AN ABSOLUTE METHOD OF DETERMINING THERMAL CONDUCTIVITY AND DIFFUSIVITY OF SOILS

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Synopsis

The most common methods for measurement of thermal conductivity of poor conductors, e.g., earth or insulating materials, are based on the fact that the sample is in a steady-state condition. For poor conductors this requires a long period before the heat flow through the sample becomes steady. During this period the heat source must be held constant. Prolonged heating aggravates such undesirable processes as moisture migration and changes in structure. Furthermore, the necessity of removing samples for measurement from their normal situation introduces uncertainties and experimental difficulties.

These methods have two more fundamental defects. First, steady-state measurements will give no information on thermal diffusivity, a constant equal in importance to conductivity in many heat-transfer problems. Second, the actual experimental devices, hot-plates, divided bars, etc., are calibrated by using materials of presumably known conductivity to either establish quantity heat flow or determine instrumental constants, such as contact resistance with the sample.

The method described here uses measurements during heating or cooling, which may be taken rapidly and will give both thermal conductivity and diffusivity. The mathematics are rigorous and therefore the effect of assumptions made in the theory can be calculated. With heat sources of appropriate shape and dimensions, the measurements may be made absolute, i.e., they are independent of the particular measuring device and are not affected by the thermal properties of the materials used to construct the source. The limits within which this condition is fulfilled can be calculated accurately. The method is applicable to a variety of different forms of apparatus. When a suitable form has been selected and a particular apparatus built, the necessary calculations may be made once and presented as graphs from which the desired results are read off as rapidly as readings are taken. A general description of the method will be followed by two particular applications, a spherical heater buried in the material and a linear heater or probe inserted into the material.

A heater, usually electric, is surrounded by the material whose thermal constants are to be measured. A temperature measuring device, usually a thermocouple or resistance thermometer, attached to the surface of this heater indicates the change in its temperature while a constant, measured, energy output is maintained. The temperature rise at two selected intervals is recorded. These intervals are pre-determined to minimize the effects of certain assumptions made in applying the theory to the particular instrument and are controlled by such factors as physical dimensions, power output, and the temperature rise considered as allowable without affecting the material under investigation.

Using tables of the appropriate functions involving the dimensions of the apparatus and the selected time intervals, a graph is constructed showing the relation between the ratio of the two temperature differences and the thermal diffusivity. If the graph is constructed to cover the range of diffusivities encountered in the type of material being tested, the diffusivity is read directly from the graph as soon as the temperature differences have been recorded and their ratio calculated. Entering another graph at the value of the diffusivity and using the measured power output and the temperature rise at either of the selected intervals, a single multiplication gives the value of the thermal conductivity. Once the graphs have been constructed, they are used with a particular apparatus and will give rapid results over a reasonable range of the thermal constants.

This simple method will give results accurate to within 3 percent. A slightly more complicated method of calculation, using the same basic principle, can be used to reduce this error, which arises in the application of the theory to actual apparatus.

The physical shape and dimensions of the heater determine the type of function used in constructing the graphs. The cases of a spherical heater treated as a point source and a cylindrical heater of small diameter treated as a linear source are discussed below. Other cases, such as a cylindrical heater treated as a cylindrical source, may be dealt with by the same method, but calculations required are more complicated and improvement in accuracy attained is of doubtful value in view of the inhomogenieties usually encountered in the substance being measured.

