MATERIALS CHARACTERIZATION—EXPERIMENTAL BEHAVIOR

John A. Deacon

Proper structural design of asphalt concrete pavement systems relies in part on a thorough understanding of the response of the constituent materials to load. Such response has two components: (a) strength, which represents the limiting condition such as fracture or slip, and (b) deformability, which represents the stress-strain-time response before the failure or limiting condition is attained.

Even though a great deal of research has been devoted to characterizing the response of these materials, there is a noticeable lack of accord as to proper test procedures, theories of behavior, and test results. This situation is readily explainable in view of the following:

1. The variety of materials encountered by the pavement designer is unlimited because of the nature of these multiphase materials and the manner in which they are manufactured from, in large part, locally occurring but often artificially processed ingredients;
2. The nature of the pavement structure in which these materials are used depends greatly on the function to be performed by the pavement and varies, for example, from an oil treatment of an unprepared soil to a substantial thickness of asphalt concrete placed on a high-quality treated base course over a carefully prepared subgrade;
3. During the service life of a pavement, a number of environmental conditions change including temperature and moisture content, and the material properties are altered because of factors such as thixotropy, aging, curing, and densification;
4. The response of pavement materials to loading is extremely complex and for the most part even under in-service stress intensities is characterized by nonlinear, inelastic, rate-dependent, anisotropic, and sometimes temperature- and moisture-sensitive behavior;
5. Until recent years, solutions to pertinent boundary value problems have been nonexistent or at least not readily available; and
6. The approach to the problem has been piecemeal at best and has involved many different researchers from many different agencies each striving for an optimal solution to a singular problem of limited scope and sometimes prejudiced intent.

The purpose of this paper is to summarize some of the experimental laboratory work that has been devoted to characterizing the behavior of pavement materials on a phenomenological basis. Attention is limited to cohesive and cohesionless soils, unbound aggregate bases, and bituminous paving mixtures and is concentrated on those test procedures that seem capable of providing the most useful results for rational pavement analysis. Pertinent variables affecting material response, an understanding of which is essential for proper interpretation of test results, is given in the following outline.

- I. Loading variables
  - A. Stress history (nature of prior loading)
    - 1. Nonrepetitive loading (such as preconsolidation)
    - 2. Repetitive loading
      - a. Nature, whether simple or compound
      - b. Number of repetitive applications
  - B. Initial stress state (magnitude and direction of normal and shear stresses)

- C. Incremental loading
    - 1. Mode of loading
        - a. Controlled stress (or load)
        - b. Controlled strain (or deformation)
        - c. Intermediate modes
    - 2. Intensity (magnitude and direction of incremental normal and shear stresses)
    - 3. Stress path (relation among stresses, both normal and shear, as test progresses)
    - 4. Time path
        - a. Static
            - (1) Constant rate of stress (or load)
            - (2) Constant rate of strain (or deformation)
            - (3) Creep
            - (4) Relaxation
        - b. Dynamic
            - (1) Impact
            - (2) Resonance
            - (3) Other, including sinusoidal (rate of loading is variable) and pulsating (duration, frequency, and shape of load curve are variables)
    - 5. Type of behavior observed
        - a. Strength (limiting stresses and strains)
        - b. Deformability
    - 6. Homogeneity of stresses
    - 7. Drainage

II. Mixture variables
- A. Mineral particles
    - 1. Maximum and minimum size
    - 2. Gradation
    - 3. Shape
    - 4. Surface texture
    - 5. Angularity
    - 6. Mineralogy
    - 7. Adsorbed ions
    - 8. Quantity
- B. Binder
    - 1. Type
    - 2. Hardness
    - 3. Quantity
- C. Water (quantity)
- D. Voids
    - 1. Quantity
    - 2. Size
    - 3. Shape
- E. Construction process
    - 1. Density
    - 2. Structure
    - 3. Degree of anisotropy
    - 4. Temperature
- F. Homogeneity

III. Environmental variables
- A. Temperature
- B. Moisture
- C. Alteration of material properties with time
    - 1. Thixotropy
    - 2. Aging
    - 3. Curing
    - 4. Densification

Test configurations are listed in the following outline.

- I. Tension
  - A. Uniaxial tension
  - B. Indirect (splitting) tension
  - C. Cohesiometer
- II. Compression
  - A. Unconfined, uniaxial compression
  - B. Triaxial compression
    - 1. Open system
      - a. Isotropic compression
      - b. Conventional triaxial compression, whether normal, vacuum, or high-pressure
      - c. Box with cubical specimen
    - 2. Closed system
      - a. Oedometer
      - b. Cell
      - c. Hveem stabilometer
- III. Flexure
  - A. Rotation
    - 1. Roating
    - 2. Nonrotating
  - B. Loading
    - 1. Cantilever
    - 2. Simple beam
      - a. Point support
      - b. Uniform support
- IV. Direct shear
  - A. Direct shear (rigid split box)
  - B. Double direct shear
  - C. Uniform direct shear (rigid caps with confined rubber membrane and split rings for lateral restraint)
  - D. Uniform strain direct-shear (hinged box)
  - E. Punching shear
- V. Torsion
  - A. Pure torsion
  - B. Triaxial torsion
  - C. Specimen shape
    - 1. Solid cylinder
    - 2. Thick-walled, hollow cylinder
- VI. Indirect
  - A. Penetration tests
  - B. Squeeze tests
  - C. Marshall stability
  - D. Angle of repose
  - E. Others

Possible specimen shapes are enumerated in the following.

- I. Rectangular parallelepiped
  - A. Short
  - B. Long
  - C. Cubic
- II. Cylinder
  - A. Solid
    - 1. Short
    - 2. Long
  - B. Thick-walled, hollow
    - 1. Short
    - 2. Long
- III. Plate
- IV. Other

STRENGTH

Strength represents the limiting or failure response of materials to load. In general, pavement materials can fail in one of three ways: (a) fracture due to excessive tensile loads (induced primarily by traffic or thermal gradients), (b) fracture due to repetitively applied tensile loads having magnitudes less than the ultimate tensile strength (fatigue), and (c) slip or relative displacement due to the action of shearing stresses. (The manner in which pavements fail or become distressed is discussed in other papers included in this Special Report.) Crushing failures involving individual aggregate particles are of no significance in pavement systems.

Tensile Strength

Only those bound components of the pavement structure are capable of withstanding significant tensile stresses without rupturing. Of these, only the asphalt paving mixture is considered here. Tensile strength of asphalt mixtures is considered important in three areas of design, including (a) fracture due to the single application of a large, normally applied load; (b) slippage, such as might be induced by large braking forces; and (c) thermal cracking.

Kennedy and Hudson (55) have reviewed tensile testing equipment and procedures that have been applied to highway materials including the direct tensile test (specimen usually cemented by an epoxy resin to end caps), beam tests (including simply supported beams and the cohesiometer), and indirect or splitting tensile tests. They concluded that the indirect test was the most appropriate for examining tensile properties of highway materials. Regardless of the test equipment used, the tensile strength is usually defined as either the peak stress on the resulting stress-strain curve or the stress at break.

Table 1 gives a number of variables that are known to influence the tensile strength of asphalt mixtures. Most notable among these variables are probably rate of loading and temperature except at low temperatures, where rate of loading and temperature increments have minor effects. Tensile strengths at low temperatures have been reported to range from about 500 to 1,400 psi (24).

Of interest also is the strain or elongation that occurs at the point of rupture. Monismith and colleagues (78) suggest that an appropriate means for examining this strain is a double logarithmic plot of rupture stress (factored by the ratio of 293 to the absolute temperature) versus strain at rupture. Their data show that a maximum rupture strain of about 1 percent occurs at an intermediate rupture stress level and that minimum rupture strains of about 0.1 percent occur at both larger and smaller rupture stress levels. For the conditions investigated by Haas and Anderson (33), the fracture strain decreased monotonically as fracture stress increased. In any case it appears that the minimum fracture strain occurs at low temperatures and is on the order of 0.1 percent (24). Table 2 gives some of the variables influencing the tensile strain at rupture.

Finally, it may be possible to relate tensile strength to mixture stiffness (a measure of deformability). Haas and Anderson (33) show that there can exist an optimum stiffness modulus that yields a maximum tensile strength.

Fatigue Strength

The repetitive application of tensile stresses having magnitudes less than the tensile strength can ultimately cause fatigue cracking in bound materials. Attention is limited here to a review of the fatigue behavior of asphalt paving mixtures.

In recent years, a number of laboratory investigations have been initiated to define the fatigue behavior of asphalt paving mixtures, to determine the causes of observed fatigue behavior, and to ascertain the influence of mixture variables on this behavior. These investigations have used a variety of specially built equipment. The range of equipment capabilities and test configurations is indicated as follows:

1. Mode of loading—controlled stress, controlled strain, and intermediate but undefined modes;

TABLE 1

TENSILE STRENGTH OF ASPHALT MIXTURES

Variable	Change in Variable	Effect on Tensile Strength	Reference	Remarks
Loading				
Rate of loading	Increase	Increase	21, 40, 42, 47, 55, 119	Effect increases as temperature increases
Type of test	Change	Change	53	Strength significantly greater for splitting tests than direct tests
Mixture				
Asphalt source	Change	Change	33	
Asphalt content	Increase	Increase	34	3.5 to 7.0 percent
	Increase	Optimum	40	
Asphalt hardness	Increase	Increase	34, 47	
	Change	Change	42	50 to 110 penetration; possible maximum
Compaction temperature	Increase	Increase	34	200 to 300 F
Mixing temperature	Increase	Increase	34	250 to 350 F
Type of compaction	Change	Change	34	Stronger for impact than for gyratory shear
Aggregate type	Change	Change	34	Stronger for crushed limestone than for rounded gravel
Mineral filler type	Change	Change	37	Asbestos mineral filler produced considerably higher strength at low temperatures
Filler-bitumen ratio	Increase	Optimum	20	
Environment				
Temperature	Decrease	Increase	21, 33, 40, 47, 53, 119, 125	Particularly significant in range of 20 to 80 F; relatively unaffected at low temperatures

2. Specimen shape—cylindrical, beam, and circular with reduced cross section, plate, and trapezoidal beam;
3. State of stress—uniaxial, biaxial, and triaxial;
4. Load type—sinusoidal and pulsating;
5. Load frequency—3 to 3,000 cycles per min; and
6. Specimen support—rigid, spring, and pressurized fluid.

TABLE 2

TENSILE RUPTURE STRAINS OF ASPHALT MIXTURES

Variable	Change in Variable	Effect on Rupture Strain	Reference	Remarks
Loading				
Rate of loading			119	No clear effect
Mixture				
Asphalt content	Increase	Increase	24, 119	
Filler-bitumen ratio	Increase	Optimum	20	
Asphalt hardness	Increase	Decrease	42	
Environmental				
Temperature	Increase	Increase	33, 42, 119	

Each particular apparatus has its special advantages and, of course, its limitations. None is sufficiently universal to warrant its adoption as a standard testing device, and all must be considered as research tools having specific and limited uses. Because of this, extreme care must be exercised for the valid interpretation of test data and for their use in pavement design.

The following represent the current state of knowledge of fatigue of asphalt paving mixtures.

1. Under conditions of constant air void and asphalt contents, the determinant of fatigue failure is the maximum principal tensile strain in the mixture (90).

