Multidimensional Choice Models: Alternative Structures of Travel Demand Models

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Extensive research in travel demand in recent years has been based on theories of individual choice. These choice theories assume a selection from a finite set of mutually exclusive and collectively exhaustive alternatives. We assume that, with qualitative or discrete alternatives, probabilistic behavior explains observations of different choices for the same set of observed independent variables. Such choice theories have been developed in the context of unidimensional choice situations. A consumer was assumed to select an alternative i out of a set of alternative choices A. If the set A includes the alternative choices of a single commodity, then the choice probability, P(i:A), is the choice analog of a demand function for a given commodity. A consumer is faced with a multidimensional choice situation in determining a consumption pattern. (The term multiple choice refers to a choice from a set of more than 2 alternatives. A choice from 2 alternatives is termed binary choice. The term multidimensional choice is used for a set of related choices, each of which can be either multiple or binary.) For example, a consumer who is selecting a residence location within the metropolitan area is choosing also among alternatives such as housing types and automobile ownership levels.

The total number of choices that a consumer makes is very large. The assumptions of a "utility tree," or a separable utility function, and negligible income effects permit the independent modeling of demand for a subset of commodities. That is, the demand functions for a subset of commodities are independent of the prices of all other commodities. [The notion of separability was introduced by Leontief (4). Separable utility functions have been developed by Muth (9) and Strotz (11, 12).]

We assume here that mobility and travel choices are such an independent branch or subset of the consumer's utility function. Choices within this subset are interdependent. This subset may be treated as a block recursive system. That is, the first block consists of the mobility choices, and the second block consists of the travel choices (assuming the mobility choices as fixed). Travel choices with respect to different trip purpose categories can also be considered independently of each other. Thus, we can model separately the set of mobility choices and the sets of travel choices for different trip purposes (assuming that mobility choices are predetermined). Yet, each of the above sets of choices represents a multidimensional choice situation.

The purpose of this paper is to extend the choice theories from unidimensional to multidimensional choice situations. In a multidimensional choice situation different assumptions about the dependencies among choices result in models with different structures. The alternative structures are identified, and their applicability to travel demand models is discussed.

PROBABILISTIC CHOICE THEORY

Choice theories are reviewed in other reports (1, 2, 3, 5, 6). The consumer is visualized as selecting the alternative that maximizes utility. The probabilistic behavior mechanism is a result of the assumption that the utilities of the alternatives are not certain, but rather random variables determined by a specific distribution.

If the utility of alternative i to consumer t is denoted as U_{it}, the choice probability of alternative i is

$$P(i{:}A_t) = \text{prob}\,[U_{it} \geq U_{jt},\ \forall j \epsilon A_t] \tag{1}$$

where A_t is the set of alternative choices available to consumer t. The utilities are essentially indirect utility functions, which are defined in theory as the maximum level of utility for given prices and income. In other words, the utility U_{it} is a function of the variables that characterize alternative i, denoted as X_i, and of the socioeconomic variables describing consumer t, denoted as S_t. Thus, we can write

$$U_{it} = U_i\,(X_i,\ S_t) \tag{2}$$

The set of alternatives A_t is mutually exclusive and exhaustive such that one and only one alternative is chosen. The deterministic equivalent of this theory is simply a comparison of all alternatives available and the selection of the alternative with the highest utility.

The mathematical form of the choice model is determined from the assumption about the distribution of the utility values.

DEPENDENCIES AMONG CHOICES

To simplify the discussion we will rely on an example of 2 choices. We consider a consumer who is making a trip for a given trip purpose, say, shopping, and is faced with the choices of destination d and mode of travel m. We distinguish between 2 types of dependencies among choices: dependency in the structural sense and dependency of the sets of alternative choices in a physical sense.

Dependency in the structural sense arises from substitution and complementary relations among choices and different choices being made with respect to the same final commodity, i.e., the utilities from different choices are not independent. For example, the choices of automobile ownership level and residence location are dependent on each other because a downtown location could be a substitute for a high automobile ownership level. The utility from an alternative location will therefore depend on the chosen car ownership level and vice versa.

The choices of mode and destination are made with respect to the same final commodity—a trip. Some of the attributes of a mode, such as travel time by bus, will be different for different destinations. Therefore, mode m to destination d is a different alternative from the same mode to destination $d'(d' \neq d)$. Similarly, some of the attributes of destination d depend on the chosen mode. Therefore, destination d reached by mode m is a different alternative from the same destination reached by mode m' $(m' \neq m)$. In other words, the utility from an alternative mode is dependent on the destination and vice versa.