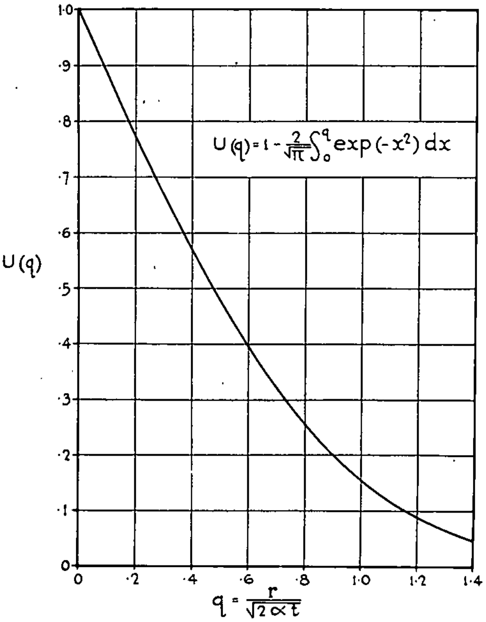

Figure 1. Values of Function U(q) for $q = r/\sqrt{2\alpha t}$ up to 1.4

It should be stressed that no novelty is claimed for the heat conduction equations used. These are to be found in the standard texts on the subject (Carlslawand Jager, Heat Conduction in Soils; Ingersoll, Zobel, and Ingersoll, Heat Conduction, McGraw-Hill, 1948). They involve no mathematical approximations affecting the accuracy of their application to cases studied. The magnitude of the errors introduced in applying the idealized theory to actual conditions can be evaluated.

Application to Spherical Heaters

Aluminum spheres 4 in. in diameter were fitted with small, centrally-located, resistance heaters and with copper constantan thermocouples soldered to their surface. These were buried to a depth of 5 ft. in the ground and allowed to attain temperature equilibrium with their surroundings. This was indicated by the constancy of the thermocouple readings over a period of 48 hours. Energy was then supplied to the heaters at a constant rate of about 25 watts and the temperature of the surface of the heater recorded every few minutes. From the graph of temperature versus time, the temperature rise after heating intervals of 1 hr. and 2 hr. were recorded. 1/ The magnitude of the temperature rise was of the order of 30 to 40 degrees F.

This case may be considered as an approximation to that of a point source of heat immersed in an infinite homogeneous medium. The limits of validity of this approximation will be discussed later.

The temperature rise in time t at a distance r from a point source of heat immersed in an infinite homogeneous medium is given by the expression

$$T = \frac{Q'}{4\pi K r}\left[1 - \frac{2}{\sqrt{\pi}}\int_0^{\frac{r}{2\sqrt{\alpha t}}} \exp(-x^2)\,dx\right] \qquad (1)$$

where
- T is the temperature rise above the initial uniform temperature
- Q' is the rate of heat energy output of the source
- K is the thermal conductivity of the medium
- α is the thermal diffusivity of the medium

$$= \frac{\text{(thermal conductivity)}}{\text{(specific heat) (density)}}$$

1/ The continuous reading of temperature was done to provide a check for certain aspects of the theory. For satisfactory calculations of conductivity and diffusivity, readings taken at 1 hr. and 2 hr. from the start of heating would be sufficient.

For convenience, equation (1) is written in the form

$$T = \frac{B}{K}\, U\left(\frac{r}{2\sqrt{\alpha t}}\right) \qquad (2)$$

where

$$B = \frac{Q'}{4\pi r}$$

and U is a function the numerical value of which has been tabulated in the texts for different values of the argument. For convenience, a graph of the function U for values of the argument from 0 to 1.4 is given in Figure 1.

Considering the temperature at the surface of the spherical source ($r = r_S$) and two specific time intervals t_1 and t_2 we obtain:

