# DAMAGE AND DISTRESS IN HIGHWAY PAVEMENTS

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An understanding of what constitutes failure in pavement structures and of the factors that contribute to damage initiation, propagation, and accumulation is important to the development of a rational method of design. This study reviews the state of the art in the area of pavement damage with a view to identifying the pertinent damage mechanisms.

The performance of a pavement structure in a given traffic and climatic environment may be defined as its ability to provide an acceptable level of serviceability with a specified degree of reliability for an assumed level of maintainability (1). The impairment or loss in the ability to provide the necessary services in a given locale may then be considered as the "failure" of the pavement. When viewed in this context, failure becomes a loss in performance; it is the extent to which the pavement has failed to render itself serviceable, and it results from an accumulation of damage during a given time period. The failure age or the life of the pavement is, then, the time during which serviceability deteriorates to an unacceptable level as determined by the users.

The question of what is the unacceptable level of serviceability at which the pavement must be considered failed is a highly subjective one. It depends on the user's evaluation of the performance, and it involves many intangible and not easily quantifiable factors such as cost, comfort, convenience, and safety. The problem is further compounded by the fact that there does not exist a usually accepted and comprehensive function describing the pavement serviceability in terms of damage parameters. At present one can only assume that pavement failure is a many-sided problem, and it is the result of a series of interacting complex processes, one of which is clearly understood.

Therefore, an integrated comprehensive picture of failure is obtained by studying in detail each of its components. From such studies, methods may be developed for analyzing the effect that each component has on the behavior of the structure. Finally, all of those methods may be grouped together in a meaningful way so that, for any given environment, the performance history of a pavement can be predicted. This study investigates failure from the "structural integrity" viewpoint and places special emphasis on load-associated factors affecting the structural integrity. It must, therefore, be emphasized that this is only one aspect of the failure problem.

## THE STRUCTURAL INTEGRITY OF THE PAVEMENT

The structural integrity of a pavement may be defined as its ability to resist destruction and functional impairment in a particular traffic and climatic environment. Under the combined destructive action of these elements, several distress mechanisms develop within the structure and propagate either independently of each other or through interacting complex processes to produce eventually any or all of these broad groups of distress: disintegration, distortion, and fracture (rupture). To develop a thorough understanding of these mechanisms, one has to trace the path from the time of initiation, through propagation, to that of global manifestation.

Although it is realized that field behavior results from a series of interacting complex processes and that all distress mechanisms must be analyzed in order to present not only a realistic but also a totally comprehensive picture of damage behavior, only the distress occurring as a consequence of mechanical loading has been investigated. The motivation for doing this stems from the fact that an extensive amount of work has been done experimentally on the modes of damage associated with mechanical loading.

This makes it possible, to a certain extent, to adopt certain assumptions as to the manner of damage propagation in this mode of loading.

This study is presented in five broad sections. The discussion begins with a close study of the conditions governing the initiation, propagation, and attainment of critical size of the defect area in engineering materials under arbitrary loading histories. It provides the background necessary for the development of the cumulative damage model.

After the conditions governing the physical failure of engineering materials in general are established, the pavement system is investigated with a view to identifying the variables that affect its performance and that subsequently bring about ultimate distress in a given environment. On the basis of the conclusions arrived at, a framework for cumulative damage is developed.

## FAILURE OF ENGINEERING MATERIALS

The intensive interest that has developed during the past several years concerning the accumulation of damage in engineering materials and structures has its roots in the following problems:

1. The life prediction of an engineering material or structure under an arbitrary load history in a given environment,
2. The amount and distribution of damage in the material or structure under the arbitrary loading spectrum mentioned previously, and
3. The manner and rate of accumulation of damage.

This section presents a concept of damage by examining the processes of fracture and flow in solid materials. It describes what the damage is, how it manifests itself, and which parameters can be employed to describe it.

Several observations are made about the distribution and propagation of damage within a material that is under an arbitrary loading history. Some well-known theories and criteria that have been postulated for the failure of engineering materials are discussed. The damage of materials in a repeated loading environment is closely examined.

### Concept of Damage

Damage may be defined as a structure-sensitive property of all solid materials; structure sensitivity is imparted to it through the influence of defects in the form of microscopic and macroscopic cracks, dislocations, and voids that may have been artificially or naturally introduced into the material, thereby rendering it inhomogeneous. Characteristically, structure-sensitive phenomena involve processes that grow gradually and accelerate rapidly once an internal irregularity or defect size exceeds a certain limit. Damage may, therefore, be said to occur in a similar fashion.

The progression of damage in an engineering material or engineering structure may occur under the application of uniaxial or multiaxial stationary or repeated loads. The damage progression has been categorized by two different conditions: ductile and brittle. The ductile condition is operative if a material has undergone considerable plastic deformation or flow before rupture. The brittle condition, on the other hand, occurs if localized stress and energy concentrations cause a separation of atomic bonds before the occurrence of any appreciable plastic flow. Note here that no mention is made of a ductile or a brittle material per se. According to von Karman (2), this implies that failure is not in itself a single physical phenomenon but rather a condition brought about by several different processes that may lead to the disintegration of a body by the action of mechanical forces. Damage may, therefore, progress within a material under the different mechanisms of fracture and flow depending on the environmental stress, strain, and temperature conditions. For instance, low carbon steel exhibits fibrous and shear types of fractures at room temperature; below -80 C brittle fracture occurs, and intergranular creep fracture is dominant in slow straining at 600 C and above (3). A material may, therefore, have several characteristic strength values when several fracture mechanisms operate at different critical levels of the stress or strain components.

Although the mechanisms of damage initiation and propagation in the two failure modes are different, they have three major points in common:

1. A particular combination of stress or strain concentration is required to create a defect nucleus;
2. A different combination of stress or strain quantities is then required for the propagation of the defect; and
3. A critical combination of stress and strain concentrations is required for the transition from relatively slow to fast propagation to catastrophic failure.

In addition, the distribution and progression of damage in solid material are a random process that is both spatial and temporal (4, 5, 6, 7), and damage in an engineering material is a statistical process brought about by the interaction of several complex mechanisms. An engineering material or structure can, therefore, fail under a given system of external loadings when either of the following two criteria is satisfied:

1. The distribution of internal flaws is such that excessive deformation is attained (usually for ductile behavior), or
2. The distribution is such that a fracture threshold is reached under an arbitrary loading history (usually for brittle behavior).

Accordingly, during the years several reasons have been advanced as explanations for the observed behavior of "damage" in an engineering material, and based on such explanations several theories have emerged. Researchers have approached the problem both deterministically and statistically from the molecular and phenomenological levels. On the molecular basis, the differences between the fracture mechanisms involved are emphasized because, at this level, the material is essentially discontinuous.

On the macro level, the criteria for fracture are basically similar and utilize the concepts of continuum mechanics. The fracture laws are generally based on either local or global energy, stress or strain concentrations within the material.

Theories like the Eyring rate process (10) developed for viscous materials, the Gnauss theory (11) for viscoelastic materials, and Weibull's theory (12) for brittle materials have attempted to explain on the basis of a statistical model some of the phenomena observed when materials like metals, textiles, concrete, and others fracture under applied stress. The basic assumption is that an assembly of unit damage processes grows in a probabilistic way to yield an observed macroscopic effect, with temperature fluctuations and activation energy distribution playing a significant role. Such statistical theories have a significant advantage over deterministic concepts because they account for the role of chance in the behavior of materials.

Nadai (15) has concluded that, whereas a particular failure theory may work well for a particular class of materials, it may often fail quite hopelessly to predict conditions of failure for another class. Examples of this are evident in the use of the maximum shear stress theory, distortional theory, and octahedral shear stress theory, all of which work very well for metals and can be justified on an atomic scale because of the mode of crystal slip in a polycrystal. However, their applications to the failure of materials such as sand, gravel, and clay are questionable because the shear stress necessary for slip in such materials depends also on hydrostatic pressure. Coulomb treated the failure of these materials as a simple frictional resistance that is proportional to pressure and developed the Mohr-Coulomb theory (16), which has met with reasonable success in soil mechanics. Although this criterion neglects the influence of the intermediate principal stress on failure, Bishop (17) and others have deemed it a satisfactory first approximation for three-dimensional situations as well.

## Parameters of Damage in the Repeated Loading Mode

In many materials, the initiation, progression, and ultimate manifestation of distress in the form of fracturing under a repeated load occurs under the action of two separate processes, crack initiation and crack growth, both of which are governed by different criteria. In metals, this behavior has been attributed to localized slip and plastic deformation (18) and to the cyclic motion of dislocations. In polymers and asphaltic mixtures, the cracks initiate from air holes, inhomogeneities, and probably molecular chain orientations and molecular density distributions (20).