Thus, the dependency among travel choices can be attributed to the commonality of the attributes. In other words, some attributes of a trip are specific to all travel choices. For example, the travel cost for shopping at a certain frequency depends on attributes such as where one shops and what mode one uses. Similarly, the travel cost of shopping at a given destination depends on how often one shops and what mode one uses. Therefore, a traveler can trade off among choices. For example, one can shop frequently at a nearby grocery store or less frequently at a distant shopping center.

The dependency, or the causality, can be assumed either in 1 direction (e.g., the utility from a mode depends on the chosen destination but the utility from an alternative destination is independent of the chosen mode) or in 2 directions (e.g., the utility from an alternative mode depends on the chosen destination and the utility from an alternative destination depends on the chosen mode). It is realistic to assume that all travel choices are interdependent. However, we consider here also alternative assumptions that result in models with different structures, as will be shown in the following sections.

If the choices of mode and destination depend on each other, then the set of alternative modes is different for different destinations and the set of alternative destinations is different for different modes. We denote the set of alternative modes for a given destination as M_d and the set of alternative destinations for a given mode as D_m.

In addition, the set of alternative modes can be physically dependent on the chosen destination and vice versa. For example, a bus service may be available to 1 destination but not to the other. Therefore, the sets of alternative modes M_d can have different numbers of alternatives for different destinations.

If 2 choices are independent, then their alternative sets will also be independent. If the choice of mode and destination is assumed to be independent, we denote the set of alternative modes as M and the set of alternative destinations as D.

OVERALL SET OF ALTERNATIVES

The consumer can be viewed as selecting an alternative destination and mode combination dm from an overall set of alternatives DM that include all possible destination and mode combinations. For example, if the number of alternative modes available to every destination is identical and equal to M and the number of alternative destinations is D, then the total number of alternatives in the overall set will be $D \times M$.

The overall set of alternatives DM can be partitioned according to modes or according to destinations. If we partition according to destination, then we can write the overall set of alternatives as follows:

$$DM = [M_1,\ M_2,\ \ldots,\ M_d,\ \ldots,\ M_D] \tag{3}$$

In this scheme we denote the set of destinations used for partitioning as D. Partitioning according to modes, we write

$$DM = [D_1,\ D_2,\ \ldots,\ D_m,\ \ldots,\ D_M] \tag{4}$$

The set of modes used for partitioning is denoted as M. If the alternative sets are independent, then

$$\begin{aligned} M_d &= M,\ \forall d \epsilon D \\ D_m &= D,\ \forall m \epsilon M \end{aligned} \tag{5}$$

ALTERNATIVE STRUCTURES

If we assume that the choices are independent, then we can write the following structural choice probabilities (the probabilities that have direct behavioral interpretation and are originally written to describe a structure are called structural probabilities):

$$\begin{aligned} P(d{:}D) &= \text{prob}\ [U_d \geq U_{d'},\ \forall d' \epsilon D] \\ P(m{:}M) &= \text{prob}\ [U_m \geq U_{m'},\ \forall m' \epsilon M] \end{aligned} \tag{6}$$

where U_d and U_m are the utilities from destination d and mode m respectively. In essence, the independence assumption implies an additive utility function:

$$U_{dm} = U_d + U_m \tag{7}$$

In words, the total utility from a destination and mode combination is equal to the utility from the destination plus the utility from the mode. Since the choices are independent, we can write the joint probability of d and m as follows:

$$P(d,\ m{:}DM) = P(d{:}D) \cdot P(m{:}M) \tag{8}$$

The structure that represents independent choices, or an independent structure, consists of marginal probabilities of the different choices.

If the choices of mode and destination are dependent on each other, then we can write the following conditional choice probabilities:

$$\begin{aligned} P(d{:}D_m) &= \mathrm{prob}[U_{d|m} \geq U_{d'|m}, \forall d' \epsilon D_m] \\ P(m{:}M_d) &= \mathrm{prob}[U_{m|d} \geq U_{m'|d}, \forall m' \epsilon M_d] \end{aligned} \tag{9}$$

where $U_{d|m}$ is the utility from destination d given that mode m is chosen and $U_{m|d}$ is the utility from mode m given that destination d is chosen. The conditional probability $P(d{:}D_m)$ is the choice probability of destination d given that mode m is chosen, and similarly $P(m{:}M_d)$ is the choice probability of mode m given that destination d is chosen.