$$T_1 = \frac{B}{K}\, U\left(\frac{r_S}{2\sqrt{\alpha t_1}}\right) \qquad (3)$$

and

$$T_2 = \frac{B}{K}\, U\left(\frac{r_S}{2\sqrt{\alpha t_2}}\right) \qquad (4)$$

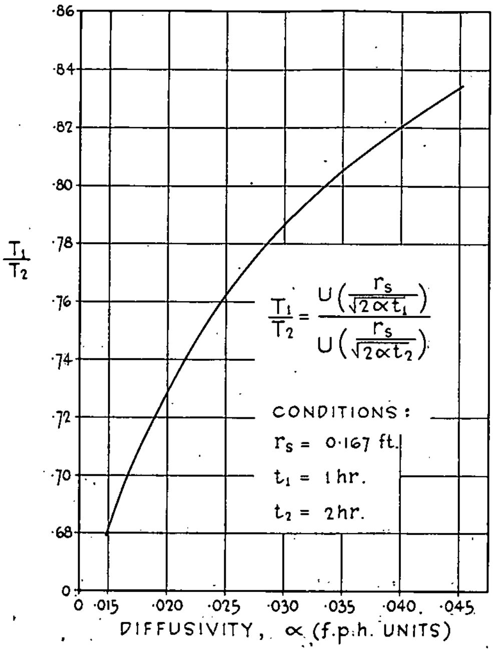

Figure 2. Values of Ratio T_1/T_2

and, by division

$$\frac{T_1}{T_2} = \frac{U\left(\frac{r_S}{2\sqrt{\alpha t_1}}\right)}{U\left(\frac{r_S}{2\sqrt{\alpha t_2}}\right)} \qquad (5)$$

By assigning particular values to α which cover the expected range, the magnitude of the right-hand side of equation (5) may be computed at a number of points and a graph drawn between α and the ratio

$$U\left(\frac{r_S}{2\sqrt{\alpha t_1}}\right) / U\left(\frac{r_S}{2\sqrt{\alpha t_2}}\right).$$

Figure 2 is this graph for the case discussed here. In use, Figure 2 is considered as showing the variation of T_1/T_2 with α. Once the temperature rise after 1 hr. (T_1) and the temperature rise after 2 hr. (T_2) have been observed, their ratio is computed and the value of α for the material surrounding the heater is read from Figure 2.

To determine the value of the thermal conductivity K, Figure 3 is used. This gives the variation of

$$U\left(\frac{r_S}{2\sqrt{\alpha t_1}}\right) \text{ with } \alpha$$

and of

$$U\left(\frac{r_S}{2\sqrt{\alpha t_1}}\right) \text{ with } \alpha \; \underline{2}/.$$

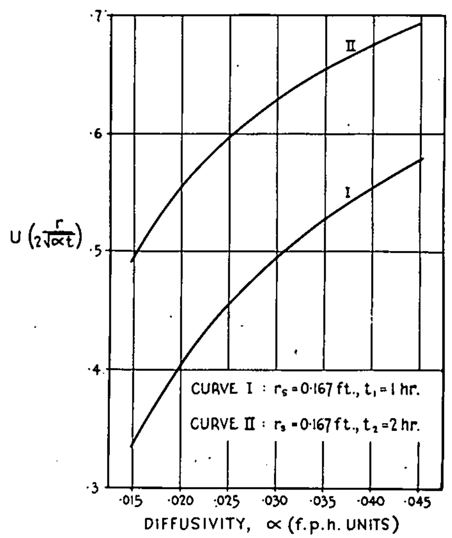

Figure 3. Values of Function $U(r/2\sqrt{\alpha T})$ for the Particular Heater Used for Range of Diffusivities

Using the value of α determined above, the corresponding magnitudes of the two functions are read from the graphs. These magnitudes,

2/ This graph requires no new computation but is simply a replotting of the values already calculated for Figure 2.

together with the observed values of T_1, T_2, and B, are substituted in equations (3) and (4) to obtain two values of K. The agreement of these two values affords a good internal check on the accuracy of the calculations.

As an illustration of the method, we consider the results of a particular test with a spherical heater for which the graphs were calculated. The observed values were $T_1 = 30$ F., $T_2 = 38$ F. with B = 40. 2 Btu. per hr. -ft.

The ratio $T_1/T_2 = 0.790$; thus, from Figure 2, $\alpha = 0.0320$ f. p. h. units

From Figure 3 $\quad U\left(\frac{r_S}{2\sqrt{\alpha t_1}}\right) = .502$

and $\quad U\left(\frac{r_S}{2\sqrt{\alpha t_2}}\right) = .634$

and $\quad B/T_1 = 1.340, \quad B/T_2 - 1.058$

Substituting in equations (3) and (4):

K = 1. 340 x . 502 = . 671 f. p. h. units
K = 1. 058 x . 634 = . 670 f. p. h. units

Application to Cylindrical Heaters

Stainless-steel or brass tubes 3/16 in. in diameter and 10-in. long were fitted with axial resistance heaters and thermocouples at the middle of their surface. The ends were closed and these probes inserted into various samples of insulating material. An adequate sample was roughly a foot or foot-and-a-half cubed. For measurement of the thermal properties of the ground more robust probes 1-1/2 in. in diameter and three feet long were used.