2. Mixture stiffness plays a dominant role in determining fatigue behavior, and any factor influencing mixture stiffness may likewise affect fatigue behavior.

3. Damaging fatigue loading causes a reduction in flexural strength (modulus of rupture) (77). Mixture stiffness is likewise decreased by fatigue loading, but the magnitude of the decrease is a function of the method for measuring stiffness (6, 14, 90).

4. Mode of loading has a tremendous effect on fatigue behavior. For identical initial stresses and strains, the fatigue life (number of repetitions to failure) is considerably larger for controlled-strain loading than for controlled-stress loading (75).

5. The roles of mixture stiffness and mode of loading are interrelated. For controlled-stress loading, specimens exhibiting the largest initial stiffnesses tend to perform most satisfactorily (largest fatigue life at a given stress level) as long as the mixture is nonbrittle and has a reasonable balance among the proportions of its constituent materials. The reverse appears to be true for controlled-strain loading (75).

6. For controlled-stress loading, the mean fatigue life, $\overline{N}$, is related to the applied stress level, σ, as follows:

$$\overline{N} = K_1(1/\sigma)^{c_1} \tag{1}$$

where K_1 and C_1 are constants. There is no evidence of an endurance limit (a limiting value of stress below which a material can endure an infinite number of stress cycles without failure) up to at least 10^8 load applications (15, 52, 90). If the mixture is linear, then the following equation is also valid

$$\overline{N} = K_2(1/\epsilon)^{c_2} \tag{2}$$

where K_2 and C_2 are constants and ϵ is the initial tensile strain. Similar equations can be used to define the simple-loading fatigue behavior under controlled-strain loading; however, the possible existence of an endurance limit under such loading is unknown (75).

7. In evaluating variables whose effects on fatigue behavior can be explained primarily in terms of associated effects on mixture stiffness, all data obtained under controlled-stress loading can often be represented by a single equation, Eq. 2. Thus, Pell and Taylor (93) found it possible to express the effects of temperature and speed of loading in the single form of Eq. 2, and Bazin and Saunier (6) and Kirk (56) used a similar procedure to evaluate the effect of temperature.

8. The constants of Eqs. 1 and 2 depend on mixture composition, conditions of testing, and the definition of failure. Recently reported values for the exponent C_1 in the controlled-stress mode of loading include 2.5 to 5.9 for an asphalt concrete depending on asphalt type (96); 5.3 to 5.9 for linear mixtures and 1.6 to 3.5 for nonlinear mixes depending on asphalt penetration (93); and 5.2 for a sandsheet mixture (6).

9. The linear summation of cycle ratios governs fatigue behavior of asphalt mixtures that are subjected to multiple strains of varying and random magnitude (15). This means that at failure

$$\sum_i (n_i/N_i) = 1 \tag{3}$$

where n_i is the actual number of applications of strain ϵ_i under the compound loading and N_i is the number of applications of strain ϵ_i that would have resulted in failure under simple loading in which ϵ_i was repetitively applied by itself.

10. Fatigue test data exhibit extreme variability as compared with other testing methods. The fatigue life of specimens tested in simple, controlled-stress loading under supposedly identical testing conditions can be approximated by the logarithmic normal probability distribution (93).

11. On the basis of limited evidence it is possible that rest periods may be beneficial for asphalt paving mixtures depending on the length of the period, the temperature, the characteristics of the mixtures, and the stress conditions existing within the mixture (6, 90). Peel (93) recently concluded that possible beneficial effects will be observed only for mixes made with very soft binders and rested at high temperatures under a compressive stress.

12. A list of factors known to influence fatigue behavior of asphalt mixtures is given in Tables 3 and 4. Monismith and Deacon (75) give explanations of most of these effects.

13. It has recently been proposed that the effects of asphalt viscosity, temperature, mode of loading, and presumably rate of loading for simple forms of testing could be expressed as

$$N_1 = K\, N_0\, (\eta_0/\eta_1)^{\alpha}\, (1/\epsilon)^{c_1} \tag{4}$$

where N_1 is fatigue life at temperature T_1; K is a constant depending on mix; N_0 is an experimentally determined fatigue life at some convenient, standard temperature, T_0; ϵ is dynamic tensile strain in mix; η_0 is asphalt viscosity at T_0; η_1 is asphalt viscosity at T_1; α varies between +1 (controlled strain) and -1 (controlled stress) depending on mode of loading; and n varies between 4 and 6 depending on the type of fatigue test used (99). Extensive future work is warranted to examine the validity of Eq. 4 and to develop it into a working design tool.

Shear Strength

Shear failures can occur in one or more pavement layers if the imposed loads are sufficiently large that the shear stresses exceed the shear strengths. Such failures are perhaps the most catastrophic of all pavement failures inasmuch as large relative dis-

TABLE 3

FATIGUE BEHAVIOR OF ASPHALT MIXTURES FOR CONTROLLED-STRESS LOADING

Variable	Change in Variable	Fatigue Life at Given Stress	Reference	Remarks
Loading				
Rate of loading	Increase	Increase	90, 93	Stiffness increased for loading conditions used
	Increase	Decrease	14	Stiffness decreased for loading conditions used
	Increase	Constant	77	Stiffness constant for loading conditions used
Load duration	Increase	Decrease	14	For pulsating loads
Mixture				
Asphalt content	Increase	Optimum	19, 51, 93	Optimum asphalt content depends on aggregate type and gradation
Air void content	Increase	Decrease	6, 14, 19, 52, 95	
Air void structure			19	Important
Asphalt type			19	Effects primarily mixture stiffness
Asphalt hardness	Increase	Increase	6, 19, 51, 72	
Aggregate type			6, 19, 51	Effect of surface texture and shape small at same asphalt content
Aggregate gradation	Increase	Increase	19, 75	Open to dense-graded
			93	No great effect
Mineral filler	Increase	Optimum	93	
Environmental				
Temperature	Increase	Decrease	75, 93	

TABLE 4

FATIGUE BEHAVIOR OF ASPHALT MIXTURES FOR CONTROLLED-STRAIN LOADING

Variable	Change in Variable	Fatigue Life at Given Strain	Reference	Remarks
Loading				
Strain reversal			77	No effects observed, maximum strain governs
			56	Effects observed, difference in maximum and minimum strains governs
Mixture				
Asphalt content	Increase	Increase	90	Mixture stiffness decreased with increasing asphalt content
Asphalt hardness	Increase	Decrease	75	
			56	No effect observed
Asphalt type	Change	Change	75	
	Change	Change	96	Asphalts yielding stiffer mixes result in reduced fatigue life
Air void content	Increase	Decrease	96	
Aggregate gradation and type			56	Negligible effect
Environmental				
Temperature	Increase	Increase	90, 96	

placements can occur along the slip surfaces. McLeod (67, 68, 69) and Hewit (44, 45, 46, 47) are among those having investigated this type of failure in a fundamental way.

Shear strength is defined as the peak or ultimate stress of a material in shear. It may be measured by direct shear tests or numerous other tests including the triaxial compression test (62, 120). The shear strength is normally interpreted by means of the Mohr-Coulomb failure law, which considers the shear strength to be composed of two parts, one of which is a frictional component that is proportional to the normal stress on the shear surface and the other of which is a cohesive component that is independent of normal stress. Stated mathematically, the Mohr-Coulomb failure law is

$$\tau_f = c + \sigma_f \tan \phi \tag{5}$$

where

τ_f = shearing strength,
σ_f = normal stress on the failure surface at failure,
c = cohesion, and
ϕ = friction angle or angle of shearing resistance.

Although other failure theories have been proposed, it seems likely that the Mohr-Coulomb theory will continue to be effectively used for some time.

The shear strength can be calculated directly from the results of direct shear tests. The shear strength in a triaxial compression test is $(\sigma_1 - \tau_3)_f/2$, where the subscript f refers to the failure condition along a shear plane. This strength is related to c and ϕ as follows (68):

$$(\sigma_1 - \sigma_3)_f/2 = \frac{\sigma_3 \sin \phi}{1 - \sin \phi} + 2c\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}} \tag{6}$$

For the unconfined compression test, the best estimate of the shear strength is $(\sigma_1)_f/2$, which is related to c and ϕ as follows (68):

$$(\sigma_1)_f/2 = 2c\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}} \tag{7}$$

The properties c and ϕ can, for some materials, be evaluated from tension and unconfined compression tests (40). They can also be evaluated from Hveem stabilometer results (89), though perhaps erroneously (40), and from direct shear tests. They are normally evaluated, however, from triaxial compression tests in which a number of identical specimens are tested at different confining pressures, and observations are made of stresses developed at the peak of the stress-strain curves. Mohr circles are constructed to represent the states of stress at failure, and the Mohr failure envelope is drawn tangent to the Mohr circles. The Mohr envelope is generally curvilinear but is usually approximated by the linear relationship of Eq. 5. The properties c and ϕ are determined graphically from the linear approximation of the failure envelope.

Cohesive and Cohesionless Soils—Lambe and Whitman (63) have presented an extensive review of the shear strengths of both cohesive and cohesionless soils. Under drained test conditions and by using effective stress principles, the behavior of these materials can be summarized as follows:

> 1. In the ultimate condition, [that is, very large shearing strains in which any effects of overconsolidation are minimized] achieved after considerable shearing strain the strength behavior of soil is that of a frictional material. That is, the failure law is
>
> $$\tau_{ff} = \bar{\sigma}_{ff} \tan \bar{\phi}_{ult}$$
>
> The ultimate friction angle $\bar{\phi}_{ult}$ is related to the clay content of the soil. The angle is greatest (about 30°) in pure sand and least (as low as 3 or 4°) in pure clay.
>
> 2. At the point of peak resistance, the strength of a normally consolidated soil is also given by a frictional type of failure law,
>
> $$\tau_{ff} = \bar{\sigma}_{ff} \tan \bar{\phi}$$
>
> The angle $\bar{\phi}$ is related to the clay content of the soil. For loose sands $\bar{\phi}$ and $\bar{\phi}_{ult}$ are equal. As the clay content increases $\bar{\phi}$ exceeds $\bar{\phi}_{ult}$ since at the peak resistance the clay platelets within the failure zone have not yet reached a fully oriented, face-to-face alignment.
>
> 3. Densification increases the peak strength of soils. For soils with a significant clay content, large stresses suffice to produce an overconsolidated soil, while stresses alone do not effectively densify predominantly granular soils and cycles of loading and unloading are necessary. The failure envelope for densified soils generally is curved, but for practical calculations the peak strength can be represented by a linear relation,
>
> $$\tau_{ff} = \bar{c} + \bar{\sigma}_{ff} \tan \bar{\phi}$$
>
> As $\bar{\sigma}_{ff}$ increases, $\bar{c}$ increases and $\bar{\phi}$ decreases.
>
> 4. Any change in effective stress changes the density at failure as well as changing the shear strength. Conversely, any action that changes the density at failure must produce a change in shear strength. For the ultimate condition, there is a unique relationship among effective stress $[(\bar{\sigma}_1 + \bar{\sigma}_3)_f/2]$, shear strength $[(\bar{\sigma}_1 - \bar{\sigma}_3)_f/2]$, and water content, such that knowledge of any of these three quantities specifies the other two quantities. [For saturated soils, this unique relationship continues to hold for all types of loading and drainage conditions.] At the peak resistance, this three way relation is not unique . . . .
>
> 5. Capillary tensions must be taken into account when determining the effective stresses within soils located above the water table. Because large capillary tensions are possible in clayey soils, such soils can exhibit a large apparent cohesion even though they produce little or no cohesion intercept $\bar{c}$. This apparent cohesion is fully explained by effective stresses.