For forecasting, however, the 2 conditional probabilities are insufficient information to compute the joint probability of destination and mode. In this case, as opposed to independent choices, the joint probability is not a product of 2 marginal probabilities since $P(m{:}M_d)$ is functionally dependent on d, i.e., $P(m{:}M_d) \neq P(m{:}M)$. If we had $P(m{:}M)$, then the joint probability is equal to its product with $P(d{:}D_m)$. However, to model the marginal probability, $P(m{:}M)$, we need to identify a utility function for an alternative mode that is independent of what destination is actually chosen. Therefore, for such a simultaneous structure, in which the choice of destination depends on the choice of mode and vice versa, we must model explicitly the joint probability $P(d, m{:}DM)$. Given the joint probability, we can derive the marginal probabilities and the structural probabilities as follows:

$$\begin{aligned} P(m{:}M) &= \sum_{d \epsilon D_m} P(d, m{:}DM) \\ P(d{:}D) &= \sum_{m \epsilon M_d} P(d, m{:}DM) \\ P(d{:}D_m) &= \frac{P(d, m{:}DM)}{P(m{:}M)} \\ P(m{:}M_d) &= \frac{P(d, m{:}DM)}{P(d{:}D)} \end{aligned} \tag{10}$$

A dependency that goes only in 1 direction results in a recursive structure. If we assume that the choice of destination is independent of what mode is actually chosen and that the choice of mode is dependent on the chosen destination, we write the following probabilities:

$$\begin{aligned} P(d{:}D) &= \mathrm{prob}[U_d \geq U_{d'}, \forall d' \epsilon D] \\ P(m{:}M_d) &= \mathrm{prob}[U_{m|d} \geq U_{m'|d}, \forall m' \epsilon M_d] \end{aligned} \tag{11}$$

This recursive structure implies the following additive utility function:

$$U_{dm} = U_d + U_{m|d} \tag{12}$$

The utility for a destination and mode combination is equal to a utility from the destination plus a utility from the mode that is dependent on the destination. In a recursive structure, the joint probability is the product of the structural probabilities.

Since we assume in this recursive structure that $P(m:M_d) \neq P(m:M)$, it is possible to derive from the joint probability a conditional $P(d:D_m)$ that is not equal to $P(d:D)$. However, this conditional probability is not causal but simply a mathematical relation derived from the model with no behavioral interpretation.

A recursive structure represents a hierarchical conditional decision structure. It is a common practice to replace a complex decision with a large number of alternatives by a recursive structure. The decision is decomposed into stages by successive partitions of the overall set of alternatives. Luce (5) noted that different partitions give different results. Therefore, a recursive structure can be viewed either as a simplifying assumption (this will require a sensitivity analysis of the partitioning scheme to determine how the results are affected) or as truly representing a sequential, or conditional, decision-making process.

SEPARABILITY OF CHOICES

Implicit in the discussion of the alternative structures was a separability-of-choices assumption. The conditional choice probability of mode given a destination was written as $P(m:M_d)$. This implies that the choice of m given d is independent of alternative modes to all other destinations $d'(d' \neq d)$, and is dependent only on the alternative modes for the given destination.

This is a reasonable assumption. It is required in order to be able to model choices separately. If we model directly a joint probability and assume a simultaneous dependency, then it appears that this assumption is not necessary. However, the interpretation of the derived conditional probabilities will not be the same as the one used here. It was also impossible to find an example of a model that does not make this assumption.

If we partition the set DM according to destinations, we can write the joint probability as follows:

$$P(d, m:DM) = P(m:M_d) \cdot P(d:D) \tag{13}$$

This equation is similar to the way in which Luce and Suppes (6) described the choice axiom,

$$P(i:A) = P(i:B) \cdot P(B:A) \tag{14}$$

for $i \in B \subset A$. The subset B corresponds to the subset of alternative modes to a given destination. However, the choice axiom is more general than the separability-of-choices assumption. It applies to any partitions of A to nonoverlapping subsets B. The separability-of-choices assumption applies only to partitions according to choices.

There is some similarity between the concept of functional separability and the separability-of-choices assumption. Functional separability is based on the idea that the marginal rate of substitution among a set of variables is independent of other variables. Separability of choices implies that a conditional probability for a given choice depends only on a part of the total utility function. The choice of mode given a destination is assumed to be dependent on $U_{m|d}$, which is the part of the utility function that for a given d varies across modes.

Hence, a separability assumption implies that, from the utility function for a destination and mode combination U_{dm}, we can identify the utility from a mode given a chosen destination $U_{m|d}$ and the utility from a destination given a chosen mode $U_{d|m}$. Clearly, their sum is not equal to U_{dm}. The separability assumption in an independent structure implies the additive utility function of Eq. 7. The separability assumption in a recursive structure where m depends on d implies the additive utility of Eq. 12.

