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CALCULATION OF DEPTH OF THAW IN FROZEN GROUND

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Synopsis

An investigation was started in 1945 under the general direction of the chief of engineers, Department of the Army, on the design and construction of airfields in arctic and subarctic regions. This investigation, which is still in progress, is being conducted by the St. Paul District, Corps of Engineers.

Observations of existing structures have been made at various airfields in Alaska since 1945. Additional test structures were constructed near Fairbanks in 1946 and 1947. Laboratory research has been carried on at the University of Minnesota to determine the thermal properties of soils and certain insulation materials. Library research has also been conducted to locate information pertaining to calculation of depth of thaw in frozen ground.

Equations for depth of freezing located in the library research were checked and modified to permit their use in calculating depth of thaw. A correction factor was developed for the relation between air temperatures and the temperature of various surfaces, such as concrete, asphalt, gravel, and vegetation. This correction factor is used in the equation for depth of thaw and in effect converts the basis of calculation from normal air temperatures to surface temperatures. Calculations of depth of thaw are made for different soil conditions and under various surfaces and structures. The data used in these calculations include those obtained in the field tests of soil density and moisture content, field observations of air temperatures, and laboratory tests of thermal conductivity of soils. The calculations can be

made for conditions where several soil strata exist in which the moisture, density, and thermal properties are different. In most cases, the calculated depths of thaw check the observed depths of thaw very closely.

The procedures used in the calculation of depth of thaw in frozen ground can also be used in computing the depth of frost penetration in thawed ground, if the factors applicable to winter conditions are included in the equation.

Observations of ground temperatures, soil conditions, and settlement of runways and structures have been made at various airfields and weather stations in Alaska since 1945. Model test structures were constructed near Fairbanks in 1946 and 1947 on which observations are being made. Laboratory research has been done at the Engineering Experiment Station at the University of Minnesota to determine the thermal properties of soils and certain insulation materials. Library searches have been made to locate information pertaining to heat transfer in ground. Equations for depth of freezing were found in articles by Berggren, Beskow, and Sumgin. The Frost Effects Laboratory, New England Division, Corps of Engineers has also developed equations for depth of freezing which are shown in "Addendum No. 1, 1945-1947" to "Report on Frost Investigation 1944-1945," published October 1949. The equations for depth of thaw described in this paper were developed and modified from the various equations for depth of freezing.

Acknowledgement is made of the assistance and valuable suggestions provided by Mr. O. M. Bjeldanes, physicist, in making theoretical studies and reviewing this paper. At the present time the Permafrost Division of the St. Paul District is under the Arctic and Subarctic Investigations Staff of which Lt. Colonel Arthur H. Lahlum, C. E., is head. Colonel L. G. Yoder, C. E., is district engineer.

The purpose of this paper is to present the development of a method of computation based on theory, laboratory experiments, and field observations which will enable the engineer to determine with reasonable accuracy the depth of thaw in frozen ground under natural or artificial surfaces or structures in areas where permafrost is encountered.

Permafrost, or permanently frozen ground, is a condition encountered in major portions of the arctic and subarctic regions. Permafrost has an important influence on the behavior of structures placed on it, especially where the soil contains clear ice or has frost heaving characteristics. Heat from a building may flow into the ground under the floor and cause settlement of the foundations if proper design precautions are not taken. Disturbance of natural surface vegetation, which normally acts as an insulator, or construction of an earth fill for a road or runway will produce a change in the thermal equilibrium of the ground. Such changes in thermal equilibrium may also cause settlements of the natural ground and superimposed fills. In order to cope with these problems, it is necessary to know how the thermal conditions in the ground change under various climatic and soil conditions.

In order to illustrate the method of computation developed, data from several runway test sections and from one large building in Alaska are discussed. Effects of moisture, density, thermal conductivity and other physical properties of the ground on heat transfer are illustrated. Ground temperatures are associated with their causes, such as solar radiation and other climatic conditions. Calculations are made for situations where the mean annual temperature of the surface is close to 32 F. For situations where the mean annual temperatures are quite removed from 32 F., several equations are suggested.

A study of available published literature revealed very few references which might be used to calculate the depth of thaw of the ground. A bibliography of literature pertinent to analysis of heat transfer in the ground is included at the end of this paper. Equations exist for the temperature within a thick body having a plane surface and having a sinusoidal temperature variation on the surface (1, 2). Generally, these equations are of small value, if any, when applied to soil which freezes and thaws since the effect of latent heat of fusion of water is not considered. Standard physics books may be used for the commonly known equations of heat flow (3).

Russian Literature

A translation from the Russian work of M. I. Sumgin and others (4) contains an equation for the depth of thaw:

$$x = h + \sqrt{\frac{2kv_h t}{\frac{Lwd}{100}} - \frac{kv_1 t}{x_1 \frac{Lwd}{100}}}$$

x = depth of thawing in meters
h = depth to plane of temperature v_h = .05 to .10 meters
k = coefficient of thermal conductivity in calories per sq. m. per deg. C. per hr. per m.
v_h = average temperature at depth h in deg. C.
t = time of thaw in hours
L = latent heat of fusion of water = 79.7 cal. per gm.
w = moisture content in percentage of dry weight of soil
d = dry weight of soil in kg. per cu. m.
v_1 = average temperature of ground at a fixed small distance x_1 below the zero ground isotherm

The preceding equation gives an approximate depth of thaw; however, it is difficult to determine certain factors such as h and v_h. In the same reference by Sumgin, an equation is given for the conditions necessary for the existence of permafrost:

$$k_f v_f t_f > k_u v_u t_u$$

f = factors during freezing
u = factors during thawing
k = coefficient of thermal conductivity
v = average temperature of the ground at depth h
t = time during freezing or thawing

It is felt that this equation is a definite contribution and it will be discussed later.

Scandinavian Literature

One of the most outstanding publications reviewed was written by Gunnar Beskow (5) and some of the more applicable concepts presented are summarized here.

> "To summarize, then, we can say that for coarse, non-frost acting soils the temperature for freezing and thawing is practically the same, and is very slightly less than 0 C. For frost-heaving soils there is quite a difference between the frost line temperature during freezing and thawing, the receding frost line temperature is 0 C. or very slightly less, while the frost line temperature during freezing is considerably lowered, being greater the finer the soil is and the faster it freezes.
>
> "Thus in a general way, heat flow in frozen soil occurs by a smoothing out of temperature differences between two constant temperature surfaces, the frost line and a constant temperature surface lower down representing the average yearly surface temperature (about 10 to 20 meters deep) below which there is no temperature variation at any level . . . .
>
> "The effect then of the ground surface temperature is that it determines the greatest possible frost depth . . . .
>
> "The effect of vegetation is, as a rule, to lower the temperature, since it is a better insulator during the warm period of the year than during the cold . . . ."

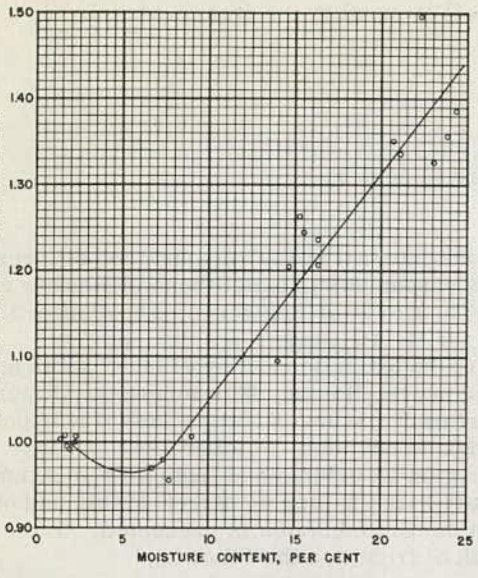


Figure 1. Ratio of Conductivity of Silt Loam Soils Below to Conductivity Above Freezing Point.