For saturated soils, the undrained strength may either exceed or be less than the drained strength depending on the type of loading and the degree of overconsolidation. However, differing values of strength (undrained versus drained) can be explained by differences in effective stresses. The undrained shear strength increases with decreasing moisture content, increasing consolidation stress, and increasing maximum past consolidation stress (63).

Tables 5 and 6 give the effects of selected sets of variables on the shear strengths of cohesive and cohesionless soils respectively.

TABLE 5

SHEAR STRENGTH OF COHESIVE SOILS

Variable	Change in Variable	Effect on Strength	Reference	Remarks
Loading				
Rate of loading	Increase	Decrease	122	Drained tests of saturated soils
	Increase	Increase	122	Undrained tests of saturated soils
Overconsolidation	Increase	Increase	63, 122	
Stress path			63	No effect for drained triaxial tests
Effective stress at failure = $(\bar{\sigma}_1 + \bar{\sigma}_3)_f/2$	Increase	Increase	63	Drained triaxial tests
Intermediate principal stress			63	Small effect on drained strength
Mixture				
Compaction energy	Increase	Increase	85	Impact compaction at constant molding moisture content
Adsorbed ions			122	Nature of ions affects strength depending on moisture content
Remolding	Increase	Decrease	122	
Plasticity	Increase	Decrease in ϕ	63	Drained tests based on effective stresses
Void ratio	Increase	Decrease	63, 122	

Asphalt Mixtures—Fundamental measures of shear strength of asphalt mixtures have been examined by numerous investigators (8, 27, 29, 39, 40, 44, 45, 46, 47, 66, 67, 68, 69, 80, 89, 112). Most of these investigators used a triaxial apparatus to evaluate the properties c and ϕ. Table 7 gives some of the more significant test results.

Indirect Strength Measures

Prior discussion has examined the strength of pavement materials from a rather fundamental point of view. Other more empirical studies and procedures have been extensively used to evaluate the relative strengths of pavement materials. These test procedures have certain advantages because of low cost, simplicity, speed, and, oftentimes, adaptability to field measurements. The tests include, among many others,

TABLE 6

SHEAR STRENGTH OF COHESIONLESS SOILS

Variable	Change in Variable	Effect on Strength	Reference	Remarks
Loading				
Type of test			63	ϕ is greater by about 2 deg from direct shear than from triaxial tests
Intermediate principal	Increase	Constant	63	
stress	Increase	Increase	26	Based primarily on theory
Rate of loading	Increase	Constant	63	
Number of load appli-	Increase	Increase	63	For loose sands only
cations	Increase	Decrease	63	For dense sands only
Confining pressure	Increase	Decrease	63, 114	
(minor principal stress)	Increase	Decrease	63, 114	
Mixture				
Void ratio	Increase	Decrease	63, 114, 122	
Aggregate gradation	Increase	Increase	63, 114	Increase means well graded
Aggregate angularity	Increase	Increase	63, 114	
Aggregate mineralogy			63	No effect unless micaceous or low crush resistance
Environmental				
Saturation			63, 114, 122	No effect

TABLE 7
SHEAR STRENGTH OF ASPHALT MIXTURES

Variable	Change in Variable	Effect on Strength	Reference	Remarks
Loading				
Rate of loading	Increase	Increase in c	8, 40, 47, 68	
	Increase	Decrease in ϕ	40, 47, 68	
	Increase	Constant ϕ	27	Theoretically constant ϕ (68)
Mixture				
Aggregate angularity	Increase	Increase in ϕ	27, 37, 40	
	Increase	Increase in c	27	
Aggregate gradation	Increase	Increase in ϕ	27	Increase means coarse to fine
Asphalt content	Increase	Optimum in c	8, 27, 40	
	Increase	Decrease in ϕ	8, 40	
Asphalt viscosity	Increase	Constant ϕ	27, 47	
	Increase	Increase in c	27, 47	
Environmental				
Temperature	Increase	Decrease in c	27, 40, 47	
	Increase	Increase in ϕ	40, 47	Slight
Immersion in water		Decrease in c	8	
		Decrease in ϕ	8	

California bearing ratio, cone penetrometer, Marshall stability, Hubbard-Field stability, and gyratory shear methods. These methods have served, and will continue to serve, in the judicious design of flexible pavements by empirical means. However, the possibility of their use in more rational design procedures is remote.

DEFORMABILITY

To enable the accurate estimation of stresses and strains in a pavement structure and the permanent cumulative deformations associated therewith, we must characterize the complex stress-strain-time behavior of pavement materials before the limiting or failure domain is reached. Of interest are two perhaps inseparable components of response: recoverable deformations and irrecoverable deformations.

Existing knowledge of the recoverable component of response is now quite well advanced and is sufficient for many useful engineering analyses of real-world problems including the dynamic response of pavements to traffic loads. At the same time, very little is known of the irrecoverable response of pavement materials to load not only for creep under long-time loading but also for the accumulation of irrecoverable deformations—both volume change (densification) and shape change—due to repetitive loading. Because rutting is one of the most prevalent forms of distress in flexible pavements and because a thorough knowledge of the irrecoverable response of pavement materials is essential to a rational analysis of the rutting problem, it would appear that the major deficiency in our current ability to characterize the mechanical behavior of these materials is in the area of irrecoverable deformations. However, recent analyses by Barksdale and Leonards (5) represent an important first step in developing an ability to predict rut depths. Operational moduli used in their linear viscoelastic analysis were derived from the results of creep tests (creep compliance as defined later) and repeated load tests (total axial strain versus number of load applications).

A second major deficiency is that most experimental investigations of mechanical response have used simplified loading patterns such as uniaxial stresses, which are quite different from the complex three-dimensional pattern to which materials are subjected in pavement structures.

Characterizing Deformability

The principal means for characterizing mechanical response is through use of a modulus defined as the ratio of stress to strain and a strain ratio, such as Poisson's ratio, for relating strains in mutually perpendicular directions. For complex materials the modulus or ratio will not be invariant but will depend on the level of stress (or strain) at which it is evaluated. Thus, additional definition is required, and the terms tangent modulus and secant modulus are often used. The tangent modulus is the slope of a tangent to the stress-strain curve; and, if the curve is not linear, the particular level of stress must be specified for the measure to be meaningful. The secant modulus is the slope of a line connecting two points of the stress-strain curve; and, to be meaningful, both points must be explicity stated. Most moduli used to characterize pavement materials are secant moduli.

Meaningful characterization of pavement materials can be accomplished in one of two ways depending on the complexity of the behavior. If the material behaves as any one of a number of idealized models, it is possible to devise suitable test procedures that evaluate fundamental material properties. These properties are sufficient in themselves to fully characterize the behavior of the material in a structure subject to any loading condition that does not induce failure. In the case of a linearly elastic isotropic material, these properties are Young's modulus, E, and Poisson's ratio, μ. Young's modulus would be evaluated by the ratio of stress to strain and would be constant for all conditions and all levels of stress, strain, and time.

On the other hand, as is the usual case for pavement materials, it may be impossible to identify an idealized model that adequately represents the measured material response. Measured moduli thus represent derived material properties for these materials. It is well to emphasize that such derived material properties are extremely useful in engineering analyses but only if the loading conditions in the structure are adequately simulated by the test procedure (or if the boundary conditions of the test closely approximate those on a small element within the structure).

In summary, then, mechanical response of pavement materials is most often specified in terms of a secant modulus that, by virtue of the complexity of these materials, is a derived property useful for analysis only when the testing procedure adequately simulates the real-world loading conditions.

Deformability Under Constant Loads—Creep tests and relaxation tests are used to examine the behavior of time-dependent materials under constant loading. In the creep test, a stress, σ_0, is applied "instantaneously" and maintained constant throughout the test duration, and the resulting strain is measured as a function of time, $\epsilon(t)$. A measure of response is the creep compliance, D(t), where

$$D(t) = \epsilon(t)/\sigma_0 \tag{8}$$

A more conventional representation of this behavior is given by the creep modulus, $E_c(t)$, where

$$E_c(t) = \sigma_0/\epsilon(t) \tag{9}$$

In the stress relaxation test, an instantaneously applied strain, ϵ_0, is maintained at a constant level, and the stress is observed as a function of time, $\sigma(t)$. The time-dependent relaxation modulus, $E_r(t)$, is calculated by

$$E_r(t) = \sigma(t)/\epsilon_0 \tag{10}$$

In general, the creep modulus and the relaxation modulus will be unequal. However, if the material response is that of a linear viscoelastic material, it is possible by means of analytical techniques to convert from one modulus to the other (30, 31, 74). Neither the creep modulus nor the relaxation modulus would seem to be in a form suitable for pavement analysis and design except possibly to predict long-term deformations under static load.

Deformability Under Uniformly Varying Loads—The uniformly varying load category includes both constant-rate-of-strain tests ($d\epsilon/dt = c$) and constant-rate-of-stress tests ($d\sigma/dt = c$). Time-dependent response to these test conditions can be given in terms of a secant modulus, $E_S(t)$, or a tangent modulus, $E_T(t)$, where

$$E_s(t) = \sigma(t)/\epsilon(t) \tag{11}$$

and

$$E_T(t) = d\,\sigma(t)/d\,\epsilon(t) \tag{12}$$

In general the moduli obtained from stress-controlled tests will not equal those determined from strain-controlled tests. If the behavior is linearly viscoelastic, however, the principle of superposition may be used to convert from one to the other or to creep or relaxation moduli. The magnitudes of these moduli will generally depend on the rate of loading.

Because neither the constant-rate-of-stress nor the constant-rate-of-strain test accurately simulates pavement loadings, direct use of these moduli for pavement analysis and design is questionable. Their use to date, as with creep and relaxation tests, has been primarily to determine the extent to which pavement materials are (a) linearly viscoelastic and (b) thermorheologically simple. These test methods have also been used to examine the viscoelastic response of asphalt paving slabs under creep loading (102) and to evaluate the constants of various mechanical models of material behavior (76).

Deformability Under Sinusoidal Loads—A particularly convenient form of laboratory loading is that in which the loads vary sinusoidally with time. Consider the case where the applied stress level σ is given by

$$\sigma(t) = \sigma_0 \sin \omega t \tag{13}$$

where σ_0 is the stress amplitude, ω is the frequency, and t is time. The strain response to such a loading pattern in the steady-state condition will likewise be sinusoidal but will lag the stress by an angle ϕ, that is,

$$\epsilon(t) = \epsilon_0 \sin(\omega t - \phi) \tag{14}$$

where $\epsilon(t)$ is the strain at time, t; ϵ_0 is the strain amplitude; and ϕ is the phase angle or lag.

Under these conditions, it is possible to relate the stresses and strains by a complex number called the complex modulus, E^*, such that

$$E^* = E' + iE'' \tag{15}$$

where E' is the real part of the complex modulus, and E'' is the imaginary part of the complex modulus, and

$$E' = \frac{\sigma_0}{\epsilon_0}\cos\phi \tag{16}$$

and

$$E'' = \frac{\sigma_0}{\epsilon_0}\sin\phi \tag{17}$$

E', the in-phase component of the modulus, represents stored recoverable energy and is called the storage modulus. E'', the in-quadrature component of the modulus, represents the energy lost by internal friction within the material and is called the loss modulus.