ESTIMATION OF ALTERNATIVE STRUCTURES

It is possible to estimate directly the conditional probabilities or to derive their estimates from the estimated joint probability. [Estimating the joint probability and then deriving the conditional probabilities are analogous to the method of indirect least squares (7).] If the purpose of the analysis is to make only conditional predictions of one choice, given that all other choices remain constant, then the conditional probabilities are all that is needed and one can estimate them directly. However, the coefficient estimates of the conditional probabilities will not necessarily be equal whether they were estimated directly or indirectly through the estimation of the joint probability.

It appears that, if estimated through the joint probability, the coefficient estimates of the conditional probabilities can gain in statistical efficiency and can be less sensitive to specification errors. (Specification errors are the consequences of an incorrect set of explanatory variables or incorrect mathematical form or both.) The basis for this statement is the possibility of incorporating restrictions across conditional probabilities and thereby using more information to estimate some coefficients in the estimation of the joint probability. As an example, consider the simultaneous structure of destination choice and mode choice described previously. It is possible that $U_{d|m}$ and $U_{m|d}$ have common coefficients. By directly estimating U_{dm} we constrain them to be equal and we use simultaneously all the information from the choice among alternative modes as well as from the choice among alternative destinations. If we directly estimate $U_{d|m}$ we can only use information on alternative destinations for the chosen mode, i.e., the alternatives in D_m. If we directly estimate $U_{m|d}$ we can only use information on alternative modes for the chosen destination, i.e., the alternatives in M_d. In estimating U_{dm} we use information on all the alternatives in the overall set DM.

Only under very restrictive conditions will direct estimates of, say, $U_{m|d}$ result in the same coefficient estimates as indirect estimation through U_{dm}. This happens when the alternatives in DM that are not in M_d do not provide additional information to that obtained from M_d alone. In other words, this happens when the variability of modal attributes for destinations $d'(d' \neq d)$ is the same as that for the chosen destination d. The exact conditions that have to be fulfilled by the data for this to occur depend on the exact specification of the choice model. However, knowledge of the exact condition seems to be unimportant because as a practical matter it never occurs. Furthermore, even if it occurs there is no reason not to estimate U_{dm} if it can only be more efficient and it is needed for forecasting anyway.

In a recursive probabilistic structure, there is no reason to estimate directly the joint probability. Therefore, it could be estimated in its structural form, as it was done (3).

A simultaneous structure could also be estimated as a recursive structure as follows: (a) estimate one conditional, say $P(m:M_d)$; (b) derive from the analytical form of the joint probability the marginal $P(d:D)$; and (c) estimate the marginal with the coefficients that are included in $P(m:M_d)$ constrained to their estimates from $P(m:M_d)$. This estimation procedure is suggested only when for some reason the direct estimation of the joint probability is computationally difficult.

MODELING THE TRAVEL CHOICES

The preceding discussion indicates that the appropriate structure for the travel choices is a simultaneous one. In the remainder of this paper we discuss alternative structures of travel demand models in more detail.

A trip taken for a specific purpose is characterized by its origin, destination, time of day, mode of travel, and route. We are interested in predicting the volume of trips V_{idhmr} from origin i to destination d during time of day h by mode m via route r. From the point of view of the individual trip-maker or the household, we consider the probability of a trip instead of a quantity or volume of trips. A trip decision consists of several choices: choice of trip frequency f (e.g., how often to go shopping), choice of destination d (e.g., where to shop), choice of time of day h (e.g., when to go), choice of mode m, and choice of route r. Hence, for an individual traveler, we are interested in predicting the joint probability:

$$P(f, d, h, m, r : FDHMR_t) \tag{15}$$

where t denotes an individual or a household in origin i and $FDHMR_t$ is the overall set of alternative trips that consists of all possible combinations of frequencies, destinations, modes, times of day, and routes available to individual t. (The choice of residence location is assumed as given. Travel demand models assume that mobility decisions are fixed.) The alternatives in this set are exhaustive and mutually exclusive. The individual t is always selecting one and only one alternative from this set. (In the following sections a notation for different subsets of FDMHR is used. This notation follows the same logic that was used to define subsets of DM and is, therefore, not explained in the text.)

For simplicity, we will write the probabilities in the remainder of this paper without denoting the set of alternatives. We write the above probability (Eq. 15) as

$$P_t(f, d, h, m, r) \tag{16}$$

A conditional probability previously denoted as $P(m:M_d)$ will now be written as $P(m/d)$. The joint probability previously written as $P(d, m:DM)$ will now be written as $P(d,m)$.

