The task of achieving control of the noise output of trucks is a systems problem of rather large dimensions. Appropriate noise regulations must be formulated and promulgated, and then suitable enforcement techniques must be applied. The process is still in the formative stage, but it appears that at some future date, through the cooperation of federal, state, and local agencies, a comprehensive system for control of in-use truck noise will evolve and will include periodic inspection, highway surveillance (covering intervals between inspections), and use of truck weighing stations. In addition there will be an adequate system of factory inspection that will control the noise emissions of newly manufactured trucks.

Devising and setting in place this system are crucial to the success of the truck noise-control program. However, this is not the problem addressed in this paper. This paper assumes that such a system is in existence and examines its effect on the number of trucks within various noise categories. A model of truck population dynamics is described. The associated analytical techniques are outlined, and the method is applied to an idealization of a practical situation. A few tentative conclusions are drawn from this analysis.

MODEL DESCRIPTION

The truck population is divided into mutually exclusive classes, $m$. These could be, for
example, the trucks present in the truck population at the start of a regulation, those manufactured after the first stage of the regulation, or those manufactured after the second stage of the regulation. Each of these classes, say, the i-th class, is associated with a rate of new-truck input (q_i trucks manufactured per year) and a rate of truck retirement (Q_i trucks retired per year per n_o of population). This input-output relation causes the truck population in the i-th class to vary with time. We assume that this is a continuous process and develop the equations describing it in the following section. The model is shown in Figure 1.

ANALYSIS OF MODEL

In a short interval of time, dt, the change in truck population in the i-th class, dN_i, is given by q_i dt, the number of new trucks injected into the population, less (N_i /n_o)Q_i dt, the number of trucks retired. That is,

\[ dN_i = q_i dt - \frac{N_i}{n_o} Q_i dt \]  

which yields the first-order linear differential equation

\[ \frac{dN_i}{dt} + \left( \frac{Q_i}{n_o} \right) N_i = q_i \]  

where

- \( N_i(t) \) = number of trucks of the i-th type in the population at time t;
- \( Q_i(t) \) = rate of retirement of the i-th class of trucks at time t and given as trucks per n_o of population retired per year, where n_o is usually taken as 1,000;
- \( q_i(t) \) = rate of injection of new trucks of i-th type into population and given as number of new-truck manufacturers per year, computed at time t.

The total population \( N(t) \) is given by

\[ N(t) = \sum_{i=1}^{m} N_i(t) \]  

General solutions for Eq. 2 are readily attainable even when \( q_i \) and \( Q_i \) are any prescribed functions of time, that is, when manufacturing rates and retirement rates are not constant. Future work is planned employing time-varying \( q_i \) and \( Q_i \) but the present paper assumes that they are constant.

Two types of solutions are obtained from Eq. 2: growth and decay. For the growth functions, \( q_i \) is not 0; for the decay functions, \( q_i = 0 \). Both functions are exponential. However, the growth functions are increasing functions of time and approach an asymptotic value as time approaches infinity. On the other hand, the decay functions are decreasing functions of time and start from some initial value and approach 0 as time approaches infinity. These are

\[ N_i(t) = \frac{n_i Q_i}{Q_i} - \frac{n_i Q_i - n_i(T_{1a})}{Q_i} \cdot e^{-\frac{Q_i}{n_o}(t - T_{1a})} \]  

for the growth function and

\[ N_i(t) = N_i(T_{1a}) \cdot e^{-\frac{Q_i}{n_o}(t - T_{1a})} \]  

for the decay function. Growth starts when \( t = T_{1a} \); decay starts when \( t = T_{1b} \). The character of the growth and decay curves is shown qualitatively in Figure 2.
Figure 1. Dynamic truck population model.

\[ N(t) = \sum_{i=1}^{m} N_i(t) \]

Figure 2. Growth and decay of truck populations.

GROWTH CURVE
\[ N_i(t) = \frac{n_{i0} q_i}{Q_i} - \left[ \frac{n_{i0} q_i}{Q_i} - N_i(T_{ib}) \right] e^{-\frac{Q_i}{n_{i0}} (t - T_{ib})} \]

DECAY CURVE
\[ N_i(t) = N_i(T_{ib}) \cdot e^{-\frac{Q_i}{n_{i0}} (t - T_{ib})} \]
Figure 3. Proposed new-truck noise regulation.

Figure 4. Truck population versus time.
APPLICATION TO PROPOSED NEW-TRUCK REGULATION

The formulas given above are applied to the truck populations, which are expected to be associated with the recent interstate motor carrier regulation and the proposed new-truck regulation. The trucks covered by these regulations are medium and heavy trucks having a gross vehicle weight rating of 10,000 lb (4500 kg) or greater. The results are approximations because data for an accurate calculation are not available. However, the character of the results is correct even though the numbers are not precise.

Figure 3 shows the proposed new-truck regulation schedule. When the regulation is formally announced, a sizable truck population will be in existence. This is labeled as an 86-dBA population. This is the motor carrier allowed level and was selected for that reason. However, for the purposes of this analysis, the number is only a label, and the value of the number does not enter into the calculation. The same is true of all the other levels shown in Figure 3; i.e., they are only labels. The schedule shows that 2 years after the announcement the new-truck level will drop to 83 dBA. During the next 4 years, 83-dBA trucks will be manufactured and injected into the truck population. At the end of this period, the new-truck level will drop to 80 dBA and stay there for 2 years. Finally, the new-truck level will reach 75 dBA and stay permanently at that value. The following numerical values were employed in the analysis:

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial truck population</td>
<td>$N_i(0) = 5.7 \times 10^6$</td>
</tr>
<tr>
<td>Annual rate of truck production</td>
<td>$q_2 = q_3 = q_4 = 4.0 \times 10^5$</td>
</tr>
<tr>
<td>Annual retirement rate of trucks</td>
<td>$Q_1, Q_2, Q_3, Q_4 = 71.9$/thousand/year</td>
</tr>
<tr>
<td>Truck retirement basis number</td>
<td>$n_c = 1,000$</td>
</tr>
</tbody>
</table>

Figure 4 shows the time history of each of the populations defined above, starting 2 years after the announcement of the regulation, i.e., at the time the 83-dBA trucks start to be manufactured.

The $N_1(t)$ curve (86 dBA) starts at about 5.7 million and decays steadily with time. However, because it has such a large initial value, it dominates the other populations for a long time. The $N_2(t)$ curve shows that the 83-dBA population increases steadily for 4 years and then decays. The $N_3(t)$ curve shows that the 80-dBA population increases for 2 years and then decays. Finally, the 75-dBA population is given by the $N_4(t)$ curve, which steadily increases and approaches an asymptotic value of $5.6 \times 10^6$, which is not shown on the curve.

SUMMARY

1. A technique for calculating truck populations has been developed. It is applicable to a rather wide variety of cases, including variable truck manufacturing rates and variable truck retirement rates.
2. The technique has been applied to an approximate model of the interstate motor carrier regulation and the proposed new-truck regulation and indicates that the population of trucks in use when the new-truck regulation is promulgated will dominate the total truck population for many years thereafter.
3. The work reported here is a first installment. Further work using variable manufacturing and retirement rates as well as a more refined estimate of the initial truck population will be undertaken.