The absolute value of the complex modulus, $|E^*|$, is

$$|E^*| = \sigma_0/\epsilon_0 \tag{18}$$

The ratio of the stress and strain amplitudes (Eq. 18) has also been called the stiffness modulus (6). The complex modulus is fully specified in terms of its absolute value, $|E^*|$, and its argument, the phase lag, ϕ.

For a linear viscoelastic material, the complex modulus is a fundamental material property that varies with the frequency of load application (87). A knowledge of the complex modulus and the complex Poisson's ratio, both of which may vary with frequency, is sufficient to characterize the behavior of a linear, isotropic, viscoelastic material. Once these properties are established, it is theoretically possible to describe the response of the material to any given loading pattern by means of Fourier and Laplace transforms. Papazian (87) has discussed the utility of such measures of response and indicated how the complex modulus can be evaluated by means of dynamic tests or static (creep) tests. The latter requires a graphical procedure for evaluation.

Another means for determining a dynamic modulus under sinusoidal loading is to measure the resonant frequency of a vibrating specimen-machine system. The modulus so determined is the real component, E', of the complex modulus and is always less than the absolute value of the complex modulus for nonzero phase lags (92). Hardin and Drnevich (35, 36) have extensively tested soils in this way and have characterized their behavior in terms of the shear modulus and the damping ratio, D, which is directly proportional to the phase lag, ϕ.

Deformability Under Pulsating Loads—Others, most notably Seed and others (106, 107), have subjected specimens to dynamic loads of a pulsating nonreversing form. Most of these repeated load tests have been of the triaxial compression variety. The measure of response is the modulus of resilient deformation, M_r, where

$$M_r = \sigma_d/\epsilon_r \tag{19}$$

In this equation, σ_d is the repetitively applied deviator stress and ϵ_r is the resilient or recoverable axial strain corresponding to a specific number of load applications.

A major advantage of this method of testing is that the loading pattern may be selected to simulate that occurring in the pavement structure. Hence, the modulus of resilient deformation, though not usually conceived as a fundamental material property, can be used directly in analytical investigations.

Some Concluding Remarks—This discussion indicates that one may characterize the behavior of pavement materials in numerous ways depending in part on the nature of the problem and in part on personal preferences. It must be emphasized, however, that in most cases pavement materials do not possess idealized properties and that the measured properties are often significantly influenced by the test procedures and equipment. It is important, therefore, for laboratory procedures to simulate to as great a degree as possible actual field loading conditions. Test procedures that result in nearly homogeneous stress and strain states are necessary to investigate the properties of a small volume element.

Cohesive Soils

Dehlen (16) states:

> A clay subjected to stress shows immediate and time-dependent recoverable and permanent strains, the immediate strains being predominant under short-duration loads, and the permanent strain per cycle decreasing to an insignificant amount after many cycles of stress. Stress history may have a significant effect on the response. The response is markedly non-linear.

The nonlinear response of cohesive soils is evidenced in two ways. First, the stiffness of these materials is dependent on the initial stress state and increases as the effective mean principal stress increases. Second, and more important, the stiffness decreases with an increase in the incremental stress amplitude (deviator stress in triaxial tests). These and other effects of testing variables on the stiffness and damping of cohesive soils are given in Tables 8 and 9. The results of most investigations of the major effects are in common accord.

TABLE 8

STIFFNESS OF COHESIVE SOILS

Variable	Change in Variable	Effect on Stiffness	Reference	Remarks
Loading				
Number of cycles	Increase	Decrease	35, 117	
	Increase	Minimum	79, 106	Minimum at 1 to 5,000 cycles
Incremental strain amplitude	Increase	Decrease	35, 50, 117	Rate of decrease depends on maximum stiffness and shear strength
Incremental stress amplitude	Increase	Decrease	16, 17, 53, 63, 79, 106	Rapid decrease at low stresses
Effective initial mean principal stress	Increase	Increase	35, 50, 53, 63	Effect depends on stress or strain amplitude
Transverse stress			16, 17	No effect
Initial octahedral shear stress			35	Effect negligible after 10 cycles
Frequency of loading	Increase	Increase	10	Effect minor above 0.1 cps
	Increase	Increase	35	
Strain rate	Increase	Increase	63	
Overconsolidation ratio	Increase	Increase	35, 63	
			50	Any effect can be explained on basis of effective pressure and void ratio
Stress path			63	Large dependency
Mixture				
Soil disturbance	Increase	Decrease	42, 63	
Void ratio	Increase	Decrease	35	
	Increase	Decrease	50	Maximum effect at low confining pressure
Dispersion	Increase	Decrease	63, 79	At small strains
Structure			35	Little effect on maximum shear modulus
Degree of saturation at compaction	Increase	Decrease	79	Modulus of resilient deformation
	Increase	Maximum	85	
Plasticity	Increase	Decrease	63	
Compaction energy	Increase	Maximum	85	Impact compaction
	Increase	Increase	42	
Environmental				
Aging	Increase	Increase	63	
Degree of saturation	Increase	Decrease	10, 35, 42, 79	
Time (thixotropy)	Increase	Increase	35, 79	Recovery after high amplitude cyclic loading or many load cycles
Densification	Increase	Increase	79	
Time (during secondary compression)	Increase	Increase	50	Bentonite

TABLE 9

DAMPING OR PHASE ANGLE OF COHESIVE SOILS

Variable	Change in Variable	Effect on Damping or Phase Angle	Reference	Remarks
Loading				
Number of cycles	Increase	Decrease	35	Up to about 50,000 cycles beyond which damping increases
Incremental strain amplitude	Increase	Increase	35	Very rapid increase but tends to reach a maximum at large strains
Effective initial mean principal stress	Increase	Decrease	35	
Initial octahedral shear stress	Increase	Increase	35	
Frequency of loading	Increase	Increase	35	Effect minor above 0.1 cps
Mixture				
Void ratio	Increase	Decrease	35	
Molding water content	Increase	Minimum	85	
Compaction energy	Increase	Minimum	85	Impact compaction
Environmental				
Time (thixotropy)	Increase	Change	35	Recovery after high amplitude cyclic loading

The following observations are also of importance in understanding the behavior of cohesive soils:

1. The cohesive soil investigated by Dehlen (17) was found to be initially cross-isotropic with the horizontal stiffness exceeding the vertical stiffness. In addition, a significant degree of stress-induced cross isotropy was observed.
2. Depending on the testing conditions, Poisson's ratio remained constant or increased slightly with increasing applied compressive stresses but was independent of the transverse stress (17).
3. The effect of the void ratio, e, on the maximum (low strain) dynamic shear modulus for void ratios less than about two is given by (35)

$$F(e) = (2.973 - e)^2/(1 + e) \tag{20}$$

4. Soil structure as affected by molding water content and type of compaction can have a significant effect on stiffness, as discussed by Monismith et al. (79):

> . . . samples compacted "wet of optimum" for a particular compactive effort by static means have similar resilience characteristics to those compacted "dry of optimum" by kneading compaction and subsequently soaked to a similar degree of saturation. . . [and hence have] resilient characteristics similar to those observed in field specimens for the same conditions of test.

5. As given in Table 8, most investigators have found the behavior of cohesive soils to be highly nonlinear. At the same time, Pagen and Jagannath (85) observed linear viscoelastic behavior in unconfined axial creep for unsaturated compacted clays up to an axial stress of 20 to 24 psi. Application of confining pressures was found to extend the range of linearity.
6. Coffman (10) found that the complex moduli obtained from dynamic tests tend to be larger than those obtained from creep tests. Although this difference was attributed to the effects of stress history, it may have been related to a disregard of inertial effects associated with the dynamic tests.

Cohesionless Soils

Regarding cohesionless soils, Dehlen (16) states:

> . . . the response of sand is primarily instantaneous and time independent. Large permanent strains may occur during the first cycle of stress, but the behavior becomes almost elastic after many cycles. The effects of stress history are generally less marked than in the case of clays, and the stress-strain response is non-linear.

Tables 10 and 11 give the effects of some testing variables on the stiffness and damping respectively of cohesionless soils. A primary loading variable is that of the initial mean effective principal stress, $\bar{\sigma}_0$, which affects the stiffness, S, as follows:

$$S = K\,\sigma_0^{\,n} \tag{21}$$

where K is a constant and n is an exponent that varies from about 0.4 to 1.0 (63). A larger exponent is observed for less dense materials (63) and for larger strain amplitudes (35). Increases in the incremental stress amplitude cause reductions in stiffness, another evidence of the nonlinearity of these materials. Effects of the deviatoric component of the initial stress state are negligible for repeated loading.

The stress-strain curve under cyclic loading is characterized by a hysteresis loop that stabilizes after 10 to 50 cycles following which there is little or no additional permanent strain for each cycle of loading (63). During the initial cycles, however, a net compressive strain is developed under triaxial compression loading (63). The hysteresis loop is an indication of the degree of damping in the material.

When sheared, a loose sand generally contracts in volume until failure is approached, at which time expansion is observed. Dense sands expand even at low strains and continue to expand as failure is approached. For sands of all initial densities, the rate of expansion increases near failure (58, 63).

Cohesionless soils are probably more isotropic than other pavement materials as evidenced in tests on Ottawa sand by Ko and Scott (59). Ko and Scott also found that

TABLE 10

STIFFNESS OF COHESIONLESS SOILS

Variable	Change in Variable	Effect on Stiffness	Reference	Remarks
Loading				
Incremental stress amplitude	Increase	Decrease	63	
Incremental strain amplitude	Increase	Decrease	35	Rapid decrease
Number of cycles	Increase	Increase	35, 63	Approaches a maximum
Load duration	Increase	Decrease	103	Pulsating loads
Loading rate or frequency	Increase	Constant	63	No effect after first few load cycles
	Increase	Constant	35, 63	0 to a few hundred cps
Initial effective mean principal stress	Increase	Increase	35, 63	
Initial octahedral shear stress	Increase	Decrease	35	Very small effect after 10 load cycles
Mixture				
Void ratio	Increase	Decrease	35, 57, 63, 121	
Environmental				
Degree of saturation	Increase	Constant	35, 63	Effective stresses must be used

TABLE 11

DAMPING OR PHASE ANGLE OF COHESIONLESS SOILS

Variable	Change in Variable	Effect on Damping or Phase Angle	Reference	Remarks
Loading				
Incremental strain amplitude	Increase	Increase	35	Very rapid increase but tends to reach a maximum at large strains
Number of cycles	Increase	Decrease	35	Up to about 50,000 cycles beyond which damping increases
Initial effective mean principal stress	Increase	Decrease	35	
Initial octahedral shear stress	Increase	Increase	35	
Frequency of loading	Increase	Constant	35	0 to a few hundred cps
Mixture				
Void ratio	Increase	Decrease	35	
Environmental				
Degree of saturation	Increase	Constant	35	

behavior in their soil test box was quite different from that observed in conventional triaxial tests (59) and in one-dimensional compression tests (57).

Finally, the influence of void ratio for ratios less than about two was found by Hardin and Drnevich (35) to be adequately described by Eq. 20.