On the disaggregate level, the travel demand function for a given trip purpose predicts the joint probability $P_t(f, d, h, m, r)$. On the aggregate level, the demand function predicts the volume V_{idhmr}. In either case, we have a complex product—a trip—with an enormous number of substitutes. Microeconomic consumer theory tells us that a demand function expresses the quantity of a product demanded as a function of its price, the prices of related commodities (substitutes and complements), and income. The complexities stem from the large number of relevant prices (i.e., price and many price-like attributes) for all the alternative trips.

ALTERNATIVE STRUCTURES OF TRAVEL DEMAND MODELS

With no further assumption, the travel demand model predicts the probability $P(f, d, m, h, r)$, or the volume V_{idmhr}, as a function of the attributes of all the alternative combinations of fdmhr. (For additional simplicity, we drop the subscript t in writing the probabilities in this section.) We denote the explanatory variables as X^1_{fdmhr}, X^2_{fdmhr}, ..., X^k_{fdmhr}, ..., X^K_{fdmhr}, or as a vector X_{fdmhr}. (The explanatory variables include all the levels of service, the spatial opportunities, and the socioeconomic variables. The socioeconomic variables are specific to an individual and not to a trip alternative. However, we assume here that they are introduced into the model as having alternative specific values.) Hence, we can write the travel demand model as follows:

$$P(f, d, m, h, r) = F[X_{fdmhr}, \forall fdmhr \in FDMHR] \tag{17}$$

where $[X_{fdmhr}, \forall fdmhr \in FDMHR]$ is a vector that includes all the variables X for all relevant combinations of the subscripts f, d, m, h, and r, and F is the demand function. Alternatively, we can write the utility function for an alternative trip as

$$U_{fdmhr} = U(X_{fdmhr}) \tag{18}$$

Clearly, this results in a very complex demand model. Without further assumptions, for a simultaneous structure this is the type of travel demand model that must be calibrated.

If, however, we make some assumptions about the travel decision-making process we can divide the overall travel demand function into several less complex functions, each including only a subset of all the explanatory variables. That is, under some assumptions we can formulate the travel demand function as a recursive or as an independent structure.

The first assumption that is required is the separability-of-choices assumption that was described earlier and is usually made also with respect to a simultaneous model. The separability assumption with respect to a certain choice says that the conditional probability of this choice given other choices is a function of only a specific subset of the explanatory variables, as depicted in the following example for route choice:

$$U_{r|fdmh} = U^r(X_{fdmhr})$$
$$P(r|f,d,m,h) = F^r[X_{fdmhr}, \forall r \in R_{fdmh}] \tag{19}$$

In words, the conditional probability of choosing a route given other choices is a function only of the explanatory variables for all routes for given fdmh. If we considered only 2 choices, say, mode and destination, then the separability assumption with respect to mode choice says that the conditional probabilities of choosing a mode given a destination is a function of the variables for all modes but for only 1 specific destination. For this example we write

$$P(d,m) = F^{dm}[X_{dm}, \forall dm \in DM]$$
$$U_{dm} = U(X_{dm})$$
$$P(d|m) = F^d[X_{dm}, \forall d \in D_m]$$
$$U_{d|m} = U^d(X_{dm}) \tag{20}$$
$$P(m|d) = F^m[X_{dm}, \forall m \in M_d]$$
$$U_{m|d} = U^m(X_{dm})$$

If we calculate the marginal probabilities P(d) and P(m), they will be a function of the vector $[X_{dm}, \forall dm \in DM]$.

An independent structure is possible only if the set of attributes is separable. That is,

$$[X_{fdmhr}] = [X_f,\ X_d,\ X_m,\ X_h,\ X_r] \tag{21}$$

where we can identify only attributes that vary only across a single choice. The independent utility function can be written as

$$U_{fdmhr} = U^f(X_f) + U^d(X_d) + U^m(X_m) + U^h(X_h) + U^r(X_r) \tag{22}$$

The independent travel demand model can be written as

$$P(f) = F^f[X_f, \forall f \in F]$$
$$P(d) = F^d[X_d, \forall d \in D]$$
$$P(m) = F^m[X_m, \forall m \in M] \tag{23}$$
$$P(h) = F^h[X_h, \forall h \in H]$$
$$P(r) = F^r[X_r, \forall r \in R]$$

and

$$P(f,d,m,h,r) = P(f) \cdot P(d) \cdot P(m) \cdot P(h) \cdot P(r) \tag{24}$$

Clearly, this is an unrealistic structure for a travel demand model.