This case may be considered as an approximation to that of a line source of heat in an infinite homogeneous medium. The rise of temperature (T) at a radial distance (r) from such an ideal source is given by

$$T = \frac{Q'}{2\pi K} I(r\,n) \qquad (6)$$

where Q' is the rate of heat energy output of the source
K is the thermal conductivity of the medium

$$I(r\,n) = \int_{r\,n}^{\infty} \frac{1}{x} \exp(-x^2)\,dx$$

$$n = \frac{1}{2\sqrt{\alpha t}}$$

α is the thermal diffusivity of the medium
t is the time from start of heating

If T_1 and T_2 are the increases in temperature after intervals of t_1 and t_2 respectively, we may proceed as in the previous application and form the ratio

$$\frac{T_1}{T_2} = \frac{I(r\,n_1)}{I(r\,n_2)} \qquad (7)$$

Values of the function I (r n) are to be found in tables so we can calculate the right hand side of equation (7) for selected values of α and plot the results as Figure 4a. Using the same calculations we also plot Figure 4b, 4c showing the variation of $I(r\,n_1)$ and of $I(r\,n_2)$ with α. The values for the particular calculation used here are $r = 3/32'' = 7.82 \times 10^{-3}$ft. , $t_1 = 4$ min. = 0. 0667 hr. and $t_2 = 10$ min. = 0. 167 hr.

The method of using these graphs is entirely similar to that described in the case of the spherical source. As an illustration we consider the results of a particular test on a sample of silica aerogel. The observed values were $T_1 = 61$ F., $T_2 = 80$ F., with Q' = 3. 29 Btu. per hr.

The ratio $T_1/T_2 = 0.762$; thus from Figure 4a, $\alpha = 0.0073$ (f. p. h. units)
From Figures 4b and 4c, $I(r n_1) = 1.44$ and $I(r n_2) = 1.90$.
Substitution of these together with Q' and the appropriate values of T in equation (6) gives the two values of
$K = 0.0123$ (f. p. h. units) for T_1 and $I(r n_1)$
and $K = 0.0124$ (f. p. h. units) for T_2 and $I(r n_2)$.

Errors Introduced by the Assumptions

There are no assumptions in the mathematical development of equations (1) and (6) which limit them to restricted ranges of application. They should hold for all values of r, α, t, Q', etc. This mathematical rigor is not present in some other methods of determining thermal constants by using heated probes.

In applying the rigorous theory to the actual conditions certain assumptions have been made. Because the theory is rigorous, the magnitude of the errors introduced by the assumptions can be calculated. The most serious assumption is that a finite heater (a sphere or cylinder) of different thermal properties from the surrounding medium may be treated as an ideal source (point or line). This assumption will undoubtedly introduce a large error at the start of the heating when the output of the heating element is largely used in raising the temperature of the heater itself. The error will be small after a longer period of heating when the heat flowing into the surrounding medium will be very nearly equal to the output of the heater, very little being used to raise the temperature of the source. The problem is to determine the period after which this error will have been reduced to allowable limits. This may be done as follows (the case of a cylindrical source is taken for illustration):

Consider a cylindrical shell in the medium just outside the probe. When the heat flow through this shell is the same for the actual probe (radius = r) as it is for an ideal line source, then the probe is producing all effects in the medium as if it were a true line source. The theory of the ideal line source which gives equation (6) may be extended to show that such a source of strength Q' is equivalent to a cylindrical source (radius = r) of strength $Q = Q' \exp(-r^2 n^2)$. For the probe described above, after 4 min., $Q = 0.98$ Q'. In other words, at 4 min. the probe was giving results within 2 percent of those which would be given by the true line source assumed in the theory. Similar reasoning may be applied to the point source case.