An equation for the depth of frost is as follows:

$$x = \sqrt{\frac{2k_f v_0 t}{P}}$$

x = depth of freeze in cm.
t = time that surface temperature is below 0 C. in hours
k_f = heat conductivity of frozen soil in cal. per cm. per sec. per deg. C.
v_0 = average of surface temperature during freezing season in deg. C.
P = frost storing capacity
$P = 79.7\,\frac{w\,d}{100} + \frac{v_0}{2}\left(0.45\,\frac{w\,d}{100} + 0.55\,d\right)$ cal. per cu. cm.
wd = weight of water per unit volume of soil
79.7 = latent heat of fusion of ice in cal. per gm.
0.45 = approximate specific heat of ice in cal. per gm.
d = dry density of the soil in gms. per cu. cm.
w = water content of the soil in percent of dry weight
0.55 = specific heat of the soil

The product v_0t is called the "freezing resistance" and is determined by measuring the area under a time-temperature curve for the air by means of a planimeter. Complicating factors in determining the depth of frost are snow cover, heat radiation, heat conduction from below, freezing point of the soil, etc. Each layer of soil parallel to a plane surface requires a defininte number of degree hours to freeze as given by the following equation:

$$F = v_0 t = \frac{P_1\, b_1^2}{k_f 2} \text{ deg. C.-hrs.}$$

P = "freezing resistance" of layer 1
b_1 = thickness of layer in cm.
P_1 = frost storing capacity in cal.per cu. cm.

In the article "Scandinavian Soil Frost Research of the Past Decade," Proceedings of the twenty-seventh Annual Meeting of the Highway Research Board (6), Beskow again described methods of computing frost depth from temperature conditions and material properties.

American Literature

For the case discussed in his article "Prediction of Temperature Distribution in Frozen Soils," Part III, Transactions of 1943, American Geophysical Union (7), W. P. Berggren develops the theory in a remarkable manner, but he points out that the Stefan equation

$$x = \sqrt{\frac{2k_f\,(32 - v_0)\,t}{L}}$$

is "almost an exact solution" when the latent heat of material is large compared to its heat capacity and when the temperature of the soil is equal to the freezing point.

x = depth of freeze
k_f = thermal conductivity of frozen soil
L = latent heat per cu. ft.
32 = freezing point of the material in deg. F.
v_o = fixed subfreezing surface temperature in deg. F.
t = time in hours

Berggren also points out that $(32 - v_o)t$ may be the integral of the temperature difference and time or the area swept by the curve. These statements parallel those of Beskow and support the theory as developed in this paper. The Corps of Engineers, Frost Effects Laboratory, has developed a graphical solution for W. P. Berggren's theory. This graphical solution is included in "Report on Frost Investigations, 1944-1945," published April 1947, New England Division, Corps of Engineers, Boston, Massachusetts, Figure 22. In "Addendum No. 1, 1945-1947" to "Report on Frost Investigation 1944-1945" published by the Frost Effects Laboratory, New England Division, October 1949, a comparison between predicted depths of frost penetration as determined from four equations, a design curve (Fig. 2, Engineering Manual, Part XII, Chapter 4, March 1946), and observed depths of freezing at several airfields in the United States is presented. The equations used in this comparison to predict depth of frost are the following:

$$83 \qquad x = \sqrt{\frac{48\,k\,F}{L}}$$

$$93 \qquad x = \sqrt{\frac{48\,k\,F}{L + C\left(v_o - 32 + \frac{F}{2t}\right)}}$$

$$154 \qquad x = \sqrt{\frac{24\,k\,F}{L + C\left(v_o - 32 + \frac{F}{2t}\right)}}$$

$$158 \qquad x = \frac{-d}{2} + \sqrt{\left(\frac{d}{2}\right)^2 + \frac{24\,k\,F}{L + C\left(v_o - 32 + \frac{F}{2t}\right)}}$$

x = depth of frost penetration in feet
k = thermal conductivity in Btu per sq. ft. per deg. F. per hr. per ft.
F = freezing index in deg. -days
L = average latent heat in Btu per cu. ft.
C = average volumetric heat in Btu. per cu. ft. per deg. F.
v_o = mean annual air temperature in deg. F.
t = duration of freezing period in days
d = thickness of insulation layer in ft.

"An average for thermal conductivity "k" = 1.3 Btu. per ft.^{2} per deg. F. per hr. per ft. is used throughout these equations."

"Value for "d" used in equation 158 is thickness of topsoil in feet."

Equation 83 is basically the same as the equation described in this paper to compute the depth of thaw except that a correction factor has been added in the latter to provide for the variation between air and surface temperatures. Equation 93 includes the amount of heat lost when each square foot of soil from the surface to maximum depth of frost is cooled from the mean annual temperature down to 1/2 the average number of degrees below 32 F. which the air temperature reaches during the freezing season. It is set up for heat conduction through the average depth of frost. Equation 154 is different from

Equation 93 in that it is set up for heat conduction during the freezing season through the maximum depth of frost rather than through the average depth. Equation 158 allows for the thermal resistance of the top soil layer assuming the top soil has no latent heat and the average thermal conductivity is the same as in the rest of the soil. It may be derived from the following equation used in a similar form in each of the depth of thaw calculations in this paper:

$$F = \frac{L_2 x_2}{24} \left(\frac{R_1 + R_2}{2} \right)$$

except that the full resistance of layer 2 is used in Equation 158 rather than the average.

$$F = \frac{L_2 x_2}{24} (R_1 + R_2) = \frac{L_2 \, x_2}{24} \left(\frac{d}{k} + \frac{x_2}{k} \right)$$

$$F = \frac{L_2 \, x_2 d}{24k} + \frac{L_2 \, (x_2)^2}{24k}$$

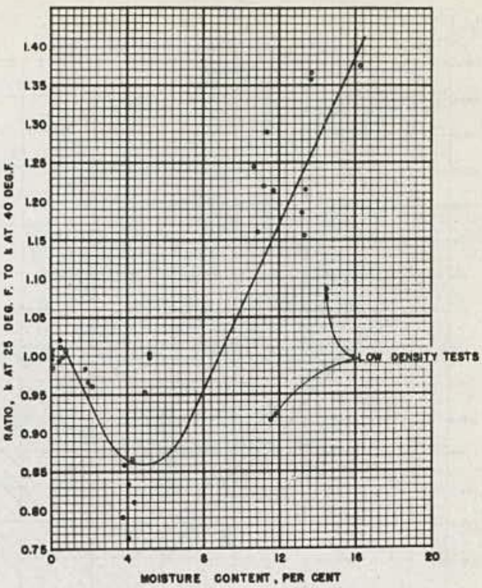


Figure 2. Ratio of Conductivity of Sand Soils Below to Conductivity Above Freezing Point.

$$24 \, k \, F = L_2 \, x_2 d + L_2 (x_2)^2$$

$$(x_2)^2 + x_2 d \; \frac{-24 \, k \, F}{L_2} = 0$$

$$x = \frac{-d \pm \sqrt{d^2 - 4 \left(\frac{-24 \, k \, F}{L_2} \right)}}{2}$$

$$x = \frac{-d}{2} + \sqrt{\left(\frac{d}{2} \right)^2 + \frac{24 \, k \, F}{L_2}}$$

When the volumetric heat capacity times average temperature change is added to the latent heat, the above equation becomes:

$$x = -d + \sqrt{\left(\frac{d}{2} \right)^2 + \frac{24 \, k \, F}{L + C \left(v_o - 32 + \frac{F}{2t} \right)}}$$

Conditions Necessary for the Existence of Permafrost

For permafrost to exist without change from year to year , the average annual temperature gradient in the ground in a homogeneous soil layer below the maximum seasonal thaw must remain the same. Since heat flows from the depths of the earth to the surface, the temperature gradient must be negative toward the surface as heat always flows from warmer toward colder regions. The depth of the bottom of permafrost is a function of the natural temperature gradient in the ground and of the mean annual surface temperature.