The mechanism of crack propagation has been explained by many researchers from a consideration of the energy balance at the crack tip, which deforms as cycling progresses. The propagation is slow when a considerable amount of plastic deformation occurs at the crack tip, which as a result of this becomes blunted. It is fast when the released portion of the stored energy exceeds the energy demand for creating new surfaces.

Erickson and Work (21) discovered that the history of load application had a significant influence on the progression of damage. When the application was a high prestress followed by a low stress, the degree of damage created was greater than when the application was vice versa. The authors explained this occurrence by suggesting that, on the first few cycles of load application, a certain number and distribution of crack sites form depending on the stress level. and the application of subsequent loads merely causes propagation from these sites. The literature contains several phenomenological and molecular theories that handle the problem of life prediction for any material in a given repeated load environment.

Molecular Theories—In some of the molecular theories, statistical mechanics principles and the kinetic reaction rates concept have been utilized. Coleman (24) and Machlin (25) employed the Eyring rate process theory to study the fatigue characteristics of nylon fibers and metals respectively.

Coleman's theory implies that for every material a constant strain level exists at which fracture will occur, but a variety of experimental results shows that this is not the case. Moreover, it does not account for progressive internal damage, inasmuch as only failure conditions are represented. Mott (26) and Orowan (3) have presented for metals fatigue theories that take into account the fact that plastic deformation and strain-hardening occur during fatigue. Mott's theory attributes the formation of microcracks to the occurrence of dislocation within the material. Orowan's theory assumes the presence of a plastic zone within which a crack forms and propagates. Both of these theories have given good agreement with experimental results at times. The discrepancy observed is mainly due to the fact that damage is a stochastic phenomenon, whereas the theories are deterministic in nature. To increase their accuracy requires a statistical approach.

Phenomenological Theories—Despite the fact that several molecular mechanisms have been shown to be operative during fatigue growth in a material, one suspects that the process itself may not be that fundamental in nature. Therefore, instead of searching for molecular theories, we can possibly get a coherent picture from the continuum mechanics approach (with certain reservations).

This has been the motivation behind several phenomenological theories of cumulative damage: Miner's (27), Corten and Dolan's (22), and Valluri's (28) to name a few. The underlying concept in these theories can be illustrated by the work of Newmark (29).

In this approach, it is assumed that when a material is in a given load and climatic environment the degree or percentage of internal damage, $D_i$, is at any time commensurate with the appropriate number of load repetitions, $N_i$ (i.e., for $0 \leq D_i \leq 1$, $0 \leq N_i \leq N_f$). With this assumption, a damage curve exists for every constant stress or strain repeated mode of loading. Because damage in effect implies that a loss in original capacity can result from either the creation and growth of plastic zones or the initiation and propagation of cracks, the strain developed in the material under load, the crack length, and the rate of crack growth can all be used as damage determinants. This reasoning is currently used in constant amplitude stress or strain fatigue tests.

Generally speaking, the combined effects of damage and recovery processes resulting from microstructural changes imply that the damage curve should have different forms for different stress levels and loading histories. Some damage theories, such as those of Miner and Williams, assume a unique degree of damage caused by a stress cycle ratio (n/N) applied at any time. Williams' theory (30) makes a similar assumption but with respect to time ratios ($t/T_f$), where t is elapsed time from start of experiment, and $T_f$ is time to failure. Both theories result in a linear summation of the ratios, and both have a major shortcoming in that prior history and sequence of events cannot be accounted for. Despite this shortcoming, Miner's theory has been successful when applied to rate-insensitive materials. Williams' theory had similar success when used for rate-sensitive materials. In Corten and Dolan's theory (22), the damaging effect under a stress cycle is considered dependent on the state of damage at any instant, and the expression for damage is

$$D_t = mrN^a \tag{1}$$

where

N = number of cycles;
r = coefficient of damage propagation, which is a function of stress level;
a = damage rate at a given stress level, which increases with the number of cycles; and
m = number of damage nuclei.

The Corten and Dolan approach is a rational attempt to modify Miner's theory. The determination of the significant parameters m and r, however, requires the performance of a considerable number of experiments. In addition, rate effects cannot be adequately accounted for. Consequently, in terms of usefulness over a wide class of materials and circumstance, Miner's theory is preferable.

Several researchers (31, 32, 33) have related the rate of crack growth to the localized energy and elastic stress conditions existing at the crack tip. The expression obtained when this fracture mechanics approach is used can be given in general form as

$$\frac{dc}{dN} = Ac^k\sigma^l \tag{2}$$

where

A and k = constants,
c = crack length,
$\sigma$ = stress at tip of crack, and
N = number of load cycles.

The constants k, $\sigma$, and l are dependent on the properties of the material tested and on the boundary conditions of the problem in question (31, 32, 33). Paris and Erdogan (32) found that the use of values 2.0 and 4.0 for k and l respectively yielded good agreement with experimental results. Paris (78), by considering the energy dissipated per cycle of load application, dw/dN, as being proportional to dc/dN, attempted to relate the rate of crack growth to the stress intensity factor, K.

It is evident that these analyses have attempted to take rate effects into account in an indirect manner. When conditions of fracture are brittle in nature, then these analyses are accurate. However, in the presence of tearing action, the property of the material changes with time and thereby affects the corresponding response behavior to application of load, and such analyses cannot account for this kind of behavior. Despite these shortcomings, analyses of this type are attractive in the sense that the fatigue process has been linked to microphenomena on a phenomenological basis.

In order to take rate effects, order effects, and prior history effects into account, Dong (35) postulated a cumulative damage theory to predict the life of a material under any arbitrary loading history. Under isothermal conditions, the mathematical expression obtained is

$$l(t) = f\left[\gamma_{ij}(\tau)\right]_{\tau=-\infty}^{\tau=t} \tag{3}$$

where

l(t) = life remaining in the material at time t after damage has accumulated during $-\infty \leq \tau \leq t$,
$\tau$ = generic time,
t = present time,

$\gamma_{ij}$ = any set of variables that can be used to describe loading history, and
f[ ] = damage functional.

This theory can account for the effect of prior history and sequence of events in damage behavior because the damage functional can be represented by an infinite series expansion of hereditary integrals of the linear and nonlinear types.

The damage behavior of any rate-dependent or rate-independent material can be predicted under any arbitrary loading history by using this approach. In the fatigue loading mode the Miner and Williams theories become special cases of Dong's concepts. This concept shows that the inability of the Miner and Williams theories to demonstrate the influence of prior history and sequence of events on failure is due to the restrictive form of their damage kernels.

The discussion on the damage created within a material in the repeated loading mode suggests that the manner of damage accumulation is a consequence of the inhomogeneity of the engineering materials and structures. Under load, various regions of stress concentration exist within the material; and because, of its inhomogeneous nature, a distribution of strengths is created such that some regions are weaker than others. When the strength of a weak region is exceeded it is quite possible that a crack may initiate and cause a redistribution of stresses with attendant crack formation in other regions. As the load is repeatedly applied, they propagate and grow to a size that eventually renders the material or structure unserviceable. When this event occurs, fatigue damage is completed.

Summary of Failure

In light of the several observations that have been made in regard to cumulative damage in the repeated loading mode, there are a few substantial points to consider in the course of developing a cumulative damage theory for highway pavements.

1. Damage is a function of the inherent inhomogeneity of materials and structures; its initiation, progression, and attainment of a critical magnitude are, therefore, stochastic processes.
2. For a given temperature, damage sites are nucleated under unique stress or strain conditions within the material. They propagate under stress or strain states different from initial conditions until a critical state is reached.
3. The state of damage at any time is a function of the material property and load history—i.e., damage is not unique; it is a function of stress level and microstructural changes within the material.
4. Assumptions have to be made regarding the manner in which damage propagates and regarding the parameters used to delineate its progression. The damage surface is essentially exponential in most materials, but the characteristics of the surface have to be determined from the stress and microstructural conditions existing in the material under load. For instance, if stress, strain, time, and temperature conditions within the material are such that brittle fracture is warranted, then the rate of damage accumulation is one of fast growth to failure. If a tearing kind of fracture is warranted, gradual accumulation of damage is experienced.

