GROUP B

SOLUTIONS TO BOUNDARY VALUE PROBLEMS

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A proper theory for use in the structural design of asphalt concrete pavement systems would provide numerical results that can be used by the design engineer to produce a design or a design procedure. Specifically, the process of development of a design procedure requires a sequence of steps. First, a theory of behavior must be promulgated. This requires that the materials involved be appropriately characterized. Once this is accomplished, the material characterization is applied to the appropriate principles of mechanics and probability theory to form a set of equations that constitute the governing mathematical relationships for the predicted behavior of the pavement system. The mathematical equations will be capable of predicting behavior only within the limits to which the components of the equations accurately represent real-world behavior.

The boundary value problem for the structural design of asphalt pavement systems is composed of the governing equations described and the boundary and initial conditions expected to occur in a real system. A properly posed boundary value problem requires both a proper set of governing equations and a proper set of boundary and initial conditions.

Once the boundary value problem is established, it must be solved to produce numerical results. Ideally the solution produced by using analytical or numerical techniques provides a means of predicting pavement performance under a wide variety of expected circumstances and conditions.

Theoretical approaches to pavement design problems have the following two functions: (a) the application of known principles of continuum mechanics and probability theory to the development of specific boundary value problems, and (b) the development of new analytically based theories of pavement response that can lead to new methods of pavement design.

In the first instance the theory is used to predict the response of a given pavement or pavement type to a given environment. To fulfill this function, one must be able to call on the proper theory, translate the theory into numerical terms, and develop a quantitative prediction for a specific engineering situation. In addition, the validity of the theory being used must be assessed. In this way limits of uncertainty can be placed on the behavioral predictions. For example, a pavement designed to carry light traffic over firm foundations, such as rock, might reasonably be analyzed within the framework of the linear theory of elasticity. On the other hand, an asphalt pavement constructed in a climate subject to temperature extremes might be analyzed by giving consideration to thermal viscoelastic theories and large deformations. Because such theories are not as yet tractable for predictive purposes, simpler linear theories must be used. It is then necessary to place bounds on the validity of the approximation.

New theories should meet three interdependent objectives:

1. The new theory must have a higher degree of predictability than the competing existing theory;
2. The new theory must be viable; and
3. The new theory must be credible.

The first objective is to develop a theory capable of predicting behavior of real materials. In reality, this means the extension of existing theories to reduce the restrictiveness of the physical assumptions of the existing theories. For example, viscoelastic and elastoplastic theories have a greater range of predictability than do elastic theories.

In addition, the new theory, as formulated in a boundary value problem, should realistically formulate the real world. This can be done deterministically by establishing bounds on response. A more realistic formulation would apply probabilistic principles to natural occurences.

To be viable, a theory must be capable of producing numerical results. This requires that consideration be given to the techniques by which numerical information can be culled from a theory, new or existing.

If a theory is to be applied to design situations, it must be credible to the design engineer. Credibility is often a subjective entity. A highly mathematical theory presented in an obtuse manner will probably frighten the design engineer. On the other hand, an oversimplification of the physical processes involved causes the theory to be less credible to engineers who are empirically knowledgeable of the physical factors involved in the action of pavement systems.

The development of techniques of solution is not confined to any particular method or level of development. It may be analytical in the sense of developing general methods of solution to a large class of boundary value problems. On the other hand, this development may be concerned with the solution to a particular boundary value problem. Furthermore, there is no restriction as to the means of solution. They may be numerical, analytical, or a combination of both methods.

In summary, the role of theory as used here is largely mathematical. Its range extends from the formulation of new theories to the implementation of existing theories. The theories used are derived from continuum mechanics and probability theory. Methods of solution are those of pure and applied mathematics, numerical analysis, and computer science; and the tools are those of calculation including the use of computers and computational systems. The output is numerical data.

CURRENT STATE OF KNOWLEDGE

The current state of knowledge of solutions to boundary value problems that have applicability to asphalt concrete pavement systems is largely the state of knowledge that exists in solid mechanics as tempered by the ability to provide numerical output to a given relevant boundary value problem. Just as solid mechanics provides a quantification of descriptive physical relationships, a computational algorithm provides numerical results for symbolic algebraic representations.

Various surveys of the literature exist (1, 2, 3). These surveys are in a continual state of updating by the Highway Research Information Service (HRIS).

Types of Problems Solved

Boundary value problems with applicability to asphalt concrete pavement systems are concerned with the response of a layered continuum to a set of defined boundary forces or displacements and environmental fluctuations or both of these. Within the context of continuum mechanics, the problems of concern are conveniently classified in terms of the constitutive laws that are adopted, as follows: linear elastic, linear viscoelastic, plasticity, and nonlinear.