Untreated Granular Aggregate

The behavior of untreated granular aggregate is quite similar to that of the smaller sized cohesionless soils. However, a comparison of Table 12 for untreated granular aggregates with Table 10 for cohesionless soils reveals some behavioral differences such as observed for the effects of incremental stress level. Whether these differences indicate fundamental differences in behavior of the two types of materials or just differences in test equipment and procedures is unknown.

In any case the effect of initial confining pressure or mean initial effective principal stress is of paramount importance. The modulus of resilient deformation, M_r, is related to the initial stress state as follows:

$$M_r = K\ \sigma_3^{\,n} \tag{22}$$

and

$$M_r = K'\ \bar{\sigma}_0^{\,n'} \tag{23}$$

where K, n, K′, and n′ are material constants, σ_3 is the confining pressure in a triaxial test, and $\bar{\sigma}_0$ is the mean initial effective principal stress (17, 48, 53, 79). The constant, n, is said to vary from 0.35 to 0.55 and n′ from 0.35 to 0.6 (17). Any nonlinearities under cremental loading would appear to be relatively insignificant as long as the stress increments are small and shear failure is not approached.

Poisson's ratio, μ, increases with (a) decreasing confining pressure, (b) increasing incremental stresses, (c) decreasing fines, and (d) decreasing degree of saturation (48). It is apparently affected little by density variations and can be estimated from an equation of the following type (48):

$$\mu = A_0 + A_1(\sigma_1/\sigma_3) + A_2(\sigma_1/\sigma_3)^2 + A_3(\sigma_1/\sigma_3)^3 \tag{24}$$

in which A_i is constant.

Asphalt Paving Mixtures

About asphalt paving mixtures, Dehlen (16) states the following:

> The response of asphalt concrete to stress is influenced to a pronounced degree by time. Asphalt concrete under stress exhibits immediate followed by time-dependent strain, both of which may be partly recoverable and partly permanent. The time-dependent response may be viscous or non-viscous. Under stresses of short duration, such as experienced under moving traffic, the in-

TABLE 12

STIFFNESS OF UNTREATED GRANULAR AGGREGATE

Variable	Change in Variable	Effect on Stiffness	Reference	Remarks
Loading				
Initial confining pressure	Increase	Increase	17, 48, 79	Triaxial compression
Initial effective mean principal stress	Increase	Increase	17, 79	
Incremental stress level	Increase	Constant	17, 79	As long as shear failure does not occur
	Increase	Increase	48	Slight increases as long as shear failure does not occur
Loading rate or frequency	Increase	Increase	10, 12, 79	Small increase
Load duration	Increase	Constant	48	0.1 to 0.25 sec
Number of cycles	Increase	Constant	48	After 50 to 100 applications of in-service stress levels
Drainage	Increase	Constant	48	
Mixture				
Void ratio	Increase	Decrease	12, 48	
	Increase	Decrease	10	At low moisture contents
	Increase	Increase	10	At high moisture contents
Angularity and surface roughness	Increase	Increase	48	
Fines	Increase	Decrease	48	
			38, 79	Minor effect
Compaction water content	Increase	Decrease	53	
Environmental				
Degree of saturation	Increase	Decrease	10, 12, 38, 48, 79	

stantaneous strains form a large proportion of the total. In a material exhibiting these time-dependent and permanent strains, stress history will influence the response. For adequately designed pavements, previously subjected to many cycles of stress, the permanent strains due to a single stress cycle are very small. The moduli of asphalt concrete vary with stress intensity, and the moduli in tension differ from those in compression. The material is thus non-linear, and this is true even at low strains. The non-linearity becomes more pronounced with increasing temperature.

These introductory remarks certainly indicate the complexity of the mechanical behavior of asphalt paving mixtures. Tables 13 and 14 give the effects of some of the variables that influence this behavior. Of particular interest and perhaps some controversy is the extent of linearity. The effects of stress amplitude as given in Table 13 are certainly inconclusive and indicate observations of both linear and nonlinear behavior.

Monismith and others (74) have observed reasonably linear behavior for uniaxial loading as long as the strains are less than 0.1 percent. Sayegh (98) observed that the domain of linearity for a particular mix was limited to deformations of less than 4×10^{-5},

TABLE 13

STIFFNESS OF ASPHALT PAVING MIXTURES

Variable	Change in Variable	Effect on Stiffness	Reference	Remarks
Loading				
Frequency of loading	Increase	Increase	6, 10, 53, 87, 90, 93, 98	Sinusoidal loading, stiffness approaches asymptote at high frequencies
	Increase	Decrease	14	Pulsating (incomplete stress relaxation)
Incremental stress amplitude	Increase	Constant	79	100 to 125 psi flexure
	Increase	Constant	53, 111	17.5 to 70 psi compression
	Increase	Constant	87	4 to 33 psi compression
	Increase	Constant	49	20 to 35 psi compression
	Increase	Decrease	53	50 to 200 psi flexure (80 F)
	Increase	Decrease	92	Rate of decrease less with increasing amplitude
	Increase	Decrease	32, 115	
Number of cycles	Increase	Decrease	53	
	Increase	Constant	33	Some effect at higher temperature (+30 C)
Strain history			92	No effect on dynamic torsional stiffness
Initial confining pressure	Increase	Increase	32, 115	
Mixture				
Air void content	Increase	Decrease	14, 19, 49, 52, 111	
	Increase	Decrease	6	Approximately linear relation between log stiffness modulus and void content
Asphalt content	Increase	Optimum	49	
	Increase	Decrease	111	4 to 6 percent asphalt at constant air voids
	Increase	Variable	111	For constant compaction, effect is variable depending on air voids and temperature
	Increase	Decrease	93	
Asphalt viscosity	Increase	Increase	93, 111	
Filler content	Increase	Increase	93, 111	
Environmental				
Temperature	Increase	Decrease	6, 10, 79, 92, 93, 98	Stiffness approaches asymptote at low temperatures

TABLE 14

DAMPING OR PHASE ANGLE OF ASPHALT PAVING MIXTURES

Variable	Change in Variable	Effect on Damping or Phase Angle	Reference	Remarks
Loading				
Frequency of loading	Increase	Maximum	87	Maximum at 4 rad/sec in range of 0.1 to 100 rad/sec
	Increase	Increase	53	4 to 14 cps
	Increase	Decrease	6	
			10	Effect variable depending on temperature
Mixture Variable				
Air voids	Increase	Decrease	6	Opposite effect observed if mix is initially weak
Environmental				
Temperature	Increase	Increase	53	Range of 40 to 70 F
	Increase	Increase	6	Maximum observed at high temperatures
			10	Variable depending on frequency

an extremely small range indeed. On the other hand, Krokosky, Tons, and Andrews (61) observed nonlinear viscoelastic behavior. Coffman, Ilves, and Edwards (11) observed that, for practical purposes, the moduli in tension and compression are equal, but others (14, 30) have found that stiffness in compression exceeds that in tension. Nonlinear behavior of the stiffening type has been observed under low levels of stress (49), short loading times (49), and compressive stresses (17). A softening nonlinearity has been observed under high levels of stress (49), long loading times (49), and tensile stresses (17). Pell and Taylor (93) aptly summarize the situation as follows:

> The non-linear behavior of a mix would appear to depend upon the properties of that mix and the environmental and loading conditions to which it is subjected, so making it difficult to give a particular value of stress or strain above which all mixes will exhibit non-linear behavior.

Table 15 gives some of the factors that affect the range of linearity of asphalt paving mixtures.

Asphalt mixtures in situ would appear to be anisotropic because of the layering and particle orientation inherent in the construction process. Dehlen and Monismith (17) concluded that the mixture they investigated was initially cross-isotropic with the horizontal stiffness exceeding the vertical stiffness. A significant degree of stress-induced cross isotropy was also observed. On the other hand, Coffman and others (11) concluded that, for practical purposes, asphalt concrete is isotropic in compression at the phenomenological level.

TABLE 15

RANGE OF LINEARITY OF ASPHALT PAVING MIXTURES

Variable	Change in Variable	Effect on Range of Linearity	Reference	Remarks
Loading				
Frequency of loading	Increase	Increase	93	
Type of loading			11	Linearity greater in compression than flexure
Mixture				
Void content	Increase	Decrease	93	
Asphalt content	Increase	Increase	93	
Asphalt viscosity	Increase	Increase	93	
Filler	Increase	Increase	93	
Environmental				
Temperature	Increase	Decrease	16, 17, 93	

Kallas and Riley (53) have demonstrated that observed moduli are sensitive to the testing procedure. They state:

> The difference between moduli determined by repeated load flexure and dynamic complex modulus test procedures may in part be attributed to different deformational responses: essentially recoverable for the complex modulus test procedures but only partially recoverable for the repeated load flexure test procedures.

Limited information is available concerning the effects of the testing variables on Poisson's ratio:

1. Poisson's ratio decreases for increasing sinusoidal frequencies above 4 cps (53, 98);
2. Poisson's ratio remains constant or increases slightly with increasing applied compressive stress but is independent of transverse stress (17);
3. Poisson's ratio is relatively insensitive to mixture type (53);
4. Poisson's ratio decreases as temperature decreases below 70 F (53, 98); and
5. Poisson's ratio is a real number that increases from 0.1 for high frequencies, low temperatures, and small deformations to 0.5 for low frequencies, high temperatures, and large deformations (98).

For a given bituminous mixture and variable temperature and frequency, there is a unique relationship between stiffness and phase angle such that there is only one phase angle corresponding to a given stiffness. This has been validated by complex modulus evaluations (6, 98).

Finally, it appears that asphalt paving mixtures are, to an engineering approximation, thermorheologically simple (6, 13, 60, 61, 78, 98). This means that there is an equivalence between time of loading and temperature, which enables the stiffness response of such a material to be presented in terms of a master curve (stiffness versus reduced time) and a shift factor curve (shift factor versus temperature). Such a property not only enables a simplified presentation of stiffness response but also enables an estimation of stiffness properties for an extended range of temperatures and times of loading beyond which it is inconvenient to perform laboratory tests.

Estimating Stiffness

Cohesive and Cohesionless Soils—By means of the equations and graphs presented by Hardin and Drnevich (36), it is possible to estimate for design purposes the dynamic shear modulus and the damping of both cohesive and cohesionless soils. Factors such as initial void ratio, overconsolidation ratio, mean effective principal stress, effective vertical stress, frequency of loading, number of loading cycles, plasticity index, and static strength parameters in terms of effective stresses ($\bar{c}$ and $\bar{\phi}$) are treated as independent variables in the estimation process.

A somewhat less sophisticated procedure for relating the dynamic stiffness, E, to an empirical strength measure, CBR, is

$$E = 100 \text{ CBR} \tag{25}$$

where E is the dynamic stiffness in kg/cm^2. Equation 25 represents a rough correlation based on field measurements and is claimed to be accurate within a factor of about two for fine-grained soils (42).

Untreated Granular Aggregate—The stiffness of an untreated granular aggregate in a pavement structure can be estimated rather crudely from a knowledge of the stiffness of supporting layers. Based on field measurements, Heukelom and Klomp (42) observed that the dynamic modulus of these materials can increase by a factor of roughly two from one compacted layer to the next. Otherwise, one could estimate the stiffness of untreated aggregate from a relationship such as that given by Eq. 23. Unfortunately, however, there is limited general knowledge concerning the relationships of the constants of Eq. 23, K' and n', to the mixture properties, and the work of individual investigators would have to be reviewed to obtain suitable estimates of these constants.