The next question that arises in view of the basic premise of this investigation is, Is it possible to develop a cumulative theory of damage for pavement structures comprising different engineering materials and utilizing the basic concepts of damage progression presented earlier? Such a theory may be able to bridge the rather wide gap between states of loading in the field and the relatively simple experiments on a mathematical model of the structure. The ability of the structure to adjust itself to these loadings should yield in symbolic terms, with some degree of reliability, the relationships between the external loadings and the physical constants that measure the competence of the system. To do this, we must know how a pavement fails in practice. The information, thus collected, must be interpreted in the light of the failure mechanisms governing the performance of the materials composing the pavement. When this is done, an adequate failure theory will begin to emerge. To this end the performance of a pavement structure in a repeated loading environment is examined in the next section within the context of internal damage development.

DAMAGE AND DISTRESS IN HIGHWAY PAVEMENTS

An engineering system in which damage or the failure of a component results in a decrease in the level of performance rather than the abrupt incidence of total failure may be called a "structure-sensitive system." For these systems, internal damage develops within an operational environment, over a given time period, and the failure is the ultimate condition that results from a loss in performance. Thus, failure is the extent of the damage that has been accumulated as a consequence of structural deterioration over a range of stress, strain, time, and temperature conditions in an operational environment.

The performance level of a structure-sensitive engineering system in an operational environment may be defined as the degree to which the stated functions of the system are executed within the environment. This level at any point in time is dependent on the history of the magnitude and the distribution of the applied load, the quality of the construction and materials used, their spatial distribution, and the extent to which proper maintenance practices are executed during the entire life of the facility. The reliability of the system consequently depends on these parameters, which are, in turn, dependent on the variabilities of nature. In other words, the performance level of the system at any instant has a probability of occurrence and a frequency distribution of values associated with it.

Figure 1 shows that the performance of the system diminishes in some way until an unacceptable level is attained. This behavior occurs as a result of the combined action of the load and the weather in a given environment. In this environment, pockets of local distress develop within the system (e.g., stress-induced inhomogeneity and corrosion), propagate in a manner that depends on the composition and spatial distribution of the structural materials, and bring about a loss in structural integrity with time.

The base level VP shown in Figure 1 represents an unacceptable level of performance, as determined by the users and engineers of the facility. The time within which the performance drops to this level characterizes the time at which the extent of damage in the structure becomes intolerable. At any instant of time $t_i$, $P_i$ represents a level of performance, and associated with it is the degree of damage $D_i$ developed within the period $t = 0$ to $t = t_i$. At time zero, the performance of the system is presumably 100 percent of its initial value, inasmuch as it has not yet been put into use. At the time when the internal damage becomes intolerable, the performance of the structure is zero percent of its initial value. The integrity level of the system at any time is, therefore, one minus the amount of damage accumulated within that time

$$P_i(t_i) = 1 - D(t_i) \tag{4}$$

In view of the previous discussion on damage it is obvious that the quantities $P_i$ and $D_i$ are probabilistic in nature. Thus, depending on the temporal and spatial distributions of damage within the structure the real instantaneous performance level will be on, below, or above the drawn curve. This means that each point on the curve has a probability of occurrence and a frequency distribution of values associated with it, and this fact must always be acknowledged.

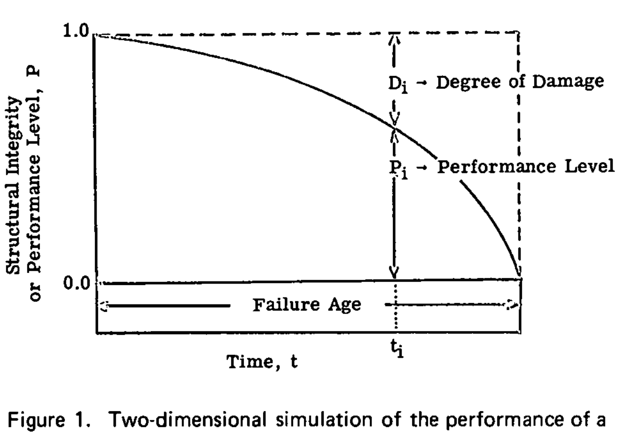

Figure 1. Two-dimensional simulation of the performance of a structure-sensitive engineering system.

The preceding discussion was conducted in the two-dimensional domain; the real picture is, however, more complex. The observed response of the structure depends on rate effects (36, 37), the position and magnitude of the applied load (38), climate (39), materials type, previous traffic history, temperature (38), and constructional variables (41). It can, therefore, be linear or nonlinear depending on the manner in which these variables combine. If the system behavior can be characterized as linearly or nonlinearly elastic, plastic, or viscoelastic, the response will have similar characteristics. Then, at any point $t_i$ in time a "performance surface" that is a function of these variables exists such that its inverse is a "damage surface."

Within this context, at any instant and at some point on the surface, a prediction of the percentage of total life already used and that remaining within the structure can be made. Therefore, the damage surface as well as the performance surface is n-dimensional parameters playing a significant role in its determination. The development of such a surface is not immediately possible. This, however, does not mean that the problem is intractable, because the possibility of reducing n may exist. In fact, such a technique is used in the development of yield surfaces for metals (42).

All the significant factors that have a role to play in internal damage progression can be generally accounted for, providing that they are translated, through the properties of the layer materials and the response behavior of the pavement structure for a given quality of construction and maintenance operations, into stress and strain quantities. In other words, the magnitude and type of stress and strain concentration (tensile or shear) within the pavement structure are a function not only of the characteristics of the applied load but also of the spatial distribution of layer material properties and local defects. A knowledge of the material properties yields information on the kind of structural response to expect. From such information postulations can be made about the manner of internal damage progression. This technique considers the two most significant structural properties, material properties and response behavior, which reflect the influence of all the others. This technique can, therefore, be used to classify pavements into three broad groups (frictional, flexural, and frictional-flexural) so that the stress-strain parameters of damage progression in each group can be identified.

The frictional type of pavement is composed of granular materials in which load transfer occurs at interparticle contact points by purely frictional action. The deformation that takes place under load is purely of the shear or flow type, and, for each application of the load, a permanent deformation results. Such pavement structures generally require a thin type of wearing course that can deflect conveniently with the rest of the structure under repeated loading. To protect the underlying materials, the wearing course should possess good ductile properties as opposed to brittle properties because toughness in this case is more important than tensile strength. However, when the deformation becomes excessive, cracks may appear in the surface because of the randomly distributed cumulative shear action in the subgrade. Therefore, in a frictional type of pavement, damage can be considered to develop as a result of shear action. Consequently, the damage parameter must somehow be associated with shear stresses and shear strains.

In a flexural type of pavement, the materials in the layers are capable of resisting the applied load through the action of tensile stresses that develop as a result of the flexing action. This implies that bending is the only mode of deformation and, upon the repeated application of load, repeated flexing results. In such a pavement, fatigue action is important; and, though the overall shear support of the components is adequate, cracks develop very early because of the accumulation of tensile strains. These propagate slowly or rapidly in a random manner depending on the porperties of the layer materials and the rate of the repeated flexing action. The fatigue properties of the materials in the layers are, therefore, a prime concern during the design state of such facilities. Damage in such pavements is propagated in the fatigue loading mode under the action of tensile stresses and tensile strains.

The third type of pavement possesses frictional and flexural materials. Its structural integrity under repeated load is impaired by the destructive tensile and shear action manifested within the layer components. It is conceivable that, if one action, tensile or shear, should dominate in creating damage within the structure, failure would occur in that mode. On the other hand, it is also possible that both actions may play a significant role during the life of the facility depending on environmental conditions. The damage parameter is, therefore, associated with both tensile and shear stresses and strains.

This classification makes possible the tractability of the damage progression within pavement structures, and one can generally say that the damage buildup occurs in three different modes, described in the following. When the behavior of the pavement structure is completely frictional, damage initiates and progresses by plastic or shear flow until the appearance of surface cracks terminates or aggravates the situation. When flexural behavior is pertinent damage, initiation, and progression occur by the development and growth of internal cracks, under the action of tensile stresses and strains. However, in the frictional-flexural type of pavement, the damage initiates and progresses by shear flow or by crack growth or by both. Consequently, a pavement structure may show signs of distress either from the independent action of excessive deformations, from the isolated action of fatigue, or from both failure mechanisms working together. This indicates that, in order to analyze the response behavior of a pavement structure and to predict the failure behavior, we must develop a number of models that would account for such behavior in a given traffic and climatic environment.

At the present time, the following three models seem to be appropriate:

1. A model is needed for the representation of the linear and nonlinear behavior of paving materials;
2. The pavement system must be modeled in terms of the geometrics of the applied load and the structure so that use of the former model within such a framework will aid in the prediction of the developed stresses and strains in a given environment; and
3. A model that must be capable of handling linear and nonlinear damage behavior should be developed.