Solutions to the linear elastic layered system problem for stress boundary conditions exist in a variety of forms. These solutions have provided numerical information on a variety of parametric effects including surface loading conditions, layer interface conditions, and material homogeneity. In addition, solution techniques and a limited amount of numerical data exist on the effects of displacement boundary conditions, thermal environments, and poroelasticity. The thermoelastic problems are all based on theories that separate the effects of load and temperature as independent activities. Thus, they use an "uncoupled" theory. No experimental or other evidence exists that bears on the question of whether the thermoelastic boundary value problem, as applied to asphalt concrete pavements, can realistically be uncoupled.

Solutions to linear viscoelastic layered system problems for stress boundary conditions also exist. Because of their comparatively recent origin and their higher level of complexity, the viscoelastic case has not been studied as extensively as the elastic case. Viscoelastic solutions have been largely used to assess the role of vehicle motion and cumulative, time-dependent deformations.

Some solutions to plasticity problems exist. These use deformation relationships and have been developed for finite element and finite difference techniques.

Some anelastic boundary value problems using incremental relationships have been solved. In general, however, the solutions are for isolated, specialized problems.

Status of Computer Programs

There are essentially no boundary value problems that apply to asphalt concrete pavement systems that do not require a procedure for a moderate-sized computer. (Machines of "moderate" size are the Burroughs B5500, the CDC 6200, the IBM 360/50, and the Univac 1106.)

The status of a useful computer program is a direct function of the availability and usability of that program in a state highway department environment. This requires consideration of the following aspects of program status:

1. Machine compatibility—the degree to which a given program is dependent on a given machine installation or machine vendor;
2. Program documentation—the written material that must accompany a program to make it understandable and usable; and
3. Program maintenance—provision of a mechanism for removing programming errors (bugs) and provision for an orderly transition of program operation as computing installations upgrade hardware and change their operating software.

It is an unfortunate fact that the state of computer programs for solving layered system problems is deplorable. There are several public domain programs for the solution of layered elastic systems. One of these programs is multivendor compatible. The state of documentation and maintenance on all of these programs is poor to nonexistent.

There is at least one proprietary program for solving layered elastic systems. Negotiation for the purchase or lease of these types of programs can include machine compatibility, documentation, and maintenance.

A single source program for the solution of layered linear viscoelastic systems exists. This program will develop numerical values for stress boundary conditions under quasi-static conditions including moving loads. This program has been implemented on a single-machine configuration. It is documented, but it is not maintained.

Applications

Layered systems analysis, as part of a design procedure, is being used by two states, Kansas and Kentucky, the U.S. Navy, and the Shell Oil Company. In all cases, an idealized version of linear elastic systems boundary value problem solutions is used. The idealizations concern the number of layers in the system and the material properties. At most, three layers are considered.

POTENTIAL FOR MODIFICATION OF CURRENT DESIGN METHODS

Of the boundary value problem solutions available or potentially available, only linear elastic and viscoelastic solutions have a potential for implementation in design procedure in the near future.

Elastic Solutions

There is a potential for the use of elastic theory to modify current design methods. The important variables that could be included in a design modification are as follows:

1. Material properties—elastic constants and thermal properties and the ratios of these properties in adjacent layers;

2. Interface conditions—consideration of interface separation and slip;
3. Surface conditions—consideration of adhesion and friction between the tire and the pavement and the effects of nonsmooth surfaces; and
4. Displacement—consideration of the relationship of mixed to stress boundary conditions.

Linear elastic theory is a wholly self-consistent theory. Thus, within its assumptions, it contains a basis for performing parametric studies and developing the stress distributions that are useful for failure studies. Furthermore, the self-consistency of the theory permits sensitivity analyses to be performed, which can be used as a basis for extrapolation.

Viscoelastic Solutions

In addition to having the attributes of elastic theory, viscoelastic theory also provides the capability of predicting accumulated deformations for loads moving at slow speeds and for temperature changes. The important additional variables to be considered are the material operators of both the pavement and the tire and the energy transfer between the tire and the pavement. Early time effects are of particular interest.

POTENTIAL FOR THE DEVELOPMENT OF NEW DESIGN METHODS

Two areas of investigation show potential for future developments of new design methods: fracture analysis and the use of stochastic techniques.

Fracture Analysis

Fracture analysis would provide estimates of crack growth, reduce the number of tests to evaluate fatigue, and assist in maintenance planning. The variables of importance are the character of the flaw, the material characteristic, and the load and environmental history.

There are two phases to a fracture analysis: the application of an uncracked stress field in a field of cracks and the consideration of the influence of cracks in a total layered system.

Stochastic Techniques

The objective of a stochastic procedure is to develop a set of output statistics that are the consequence of a set of input statistics. Hardly any of the entities involved in the analysis and design of pavement systems are deterministic. All of these entities are subject to random variations of one form or another.