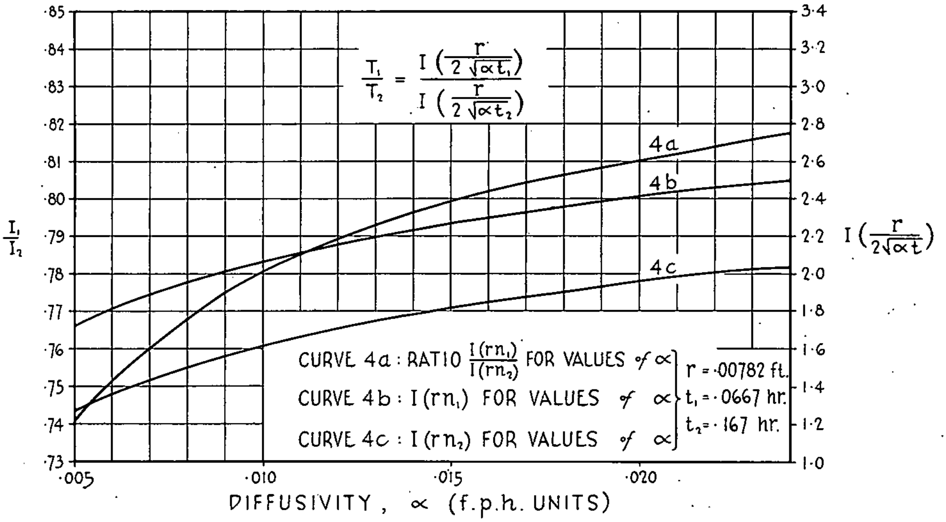

Figure 4. Values of Function I(rn) for the Particular Linear Heater Described for a Range of Diffusivities

The time interval after which the ideal and the actual sources give sufficiently close agreement may be determined by another method which will have more appeal to those who prefer experimentally determined limits of error. If a complete heating curve has been obtained (not just two readings at selected times), this may be plotted. It represents the behavior of the actual source. By using the two selected values and the above theory (ideal source behavior) values of the thermal constants are calculated. By substituting these values in the appropriate equation, (1) or (6), a second heating curve (ideal) can be calculated. A comparison of the agreement between these two curves quickly shows whether or not the selected intervals for measurement have been chosen to give a sufficiently good approximation to the ideal conditions assumed. Such a comparison for the case of the spherical heater is shown in Figure 5. The selected intervals of 1 hr. and 2 hr. are well within the range of good agreement between the two curves. Once such a check has been made, the selected intervals may be used for any other substances which do not differ too greatly in thermal properties.

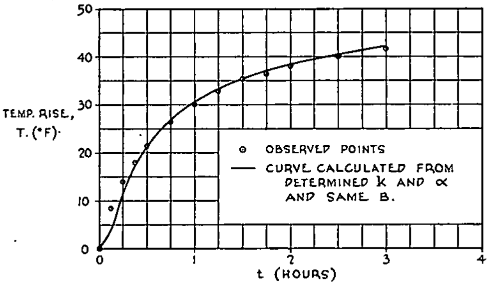

Figure 5. Comparison Between Observed and Theoretical Heating Curves from a Spherical Source

The theory is developed for a medium infinite in extent which is obviously not the case in practice. However we can calculate the thermal effects (temperature rise, heat flow, etc.) at any point in the ideal infinite medium for any period of heating. When these effects are negligible at the distances corresponding to the actual physical boundaries of the substance being studied, the behavior of interior points is the same as if the substance were infinite in extent. With substances of low conductivity and with the short heating periods used in this method, this requirement is satisfied by quite small samples. The validity of this assumption may be proved experimentally as well. In the case of the spherical heaters buried in the ground, a thermocouple was placed 2 ft. from the heater. This thermocouple gave no indication of a temperature change greater than 0.05 deg. F. during the 3-hr. heating period. In a continuous run for 72 hr. the temperature rise at 2 ft. was only 2.0 F. It is therefore safe to assume that the surface at a distance of 5 ft. does not affect the temperature rise at the heater during a short heating period of a few hours.