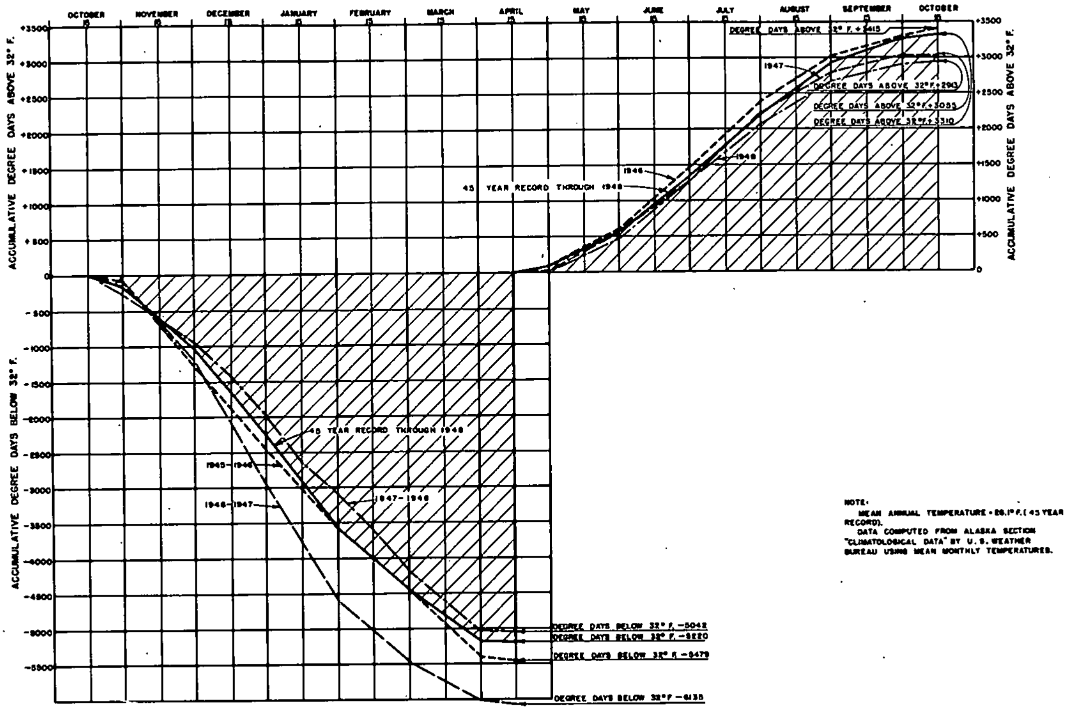


Figure 3. Degree Days Above and Below 32 F., Fairbanks, Alaska.

When the mean annual surface temperature and the temperature gradient in the soil are known, a projection of the gradient to the freezing point gives the approximate depth of permafrost, provided that the soil characteristics in the projected depth are the same as in the known depth.

If the mean annual temperature of the surface or any soil layer below the surface is raised, the heat balance is destroyed and the natural temperature gradient is changed. The new temperature gradient from the bottom to the top of permafrost decreases the heat flow from the bottom and, since the heat flow from the depths of the earth does not change, less heat flows to the ground surface from the bottom of permafrost than is received there, resulting in thaw at the latter plane. In other words, raising the mean annual temperature of the earth's surface results in decreasing the thickness of permafrost. This thawing at the bottom of permafrost is very slow, but it continues until the heat flowing to and from the bottom of permafrost is in equilibrium.

During the annual cycle of temperature change in the top layers of the earth, heat flows into the earth as long as the surface temperature is higher than that of the layer adjacent to the surface and heat flows out when the reverse is true. This means that heat flows into the ground from shortly after the time of coldest surface temperature to shortly after the time of warmest temperature. To maintain the same mean annual temperature, the surface possesses the same amount of heat energy on the average from year to year. Expressed another way, the heat lost by the surface by any method whatsoever must equal the heat gained.

Heat is lost by the surface to the atmosphere and outer space by surface conductance and radiation during all of the year. An increase in wind velocity produces an increase in surface conductance. When a cold wind blows, the so-called "chill effect" is noticeable; objects and people are cooled rapidly. When air is moist, its specific heat is greater, which means that a cool, moist wind can lower the temperature of a ground surface more than a cool, dry wind because the heat that passes from the surface to the air does not raise the temperature of the moist air as much as it would the dry air. Since the energy radiated from a body is a function of the fourth power of its absolute temperature, the radiation from the ground surface is higher in the summer than in the winter. Surface conductance and radiation tend to equalize surface temperature and air temperature. However, other factors such as moving air masses, evaporation, transpiration, daily radiation cycles, snow cover, latent heat, type of surface, localized shading, reflected radiation, cycles of low and high atmospheric pressure, and so forth, will make surface temperatures somewhat different from the air temperatures as observed by the Weather Bureau.

As long as there is a surface temperature higher than 32 F., it is possible that frozen soil is melting below the surface. Even though the surface temperature is below 32 F. at the end of the thawing season, there may be heat stored in the ground between the surface and the thawing layer which causes thawing to continue for some time. As long as the surface temperature is below 32 F., it is possible that soil is freezing.

Near the end of the freezing season, in regions of permafrost, the seasonally thawed layer generally has been completely frozen, and the ground is being cooled below the freezing point. To maintain the level of the permafrost table, it is only necessary to freeze back in winter that which has thawed during the previous summer. However, if the mean annual temperature of the surface is raised, permafrost will melt upward from the bottom because of the decreased gradient in the permafrost. Assuming that the temperature of the soil below the level of seasonal freeze is 32 F., that the temperature gradients in homogeneous ground in the active layer during the thawing season and the freezing season are approximately straight lines (an assumption which ground temperature readings show to be reasonable), and that the depth of the frozen ground at the end of the freezing season is equal to the depth of the ground thawed during the thawing season, then the approximate amount of heat which flows into the ground during the thawing season per square foot of level ground is:

$$Q_u = \frac{24\, k_u G_u t_u}{x/2} = \frac{48\, k_u I}{x} \qquad (1)$$

Q_u = total amount of heat in Btu's which enters the ground during the period the ground thaws
k_u = thermal conductivity of the thawed soil
G_u = average surface temperature in deg. F. during the thawing season minus 32.
t_u = number of days the surface temperature is above 32 deg. F.
x = maximum depth of thaw in feet
$x/2$ = average depth of thaw during season
24 = number of hours in a day
$I = G_u t_u$ = thawing index or number of deg.-days of thaw

Similarly, the equation for the heat flow during the freezing season may be written:

$$Q_f = \frac{24\, k_f G_f t_f}{x/2} = \frac{48\, k_f F}{x} \qquad (2)$$

Q_f = total amount of heat which flows out of the ground during the freezing season
k_f = thermal conductivity of the frozen soil
G_f = 32 minus the average surface temperature during the freezing season
t_f = number of days the surface temperature is below 32 deg. F.
$F = G_f t_f$ = freezing index or number of degree days of freeze

Since the heat necessary to thaw layer x is equal to the heat given up in freezing it, and any other heat involved is neglected as an approximation, the preceding equations may be combined:

$$k_u I = k_f F \quad (3)$$

This means that, under the conditions given by the equation, no permafrost exists and seasonal thaw is equal to the seasonal freeze.

When seasonal thaw does not melt all of the frozen ground, permafrost does exist, and,

$$k_f F > k_u I$$

$$\frac{k_f}{k_u} > \frac{I}{F}$$

For Fairbanks Silt Loam, conductivity values determined by experiment are:

Dry Density lb. per cu. ft.	Moisture Content Percent	Mean Temp. Deg. F.	Thermal Conductivity Btu per sq. ft. per hr. per deg. F. per in.
93.3	24.4	40.0	9.55
93.3	24.2	24.9	13.23

Then $\frac{k_f}{k_u} = \frac{13.23}{9.55} = 1.39$, and $\frac{I}{F}$ must be less than 1.39 for permafrost to exist. It should be noted that the ratio can be greater than 1.00. In other words, permafrost can exist even though the thawing index is greater than the freezing index.