A realistic view of the constitutive relationships must assign a degree of random variation to the numerical values for the material properties and the form of the constitutive relationships. This randomness may be due to quality control and is thus a property of the specific material. It also has an areal distribution connected with geologic variations, the methods of construction, and the location of aggregate supplies.

The boundary conditions are also subject to randomness. Traffic patterns are not deterministic. In addition, environmental conditions such as climactic and water conditions can only be established in a proper manner by considering their random nature.

A stochastic technique would include such probabalistic entities in the appropriate boundary value problem. The solution that is developed will provide stress, strain, and displacement as statistical entities. That is, each numerical value of pavement response would carry a probability of occurrence. Thus, a theory of pavement behavior would assign a level of confidence (or probability) to a particular predicted occurrence. This carries over to design in which the confidence of behavior is applied to a confidence in the design.

RESEARCH DIRECTIONS AND NEEDS

To develop research needs requires an assessment of current directions of research.

Current Research Directions

The directions of current research are as follows:

1. Development of solutions and solution techniques for boundary value problems that use increasingly complex constitutive equations (the constitutive equations being used are more realistic than their predecessors);
2. Development of solutions and solution techniques for boundary value problems that use increasingly complex geometric configurations;
3. Development of computational techniques involving new methods for solving old problems and newly formulated problems and techniques for information handling (data structuring) by computer to provide more efficient machine codes;
4. Development of comprehensive user-oriented man-machine computer systems including interactive computer graphics techniques;
5. Development of global methods that examine the response of the pavement system as a total entity; and
6. Evaluation of the predictive capability of existing theories.

Areas of Needed Research

The areas of needed research are divided into long- and short-range topics. No attempt is made to define the time required to develop these tasks. This is a question of indeterminate variability. Research with short-term goals includes the following:

1. Extension of currently available mechanistic methods to be within the reach of practicing highway engineers by determining the predictive capability of currently available mechanistic methods and by documenting and maintaining computer programs that are machine-independent;
2. Evaluation of the payoff potential resulting from the use of mechanistic methods of analysis in the design process; and
3. Evaluation of the effect on the potential payoff of the use of mechanistic methods with regard to variations of the methods of analysis and variations in the models used and in parameters that influence behavior.

Needed research with long-term goals includes the following:

1. Consideration of stochastic variables;
2. Consideration of more complex (and perhaps more realistic) models of material behavior, including continuum and particulate mechanics problems; and
3. Study of fracture mechanics.

It is recognized that much work is under way in all these fields. The work, however, is not organized to assist in the solution of problems directed toward asphalt concrete pavements. It is hoped that, by publicizing the need and challenge, competent researchers will direct their attention to these problems.

DISCUSSION OF SOLUTIONS

There are two major points of discussion germane to the solution of boundary value problems. First, the relationship of boundary value problem solutions to the overall picture of pavement design is of concern. Second, the techniques that may be employed in solving a given boundary value problem are of direct concern inasmuch as they control the types of problems that can be solved.

Relationship to the Design Process

The design community will not accept a boundary value problem solution unless there is convincing evidence that such a solution will provide an improvement to current practice. Improvement can be defined in many ways. One criterion for improvement is economics; a larger benefit (also to be defined) for a smaller cost. Another criterion is that a boundary value problem solution provides predictability where other methods do not. A third criterion for improvement is that a boundary value problem solution lead to the design of a longer lived pavement without increased cost. A fourth criterion is an increased margin of highway safety.

In addition to these criteria, there is a belief by some engineers that present design procedures are vague, are inconsistent, and lack credibility. A design method based on the solution to one or more boundary value problems will, by its consistency, provide a rational basis for extrapolation and for analysis of field observations.

If a design method based on boundary value problem solutions is proposed for adoption, it must prove itself under prototype conditions. A systematic and well-documented field measurement program is thus requisite for the final adoption of the proposed method.

The solution of boundary value problems is a single element in a total pavement design system. It is strongly suggested that attention be directed to the total system and its requirements as a prerequisite to the investigation of solutions to boundary value problems.

Problem-Solving Techniques

The present state of knowledge and technology is such that most relevant boundary value problems in linear elasticity or viscoelasticity are capable of solution by numerical methods. It is noted, however, that many problems involving more than two dimensions are not capable of economic solution. The size of computer and the amount of machine time required are much too large.

Unfortunately, a considerable amount of naiveté exists concerning the state of operational programs. Most of the programs referred to are operational only in a limited sense. Documentation is generally poor to nonexistent; maintenance does not exist. If boundary value problem solutions are to be useful, there must be an organized, before-the-fact development of documentation and maintenance procedures. There also must be recognition of the fact of unbundling and the existence of useful proprietary programs.