The assumption that the medium is homogeneous is required by the theory but is most certainly not the case in practice. However, the results obtained must be interpreted as those for a homogeneous medium which would show the same average thermal properties as the actual substance used. This places no restriction on the usefulness of the method, since it is precisely the result desired in heat transfer considerations. In effect, the measurements give the average thermal constants for a limited region surrounding the heater. Averages for larger samples must be obtained by measurements at a number of locations.

Discussion

The particular merit of the method is that it provides a rapid and absolute measurement of both thermal conductivity and thermal diffusivity. The degree of error of the results may be assigned a mathematically rigorous upper limit without knowledge of the thermal properties of the apparatus used. For extremely accurate work this error may be reduced by more detailed calculations than those given here, but the simple method is apparently accurate to within 2 or 3 percent which compares favorably with other methods.

The construction of the graphs used is not particularly laborious and once obtained they are used for the life of the particular heater. The size and shape of heater is a matter of choice, influenced to a large extent by the type of material tested, the degree of accuracy desired, and the time available for an individual measurement.

Wide variations are possible in the suitable application of the method. For instance, if it is desirable to have a very small increase of temperature, say 8 to 10 F. rather than the 40 to 50 F. used in the applications above, this may be done with no loss of accuracy provided the sensitivity of the temperature measuring equipment is suitably increased. If very rapid readings are desired, a heater design which gives a rapid approximation to the ideal case can be used. If a value which is representative of a large volume of the sample is desired, a heater which has a low output and which can be operated for a long period without giving excessive temperature increases is used.

The general method, here illustrated by the point source and the line source, can be extended to other types of sources if desired. Because of its flexibility and the rapidity with which results may be obtained, it is being further developed for studies of thermal properties of poor conductors.

THE THERMAL CONDUCTIVITY PROBE

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Interest in the thermal properties of soils has recently increased because of the introduction of the ground-coil heat pump, and through an awakening to the necessity for an accurate understanding of heat flow in soil freezing and associated problems. However, the determination of thermal conductivity and of thermal diffusivity, the two thermal properties of principal interest, is complicated in the case of natural soils by two factors peculiar to this material. First, soils normally occur in a moist condition and are subject to large seasonal and locational variations in their moisture content. Second, soils have a definite structure which, once disturbed, is difficult to restore. These factors cause test methods adequate for the testing of manufactured bulk materials to be unsatisfactory when applied to soils.

The difficultires associated with obtaining structurally undisturbed soil samples of suitable size and shape for laboratory apparatus are apparent. The difficultires arising from the presence of moisture require some explanation.

When a temperature difference exists between two points in a moist soil, a vapor-pressure difference will also exist. Water will tend to vaporize in the warmer position, flow or diffuse to the cooler position where the vapor pressure is lower, and condense in the cooler position. Thus, a migration of moisture will occur which will not only continuously alter the distribution of the moisture within the soil by drying the warmer position and wetting the cooler position but will also account for a separate mechanism of heat transmission by virtue of the latent heat carried by the vapor. Any apparatus depending upon a steady-state heat-flow principle will not be able to yield a result until a moisture equilibrium has been established, at which time the specimen will not be uniformly wetted and the moisture migration mechanism of heat transmission will not be operative.

To overcome these and other difficulties, the thermal conductivity probe has been developed at Toronto. Because the new instrument is portable, it can be carried to the site and no disturbance of the soil is involved. By utilizing a transient heat-flow principle, the tests are accomplished in a few minutes, before the moisture migration has significantly disturbed the original distribution, while at the same time including this mechanism as a contributing factor in the measured properties.

The Instrument

The thermal conductivity probe is detailed in Figure 1 and the measuring circuit in Figure 2. The probe itself consists of an aluminum tube approximately 18 in. in length. Inside of the tube is stretched an axial constantan resistance wire which serves as a constant strength heat source. Near the center of the tube length in contact with the inner wall are the hot junctions of several thermocouples arranged in series with external cold junctions. The tube is closed by a steel tip at the lower end and the wiring