Asphalt Paving Mixtures—Bazin and Saunier (6) have observed that:

> Within the range of linear behavior in bending, for mixes with correct binder contents and normal void contents (4-8%) all mix variables tested had only a small effect on the complex modulus as compared to the effects of binder type, temperature, and rate of loading.

Observations such as this have led investigators to search for procedures to enable prediction of the dynamic modulus of asphalt paving mixtures based on routine test properties.

The work of Van der Poel (125, 126) is certainly notable in this respect. Based on extensive static and dynamic testing, Van der Poel developed a nomograph useful for predicting the stiffness of pure bitumens, S_{bit}, such that

$$S_{bit} = f\ (\text{frequency of loading, temperature, penetration of extracted bitumen, and ring-and-ball softening point of extracted bitumen}) \qquad (26)$$

where f is some function. A second nomograph enabled the estimation of mixture stiffness, S_{mix}, from S_{bit} and the volume concentration of mineral aggregate, C_v.

Heukelom and Klomp (43) slightly modified Van der Poel's nomograph for obtaining S_{bit} and suggested that S_{mix} could be obtained as follows:

$$S_{mix}/S_{bit} = \{1 + 2.5\ C_v/[n(1 - C_v)]\}^n \qquad (27)$$

where

S_{mit} = mixture stiffness in kg/cm^2,
S_{bit} = bitumen stiffness in kg/cm^2 obtained from nomograph at desired temperature and time of loading,
C_v = volume concentration of aggregate in the mixture (ratio of volume of compacted aggregate to volume of aggregate and bitumen), and

$$n = 0.83 \log_{10} (400{,}000/S_{bit}) \qquad (28)$$

Heukelom and Klomp's method was limited to mixtures having air void contents on the order of 3 percent and C_v between 0.7 and 0.9. Van Draat and Sommer (127) have suggested that air void contents of greater magnitude can be appropriately considered by using a corrected volume concentration of aggregate, C_v', such that

$$C_v' = C_v/(1 + H) \qquad (29)$$

where H is the difference between the actual air void content and 3 percent, expressed as a decimal.

Bazin and Saunier (6) have presented a nomograph similar to Van der Poel's that is valid for linear deformations in bending and that uses binder properties determined before mixing. Independent variables that are recognized include time of loading, temperature, bitumen type, and mixture void content.

Others have used standard linear regression techniques to relate the absolute value of the complex modulus (111) and the modulus of resilient deformation (25) to various properties of the mixture and its constituent materials.

Additional Investigations

The discussion has concentrated on more conventional approaches to the testing and characterization of pavement materials. It is well to point out two rather recent investigations that deviate somewhat from the conventional format and offer means for possibly more fundamental studies of mechanical behavior.

The first of these is the investigations of Ko and Scott (57, 58, 59). These investigators have developed a soil test box capable of testing a cubical sample by applying three different normal pressures to its sides. The principal stresses can be varied independently or by means of a stress control device that is a mechanical-hydraulic analog of an octahedral plane in the principal stress space. With this equipment, it is possible to vary only the octahedral shear stress while maintaining a constant hydrostatic stress to study the true response to shearing stresses. The three principal strains are measured with this device, and an independent measurement is made of the volume change of the soil sample. Reputed advantages of this device include the following: (a) a homogeneous stress state is produced, (b) the nature of the stress path that can be developed is unlimited, (c) the device is stress-controlled rather than strain-controlled, and (d) it is applicable for both loading and unloading tests. Although the adoption of this equipment for routine testing is difficult to envision, it does offer a means for research investigations into the three-dimensional response of pavement materials.

The second investigation is that of Dehlen and Monismith (16, 17). These investigators used a triaxial testing procedure in which repeated axial stress and repeated radial stress could be independently varied and superposed over varying and unequal constant stresses in the axial and radial directions. The results of their investigations were expressed as incremental stress-strain coefficient matrices at varying reference stress states.

For axisymmetric incremental stresses the coefficient matrix relates the incremental strains, ϵ' and γ', and stresses, σ' and τ', as follows:

$$\begin{Bmatrix} \epsilon'_{rr} \\ \epsilon'_{\theta\theta} \\ \epsilon'_{zz} \\ \gamma'_{rz} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{13} & 0 \\ B_{12} & B_{11} & B_{13} & 0 \\ B_{31} & B_{31} & B_{33} & 0 \\ 0 & 0 & 0 & B_{44} \end{bmatrix} \begin{Bmatrix} \sigma'_{rr} \\ \sigma'_{\theta\theta} \\ \sigma'_{zz} \\ \tau_{rz} \end{Bmatrix} \tag{30}$$

This incremental formulation of the constitutive equation of a nonlinear elastic material is valid for (a) small stress and strain increments, (b) an initially cross-isotropic material with an axis of symmetry coinciding with the vertical or Z-coordinate axis, (c) a cross-isotropic stress-induced anisotropy, and (d) no coupling between shear and volumetric stresses and strains. For triaxial testing procedures Eq. 30 reduces to

$$\begin{Bmatrix} \epsilon'_{r} \\ \epsilon'_{z} \end{Bmatrix} = \begin{bmatrix} B_{11} + B_{12} & B_{13} \\ 2B_{31} & B_{33} \end{bmatrix} \begin{Bmatrix} \sigma'_{r} \\ \sigma'_{z} \end{Bmatrix} \tag{31}$$

The coefficient matrix B depends on the reference stress state at which it is determined. It was found to be approximately symmetric at hydrostatic reference stress states but significantly nonsymmetric otherwise.

The main limitation of this means for characterizing the response of nonlinear elastic materials would appear to be the limited range of stress states possible with the triaxial test procedure. That is, the triaxial test adequately defines the constitutive relations only for materials located beneath an axis of symmetry because it permits only two normal stresses to be varied independently. Complete characterization of materials outside an axis of symmetry requires investigation under three normal stresses and one shearing stress.

In addition to the investigations described here, a great deal of work has been reported involving static testing of pavement materials. Characteristic of this work are investigations by Saada (94) on compacted clays and Monismith et al. (74) on asphalt concrete. The intent of most of this testing has been to evaluate the rheological behavior of these materials under simple forms of stress. Most of the tests have been relaxation or creep tests, though some constant-rate-of-deformation or constant-rate-of-load tests have been performed. The results of these tests have been analyzed largely with the intent to (a) establish the extent of linearity, (b) examine the extent to which the material is thermorheologically simple, (c) determine values of rheological constants such as relaxation moduli and creep compliances, (d) ascertain the complexity of mechanical models necessary to accurately describe the observed behavior and assess the values of the associated constants, and (e) ascertain the applicability of the superposition principle.

Finally, it must be observed that, although this review has been limited almost solely to laboratory investigations, rather extensive field investigations have also been conducted including test pits, test tracks, and pavements in service. These field investigations serve to complement, verify, and extend the results of the laboratory investigations.

CONCLUDING REMARKS

Practitioners in the field of pavement materials characterization would probably be quick to agree that the most pressing need today is the development of a universally accepted standard means for characterizing and testing these materials. At the same time, it must be realized that the factors that have retarded the development of such standards in the past are most likely to continue to be operative in the near future. With this deficiency in mind, then, I consider the following list to represent some of the more specific, current deficiencies in our ability to adequately characterize pavement materials:

1. Understanding of what material properties are of fundamental importance to the performance of asphalt concrete pavements and what properties must be known or estimated before a rational analysis of this performance is feasible, considering the planned use of the materials and the available methods of analysis;
2. Commonly accepted and explicitly recognized criteria that would allow assessments of the utility of various test equipment and various means for characterizing behavior as a part of a rational pavement analysis procedure;
3. Knowledge of the irrecoverable or permanent component of deformation and of possible means for characterizing it for all pavement materials;
4. Understanding of the three-dimensional response of all pavement materials under realistic in-service stress states and of the degree to which behavior under simplified and commonly used stress states is similar to that under more realistic conditions (this is as true of strength measures, particularly tensile and fatigue strength of asphalt paving mixtures, as it is of deformability measures);
5. Knowledge of the possible development of slip surfaces in shear due to load repetitions of a stress level less than the shear strength (particularly for unbound granular aggregate and asphalt paving mixtures);
6. Firm basis for understanding the possibly important effects of mode of loading on fatigue behavior of asphalt paving mixtures, and ability to vary mode of loading in the laboratory in order to simulate realistic in-service loading;
7. Adequately verified comprehensive equation that allows estimates of fatigue life for asphalt paving mixtures without extensive laboratory testing;
8. Knowledge of the strength and deformation behavior of treated materials;
9. Ability to characterize in a fundamental way the volume changes in all pavement materials induced by environmental factors superimposed over stress states representative of in-service conditions;
10. Knowledge of the fatigue behavior of asphalt paving mixtures when load repetitions are combined with cyclic environmental changes (predominantly temperature and specimen support); and
11. Ability to characterize untreated granular aggregate.

REFERENCES

1. Alexander, R. L. Limits of Linear Viscoelastic Behavior of an Asphalt Concrete in Tension and Compression. Institute of Transportation and Traffic Engineering, Univ. of California, Berkeley, Graduate Research Rept., 1964.
2. Anderson, K. O., and Hahn, W. P. Design and Evaluation of Asphalt Concrete With Respect to Thermal Cracking. Proc., AAPT, Vol. 37, 1968, pp. 1-31.
3. Balmer, G. G. Shear Strength and Elastic Properties of Soil-Cement Mixture Under Triaxial Loading. Proc., ASTM, Vol. 58, 1958, pp. 1187-1204.