The thermal conductivity of ice is approximately 16 Btu per sq. ft. per hr. per deg. F. per in. and of water 4 Btu per sq. ft. per hr. per deg. F. per in. which makes the ratio of the two 4 to 1. The conductivity of frozen soil is a function of the moisture content. Under most conditions, the greater the moisture content, the greater is the ratio of the thermal conductivity of the soil frozen to that of the soil thawed. The preceding statements being true, the ratio of thawing index to freezing index may be larger for higher moisture contents than in the above example and not violate the conditions necessary for the existence of permafrost. Figures 1 and 2 show how the ratio of the thermal conductivity at 25 F. to the thermal conductivity at 40 F. varies with moisture content in silt loam and sandy soils, respectively. It may be seen that moisture content is definitely a factor in causing permafrost.

It has been found by study of the Weather Bureau records at Fairbanks, Alaska that the ratio of $\frac{I_a}{F_a}$ based on air temperatures was $\frac{3055}{5042}$ or 0.61 for the summer of 1947 and the winter of 1947-1948. See Figure 3. For bituminous surfaces, the corrected thawing index is 2.19 x 3055 = 6690, and from available information, it is known that the corrected freezing index is 0.72 x 5042 = 3630. The ratio $\frac{I}{F} = \frac{6690}{3630} = 1.84$. To maintain permafrost in silt loam soils under these conditions, very high moisture contents are required.

Assuming the thickness of thaw in the saturated gravel to be layer x and the heat lost during the freezing season to be just sufficient to freeze back layer x, the approximate amount of heat conducted per square foot of plane surface may be written:

$$Q_f = \frac{G_f t_f (24)}{R_f} = \frac{24\ F}{r_1 + r_2 + \frac{r_f}{2}} = \frac{24\ F}{\Sigma r + \frac{x}{2k_f}} \quad (4)$$

$$Q_u = \frac{G_u t_u (24)}{R_u} = \frac{24 I}{r_1 + r_2 + \frac{r_u}{2}} = \frac{24\ I}{\Sigma r + \frac{x}{2k_u}} \quad (5)$$

$$Q_f = Q_u \quad (6)$$

$$\frac{24\,F}{\Sigma r + \frac{x}{2k_f}} = \frac{24\,I}{\Sigma r + \frac{x}{2k_u}}$$

$$\frac{I}{F} = \frac{\Sigma r + \frac{x}{2k_u}}{\Sigma r + \frac{x}{2k_f}} \tag{7}$$

Σr = thermal resistance down to the thawing layer
x = thickness of the thawing layer (or freezing layer)
k_u = thermal conductivity of layer x unfrozen
k_f = thermal conductivity of layer x frozen

For permafrost conditions: $Q_f > Q_u$ and $\dfrac{\Sigma r + \frac{x}{2k_u}}{\Sigma r + \frac{x}{2k_f}}$ should be greater than $\dfrac{I}{F}$.

In general, the natural temperature gradient in the ground is the best means of estimating the thickness of permafrost. Permafrost melts upward from the bottom when the gradient in the ground is decreased. Any change in materials at the surface affects the ratio of thermal conductivities and indexes and changes the temperature gradient. (Depth of thaw is a function of the amplitude of surface temperature oscillation). An estimate of the maximum rate of thaw of permafrost upward from the bottom may be made. Under natural conditions, it may be assumed that the temperature gradient in the permafrost from about 30 ft. down is stable and that the heat flow to the surface of the earth is the same as the heat flow from lower depths to the bottom of permafrost. It is recognized that this assumption is not strictly true because of the changes being made in the earth's climate and in geologic changes in the surface. When the thermal gradient in permafrost is diminished to zero by causing thaw to exceed freeze at the surface, a condition for maximum rate of thaw at the bottom has been brought about. All the heat which flows to the bottom of permafrost from below thaws the frozen ground, and this heat is equal to:

$$dQ_1 = dQ_2 = \frac{dT}{dx}\,kAdt = Ldx_1 = 1.434\ wddx_1 \tag{8}$$

dQ_1 = heat flow to the bottom plane surface of permafrost from below
dQ_2 = heat flow to the surface of the earth from the bottom, plane surface of permafrost when gradients are stable
$\frac{dT}{dx}$ = thermal gradient in permafrost
k = thermal conductivity of permafrost at point where thermal gradient is determined
A = area = 1 sq. ft. generally
dt = time in hours
143. 4 = latent heat of fusion of 1 lb. of ice in Btu's
w = moisture content of soil in percent at the bottom, plane surface of permafrost
d = dry density of soil at the bottom, plane surface of permafrost

The rate of thaw may be written:

$$\frac{dx_1}{dt} = \frac{k}{1.434\ wd}\,\frac{dT}{dx} \qquad x_1 = \frac{k}{1.434\ wd}\,\frac{dT}{dx}\,t \tag{9}$$

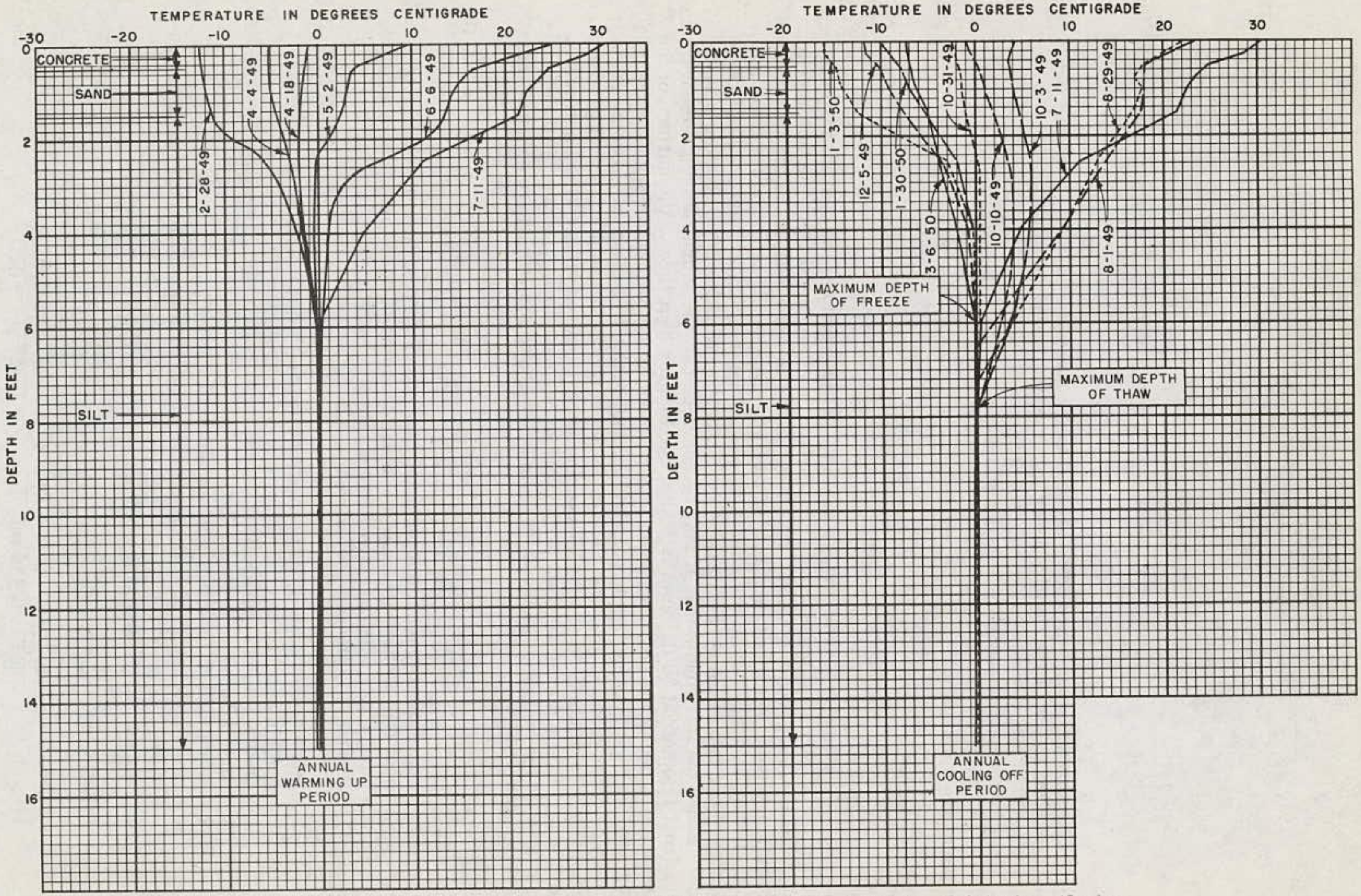


Figure 4. Temperature Gradients Under Black Surface Concrete, Fairbanks, Alaska.