Finally, to achieve realistic predictions requires that these models be combined in a probabilistic manner, inasmuch as the progression of damage as has been demonstrated is stochastic in nature.

Because the objectives of this study are to provide a better understanding of mechanisms of damage and distress in pavement structures, the remainder of the discussion is devoted to item 3 and the stochastic nature of the problem.

## Models for Pavement Damage

The pavements discussed in this section belong to the frictional-flexural group and are therefore representative of many current pavement sections. In this type of structure fatigue damage may occur in the surface layer when it behaves in a flexural manner.

The occurrence of fatigue in pavements has been observed or noted for a considerably long period of time. Porter (55) in 1942 observed that pavements do, in fact, undergo fatigue. In 1953, Nijboer and Van der Poel (56) related fatigue cracks to the bending stresses caused by moving wheel loads. Hveem (57) also correlated the performance of flexible pavements with deflections under various repeated axle loads. The AASHO and WASHO tests (39) confirmed these observations by relating the cracking and initial failure of pavements to repeated loading of the type discussed by Seed et al. (58).

The field observations of this kind of behavior led to laboratory investigation. Many researchers have conducted laboratory experiments to determine the fatigue properties of paving materials and to investigate the possibility of extrapolating laboratory results to existing field conditions. To this end, Hennes and Chen (59) conducted tests on asphalt beams resting on steel springs and subjected to sinusoidal deformation with a variety of constant amplitude magnitudes. They discovered that, as the frequency of application is increased, the creep-rupture compliance of the material decreases. Similar tests conducted by Hveem (57) on beams cut from actual pavements yielded the same results.

Monismith (60) in his tests on asphalt beams supported on flexible diaphragms mounted on springs under constant stress amplitudes discovered that increases in the stiffness of the material resulted in corresponding increases in fatigue life. Sall and Pell (61) conducted similar tests from which the tensile strain to failure, $\epsilon_T$, versus the number of cycles to failure or fatigue life, $N_f$ relation, was found to be $N_f = 1.44 \times 10^{-16} (1/\epsilon_T)^6$. They further found that this expression does not vary with temperature, rate of loading, and type of asphalt. These results are not surprising, because one should expect such factors to affect the developed stresses and not the strains through the stiffness of the material. For the mixtures tested, no endurance limit was observed up to $10^8$ application, as is to be expected, because the mode of failure is one of crack initiation and propagation to failure at each stress level. The general conclusion arrived at by several authors from such tests indicates that the fatigue life of an asphaltic paving material is a function of several variables: tensile strain level to which the specimen is subjected, amount of asphalt, and age, temperature, stiffness, density, and void ratio of the mixture.

Another important factor in such tests is the mode of loading. In controlled stress tests, for example, fatigue life increases not only as the stiffness of the sample increases but also as the temperature decreases. However, in strain-controlled tests, the fatigue life decreases as stiffness increases. For this test, at low temperatures no change is observed in fatigue life, and as temperature increases the fatigue life increases as well (63, 64, 65). Controlled stress and strain behavior can be explained from a consideration of either the time-temperature superposition principle or the amount of energy stored in the sample when such tests are performed. In controlled-stress tests the minimum energy stored per load repetition can be achieved by minimizing deflection and causing a resultant increase in fatigue life. In controlled strain tests, the reverse is true. This implies that, for a specimen of a given initial stiffness and initial strain, failure under a controlled stress mode of loading will occur sooner. Therefore, when extrapolating laboratory results to field conditions such considerations play a significant role. Monismith (67) through the use of a mode factor suggested that, for surface layers less than 2 in. thick, the controlled strain mode of loading results; whereas for those layers 6 in. thick or greater, the controlled stress mode of loading is applicable. For thickness between these, an intermediate mode of loading is appropriate.

Tests have also been performed on granular and other paving materials to determine the significant characteristics of their behavior under repeated loading (58, 66). The approach is empirical; however, it points up the important fact that the response of granular and treated materials in pavement sections depends on the characteristics of the applied loading, the material, and the existing confining stress.

In an attempt to apply the experimental results to predict the occurrence of damage in a real pavement, Deacon and Monismith (69) suggested a modification of the Miner theory of linear summation of cycle ratios. They pointed out that such an approach has the desirable features of procedural simplicity and a wide range of applicability to different types of compound loading. Their analysis, however, is rather difficult to interpret; also, the sequence of events and prior history cannot be accounted for in such an approach.

Majidzadeh et al. (82) in their recent work have shown that crack initiation and propagation in asphaltic mixtures can be predicted by using the fracture mechanics approach. They have found that the critical stress intensity factor, which is a function of the material's elastic properties and the driving force at the tip of the crack, is an inherent property of the asphaltic mixtures at low temperatures. Because the crack geometry affects the value of the stress intensity factor, the authors suggest an analytical technique for its calculation for some practical cases.

## Stochastic Nature of Damage

Factors contributing to the initiation, propagation, and accumulation of pavement damage can be divided into three categories: (a) material properties and pavement geometry, (b) loading variables, and (c) climatic conditions. A substantial variability is associated with the measurement or specification of each of these factors, and as a result the pavement response is stochastic. To account for these variabilities requires that the damage model be able to yield statistical estimates of the pavement response. In other words, the model should be able to estimate the probability that damage will occur in a certain mode. To achieve this, we can use simulation techniques. The following discussion describes the possibility of using one such technique, the Monte Carlo method, to predict the stochastic behavior of damage accumulation.

## Materials and Environmental Variabilities

The behavior of a material in a given environment can be represented by a set of responses, $R_i$. The material itself is defined by a set of relevant properties, $Y_k$, and the environment can be specified by a set of conditions, $X_j$.

In the deterministic approach, it is usually assumed that a functional relationship exists between each response term and the associated material properties and environmental condition. Material properties will also vary systematically with environment. So,

$$R_i = g_i(Y_1, \ldots, Y_k, \ldots, \mid X_1, \ldots, X_{ji}, \ldots, X_n) \quad (5)$$

$$Y_k = \nu_k(X_1, \ldots, X_j, \ldots, X_n) \quad (6)$$

However, both material properties and environmental conditions are subject to considerable variability in a random manner over fairly wide ranges. When environmental conditions are correlated, i.e., when there is an interaction between these parameters such as the interaction of moisture and temperature and the effect of one on the other, their joint frequency distribution $f(X_1, X_2, \ldots, X_n)$ will yield the density function as the necessary data to be input for the environmental conditions. If they are not correlated, their independent frequency distributions could sufficiently define the environment. The vectors $X_j$ are, therefore, treated as random variables with probability density functions, $f_{x_j}$, and associated cumulative distributions, $F_{x_j}$.

Material properties are inherently variable, and the terms, $Y_k$, are also considered to be random variables with density functions, $f_{Y_k}$, and cumulative distributions, $F_{Y_k}$.

Because the material properties are dependent on the environmental conditions, statistical correlation is implied by Eq. 6. Complete information of inherently correlated material properties can be given by the joint density function $fY_1, Y_2, \ldots, Y_L$ rather than by the density functions $f_{Y_k}$.

Variability in material properties and environmental conditions implies variability in the material behavior or response. Variability in material behavior is represented by a set of density functions, $f_{R_i}$, or alternatively by the cumulative distribution, $F_{R_i}$.

To evaluate $f_{R_i}$, one should have data available about the density functions $f_{x_j}$ and $f_{Y_k}$. Even if these density functions are somehow evaluated, considerable difficulty can arise in determining $f_{R_i}$ by analytical methods. Such difficulties can be encountered if $f_{x_j}$ and $f_{Y_k}$ are not normal and the equations giving the functional relationships of the three basic vectors are not linear. In these cases a numerical solution can be obtained by the Monte Carlo method.

## Monte Carlo Simulation

The Monte Carlo technique is a simulation method for the evaluation of the cumulative distribution, $F_{R_i}$, in an algorithmic form suitable for computer programming (83). The method is probabilistic in its approach and considers a conditional probability distribution of the form

$$F_{Y_k} \mid x_j \; (Y_k \leq y_k \mid X_j = x_{ji}),\; j = 1, 2, \ldots, p \quad (7)$$

where $x_{ji}$ is any set of values of $X_j$ from populations with cumulative distributions $F_{x_j}$. Similarly, the inherent interdependence of material properties can be taken into account in the same algorithm; i.e.,

$$F_{Y_k} \mid x_j \; (Y_k \mid X_j) \times F_{x_j} \; (X_j) = F_{Y_k} \; (Y_k) \quad (8)$$

The algorithm involves an iteration process from which n number of samples is drawn for the values of $R_i$. From the sample of n, simulations, histograms, means, variances, and percentage points can be obtained. If the number n of the simulation is very large, the histogram can accurately represent the continuous distribution of the parent population.