A recursive structure requires the assumption of a sequential decision-making process or a hierarchy of conditional decisions. The sequence is expressed in a recursive travel demand model in 2 ways. The first is the manner in which the set of all trip alternatives is partitioned. In a recursive model of mode and destination choices where mode choice is conditional on the chosen destination, the set of all alternative combinations of mode and destination is partitioned according to destination. The second way is the composition of explanatory variables. For the same example, the problem is how to include in a model of the marginal probability of destination choice the variables, such as travel time and fare, that are defined by destination and mode. The way this is handled is to construct a composite variable that combines the above variable across modes to create a variable that is specific only to a destination. Consider for example the following recursive structure:

$$\begin{aligned} U_{fdmhr} &= U_f + U_{d|f} + U_{m|fd} + U_{h|fdm} + U_{r|fdmh} \\ &= U^f(X_f) + U^d(X_{fd}) + U^m(X_{fdm}) + U^h(X_{fdmh}) + U^r(X_{fdmhr}) \end{aligned} \tag{25}$$

and

$$\begin{aligned} P(f) &= F^f[X_f, \forall f \in F] \\ P(d|f) &= F^d[X_{fd}, \forall d \in D_f] \\ P(m|f,d) &= F^m[X_{fdm}, \forall m \in M_{fd}] \\ P(h|f,d,m) &= F^h[X_{fdmh}, \forall h \in H_{fdm}] \\ P(r|f,d,m,h) &= F^r[X_{fdmhr}, \forall r \in R_{fdmh}] \end{aligned} \tag{26}$$

where each variable is defined as follows:

$$\begin{aligned} X_{fdmh} &= [X_{fdmhr}, \forall r \in R_{fdmh}] \\ X_{fdm} &= [X_{fdmh}, \forall h \in H_{fdm}] \\ X_{fd} &= [X_{fdm}, \forall m \in M_{fd}] \\ X_f &= [X_{fd}, \forall d \in D_f] \end{aligned} \tag{27}$$

If we keep the variables in their original form, then the model for P(f) will include all the explanatory variables $[X_{fdmhr}, \forall fdmhr \in FDMHR]$. The definition of composite variables allows the treatment of X_{fdmh}, X_{fdm}, X_{fd}, and X_f as single variables. In other words, these variables are expressed as a specific function of their elements. For example, we express

$$X_{fdmh} = g[X_{fdmhr}, \forall r \in R_{fdmh}] \tag{28}$$

where g is the composition function. The functional form of the composition rule requires further assumptions.

There are a variety of possible composition schemes. One such scheme that was derived from an assumption of additive utility function (3) is as follows:

$$X_{fdmh} = \sum_{r \in R_{fdmh}} X_{fdmhr} \cdot P(r|f,d,m,h) \tag{29}$$

This composition scheme is essentially a computation of the expected value of the original variable. Another way to observe this is to rewrite Eq. 29 and use the definition of conditional probability as follows:

$$X_{fdmh} \cdot P(f, d, m, h) = \sum_{r \in R_{fdmh}} X_{fdmh} \cdot P(f, d, m, h, r) \tag{30}$$

Thus, the composite variable as defined by Eq. 29 is in accordance with a consistency requirement that the expected value of a variable is maintained. If X is a price variable, then Eq. 30 says that the expected expenditure is consistent in the different stages of a recursive model.

Clearly, there are many other schemes of creating the composite variables, among them a simple sum,

$$X_{fdmh} = \sum_{r \in R_{fdmh}} X_{fdmhr} \tag{31}$$

or the value for the "best" route (10),

$$X_{fdmh} = X_{fdmhb} \tag{32}$$

where r = b is the best route according to some criteria.

Often, several price variables are combined to form a generalized price. Then, the composite variable is a composition of the generalized price instead of each variable separately (3, 8).

Constructing a composite variable from several explanatory variables together amounts to maintaining equal marginal rates of substitution among those variables in the different probabilities of a recursive structure.

Thus, given a separability assumption, a specific sequence assumption, and an assumption on the mathematical form of the composite variables, the overall travel demand model can be formulated as a recursive structure.

A simultaneous structure requires the estimation of an equation that includes a large number of explanatory variables. On the other hand, each equation in a recursive structure includes only a subset of the explanatory variables that are included in a simultaneous model. In addition, the number of variables is reduced by the construction of composite variables. Therefore, a recursive model can be easier to implement, computationally and analytically, than a simultaneous model.