At Umiat, these approximate data were estimated:

$$\frac{dT}{dx} = \frac{1 \text{ deg.}}{58'} \quad \text{(from temperature observations)}$$

w = 30 percent (by estimation)
d = 85 lb. per cu. ft. (by estimation)
k = 1.08 Btu. per sq. ft. per hr. per deg. F. per ft. (from chart -Fig. 9)

Approximately

$$x_1 = \frac{1.08}{1.434\ (30)\ (85)} \frac{1}{(58)} (365)\ (24) = 0.044 \text{ ft.}$$

which means that the approximate maximum rate of thaw is 0.044 ft. per yr. or that it would take at least 22.7 yr. to thaw one foot of permafrost upward from the bottom by means of heat from the earth. This value is meant only to give the order of magnitude because of the nature of the data.

Depth of Thaw in Frozen Ground

The depth of thaw in frozen ground is dependent upon such location factors as latitude, altitude, direction of exposure to the sun, shading or decrease in radiation, reflection or intensification of radiation, proximity to large bodies of water, and proximity to local sources of heat. The surface of the earth receives heat energy from the sun and from the center of the earth. During the years 1936 to 1947 inclusive, the solar radiation received annually at Fairbanks, Alaska averaged 303,000 Btu's per sq. ft. However, not all of the radiation received is absorbed since the earth's surface radiates heat into outer space. During periods when the radiation received is greater than the radiation given off an increase in ground temperature results. The amount of heat transmitted from the center of the earth is very small compared to that received from the sun.

Surface temperature is one of the factors in the equation for depth of thaw. It is dependent upon the solar radiation received and emitted, heat lost to the air by conduction and convection, heat lost or gained due to evaporation or condensation of surface moisture, thermal diffusivity of the soil below the surface, and heat transferred from the interior of the earth. Surface temperature data are not commonly available for most areas. However, air temperatures which are observed at all weather stations are related to surface temperatures. Normally, the surface of the earth heats the air so the surface is warmer than the air. Vegetation affects air and surface temperature because of shading and transpiration. The effect of vegetation is to lower the mean annual temperature of the surface and to preserve permafrost. Vegetation may be a very important factor when it is the purpose of the engineer to maintain permafrost. Since many factors affect surface temperature, it is necessary to measure the temperature or to estimate it from air temperatures for use in the succeeding equations.

The following equation is a standard heat equation for parallel plane surfaces:

$$Q = \frac{k(v_1 - v_2)At}{x} = \frac{(v_1 - v_2)At}{x/k} = \frac{T}{R} At \qquad (10)$$

Q = total amount of heat transferred in Btu's
T = temperature difference between surfaces in deg. F.
A = area in sq. ft.
R = $\frac{x}{k}$ = thermal resistance in deg. F. per Btu per sq. ft. per hr.
v_1 = temperature of one face of conducting layer
v_2 = temperature of other face of conducting layer
t = time in hours

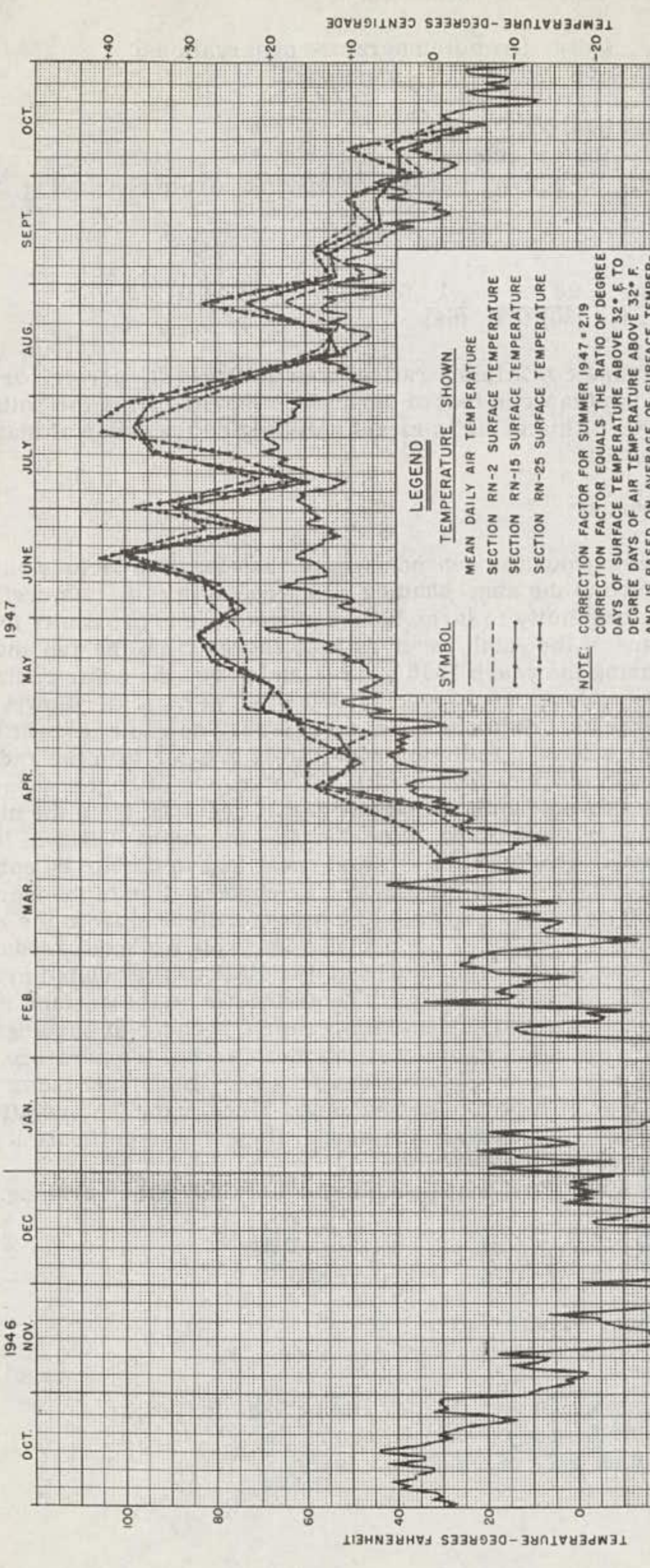


Figure 5. Air and Bituminous Surface Temperatures, Fairbanks, Alaska, Summer-1947.

x = thickness of layer in ft.
k = thermal conductivity in Btu per hr. per deg. F. per sq. ft. per ft. of thickness

When $A = 1$ and $t = 1$, $Q = \frac{T}{R}$

A large plane surface is assumed and edge effects are ignored. When time is measured in days and a one sq. ft. area is considered, Equation (10) may be written:

$$Q = \frac{24\ Tt}{R} = \frac{24I}{R} \tag{11}$$

Q = total number of Btu's per thawing season
$I = T \cdot t$ = maximum number of deg. -days of thaw based on surface temperature

"I" is the thawing index and is determined by a summation of the deg. -days of thaw based on the average daily temperatures of the surface during the thawing season.