This simulation method is a simple numerical method that gives statistical answers to specific problems that are not amenable to analytical procedures because of their inherent complexity and interacting factors. This method is approximate in nature, but a high degree of accuracy can be achieved if the number of simulations is sufficiently large. The sufficiency conditions here depend on the statistical data available or required.

In this case the decision as to how many samples to be drawn out should be preceded by something like sensitivity studies. Several techniques have been developed based on such studies. These are basically variance reducing techniques to increase information in the "interesting regions" of the distribution $F_{R_1}$ and, hence, to decrease the information in the noninteresting regions or ranges.

The other factors that have an influence on the cumulative frequency distribution are the probability density functions, interactions, and correlations of the parameters involved, i.e., the environmental variables and the material properties. In the case of interacting parameters, a joint density function can be used instead of the single density functions.

The probability density functions of the different parameters that are being simulated can generally be obtained by some statistical tests. Sampling from the real, statistically measured distribution is preferable to obtaining samples from the assumed distribution because the more realistic these density functions are, the better the results of simulation are. However, if the statistical data are lacking for density functions of the parameters under consideration, special care should be given to making assumptions for such density functions. This could be done by looking into the literature for statistical representation of the same or similar parameter.

### Summary of Pavement Damage

The results of the review of literature on pavement damage due to the mechanical loading indicate that, in order to develop a general damage function for pavements, the pavement system may be considered as a black box that possesses some unknown properties and is subjected to some variable inputs (load and environment). The properties of the black box are dependent not only on the distribution of the inputs but also on the materials and geometrical configuration of the structure. The outputs are the responses of interest such as the deflection, the extent of cracking, and so forth.

In such structures, damage accumulates, and either a modified form of Dong's general theory or a modified form of Miner's linear law is needed to account for accumulation of damage. Any damage concept developed for highway pavement should be adaptable to account for the stochastic nature of the problem. A direct probabilistic method should be used if possible; otherwise, a method using simulation techniques should be developed.

## PAVEMENT CUMULATIVE DAMAGE MODEL

### Formulation of Damage Law

To develop a general damage law one must relate the input variables (loads, temperature and time) to output variables (deflections and cracks). The relationship sought is generally difficult to obtain, and its application to each specific case requires a great many modifications. The complexity of the required general relation can be simplified by handling the damage formulation in two steps. The first step yields the stress and strain fields within the system. Stress, strains, and displacements are called the primary responses and are then used as inputs to the second step in the model, which yields damage parameter or limiting responses. This separation of the damage formulation into two independent parts assumes that the limiting responses are dependent solely on the primary responses, a fact that seems to be supported in the literature.

Primary Response—The geometrical model that is used in this study is a semi-infinite half space consisting of three distinct layers. It is assumed that each layer has distinct material properties that can be characterized as linear elastic or linear viscoelastic. The load is considered to be uniform, normal to the surface, and acting over a circular area.

An assumption of incompressibility is made so that one constitutive relationship is sufficient to define the viscoelastic equation of state for each layer. This constitutive equation is assumed in terms of a viscoelastic equivalent to the elastic compliance. That is, for the i th layer,

$$\frac{1}{E}:\ (\text{equiv.}) = \left[ D_i(o)\,(\ ) - \int_o^t (\ )\,\frac{\partial D_i(t-\tau)}{\partial \tau}\, d\tau \right] \tag{9}$$

where $D_i(t)$ = the creep compliance of the i th layer.

To obtain the viscoelastic solution for the stresses and displacements for other loading conditions, we use the correspondence principle (72). Using this method, Elliott and Moavehzadeh (71) have analyzed three loading conditions: a stationary load, a repeated load, and a load moving at a constant velocity. In each of these three cases, the environmental conditions were assumed to remain constant. In this study the results of their study are used and extended to account for varying environmental conditions.

Limiting Response—The structural damage of a pavement structure is divided into two parts that are not necessarily independent of each other: excessive deformation and cracking. The excessive deformation can be directly predicted by any primary model of a multilayer system that has accumulative capabilities (71), provided that it is modified to account for the stochastic nature of the environmental factors. Cracking is assumed to arise mainly from fatigue behavior, and its accumulation thus can be measured in different manners. For example, one can relate the crack nucleation and growth to specific combinations of the primary responses such as the stress intensity factor K (82) and then evaluate the probabilities of having a given distribution of cracks. Another method is to use a more phenomenological approach and represent the accumulation of cracking by a damage functional F(t), which depends on the past histories of the stress and strain tensors. With some assumptions of continuity this functional can be expanded into a series of multiple integrals (35):

$$F(t) = \int_o^t K_1(t, s)V(s)ds + \int\int_o^t K_2(t, s_1, s_2)V(s_1)V(s_2)ds_1ds_2 + \ldots + \int \ldots \int_o^t K_n(t, s_1, \ldots, s_n)V(s_1)\ldots V(s_n)ds_1, \ldots, ds_n \tag{10}$$

The measure of damage F(t) is not uniquely defined. The damage may be measured by the density of cracking or by the value of dynamic modulus of the layer materials at a given frequency because this modulus decreases as the density of cracking increases (87). The creep compliance of the material can be used as a measure of crack propagation (82, 92), or the number of remaining cycles before complete failure under a given mode of loading can be used for this purpose (85). Any of these measures can be used, and it is convenient to normalize them so that the damage functional equals zero when the material is intact and increases to one at failure. In Eq. 10 V(s) is a function involving stress or strain invariants or both, and s is an arbitrary parameter that may have a meaning of time or cycles. This representation of the damage functional is general and accounts for accumulation of damage as well as recovery processes such as healing and accumulation of aging effects.

The review of literature showed that for various asphaltic and bituminous mixtures failure envelopes were related to a strain measure. In the general case of triaxial loading conditions the strain measure should be expressed as a combination of invariants. In the absence of results of triaxial tests, we will use the derivative of the major principal strain as a strain measure in the damage functional. Thus,

$$F(t) = \int K_1(t, s)\dot{\epsilon}(s)ds + \int_o\int^t K_2(t, s_1, s_2)\dot{\epsilon}(s_1)\dot{\epsilon}(s_2)ds_1ds_2 + \ldots \tag{11}$$

where ( ) represents a differentiation with respect to the argument. When s has a meaning of a time, this expansion is similar to the representation of the time response of a nonlinear viscoelastic material; whereas when s has a meaning of a cycle, it may be related to the dynamic representation of a nonlinear viscoelastic material and may be determined as a transfer function of a system subjected to a cyclic loading. Dong (35) has shown that, in the latter case, making these integrals discrete results in Miner's law (27).

This expansion is simplified by making an assumption that three different damage processes may be recognized: a damage process depending on the number and amplitude of cycles, a healing process (or a recovery process) depending on the elapsed time since the damage was created, and an aging process wherein the materials properties are changing with time. The damage functional may now be written as

$$F(t) = \int_0^t K_1(s, t-s)\epsilon(s)ds$$

$$+ \int\int_0^t K_2(s_1, t-s_1, s_2, t-s_2)\epsilon(s_1)\epsilon(s_2)ds_1ds_2 + \dots \quad (12)$$

This equation implies that the kernels are functions of the running time s (cumulative and aging processes) and of the lapse of time t - s (recovery process).

In a first approach to the problem, the second and higher order kernels will be neglected in the damage expression. We will further assume that the first order kernel sought may be factorized as

$$K_1(s, t-s) = K_{CUM}(s)\, K_{REC}(t-s, s) \quad (13)$$

i.e., the cumulative and recovery processes are independent. The aging process is included in both $K_{CUM}$ and $K_{REC}$ through the dependency on the time s.

In order to determine these kernels we must choose a measure for the damage and normalize it as mentioned. Let N be the number of cycles to failure (i.e., inadmissible density of cracking) under a given type of random load during a relatively short time (no aging or recovery takes place). A damaged material will undergo only N′ cycles under the same conditions before failing. The amount of damage is represented by (N - N′)/N. N and N′ can be measured on control specimens. Note that in this case F(t) is not a measure of the amount of cracking but is a function of it.