4. Barenberg, E. J. A Structural Design Classification of Pavements Based on an Analysis of Pavement Behavior, Material Properties, and Modes of Failure. Univ. of Illinois, Urbana, PhD dissertation, 1965.
5. Barksdale, R. D., and Leonards, G. A. Predicting Performance of Bituminous Surfaced Pavements. Proc., Second Internat. Conf. on Structural Design of Asphalt Pavements, 1967, pp. 321-340.
6. Bazin, P., and Saunier, J. B. Deformability, Fatigue and Healing Properties of Asphalt Mixes. Proc., Second Internat. Conf. on Structural Design of Asphalt Pavements, 1967, pp. 553-569.
7. Breen, J. J., and Stephens, J. E. Split Cylinder Test Applied to Bituminous Mixtures at Low Temperatures. Jour. of Materials, Vol. 1, No. 1, March 1966, pp. 66-76.
8. Carpenter, C. A., Goode, J. F., and Peck, R. A. An Improved Triaxial Cell for Testing of Bituminous Paving Mixtures. Proc., AAPT, Vol. 20, 1951, pp. 154-179.
9. Chang, T. Y., Ko, H. Y., Scott, R. F., and Westmann, R. A. An Intergrated Approach to the Stress Analysis of Granular Materials. California Institute of Technology, Soil Mechanics Laboratory Report, 1967.
10. Coffman, B. S. Pavement Deflections From Laboratory Tests and Layer Theory. Proc., Second Internat. Conf. on Structural Design of Asphalt Pavements, 1967, pp. 819-862.
11. Coffman, B. S., Ilves, G., and Edwards, W. F. Isotropy and an Asphaltic Concrete. unpublished.
12. Coffman, B. S., Kraft, D. C., and Tamayo, J. A Comparison of Calculated and Measured Deflections for the AASHO Road Test. Proc., AAPT, Vol. 33, 1964, pp. 54-91.
13. Davis, E. F., Krokosky, E. M., and Tons, E. Stress Relaxation of Bituminous Concrete in Tension. Highway Research Record 67, 1965, pp. 38-58.
14. Deacon, J. A. Fatigue of Asphalt Concrete. Institute of Transportation and Traffic Engineering, Univ. of California, Berkeley, Graduate Report, 1965.
15. Deacon, J. A., and Monismith, C. L. Laboratory Flexural-Fatigue Testing of Asphalt Concrete With Emphasis on Compound-Loading Tests. Highway Research Record 158, 1967, pp. 1-31.
16. Dehlen, G. L. The Effect of Non-Linear Material Response on the Behavior of Pavements Subjected to Traffic Loads. Institute of Transportation and Traffic Engineering, Univ. of California, Berkeley, Graduate Rept., 1969.
17. Dehlen, G. L., and Monismith, C. L. The Effect of Nonlinear Material Response on the Behavior of Pavements Under Traffic. Highway Research Record 310, 1970, pp. 1-16.
18. Dunlap, W. A. A Report on a Mathematical Model Describing the Deformation Characteristics of Granular Materials. Texas Transportation Institute, Texas A&M Univ., Tech. Rept. 1, 1963.
19. Epps, J. A., and Monismith, C. L. Influence of Mixture Variables on the Flexural Fatigue Properties of Asphalt Concrete. Proc., AAPT, Vol. 38, 1969, pp. 423-464.
20. Eriksson, R. Deformation and Strength of Asphalts at Slow and Rapid Loadings. Statens Vaginstitut, Moddelance 82, Stockholm, 1951.
21. Eriksson, R. Tension Tests on Sheet-Asphalt. Statens Vaginstitut, Moddelance 82, Stockholm, 1954.
22. Felt, E. J., and Abrams, M. S. Strength and Elastic Properties of Compacted Soil-Cement Mixtures. ASTM, Spec. Tech. Pub. 206.
23. Ferrari, P. The Behavior of Asphalt Pavements Under Variable Repeated Loads. Proc., Second Internat. Conf. on Structural Design of Asphalt Pavments, 1967, pp. 169-174.
24. Finn, F. N. Factors Involved in the Design of Asphaltic Pavement Surfaces. NCHRP Rept. 39, 1967.
25. Finn, F. N., Hicks, R. G., Kari, W. J., and Coyne, L. D. Design of Emulsified Asphalt Treated Bases. Highway Research Record 239, 1968, pp. 54-75.

26. Finn, W. D. L., and Mittal, H. K. Shear Strength of Soil in General Stress Field. ASTM, Spec. Tech. Pub. 361, 1964, pp. 42-51.
27. Goetz, W. H. Comparison of Triaxial and Marshall Test Results. Proc., AAPT Vol. 20, 1951, pp. 200-245.
28. Goetz, W. H. Sonic Testing of Bituminous Mixes. Proc., AAPT, Vol. 24, 1955, pp. 332-355.
29. Goetz, W. H., and Chen, C. Vacuum Triaxial Technique Applied to Bituminous-Aggregate Mixtures. Proc., AAPT, Vol. 19, 1950, pp. 55-81.
30. Gradowczyk, M. H., and Moavenzadeh, F. Characterization of Linear Viscoelastic Materials. Trans., Society of Rheology, Vol. 13, No. 2, 1969, pp. 173-191.
31. Gradowczyk, M. H., Moavenzadeh, F., and Soussou, J. F. Characterization of Linear Viscoelastic Materials Tested in Creep and Relaxation. Jour. of Applied Physics, Vol. 40, No. 4, March 1969, pp. 1783-1788.
32. Gregg, J. S., Dehlen, G. L., and Rigden, P. J. On the Properties, Behavior, and Design of Bituminous Stabilized Sand Bases. Proc., Second Internat. Conf. on Structural Design of Asphalt Pavements, 1967, pp. 863-882.
33. Haas, R. C. G., and Anderson, K. O. A Design Subsystem for the Response of Flexible Pavements at Low Temperatures. Proc., AAPT, Vol. 38, 1969, pp. 179-223.
34. Hadley, W. O., Hudson, W. R., Kennedy, T. W., and Anderson, V. L. A Statistical Experiment to Evaluate Tensile Properties of Asphalt-Treated Materials. Proc., AAPT, Vol. 38, 1969, pp. 224-241.
35. Hardin, B. O., and Drnevich, V. P. Shear Modulus and Damping in Soils—I: Measurement and Parameter Effects. College of Engineering, Univ. of Kentucky, Tech. Rept. UKY 27-70-CE2, July 1970.
36. Hardin, B. O., and Drnevich, V. P. Shear Modulus and Damping in Soils—II: Design Equations and Curves. College of Engineering, Univ. of Kentucky, Tech. Rept. UKY 27-70-CE3, July 1970.
37. Hargett, E. R. Basic Material Properties for the Design of Bituminous Concrete Surfaces. Proc., Internat. Conf. on Structural Design of Asphalt Pavements, 1963, pp. 606-610.
38. Haynes, J. H., and Yoder, E. J. Effects of Repeated Loading on Gravel and Crushed Stone Base Course Materials Used in the AASHO Road Test. Highway Research Record 39, 1963, pp. 82-96.
39. Hennes, R. G., and Wang, C. C. Physical Interpretation of Triaxial Test Data. Proc., AAPT, Vol. 20, 1951, pp. 179-199.
40. Herkenhoff, P. G. A Determination of the Shear Strength Properties of Bituminous Mixtures by the Use of Tension and Unconfined Compression Tests. Proc., AAPT, Vol. 33, 1964, pp. 363-405.
41. Heukelom, W. Observations on the Rheology and Fracture of Bitumens and Asphalt Mixes. Proc., AAPT, Vol. 35, 1966, pp. 358-399.
42. Heukelom, W., and Klomp, A. J. G. Dynamic Testing as a Means of Controlling Pavements During and After Construction. Proc., Internat. Conf. on Structural Design of Asphalt Pavements, 1962, pp. 667-679.
43. Heukelom, W., and Klomp, A. J. G. Road Design and Dynamic Loading. Proc., AAPT, Vol. 33, 1964, pp. 92-125.
44. Hewitt, W. L. Analysis of Flexible Paving Mixtures by Theoretical Design Procedure Based on Shear Strength. Highway Research Record 104, 1965, pp. 78-104.
45. Hewitt, W. L. Analysis of Stresses in Flexible Pavements and Development of a Structural Design Procedure. HRB Bull. 269, 1960, pp. 66-74.
46. Hewitt, W. L. Continuation of a Study on the Theoretical Design of Flexible Pavements Based on Shear Strength. Proc., Internat. Conf. on Structural Design of Asphalt Pavements, 1962, pp. 332-338.
47. Hewitt, W. L., and Slate, F. O. The Effects of the Rheological Properties of Asphalt on Strength Characteristics of Asphalt Concrete. Proc., Second Internat. Conf. on Structural Design of Asphalt Pavements, 1967, pp. 757-767.

48. Hicks, R. G. Factors Influencing the Resilient Properties of Granular Materials. Institute of Transportation and Traffic Engineering, Univ. of California, Berkeley, Dissertation Series, May 1970.
49. Huang, Y. H. The Deformation Characteristics of Sand-Bitumen Mixtures Under Constant Compressive Stresses. Proc., AAPT, Vol. 34, 1965, pp. 80-124.
50. Humphries, W. K., and Wahls, H. E. Stress History Effects on Dynamic Modulus of Clays. Jour., Soil Mech. and Found. Div., ASCE, Vol. 94, No. SM2, March 1968, pp. 371-389.
51. Jimenez, R. A. An Apparatus for Laboratory Investigations of Asphaltic Concrete Under Repeated Flexural Deformation. Report submitted to the Texas Highway Department and the Texas Transportation Institute, Texas A&M Univ., 1962.
52. Jimenez, R. A., and Gallaway, B. M. Behavior of Asphaltic Concrete Diaphragms to Repetitive Loadings. Proc., Internat. Conf. on Structural Design of Asphalt Pavements, 1962, pp. 339-344.
53. Kallas, B. F., and Riley, J. C. Mechanical Properties of Asphaltic Pavement Materials. Proc., Second Internat. Conf. on Structural Design of Asphalt Pavements, 1967, pp. 931-952.
54. Kasianchuk, D. A. Fatigue Considerations in the Design of Asphalt Concrete Pavements. Univ. of California, Berkeley, PhD dissertation, 1968.
55. Kennedy, T. W., and Hudson, W. R. Application of the Indirect Tensile Test to Stabilize Materials. Highway Research Record 235, 1968, pp. 36-48.
56. Kirk, J. M. Results of Fatigue Tests on Different Types of Bituminous Mixtures. Proc., Second Internat. Conf. on Structural Design of Asphalt Pavements, 1967, pp. 571-575.
57. Ko, H. Y., and Scott, R. F. Deformation of Sand in Hydrostatic Compression. Jour., Soil Mech. and Found. Div., ASCE, Vol. 93, No. SM3, May 1967, pp. 137-156.
58. Ko, H. Y., and Scott, R. F. Deformation of Sand in Shear. Jour., Soil Mech. and Found. Div., ASCE, Vol. 93, No. SM5, Sept. 1967, pp. 283-310.
59. Ko, H. Y., and Scott, R. F. A New Soil Testing Apparatus. Geotechnique, Vol. 14, No. 1, March 1967, pp. 40-57.
60. Krokosky, E. M., and Chen, J. P. Viscoelastic Analysis of the Marshall Test. Proc., AAPT, Vol. 33, 1964, pp. 406-436.
61. Krokosky, E. M., Tons, E., and Andrews, R. D. Rheological Properties of Asphalt-Aggregate Compositions. Proc., ASTM, Vol. 63, 1963, pp. 1263-1286.
62. Laboratory Shear Testing of Soils. ASTM, Spec. Tech. Pub. 361, 1964.
63. Lambe, T. W., and Whitman, R. V. Soil Mechanics. John Wiley and Sons, New York, 1969.
64. Livneh, M., and Shklarsky, E. The Splitting Test for Determination of Bituminous Concrete Strength. Proc., AAPT, Vol. 31, 1962, pp. 457-476.
65. Lundgren, R., Mitchell, J. K., and Wilson, J. H. Effects of Loading Method on Triaxial Test Results. Jour., Soil Mech. and Found. Div., ASCE, Vol. 94, No. SM2, March 1968, pp. 407-419.
66. McDowell, C. Triaxial Tests in Analysis of Flexible Pavements. HRB Res. Rept. 16-B, 1954, pp. 1-28.
67. McLeod, N. W. A Rational Approach to the Design of Bituminous Paving Mixtures. Proc., AAPT, Vol. 19, 1950, pp. 82-224.
68. McLeod, N. W. The Rational Design of Bituminous Paving Mixtures. HRB Proc., Vol. 29, 1949, pp. 107-159.
69. McLeod, N. W. An Ultimate Strength Approach to Flexible Pavement Design. Proc., AAPT, Vol. 23, 1954, pp. 119-236.
70. Metcalf, C. T. Use of Marshall Stability in Asphalt Paving Mix Design. HRB Bull. 234, 1959, pp. 12-22.
71. Mitchell, J. K., and Chen, C. K. Soil-Cement Properties Determined by Repeated Loading in Relation to Bases for Flexible Pavements. Proc., Second Internat. Conf. on Structural Design of Asphalt Pavements, 1967, pp. 427-451.
72. Monismith, C. L. Effect of Temperature on the Flexibility Characteristics of Asphaltic Paving Mixtures. ASTM, Spec. Tech. Pub. 277, 1960, pp. 89-108.