It is recognized that Equation (10) is true only when the thickness of the layer and the temperature difference between surfaces are constants. However, a study of the temperature gradients in the ground during the thawing season (or freezing season) reveals the close approximation of the gradient to a linear condition during most of the thawing season and especially so during the height of the season which makes it possible to adapt the equation to estimating heat flow in the ground. See Figure 4. This drawing also shows that a concave gradient at the beginning of the season at Fairbanks, Alaska is balanced, more or less, by a convex gradient at the end of the season, that the permafrost is very close to 0 C., that during most of the time that the ground is thawing heat is flowing directly from the surface to the thawing layer, and that during most of the time that the ground is freezing heat is flowing directly from the freezing layer to the surface. Figure 4 may be used to determine what heat in addition to latent heat is absorbed on the average by each cubic foot of soil in changing temperature during the thawing season.

Since air temperatures are generally available and surface temperatures are not, a study has been mace of data from Fairbanks, Alaska to determine the relation between a thawing or freezing index calculated from air temperature and indexes calculated from the temperatures of different types of surfaces. This study has resulted in correction factors for various types of surfaces. Data available at present indicate values for the correction factor as shown in Table 1 for the horizontal surfaces investigated. The correction factors for the various surfaces investigated are determined from data such as that shown in Figures 5 and 6. Air temperatures are based on data collected by the U. S. Weather Bureau at Weeks Field in 1947 and early 1948. Surface temperatures are those obtained at test sections in the Permafrost Research Area during the same period. The correction factor is merely the ratio of the deg. -days above or below 0 C. for the surface temperature to the deg. -days of the air temperature during the same period. The deg. -days are most simply obtained by planimetering the area between the 0 C. line and the air or surface temperature curve. These data indicate that, on the average, the ground surface temperature under a cover of trees, brush and moss is only one third as high above freezing as the air temperature during the thawing season. Bituminous surface temperatures average twice as high above freezing as the air temperatures during the same period. Snow was removed from the bituminous concrete and gravel surfaces and left in place on the other surfaces.

The heat which flows into the ground melts ice and raises the temperature of both thawed and frozen ground. In subarctic regions where the mean annual temperature is close to the freezing point, the heat which raises the soil temperature can be ignored. This procedure introduces a small error which is one of safety as far as design is concerned since it results in a calculated depth of thaw greater than the actual thaw. Considering only the latent heat of fusion, the number of Btu's per sq. ft. of area required to thaw the soil to depth "x" is:

TABLE 1

COMPUTATION OF CORRECTION FACTORS FOR VARIOUS SURFACES IN FIELD RESEARCH AREA
SUMMER 1947 & WINTER 1947-48

Type of Surface	Area and Section Used	Summer 1947 Degree-Days Above 32° F. Air Temp.	Period Covered	Surface Temp.	Period Covered	Correction Factor	Winter 1947-48 Degree-Days Below 32° F. Air Temp.	Period Covered	Surface Temp.	Period Covered	Correction Factor
1. Spruce trees, brush & moss over peat soil	Area 1- Sect. A	3055	4-13-47 to 9-30-47	1130	5-12-47 to 10-6-47	.37	5042	9-30-47 to 5-3-48	1430	10-6-47 to 5-10-48	.29
2. Cleared of trees & brush but with moss in place over peat soil.	Area 1- Sect. B	"	"	2220	4-28-47 to 10-6-47	.73	"	"	1245	10-6-47 to 4-22-48	.25
3. Silt loam cleared & stripped of trees & vegetation.	Area 1- Sect. C	"	"	3720	4-21-47 to 10-6-47	1.22	"	"	1660	10-6-47 to 5-3-48	.33
4. Gravel	Area 2- Sect. RN-3	"	"	6080	4-11-47 to 10-14-47		"	"	3840	10-14-47 to 5-3-48	
	Sect. RN-26			6140	4-6-47 to 10-10-47				3160	10-10-47 to 4-28-48	
			Av. =	6110				Av. =	3500		.70
5. Concrete	Area 2- Sect. RN-16	"	"	6320	4-6-47 to 10-11-47		"	"	3730	10-11-47 to 4-27-48	
	Sect. RN-17			5650	4-4-47 to 10-11-47				3800	10-11-47 to 4-29-48	
	Area 2- Sect. RN-18		"	6160	4-3-47 to 10-11-47		"	"	4090	10-11-47 to 4-13-48	
	Sect. RN-23			6520	4-5-47 to 10-11-47				4280	10-11-47 to 4-13-48	
	Sect. RN-24			6360	4-3-47 to 10-11-47				3500	10-11-47 to 4-25-48	
			Av. =	6202		2.03		Av. =	3880		.77
6. Bituminous	Area 2- Sect. RN-2	"	"	6560	4-8-47 to 10-13-47		"	"	3650	10-13-47 to 4-29-48	
	Sect. RN-15			6450	4-10-47 to 10-10-47				3950	10-10-47 to 4-29-48	
	Sect. RN-25			6980	3-29-47 to 10-12-47				3290	10-12-47 to 4-4-48	
			Av. =	6663		2.19		Av. =	3630		.72

Note: Snow was not removed from Surfaces 1, 2, and 3.
Snow was removed from Surfaces 4, 5, and 6.

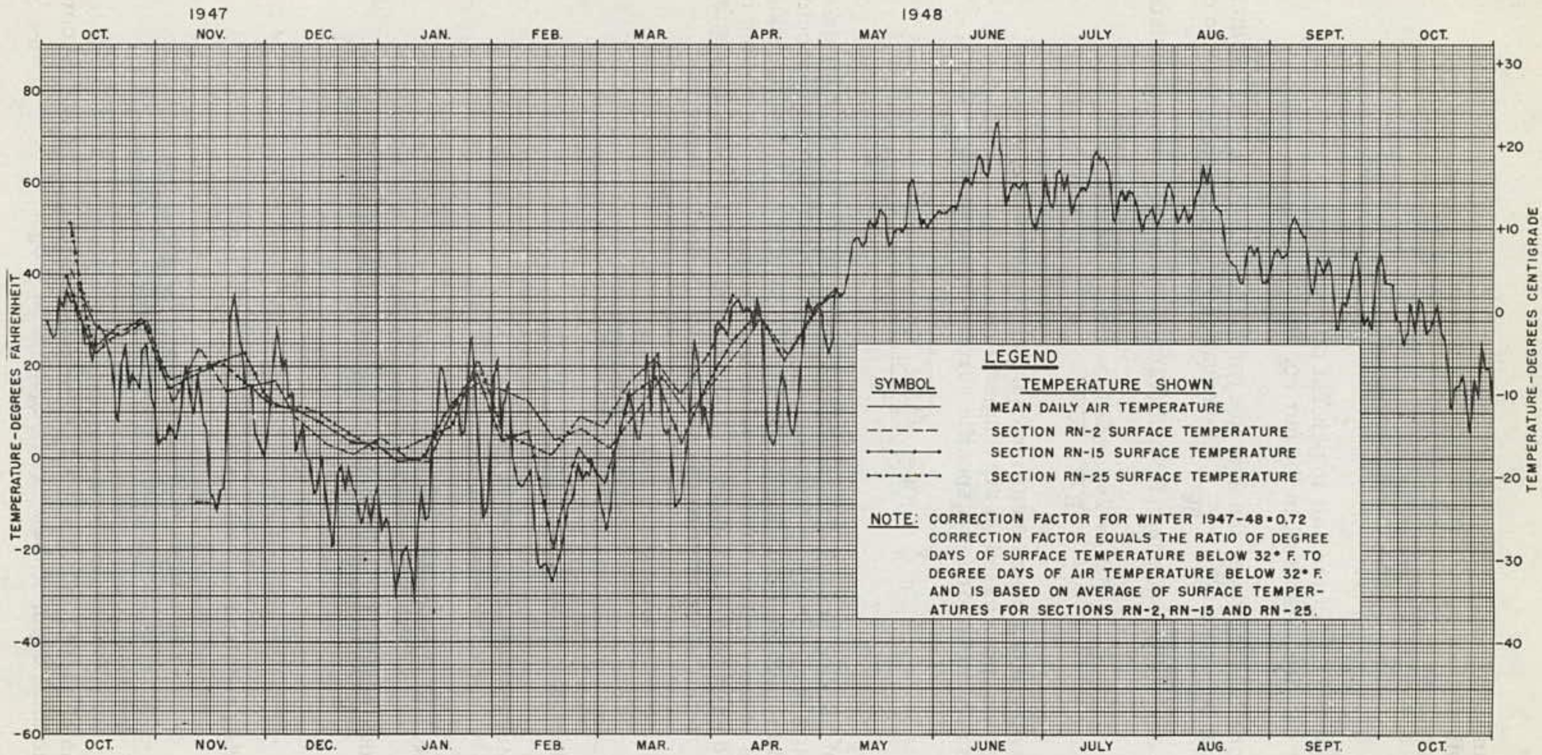


Figure 6. Air and Bituminous Surface Temperatures, Fairbanks, Alaska, Winter - 1947-1948.