There are two types of computing systems that are of concern: (a) those programs designed to solve given boundary value problems and usually (but not always) characterized by relatively simple data bases accompanied by complex computational algorithms, and (b) data management systems, which are usually (but not always) characterized by large data bases (both numeric and nonnumeric) and by relatively simple algorithms. A pavement management system will certainly encompass both types. Thus, any consideration of computer and computing techniques must be concerned with both data structuring and algorithm development.

There are three factors of concern in the development of a computational system: machine compatibility, program documentation, and program maintenance.

Machine compatibility is a function of the computer language. If a system is written in Assembly Language, there is little hope that conversion can be made from one machine type to another without complete reprogramming. If a system is designed to be machine- or vendor-independent, the system must be written in FORTRAN or COBOL. Even under these conditions machine independence is a myth. USASI FORTRAN IV is such a limited subset of FORTRAN IV as to be relatively useless. Beyond this, different machines have different characteristics. For the CDC 6000 series machines, FORTRAN IV is a superset of FORTRAN IV for IBM 360 machines. A downward conversion is a nontrivial task. The reverse is true for COBOL.

Recent efforts in computer languages have centered around the development of macroprocessing languages. These languages are mobile in that they can be moved from machine to machine. They also have the machine efficiencies of assembly language.

Adequate documentation of even a moderately complex system should have the following four components:

1. An analysis and algorithm manual that provides a detailed exposition of the analysis used and the computational algorithms.
2. A user's manual that provides detailed instructions on the use of the system and that contains a set of example problems with results providing the user with check results when he sets the system up on machines other than the prototype computer.

3. A reference manual that includes a fully commented, machine-generated listing of the system and that contains write-ups of each subsystem, subprogram, and subroutine as well as a complete dictionary of all variables (macro and micro flow charts of the system and its components are essential).
4. An engineering manual that provides the engineer with a number of examples of the use of the system and that combines the problem-solving capabilities of the system with its field use.

The machine-generated listing mentioned with the reference manual should not be a straight listing but should be a system printout generated with a successful run. There is no such thing as a machine-independent language. Consequently, the reference manual should contain information on the hardware used, the language, the operating system, storage requirements (both central memory and peripherals), and other machine characteristics of importance.

No large- or even moderate-sized computational system is free of programming errors (bugs). Furthermore, the frequent changes in operating systems often necessitate changes in the computational system. Thus, there is a long-range responsibility in maintaining the computational system. The maintenance of a system is something more than a technical problem. Unless a maintenance responsibility is clearly understood and defined at the outset, many serious areas of dispute can occur.

There is an increasing reliance on computer programs in the pavement design process. Research and development efforts in this area will have, as an end product, a computer-based system. It is thus required that any computer programs that are developed have two major attributes:

1. Portability—the capability of moving a program from one machine environment to another with a minimum of effort; and
2. Adaptability—the capability of facile modification of a system to relate to specific user needs. For example, a program designed for a UNIVAC 1108 should be designed with the possibility that a subset of the program may be adapted for an IBM 1130. Also, programs originally designed for batch process operation should be adaptable to timesharing.

The development of techniques for portability and adaptability is a currently active research area in computer science and technology. These efforts, to date, have shown that the initial construction of a portable, adaptable system must clearly separate the input/output structure, the algorithms, and the data structure. For example, an algorithmically based system is potentially portable if the algorithm is embedded in the input/output structure. Portability is not feasible if the input/output structure is embedded in the algorithm.

In addition to program development, it is essential that program maintenance principles be established. Investigators who develop programs are generally not inclined to maintain a system once it is released. It is a tedious, technically unrewarding task. On the other hand, an unmaintained system will very shortly pass out of existence because of the inability of the user to make the system work. To avoid future problems requires that a maintenance plan be clearly established at the outset of the programming effort.

One of the established efforts within the Federal Highway Administration is TIES (total integrated engineering system). This is a massive effort dedicated to the development, use, and maintenance of computer programs for highway engineering. It is recommended that the computer systems aspects of asphalt concrete pavement systems be developed within the concepts of the TIES program.

RELEVANCY OF SOLUTIONS

The solutions to boundary value problems are relevant to the pavement design problem as long as it can be demonstrated that they provide an improvement to the design process, as was discussed previously. Questions of usefulness and economy are also dependent on improvement. In this case the degree of improvement is a governing criterion. It is not clear that this question has been, or even can be, answered directly.

There is no evidence in the field of pavements. There is, however, ample evidence in other engineering disciplines that significant improvements can be and have been achieved by "upgrading" the design process by use of "higher level" boundary value problem solutions.

When the question of relevancy is viewed in its broader context, it is legitimate to be concerned with the relationship of pavement design improvement to other national efforts. An effort in this direction, at this time, may well divert critically needed manpower and other resources from projects of higher priority. On the other hand, an effort of this nature may be within the national goals of our society. This is a higher order question and concerns the types and levels of national priorities.

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