The separability and the sequence assumptions required by a recursive travel demand model are equivalent to an assumption of a conditional decision structure. The choice of a particular fdmhr combination is made from a relatively large set of alternatives. It makes sense to partition the set of all alternatives into collections of nonoverlapping subsets. Consider, for example, 2 choices: destination and mode. The set of all alternative combinations of d and m, DM, is large. We can partition DM into the subsets M_1, M_2, ..., M_d, ..., M_D, where each subset includes all the alternative modes to a specific destination. The assumption is that the traveler is, first, choosing among these subsets or choosing a destination and, second, choosing within the chosen subset or choosing a mode. The choice of mode is now a function of only the characteristics of available modes to a given destination. The choice of destination depends on some measure of the expected attributes of all modes to a given destination. The utility function of a dm combination is assumed to consist of 2 parts: one for each choice. The choice of destination is based on the utility of the destination, which is also dependent on the expected attributes from the modes available to this destination.

However, we can also partition the set DM according to modes as follows: D_1, D_2, ..., D_m, ..., D_M. When we apply choice models to this or the previous sequence we do not expect the predictions to be the same. The problem is, therefore, to know when the consumer decomposes his or her decision into stages and what partitions are used.

If we modeled the choice of an fmdhr combination as a deterministic optimization problem, it would not be important what partitions were used. The reason that we expect different partitions to give different results is due to the probabilistic choice mechanism and the computation of expected attributes from lower stages.

The problem with travel decisions is that we cannot find a unique natural sequence of partitions that will be generally applicable. Therefore, a simultaneous structure is superior to a recursive structure. In general, the simultaneous structure of a travel demand model consists of the following conditional probabilities:

$$\begin{gathered} P(f \mid d, m, h, r) \\ P(d \mid f, m, h, r) \\ P(m \mid f, d, h, r) \\ P(h \mid f, d, m, r) \\ P(r \mid f, d, m, h) \end{gathered} \tag{33}$$

Under particular behavioral assumptions we can place restrictions on this general structure and obtain alternative simultaneous structural forms. Consider the following simultaneous structure:

$$\begin{gathered} P(f \mid d, m, h, r) \\ P(d \mid f, m, h) \\ P(m \mid f, d, h) \\ P(h \mid f, d, m, r) \\ P(r \mid f, d, m, h) \end{gathered} \tag{34}$$

The conditional probabilities of mode choice and destination choice are not conditional on the chosen route because we cannot generally identify alternative modes or destinations for a given route.

The choices that are conditional on f in either a simultaneous or a recursive structure are defined only for $f > 0$ because it does not make sense to define alternative trips when no trip is taken. It may be argued that for some trip purposes the choice of trip frequency is based on some measure of expected accessibility and is not dependent on the actual values of d, m, h, and r. Therefore, it is natural to partition according to f and, for each f, have all possible combinations of mdhr.

If for some trip purpose the choice of time of day is constrained or limited to alternative times for which the traveler can be assumed to be indifferent, then it is possible to partition according to f and, for each f, have all possible combinations of m and d. Then, partitioning according to dm combinations creates the sets of alternative routes for a given trip. This decomposition implies the following structural probabilities:

$$\begin{gathered} P(f) \\ P(d \mid f, m) \\ P(m \mid f, d) \\ P(r \mid f, d, m) \end{gathered} \tag{35}$$

The choices of mode and destination are simultaneous, but recursive with respect to f. The choice of route is recursive with respect to f, d, and m. This is essentially the structure that is assumed in the empirical study reported elsewhere (1). Time of day was excluded because the sample included only off-peak shopping trips.

It should be clear that in simultaneous and recursive structures we can derive any conditional or marginal probabilities. (However, only the structural probabilities are causal.) Therefore, for forecasting, it is possible to use the joint probability directly or any combination of marginal and conditional probabilities provided that their product is equal to the joint probability. For example,

$$
\begin{aligned}
P(f,d,m,h,r) &= P(f) \cdot (Pd|f) \cdot P(m|f,d) \cdot P(h|f,d,m) \cdot P(r|f,d,m,h) \\
&= P(f) \cdot P(h|f) \cdot P(m|f,h) \cdot P(d|f,h,m) \cdot P(r|f,h,m,d) \qquad (36) \\
&= P(f) \cdot P(h,m,d|f) \cdot P(r|f,h,m,d)
\end{aligned}
$$

DIRECT AND INDIRECT TRAVEL DEMAND MODELS

A distinction was made between simultaneous, recursive, and independent travel demand models. It was based on the behavioral assumptions of the model. Another distinction that is often made is between direct and indirect travel demand models (8). This distinction, however, is based on the way that the travel demand model is used for forecasting.