Cumulative Kernel—For a small period of time during which there is neither aging nor damage recovery we have for the increment of damage, $\Delta F(\tau)$

$$\Delta F(\tau) = \int_0^S K_{CUM}(S-s)\frac{\partial\epsilon}{\partial s}\,ds \quad (14)$$

Dong (35) proved that, if s is the number of cycles of a given strain amplitude, Eq. 14 is identical to Miner's representation; i.e.,

$$\Delta F(\tau) = \sum_{i=1}^{m} \frac{dn_i(\tau)}{N_i(\tau)} \quad (15)$$

where $dn_i(\tau)$ is the number of cycles of amplitude $\Delta\epsilon_i$ that are applied at time $\tau$, and $N_i(\tau)$ is the number of cycles $\Delta\epsilon_i$ that would cause failure. Hence, the general expression for the damage becomes

$$F(t) = \int_0^t \Delta F(\tau)\, K_{REC}(t-\tau, \tau)\, d\tau$$

$$F(t) = \int_{0}^{t} K_{REC}(t - \tau, \tau) \left( \sum_{i=1}^{m} \frac{dn_i(\tau)}{N_i(\tau)} d\tau \right) \quad (16)$$

The aging process is accounted for in the dependency of $K_{REC}$ and N on the time $\tau$.

For a uniaxial case we will consider a random stress history to be composed of a mean value and a cyclic component. In the triaxial case these may be replaced by a mean value of the hydrostatic stress and a cyclic component of the octahedral stress. If sinusoidal stresses with constant amplitude are applied to various specimens of a material, a fatigue envelope (S-N curve) is usually obtained in the form of a stress or strain amplitude versus the number of cycles to failure. Miner's law may be applied to such diagrams to predict the results under varying amplitudes. Because this envelope is found to be generally independent of temperature and rate of loading when it is given in the form of strain amplitude versus number of cycles to failure, we will concentrate on the use of such diagrams. These diagrams and Miner's law, however, do not account readily for the order in which successive loads are applied and the effects of varying amplitudes. The order in which the loads are applied will be accounted for implicitly because strain amplitudes are obtained as the primary responses of the three-layer viscoelastic model, and thus they are functions of the sequence of loading. To take into consideration the effects due to varying amplitudes and nonzero mean, we can use an approach suggested by Freudenthal and Heller (84) and based on the statistical character of fatigue. The basis of this statistical evaluation of the results is the three-parameter distribution function of the smallest values (84). This function derives from the assumption of a distribution function of the strength (or strain at failure) of cohesive bonds in the material and a distribution function of the induced stresses (or strains) when a macroscopic stress (or strain), S, is applied to the structure. The probability function, P(S), of the strength (or strain at failure) of the material is obtained as a convolution of the two previous distribution functions. The relation between the mode of loading P(S) and N determines the trend of a theoretical S-N diagram. Based on the determination of a single S-N diagram obtained for a given mode of loading (e.g., sinusoidal with constant amplitudes), we can predict the probable S-N diagrams for other modes of loadings. Freudenthal and Heller (84) related the effect of the change in the load spectrum to the changes in the S-N diagram.

This results in a modification of Miner's law to include an interaction factor $\omega_i > 1$ to account for interaction effects of various stress amplitudes. Thus, Miner's law becomes

$$\sum_{i=1}^{m} n_i/\omega_i N_i = 1 \quad (17)$$

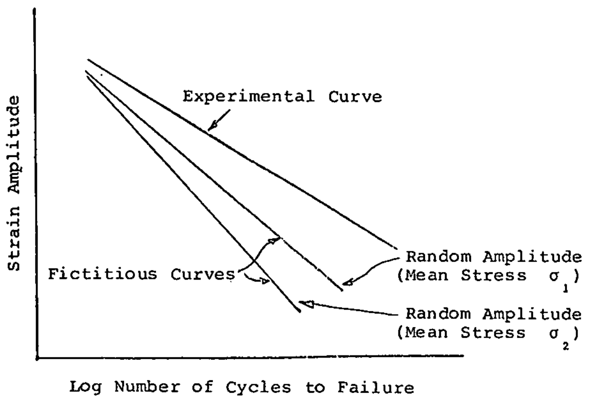


Figure 2. Failure envelopes.

where $\omega_i$ depends on the load spectrum and results in the fictitious envelope shown in Figure 2. The cumulative process can therefore be given by an expression such as

$$\sum_{i=1}^{m} \frac{dn_i}{\omega_i N_i \, [\Delta\epsilon(\tau), \tau]} \quad (18)$$

where $\tau$ indicates that the number of cycles to failure may vary because of aging and that the envelope is to be determined for different values of $\tau$. The increment of damage is also a function of the average strain amplitude applied during the increment of time $\tau$.

The determination of $N_f(\Delta\epsilon)$ can also be derived from the knowledge of the mechanisms of cracking. For example, it can be determined through the use of the concept of stress intensity factors (82).

Recovery Kernel—The recovery kernel, $K_{REC}(t-\tau, t)$, is a function of the time, $(t-\tau)$, elapsed since the application of the damage increment and of the age of the material. From Bazin and Saunier's paper (85) it is apparent that healing requires the presence of a minimum compressive stress. Thus, we will assume that the argument $t-\tau$ can be replaced by $t^* - \tau$ where

$$t^* = \int_{\tau}^{t} H[\sigma(s) - \sigma_{min}]ds \qquad (19)$$

H[ ] is the Heaviside step function, which is equal to one when its argument is positive and equal to zero elsewhere. $\sigma_{min}$ is the minimum compressive stress that triggers healing. Thus, $t^* - \tau$ is the accumulated time during which a minimum compressive stress is present.

To determine $K_{REC}(T)$ we should give two identical specimens (or sets of specimens) the same amount of damage, F. F is determined by testing one of the two specimens (control specimen) and measuring the amount of damage that should still be applied to fail the specimen. The second specimen is left to rest for a time, T, and then failed to determine the amount of recovery, $K_{REC}(T)$.

Aging—Aging is accounted for through changes in the characteristics of the constitutive equation as well as in the cumulative and recovery kernels.

Input Requirements

Materials Characterization—For the determination of the damage function, F, it is important to determine the stress or strain invariants or both or, as in Eq. 19, to determine the major principal strains. To do this, we must determine the properties of the materials in the layers. These properties are generally dependent on the manner in which they are prepared and constructed, their thickness and confining stress, the rate of loading, and the history of the environmental variables. Because all of these factors are statistical quantities, we must expect the properties to also be statistically distributed within the layers.

The materials properties assumed to be pertinent here are the compliances or creep functions, Poisson's ratio, and the height of the layered materials. Poisson's ratio and the height of each layer are considered as deterministic quantities. Although the height of the layers can change with different structural sections, Poisson's ratio is set equal to one half for all sections. The properties of the material for each layer will be represented by a creep compliance function for a viscoelastic layer and by a creep compliance for an elastic layer. For a viscoelastic layer the following representation will be assumed

$$D_j(t) = D_{E_j} + \sum_{i=1}^{n} G_i^j e^{-t\delta_i} \quad j = 1, 2, 3 \qquad (20)$$

where

$j$ = layer number;
$D_j(t)$ = value of creep function at time t;
$D_{E_j}$ = zero time value of the creep function, i.e., $\sum_{i=1}^{n} G_i^j = 0$;
$G_i^j$ = coefficient in Dirichlet series $\sum_{i=1}^{n} G_i^j e^{-t\delta_i}$;
$\delta_i$ = exponent in exponential term corresponding to coefficient $G_i^j$; and
$\delta_n = 0, \ldots, D(\infty) = D_{E_j} + G_n^j$.

To include the statistical characteristics of the properties in the analysis requires that a method similar to that described by Soussou and Moavenzadeh (86) be used. In this method, a random loading is used as an input in the tests designed to determine the creep compliances. The resulting functions are least square approximations of the social properties.

History of the Environment—The main measurable quantities reflecting the influence of the environment are temperature and humidity. The temperature within the system will be assumed to be uniformly distributed. Later stages of the study may introduce the spatial distribution of temperature as a function of the atmospheric temperatures and wind condition. Data are available on such distributions, and analytical means of computation exist (88), but the present models for viscoelastic layered systems do not have this capability. The same observations as made for temperature can be made for the moisture or humidity distribution within the system. The history of environment will be generated in a random manner so as to approximate the climate in a given area.