$$Q = xL \tag{12}$$

Q = quantity of heat per sq. ft. in Btu's
x = depth of thaw in ft.
L = latent heat of fusion of water per cu. ft. of soil

$$L = 1.434 \, wd$$

w = water content of the soil in percent of dry weight
d = dry density of the soil in lb. per cu. ft.

In the case of one thawing layer the average resistance during the period that thaw is taking place may be written $\frac{R}{2}$ or $\frac{x}{2k}$. The equation for the depth of thaw in one homogeneous layer is derived by equating (11) and (12) and using the average resistance:

$$Q = xL = \frac{24I_g}{R/2} = \frac{24I_g}{x/2k} = \frac{48kI_g}{x} = \frac{48k.\, I_a.\, C}{x} \tag{13}$$

I_g = thawing index based on ground surface temperature
I_a = thawing index based on air temperature
C = correction factor for a specific surface in equation $I_g = C \cdot I_a$.

$$x = \sqrt{\frac{48kI_a\, C}{L}} \tag{14}$$

The depth of thawing in ground composed of one or more different strata of materials may be computed very closely by determining the part of the annual corrected thawing index required to melt the ice in the voids of each stratum. The summation of these partial indexes in the various strata, equal to the annual corrected thawing index for the locality and existing ground surface, may be used to determine the depth of thaw. From (13) the partial index required to melt the ice in the top layer is:

$$I_1 = \frac{L_1 b_1}{24} \cdot \frac{R_1}{2} \tag{15}$$

where L_1 = latent heat of water per cu. ft. of soil

b_1 = thickness of soil layer in feet
$R_1 = \frac{b_1}{k_1}$ = thermal resistance of the soil layer
k_1 = thermal conductivity of the soil layer

The partial index required to melt the ice in the second layer is:

$$I_2 = \frac{L_2\, b_2}{24} \cdot \left(R_1 + \frac{R_2}{2}\right) \tag{16}$$

The partial index required to melt the ice in the "n"th layer is:

$$I_n = \frac{L_n\, b_n}{24} \left(\Sigma R + \frac{R_n}{2}\right) = \frac{L_n \cdot b_n}{24} \left(\Sigma R + \frac{b_n}{2 \cdot k_n}\right) \tag{17}$$

The summation of partial indexes $I_1 + I_2 + - - - I_n$ is equal to the annual corrected thawing index. The total depth of thaw "x" is equal to $b_1 + b_2 + - - - - - - + b_n$. The term b_n

TABLE 2

COMPUTED DEPTH OF THAW

RUNWAY TEST SECTION RN-4 - FAIRBANKS RESEARCH AREA

Layer Material	"b" Thickness of Layer in ft.	"d" Dry Density of Soil lb. per cu. ft.	"w" Water Content of Soil in Percent of Dry Wt.	"L" 1.434 wd Volumetric Latent Heat of Fusion	"k" Thermal Conductivity Btu/sq. ft./hr/°F/ft.	$R = \frac{b}{k}$ Thermal Resistance	ΣR	$\Sigma R + \frac{R_n}{2}$	I = Thawing Index* Increment	Summation
Thawing Index for 1947 based on air temperature = 3055. Corrected = 3055 · 2.19 = 6690.										
1 Asphalt	0.4	150	0	0	$\frac{10.3}{12} = 0.86$	0.47	0	0.24	0	0
2 Gravel (GW)	3.8	143	3.7	759	$\frac{22.0}{12} = 1.83$	2.08	0.47	1.51	$\frac{759 \cdot 3.8 \cdot 1.51}{24} = 181$	181
3 Silt (MH)	2.5	99	27.7	3932	$\frac{10.0}{12} = 0.83$	3.02	2.55	4.06	$\frac{3932 \cdot 2.5 \cdot 4.06}{24} = 1665$	1846
4 Peat	1.5	25	81.9	2936	$\frac{2.0}{12} = 0.17$	8.80	5.57	9.97	$\frac{2936 \cdot 1.5 \cdot 9.97}{24} = 1824$	3670
5 Silt and Peat	1.0	62	50.0	4445	$\frac{6.0}{12} = 0.50$	2.00	14.37	15.37	$\frac{4445 \cdot 1.0 \cdot 15.37}{24} = 2840$	510
6 Silt and Peat	x = 0.05 9.25	78.2	39.5	4429	$\frac{7.5}{12} = 0.63$	$\frac{x}{0.63}$	16.37	$16.37 + \frac{x}{1.26}$	$\frac{4429 \cdot x \cdot (16.37 + \frac{x}{1.26})}{24} = 180$	6690

$$^{*}I_1 = \frac{L_1 b_1}{24} \cdot \frac{R_1}{2} \qquad I_2 = \frac{L_2 b_2}{24}\left(R_1 + \frac{R_2}{2}\right) \qquad I_3 = \frac{L_3 b_3}{24}\left(R_1 + R_2 + \frac{R_3}{2}\right) \qquad I_n = \frac{L_n b_n}{24}\left(\Sigma R + \frac{R_n}{2}\right)$$

Solving for x in Layer 6 $I_x = \frac{L_x b_x}{24}\left(\Sigma R + \frac{b_x}{2k}\right)$