73. Monismith, C. L. Some Applications of Theory in the Design of Asphalt Pavements. Proc., Fifth Annual Nevada Street and Highway Conf., 1970, Section 4.
74. Monismith, C. L., Alexander, R. L., and Secor, K. E. Rheologic Behavior of Asphalt Concrete. Proc., AAPT, Vol. 35, 1966, pp. 400-450.
75. Monismith, C. L., and Deacon, J. A. Fatigue of Asphalt Paving Mixtures. Transportation Engineering Jour., ASCE, Vol. 95, No. TE2, May 1969, pp. 317-346.
76. Monismith, C. L., and Secor, K. E. Viscoelastic Behavior of Asphalt Concrete Pavements. Proc., Internat. Conf. on Structural Design of Asphalt Pavements, 1962, pp. 476-498.
77. Monismith, C. L., Secor, K. E., and Blackmer, E. W. Asphalt Mixture Behavior in Repeated Flexure. Proc., AAPT, Vol. 30, 1961, pp. 188-222.
78. Monismith, C. L., Secor, G. A., and Secor, K. E. Temperature Induced Stresses and Deformations in Asphalt Concrete. Proc., AAPT, Vol. 34, 1965, pp. 248-285.
79. Monismith, C. L., Seed, H. B., Mitry, F. G., and Chan, C. K. Prediction of Pavement Deflections From Laboratory Tests. Proc., Second Internat. Conf. on Structural Design of Asphalt Pavements, 1967, pp. 109-140.
80. Monismith, C. L., and Vallerga, B. A. Relationship Between Density and Stability of Asphaltic Paving Mixtures. Proc., AAPT, Vol. 25, 1956, pp. 88-108.
81. Nijboer, L. W. Mechanical Properties of Asphalt Materials and the Structural Design of Asphalt Roads. HRB Proc., Vol. 33, 1954, pp. 185-200.
82. Nijboer, L. W. Plasticity as a Factor in the Design of Dense Bituminous Carpets. Elsevier Publishing Co., New York, 1948.
83. Pagen, C. A. Rheological Response of Bituminous Concrete. Highway Research Record 67, 1965, pp. 1-26.
84. Pagen, C. A. A Study of the Temperature-Dependent Rheological Characteristics of Asphaltic Concrete. Highway Research Record 158, 1967, pp. 116-143.
85. Pagen, C. A., and Jagannath, B. N. Mechanical Properties of Compacted Soils. Highway Research Record 235, 1968, pp. 13-26.
86. Pagen, C. A., and Ku, B. Effect of Asphalt Viscosity on Rheological Properties of Bituminous Concrete. Highway Research Record 104, 1965, pp. 124-140.
87. Papazian, H. S. The Response of Linear Viscoelastic Materials in the Frequency Domain With Emphasis on Asphaltic Concrete. Proc., Internat. Conf. on Structural Design of Asphalt Pavements, 1962, pp. 454-463.
88. Papazian, H. S., and Baker, R. F. Analyses of Fatigue Type Properties of Bituminous Concrete. Proc., AAPT, Vol. 28, 1959, pp. 179-210.
89. Paquette, R. H. The Effect of Aggregate Gradation on Bituminous Concrete in the State of New York. Cornell Univ., unpublished master's thesis, 1956.
90. Pell, P. S. Fatigue Characteristics of Bitumen and Bituminous Mixes. Proc., Internat. Conf. on Structural Design of Asphalt Pavements, 1962, pp. 310-323.
91. Pell, P. S. Fatigue of Asphalt Pavement Mixes. Proc., Second Internat. Conf. on Structural Design of Asphalt Pavements, 1967, pp. 577-593.
92. Pell, P. S., and McCarthy, P. F. Amplitude Effect on Stiffness of Bitumen and Bituminous Mixes Under Dynamic Conditions. Rheologica Acta, Vol. 2, No. 2, 1962, pp. 174-179.
93. Pell, P. S., and Taylor, I. F. Asphaltic Road Materials in Fatigue. Proc., AAPT, Vol. 38, 1969, pp. 371-422.
94. Saada, A. S. A Rheological Analysis of Shear and Consolidation of Saturated Clays. HRB Bull. 342, 1962, pp. 52-89.
95. Saal, R. N. J., and Pell, P. S. Fatigue of Bituminous Road Mixes. Kolloid-Zeitschrift, Darmstadt, Vol. 171, 1960, pp. 61-71.
96. Santucci, L. E., and Schmidt, R. J. The Effect of Asphalt Properties on the Fatigue Resistance of Asphalt Paving Mixtures. Proc., AAPT, Vol. 38, 1969, pp. 65-97.
97. Sauer, E. K., and Monismith, C. L. The Influence of Soil Suction on the Behavior of a Glacial Till Subjected to Repeated Loading. Highway Research Record 215, 1968, pp. 8-23.

98. Sayegh, G. Viscoelastic Properties of Bituminous Mixtures. Proc., Second Internat. Conf. on Structural Design of Asphalt Pavements, 1967, pp. 743-755.
99. Schmidt, R. J., and Santucci, L. E. The Effect of Asphalt Properties on the Fatigue Cracking of Asphalt Concrete on the Zaca-Wigmore Test Project. AAPT, Vol. 38, 1969, pp. 39-64.
100. Secor, K. E., and Monismith, C. L. Analysis and Interrelation of Stress-Strain-Time Data for Asphalt Concrete. Trans., Society of Rheology, Vol. 8, 1964.
101. Secor, K. E., and Monismith, C. L. Analysis of Triaxial Test Data on Asphalt Concrete Using Viscoelastic Principles. HRB Proc., Vol. 40, 1961, pp. 295-314.
102. Secor, K. E., and Monismith, C. L. The Viscoelastic Response of Asphalt Paving Slabs Under Creep Loading. Highway Research Record 67, 1965, pp. 84-97.
103. Seed, H. B., and Chan, C. K. Effect of Duration of Stress Application on Soil Deformation Under Repeated Loading. Proc., 5th Internat. Conf. on Soil Mech. and Found. Eng., Paris, Vol. 1, 1964, pp. 340-345.
104. Seed, H. B., and Chan, C. K. Effect of Stress History and Frequency of Stress Applications on Deformations of Clay Subgrades Under Repeated Loading. HRB Proc., Vol. 37, 1958, pp. 77-87.
105. Seed, H. B., and Chan, C. K. Thixotropic Characteristics of Compacted Clays. Trans., ASCE, Vol. 124, 1959, pp. 894-925.
106. Seed, H. B., Chan, C. K., and Lee, C. E. Resilient Characteristics of Subgrade Soils and Their Relation to Fatigue Failures in Asphalt Pavements. Proc., Internat. Conf. on Structural Design of Asphalt Pavements, 1962, pp. 611-636.
107. Seed, H. B., and Fead, J. W. N. Apparatus for Repeated Load Tests on Soils. ASTM, Spec. Tech. Pub. 254, 1960, pp. 78-87.
108. Seed, H. B., Mitry, F. G., Monismith, C. L., and Chan, C. K. Factors Influencing the Resilient Deformations of Untreated Aggregate Base in Two-Layer Pavements Subjected to Repeated Loading. Highway Research Record 190, 1967, pp. 19-57.
109. Seed, H. B., and Monismith, C. L. Moderators' Summary Report of Papers Presented for Discussion at Session V. Proc., Internat. Conf. on Structural Design of Asphalt Pavements, 1962, pp. 551-580.
110. Schiffman, R. L. The Use of Visco-Elastic Stress-Strain Laws in Soil Testing. ASTM, Spec. Tech. Pub. 254, 1960, pp. 131-155.
111. Shook, J. F., and Kallas, B. F. Factors Influencing Dynamic Modulus of Asphalt Concrete. Proc., AAPT, Vol. 38, 1969, pp. 140-178.
112. Smith, V. R. Triaxial Stability Method for Flexible Pavement Design. Proc., AAPT, Vol. 18, 1949, pp. 63-94.
113. Sowers, G. F. Strength Testing of Soils, ASTM, Spec. Tech. Pub. 361, 1964, pp. 3-31.
114. Taylor, D. W. Fundamentals of Soil Mechanics. John Wiley and Sons, New York, 1948.
115. Terrel, R. L. Factors Influencing the Resilient Characteristics of Asphalt Treated Aggregates. Univ. of California, Berkeley, PhD dissertation, Aug. 1967.
116. Terrel, R. L., and Monismith, C. L. Evaluation of Asphalt-Treated Base Course Materials. Proc., AAPT, Vol. 37, 1968, pp. 159-199.
117. Thiers, G. R., and Seed, H. B. Cyclic Stress-Strain Characteristics of Clay. Jour., Soil Mech. and Found. Div., ASCE, Vol. 94, No. SM2, March 1968, pp. 555-569.
118. Thompson, M. R. The Split-Tensile Strength of Lime-Stabilized Soils. Highway Research Record 92, 1965, pp. 11-23.
119. Tons, E., and Krokosky, E. M. Tensile Properties of Dense Graded Bituminous Concrete. Proc., AAPT, Vol. 32, 1963, pp. 497-529.
120. Triaxial Testing of Soils and Bituminous Mixtures. ASTM, Spec. Tech. Publ. 106, 1951.
121. Trollope, D. H., Lee, I. K., and Morris, J. Stresses and Deformations in Two Layer Pavement Structures Under Slow Repeated Loading. Proc., Australian Road Research Board, Vol. 1, Part 2, 1962, pp. 693-721.

122. Tschebotarioff, G. P. Soil Mechanics, Foundations, and Earth Structures. McGraw-Hill Book Co., New York, 1951.

123. Vallerga, B. A., Finn, F. N., and Hicks, R. G. Effect of Asphalt Aging on the Fatigue Properties of Asphalt Concrete. Proc., Second Internat. Conf. on Structural Design of Asphalt Pavements, 1967, pp. 484-510.

124. Vallerga, B. A., Seed, H. B., Monismith, C. L., and Cooper, R. S. Effect of Shape, Size and Surface Roughness of Aggregate Particles on the Strength of Granular Materials. ASTM, Spec. Tech. Pub. 212, 1957, pp. 63-76.

125. Van der Poel, C. A General System Describing the Viscoelastic Properties of Bitumens and Its Relation to Routine Test Data. Jour. of Applied Chemistry, May 1954, pp. 221-236.

126. Van der Poel, C. Road Asphalt. In Building Materials, Their Elasticity and Inelasticity (Reiner, M., ed.), Interscience Publishers, New York, 1954, pp. 361-443.

127. Van Draat, W. E. F., and Sommer, P. Ein Gerät zur Bestimmung der dynamischen Elastizitatsmoduln von Asphalt. Strasse und Antobahn, Vol. 35, 1965, pp. 206-211.

128. Wood, L. E., and Goetz, W. H. The Rheological Characteristics of a Sand-Asphalt Mixture. Proc., AAPT, Vol. 28, 1959, pp. 211-229.