Many viscoelastic materials were shown to be "thermorheologically simple," i.e., to fulfill the time-temperature superposition principle. Similar principles of superposition were found to be true for other types of environmental changes (89). Thus, if $\phi(t)$ is the value of the environment at time t and $\phi_0$ is the reference value of the environment, viscoelastic properties at $\phi(t)$ may be derived from viscoelastic properties at $\phi_0$ through different operations of scaling. A general expression is

$$D[t, \phi(t)] = \alpha[\phi(t)] + \beta[\phi(t)]\, D\{\gamma[\phi(t)]\, t, \phi_0\}$$

where $\phi(t)$ is a function of the values of the temperature or moisture content, $\alpha$ is a vertical shifting of the creep compliance, $\beta$ is a vertical scaling of the transient response, and $\gamma$ corresponds to a horizontal shifting for a semilog plot (change in time scale). These techniques of scaling apply well to a wide variety of viscoelastic materials, and they allow for an easy introduction of the environment effects in viscoelastic analyses.

This representation is used to describe the effect of changes of properties with temperature and moisture. Moreover, because the repeated load program can be directly related to the results of the stationary load program, these scaling techniques will be used to represent the response of the stationary load program for different values of the environment variables.

History of Load Applications—The traffic load intensity on a pavement system is statistically distributed in time, space, and magnitude. In this investigation, a single wheel load applied over a circular area is considered to be appropriate on the assumption that the equivalent-single-wheel-load concept is valid. The rate of load application and the magnitude of the load can be varied. The loading function is assumed to be a Heaviside characterized by a load duration and a period that is the time between two consecutive load applications. At this stage of the study, the load magnitude and the duration of the load are assumed to be constant. The period of the load is constant during a time period (day, week, month, and so on) and is a function of the number of load applications during that period.

## Application to Highway Pavements

Primary Response—Because a major part of the problem of determining the damage functionals is to predict stresses, strains, and deflections under a random load and environment histories, it was necessary to modify the repeated load program (71) to give it this capability. The response, $P_s(t)$, of a linear viscoelastic system to any varying load, P(t), may be written as

$$P_s(t) = \int_{-\infty}^{t} SR(t - \tau) \frac{\partial}{\partial \tau} P(\tau) d\tau$$

where SR(t) is the response to a step load (stationary load program). If we want to introduce the effect of the history of the environment, $\phi(t)$ where $\phi(t)$ may be the history moisture, and other environmental variables, the response is given by (71)

$$P_s(t) = \int_{-\infty}^{t} SR\left[t-\tau, \overset{t}{\underset{s=\tau}{\phi}}(s)\right] \frac{\partial}{\partial\tau} P(t)d\tau$$

and

$$SR\left[t-\tau, \overset{t}{\underset{s=\tau}{\phi}}(s)\right] = \alpha[\phi(t)] + \beta[\phi(t)]$$
$$\times SR\left\{\int_{\tau}^{t} \gamma[\phi(\theta)]d\theta, \phi_o\right\} - \int_{\tau}^{t} SR\left\{\int_{\tau}^{t} \gamma[\phi(\theta)]d\theta, \phi_o\right\} d\beta[\phi(s)]$$

where $\alpha$, $\beta$, and $\gamma$ are scaling factors that were described earlier and that are functions of the environment $\phi(t)$; and $\phi_o$ is the reference value for the environment. The determination of $\alpha$, $\beta$, and $\gamma$ is made by curve-fitting techniques (86).

When SR(t) is given in form of a series of exponentials

$$SR(t) = \sum_{i=1}^{N} G_i e^{-t\delta_i}$$

for a step load under a constant value $\phi(t)$, the response becomes

$$SR[t, \phi] = \alpha[\phi] + \beta[\phi]\left(\sum_{i=1}^{N} G_i e^{-t\delta_i[\phi]}\right)$$

whereas in the more general case of a variable environment history it becomes

$$SR\left[t-\tau, \overset{t}{\underset{s=\tau}{\phi}}(s)\right] = \alpha[\phi(t)] + \beta[\phi(t)]$$
$$\times\left(\sum_{i=1}^{N} G_i e^{-\delta_i t^*}\right) - \int_{\tau}^{t}\left(\sum_{i=1}^{N} G_i e^{-\delta_i s^*}\right) d\beta[\phi(s)]$$

The notation $x^*$ is defined as

$$x^* = \int_{\tau}^{x^*} \gamma[\phi(\theta)]d\theta$$

At the present time only the temperature is considered in the variable $\phi(t)$, and it is assumed that the temperature remains constant for a time period and changes as a step function at the end of every time period. Thus,

$$SR\left[t-\tau, \overset{t}{\underset{s=\tau}{\phi}}(s)\right] = \alpha[\phi(t)] + \beta[\phi(t)]\left(\sum_{i=1}^{N} G_i e^{-\delta_i t^*}\right)$$
$$- \sum_{m=1}^{j} \sum_{i=1}^{N} \left(G_i e^{-\delta_i s^*_m}\right) \times \left[\frac{T(t_m) - T(t_{m-1})}{T_o}\right]$$

This formulation can be used to compute the primary responses due to arbitrary histories of loads and environment variables. In the present work we have adopted the concept of equivalent single wheel loads where the magnitude of the applied load is maintained constant. In the more general case the magnitude of the applied load will also present a statistical distribution. In the present analysis, however, the magnitude of the load, as well as the duration of its application, is considered to be constant. The load is described by

$$P(\tau) = \text{sine}^2\omega\tau[H(0) - H(\text{duration}) + H(1 \times \text{period}) - H(1 \times \text{period} + \text{duration}) + H(2 \times \text{period}) - H(2 \times \text{period} + \text{duration}) + \ldots]$$

where H is the Heaviside step function. Hence,

$$\frac{\partial P(\tau)}{\partial \tau} = \begin{cases} \omega \sin \omega\tau \text{ while a load is applied} \\ 0 \text{ otherwise} \end{cases}$$

The unit step response is described by

$$SR\left[t-\tau, \overset{t}{\underset{s=\tau}{\phi}}(s)\right] = \alpha[\phi(t)] + \beta[\phi(t)]\left(\sum_{i=1}^{N} G_i e^{-\delta_i t^*}\right) - \sum_{m=1}^{j}\sum_{i=1}^{N}\left(G_i e^{-\delta_i s^*{}_m}\right) \times \left[\frac{T(t_m) - T(t_{m-1})}{T_o}\right]$$

Based on results obtained by Glucklich (92), we will use the following as typical values in the computer program:

$$\alpha[\phi(t)] = 0$$
$$\beta[\phi(t)] = T(t)/T_o$$
$$\gamma[\phi(t)] = 1/a_T(t) = 10\left\{10{,}000\left[\frac{1}{T(t)} - \frac{1}{T_o}\right]\right\}$$

where T(t) and $T_o$ are respectively the present temperature and the reference temperature in degrees Kelvin, and $a_T(t)$ is the present value of the shift factor. In this case we can write

$$SR\left[t-\tau, \overset{t}{\underset{s=\tau}{\phi}}(s)\right] = (T_j/T_o)\left(\sum_{i=1}^{N} G_i e^{-\delta_i t_j^*}\right) - \sum_{m=1}^{j}\sum_{i=1}^{N}\left(G_i e^{-\delta_i s^*{}_m}\right) \times \left[\frac{T(t_m) - T(t_{m-1})}{T_o}\right]$$

where $t_j^*$ and $s_m^*$ reduce to

$$t_j^* = \int_0^{t_j} \gamma(\tau)d\tau$$

and

$$s_m^* = \int_0^{s_m} \gamma(\tau)d\tau$$

Limiting Responses—The limiting responses that will be considered for a pavement are rutting, slope variance, and cracking. The first two are directly predictable from the primary response, but the latter requires the development of a damage model. Although cracking may result from a single load application, it is more often created by repeated loading, i.e., fatigue. The amount of damage created by fatigue is represented by the function F(t) defined previously.

The evaluation of F(t) requires the knowledge of the kernels involved. The form of these kernels was suggested in the review of the literature. The cumulative kernel is given by the fatigue envelope relating strain amplitudes and number of cycles to failure. This fatigue envelope is defined by a relationship of the form $N = K\,(1/\Delta\epsilon)^n$ (88), where N is cycles to failure at a particular strain level, K is of the order of $10^{-6}$ to $10^{-10}$ for various asphalt concrete mixtures, and n varies between 2.8 and 5. $\Delta\epsilon$ will be defined as the average difference between two consecutive peaks and valleys in the strain function. The strain level is the average strain level for the whole period K, and n can be given in terms of mean strain level (as is done commonly for metals). The recovery kernel is more difficult to obtain because of the scarcity of data. It is possible to use some experimental results such as those of Kasianchuk's (88), who reports the rate of healing and recovery of some asphaltic mixes.

Still fewer data are available for evaluating the effects of aging. This process is important to include because it accounts for some of the non-load-associated failures.