A direct demand model predicts directly the joint probability $P_t(f,d,m,h,r)$, or the volume V_{idmhr}, as a function of all the explanatory variables. In an indirect travel demand model the joint probability, or the volume, is predicted with several intermediate steps. Each step corresponds to a single choice or to a single subscript of the volume. For example, one equation can predict the number of trips taken by the household, another equation will distribute trips among the various destinations, and so forth. Hence, in a direct model a forecast is made with a single equation, while in an indirect model a forecast is made by a multiequation model.

There are a variety of possible indirect models in which an intermediate step may predict directly more than one choice. For example, one equation can predict the number of trips taken by the household to a certain destination, another equation will split these trips among the various modes of travel, and so forth.

From the forecasting point of view it makes no difference whether we use a model as direct or as indirect. The way a model is used for forecasting should be determined only on the basis of computational efficiency considerations.

The sequence used for forecasting does not necessarily have a behavioral interpretation. Even a recursive model could in principle be used for forecasting in an indirect fashion that does not correspond to the structural sequence.

In this paper we are concerned with the behavioral structure of travel demand models. However, we can express any given model in many different ways. Therefore, an obvious question to ask is, How can the behavioral structure of a given model be recognized?

In general, the answer to this question is that the behavioral structure cannot be determined unless the model is written in its structural form. This answer could be explained by the analogy of a structure of simultaneous equations. Given a reduced form, which is used for forecasting, it is impossible to determine the original structure. (A reduced form of a system of simultaneous equations is the solution of endogenous variables in terms of the exogenous ones.) However, in travel demand models that were structured with composite variables, the structure may be discerned. It is possible to recognize the sequence through the order of composition (e.g., order of summation) that is maintained in a composite variable no matter how the model is expressed.

EMPIRICAL PROBLEM

As mentioned earlier, the complexity of the overall travel demand function stems primarily from the large number of alternatives and attributes that call for a large number of variables. To appreciate the dimensions of the overall travel demand function, consider the following example of travel choices.

Suppose that for a certain trip purpose a person has the following options: 2 daily trip frequencies (1 trip or no trip), 4 destinations, 2 modes of travel, and 2 times of day (peak or off-peak). The total number of alternatives facing the decision-maker is 17 (16 one-trip alternatives and 1 no-trip alternative). Suppose that for each 1-trip alternative there are only 2 price variables, travel time and travel cost. (The price of a no trip is 0.) The total number of price variables is 32. If we increment each choice by 1 additional option, we have 91 alternatives and 180 price variables.

It appears that the joint probability may be too complex and the number of variables too large to be condensed into a single relation. The most important question is whether we can calibrate a choice model with such large numbers of alternatives and variables. Using a recursive structure, we will have to calibrate 4 choice models but with the number of alternatives in each model equal to the number of options for the corresponding choice. The data requirements are identical for both structures unless further assumptions are made.

It is not clear whether it is less expensive to calibrate 4 models each with a small number of alternatives rather than 1 model with many alternatives (assuming, of course, that estimation of a joint probability is feasible).

Under the presumption that the implementation of a recursive model is easier and less expensive, is the additional expense to implement a simultaneous model justified? The answer is unclear. Costs can be compared only together with the benefits. Therefore, we need to know how the simplifying assumptions of a recursive model affect the results of the prediction process.

These are critical issues that can only be addressed by an empirical study. The evidence from the calibration of alternative structures in another study (1) indicates that (a) it is feasible to calibrate the simultaneous model and (b) the calibration results are highly sensitive to the assumed structure. This empirical evidence is not absolutely conclusive, however, because it is based on a small sample and only on a subset of the travel choices for a single trip purpose. Future research is needed to extend the empirical evidence to different data sets, larger samples, and a complete set of travel choices for all trip purpose categories.

SUMMARY AND CONCLUSION

A multidimensional choice situation can be represented by a simultaneous or recursive model structure. The paper described assumptions of each structure and argued that, in the absence of restrictive assumptions about behavior, travel decisions are more realistically represented by a simultaneous model structure. It is simple to estimate a recursive structure, for each choice model contains fewer alternatives and variables. The primary issues in the selection of a strategy for calibration are (a) whether calibrating the simultaneous model is feasible and (b) what effect the use of a recursive rather than a simultaneous model structure has on the estimated parameters.

In particular, the calibration strategy is independent of the method of prediction to be used. That is, both the simultaneous and recursive models can be used as direct prediction models based on the joint probabilities or as indirect prediction models by deriving any desired set of marginal and conditional probabilities.

Empirical evidence for a 2-dimensional choice situation indicates that calibration of the simultaneous choice model is feasible and equally important and that calibration as a recursive structure leads to different parameter estimates, which are very sensitive to the order of decision-making assumed. Additional research is required to verify these results and to extend them to more complex choice situations.

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