta_f}\, F\left(\frac{2a}{L}, b_{crit}\right) \tag{26}$$

When we insert the numerical values that are generally used in the formula, i.e., $k = 2.414$ kcal/m.h. deg C; $\gamma = 1{,}000$ kg/m^3; $c = 1$ kcal/kg deg C; $\theta_f = 0$ C; $\theta_c = -20$ C; $\theta_\infty = 18$ C; $2a/L = 0.1270$; and from Figure 7, $F(2a/L, b_{crit}) = 0.983$, we obtain

$$U_{\infty crit} = \frac{\pi \times 2.414}{r \times 1000 \times 1} \times \frac{20}{18} \times 0.983 \times 24 = \frac{0.2}{r}\ \text{m/day}$$

In the case of comparatively rough soils that induce a seepage flow,

$$\frac{A\nu}{A} = 0.1 \sim 0.2$$

and so for the directly measured velocity we obtain

$$U_{crit} = \frac{1}{r} \sim \frac{2}{r}\ \text{m/day}$$

We observe that the ice-soil cylinders can join in the freezing area with a radius of 2 m or so in the application of the method of artificial freezing of soils, even if the seepage velocity by direct measurement is $0.5 \sim 1.0$ m/day, but the velocity of even 0.2 m/day causes difficulties when the radius r is enlarged to 10 m.

The Case of a Plane Ice-Soil Curtain

As another simple example, we will consider the case where the seepage flows perpendicular to the frozen-soil curtain as shown in Figure 2(a). In this case, because the dam-up head is represented by Eq. 5, by the same viewpoint as previously, the critical velocity $U_\infty \mathrm{crit}$ is obtained in the form

$$U_\infty \mathrm{crit} = \frac{2\pi k}{\ell\gamma c}\,\frac{\theta_f - \theta_c}{\theta_\infty - \theta_f} \cdot F\left(\frac{2a}{L}, b_{\mathrm{crit}}\right) \tag{27}$$

These two instances are the simplest ones in the case of forming an ice-soil curtain in uniform soils, and every critical flow velocity $U_\infty \mathrm{crit}$ is in inverse ratio to the representative length of the frozen-soil curtain r, and ℓ. In other words, the larger the area of freezing soils is, the more difficult formation of an ice-soil curtain becomes.

CONCLUSION

Although this analysis depends on many assumptions, it nevertheless clarifies the mutual relationship between critical velocity, representative length of freezing zone, permeability coefficient of soils, temperature of stream and coolant, distance between freezing pipes, and radius of freezing pipe. This makes it possible by preliminary investigation to decide more precisely whether an ice-soil curtain can be achieved at a site of previous soil strata where the permeability coefficient is more than 10^{-3} cm/sec, knowing the permeability coefficient in the region and the hydraulic gradient or the velocity.

In the case where freezing temperature and the distance between the freezing pipes is normal and the critical velocity (or the critical dam-up head) is smaller than the seepage velocity (or the dam-up head), by changing the freezing temperature and the distance to the economical limit, we can make the critical velocity larger. But if the purpose is still not accomplished, we must try four methods as follows for countermeasures:

1. Make the permeability coefficient smaller by improving the soil properties between the freezing pipes, and enlarging the critical dam-up head (8);
2. Make the dam-up head smaller by installing a well for observation of water level and for pumping on both the upstream and downstream side (3);
3. Lower the freezing temperature θ_c temporarily by using liquified nitrogen refrigerant on the part of the frozen-soil curtain that remains open;
4. Accelerate the formation of the ice-soil curtain by adding a row of freezing columns.

The fourth of these countermeasures is more efficient than shortening the distance between the pipes or enlarging their radius. This is because the critical dam-up head is in inverse ratio to the distance and proportional to the radius of pipes, and by adding the row of freezing pipes, the path resistance of the seepage increases and the quantity decreases and, moreover, θ_∞ of the second row becomes small.

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