Systems Simulation—Figure 3 shows the steps involved in modeling the pavement system and simulating load and environmental histories. This section will describe the different steps involved in the formulation of this model.

The first stage in the model consists of dividing the time into periods during which the environment variable (i.e., temperature) is assumed to be constant. These time periods may be days, weeks, or months, depending on the assumptions made for the analysis. The duration of the load and its magnitude are assumed to be constant. The number of load applications for each time period is generated randomly so that it has given statistical characteristics, e.g., uniform distribution over a given range. Similarly, the value of the temperature for each time period is also a random variable that has a specific frequency distribution such as a normal distribution over a given range of temperatures.

The characterization of the materials properties yields environmental scaling factors $\alpha_i$, $\beta_i$, and $\gamma_i$ for the ith layer. Then the responses of the three-layer viscoelastic model to static loads at different values of the temperature, T, are curve-fitted to obtain the unit step response of the system at a reference temperature $SR(t, T_{ref})$, as well as scaling factors $\alpha(T)$, $\beta(T)$, and $\gamma(T)$ for the overall system.

In the present analysis, because there were no particular assumptions on the values of these coefficients for each layer, the values of $\alpha(T)$, $\beta(T)$, and $\gamma(T)$ were assumed to be those found for a particular sand-asphalt mixture (87). These values were assumed to be the same for both the deflection and the strain unit step response of the layered system. Note that the variability of the materials properties is not included, but this can be done by associating frequency distributions for each of $\alpha$, $\beta$, and $\gamma$ and the coefficients describing $SR(t, T_{ref})$.

The simulation program proceeds then to compute the total residual deflection at the end of the jth time period as well as the mean circumferential strain and average circumferential strain amplitudes during the jth time period. The strains are computed at the bottom of the top layer, and the strain amplitudes are computed as half the difference between successive peaks and valleys of the resulting strain.

These results are then readily related to the three principal measures of damage: rutting, slope variance, and cracking.

Rutting is measured by the amount of residual deflection. A large number of simulation runs yield a probability of occurrence of a maximum deflection before a given number of time periods. The variability of the materials properties can be easily accounted for in such results; however, for the present time deterministic values were chosen for the materials properties in order to keep the required number of simulation runs at a minimum.

Slope variance is measured by differential deflections. These differential deflections occur mainly because of variabilities in the system's properties. Therefore, it is

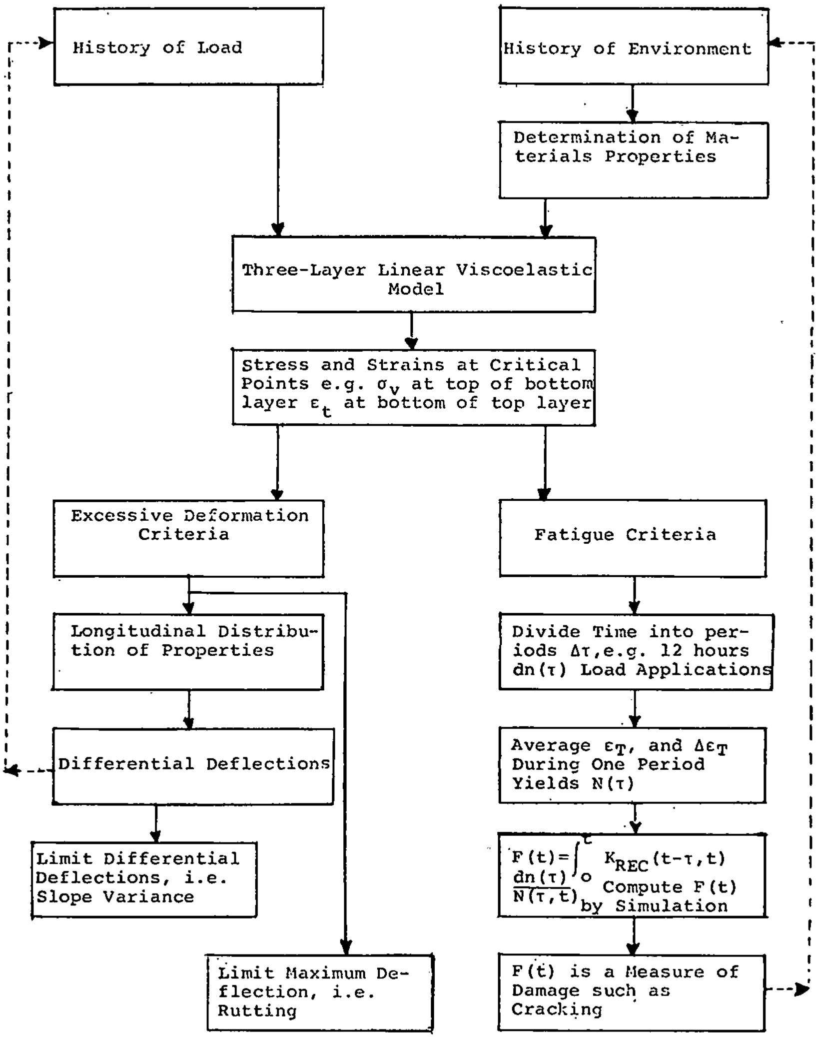


Figure 3. Modeling pavement system and simulating histories.

important to evaluate the correlations between the properties of the system at two points separated by a distance d (e.g., d = 2 ft). The knowledge of these correlation coefficients allows determination of the probabilities of having a given amount of differential settlement before the j th time period. This determination is done through a series of simulation runs representing each of the two points. Each of the points is assumed to be a three-layered, half-space system (i.e., the differential settlement is not accounted for) directly in the mathematical model.

Cracking is measured by the function F(t) as essentially a fatigue phenomenon. At the end of each time period, the increment of damage $\Delta F(\tau)$ is evaluated by using a Miner's type of law. The number of cycles applied during that time period is known, and $N(\tau)$ is obtained from the value of the average amplitude of the strain $\Delta\epsilon_t$ and its mean value $\epsilon_t$. The total value of F(t) is obtained by convolution of $\Delta F(\tau)$ with the recovery kernel $K_{REC}(t - \tau)$. The effect of aging is included by changing at different stages of the simulation the functions describing $N(\tau)$ and $K_{REC}(t - \tau)$. Hence, a series of simulation runs yields the probabilities of obtaining F = 1 before a given time $t_j$. At a later stage of the study, a feedback loop from the value of F(t) to the history of the environment can be added. Such a loop would account for facts such as the changes of moisture content of the subgrade due to moisture infiltration through newly formed cracks.

Summary of Cumulative Damage Model

This section presented the framework for a pavement cumulative damage model. This model uses the primary responses as an intermediary step in the process of computing the limiting responses. The primary responses are obtained for linear elastic and viscoelastic layered systems under varying loads and environmental conditions. These primary responses are used to predict three components of the damage: rutting, slope variance, and fatigue cracking. The damage functional assumes three independent mechanisms: a cumulative fatigue process, a healing or recovery process, and an aging process. The model can include some of the nonlinearities of the damage functional in feedback loops, which account for interactions between the output variables (cracks and deflections) and the input variables (loads and environmental variables). Simulation techniques are applied to this model in order to account for the stochasticity of the input variables.

## SUMMARY AND CONCLUSIONS

The objective of this review of literature is to identify the modes of damage and their initiation, propagation, and accumulation in the flexible pavement structures. The review is limited to only load-associated damage and its influence on the structural integrity of the pavement. The study is performed by first reviewing the concept of damage in engineering material, thus providing the necessary background work for the development of a damage concept in structure-sensitive engineering systems. Then the pavement system and its modes of damage are reviewed with special emphasis on the mode of damage associated with the repeated loading.

Finally, the framework of a comprehensive model for analysis of damage in highway pavement is presented. This framework consists of a three-layer viscoelastic model and a cumulative damage concept used in conjunction with a simulation technique.

The results of this study substantiate the following conclusions:

1. Pavement failure is a many-sided problem, and it is the result of a series of interacting complex processes, none of which is completely understood. The question of what constitutes the failure is highly subjective and depends on the user's evaluation of the facility.
2. The damage in pavement structures is accumulative and depends on the external excitations, loading and environmental variables, and the physical factors that measure the competence of the system.
3. The input variables and the capabilities of the pavement to resist the initiation and growth of damage can at best be represented in a stochastic manner.
4. Development of any comprehensive model for analysis of damage in a pavement structure should take into account (a) the subjective nature of definition of failure, (b) the cumulative nature of damage, and (c) variabilities present in materials properties, environmental factors, and load.

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