$$180 = \frac{4429x}{24}\left(16.37 + \frac{x}{1.26}\right)$$

x = 0.05' Computed Depth = 9.25. Actual Depth = 10.2

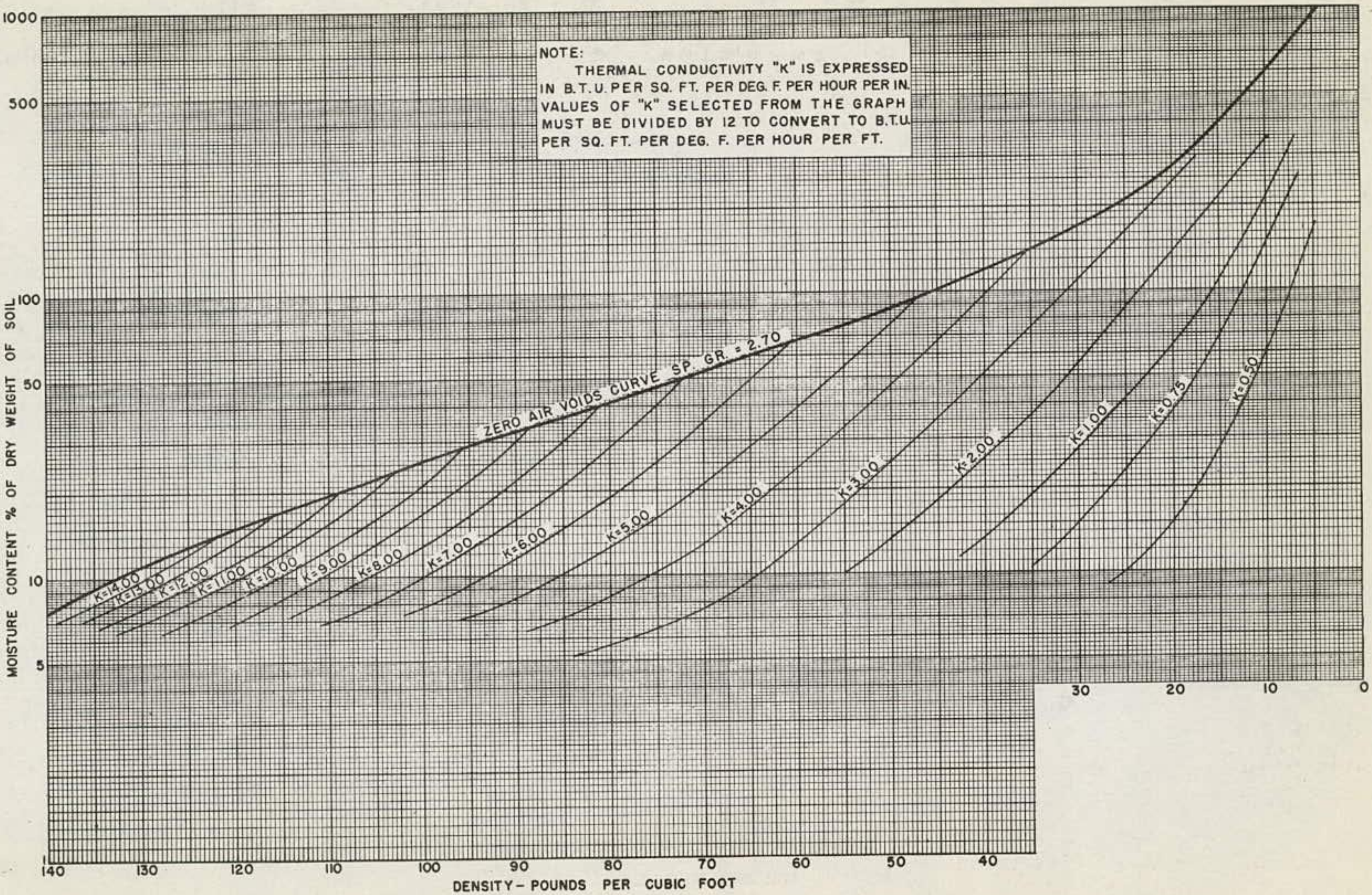


Figure 7. Average Thermal Conductivity for Silt, Clay, and Peat - Unfrozen, Mean Temperature 40 F.

may be equal to or less than the thickness of the final layer, since all of the final layer may or may not be thawed.

A sample computation of depth of thaw in ground composed of several different soil layers is given in Table 2. The thawing index based on air temperatures at Fairbanks during 1947 was 3055. The correction factor for a bituminous surface is 2.19. Thus, the thawing index based on surface temperatures was (3055) (2.19) = 6690. Values for thickness, density, and moisture content of the various soils are based on field tests. Values of thermal conductivity were obtained from Figures 7 and 8 which were derived from tests of thermal properties made at the University of Minnesota under a contract with the St. Paul District, Corps of Engineers. The computed and actual field test data are plotted in Figure 11.

The above analysis does not take into account the heat necessary to raise the ground temperature above the freezing point. In most cases, this additional heat is of relatively small importance as compared with the heat required to melt the ice in the soil. However, under those conditions where the soil contains little moisture for a considerable depth, the available heat energy is used principally in raising the soil temperature. Such a condition can occur where a porous gravel fill is placed above the natural ground-water table. Ordinarily a relatively small amount of heat is expanded in heating the dry gravel while a large amount of heat is expended in melting ice below the ground-water table. The amount of heat used in raising the temperature of unfrozen soil with a surface area of one sq. ft., one deg. F. is:

$$Q_1 = 1.0\frac{(w)}{100}(d)(x) + (c)(d)(x) \tag{18}$$

x = thickness of the soil layer in ft.
w = moisture content of the soil in percent of dry weight
d = dry density of the soil in lb. per cu. ft.
c = specific heat of the soil (approx. 0.19 Btu. per lb.)
1.0 = specific heat of the water

The amount of heat required during the thawing season to raise the temperature of the thawed soil to the mean temperature during the period is equal to:

$$Q_2 = \frac{(I)(d)(x)}{2t}\left(\frac{w}{100} + c\right) \tag{19}$$

I = thawing index in degree days
t = number of days that the surface temperature is above 32 F.

The amount of heat required to raise the temperature of frozen soil from the mean annual temperature of the soil up to 32 F. is:

$$Q_3 = (32 - M)(0.5\ \frac{wd}{100}x + cdx) \tag{20}$$

32 F = freezing point of water
M = mean annual temperature of soil surface
0.5 = specific heat of ice

A more exact calculation of depth of thaw would include the above factors in Equation (13).

Heat absorbed = heat conducted (per sq. ft.)

$$xL + \frac{Id}{2t}\left(\frac{w}{100} + c\right) + x(32 - M)\left(0.5\,\frac{wd}{100} + cd\right) = \frac{48kI_aC}{x}$$

and

$$x = \sqrt{\frac{48kI_aC}{L + \frac{Id}{2t}\left(\frac{w}{100} + c\right) + (32 - M)\left(0.5\,\frac{wd}{100} + cd\right)}} \qquad (21)$$

Computed depths of thaw for uniform layers of peat, silt loam, sand, and gravel from Northway and the Permafrost Research Area near Fairbanks, Alaska, are shown on Figure 12. It may be noted that for a given density condition, large variations in moisture content have very little effect on the depth of thaw in peat and silt loam and that the variation in depth between the high and low density conditions is only a few feet. From the knowledge of the fact that thermal conductivity increases generally as the moisture content increases, it may appear strange that the depth of thaw does not also increase as the moisture content is increased. However, the latent heat capacity of the soil varies in proportion to the moisture content and, in the case of the peat and silt loam studied, acts to very nearly compensate for the increase in thermal conductivity. In the cases of sand or gravel, the latent heat capacity increases faster than the effect of thermal conductivity and, as a result, the depth of thaw is less in wet than in dry sand or dry gravel. The effect of density on depth of thaw is very pronounced in sand or gravel with a range of about 15 ft. from maximum to minimum. High density in either the silt loam or gravel causes thawing to greater depths than does low density. It is evident that greater depths of thaw are likely to occur in corase-grained material such as gravel than in fine-grained material such as silt loam.

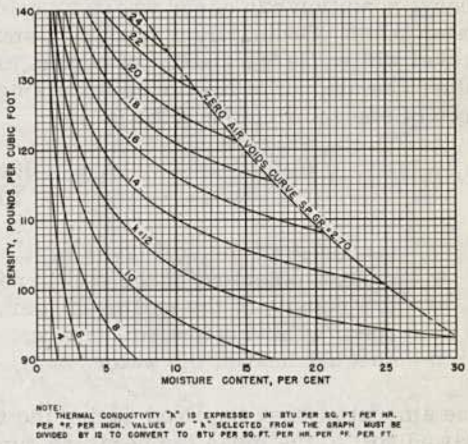


Figure 8. Average Thermal Conductivity for Sandy Soils Unfrozen, Mean Temperature 40 F.

In normal construction operations, it is usual practice to place fills at the highest practicable density to support structural loads. Such fills, which are generally coarse-grained, will produce conditions most suitable for deep thawing. The tests and studies reported on herein indicate that it is impracticable to construct fills under runways or highways to sufficient depth to entirely prevent thawing below the fills. Frost heaving in itself is not generally damaging to a runway or highway if it is uniform. However, where the subgrade is not uniform in soil characteristics, differential heaving can be expected. Although it may be somewhat difficult to operate traffic over pavements where differential heaving has taken place, the most serious effect of the frost action is due to the loss in bearing capacity of the subgrade during the period that the ice lenses are melting. Since frost action in soils depends, among other things, on the grain size, available water supply, and capillarity of the soil, it is possible, according to Beskow (1), to reduce frost action by constructing in the fill a well-drained layer of corase-grained soil having low capillarity. The effect of this layer is to break the capillary flow of water to any fine-grained soil placed above it. In general, the fine-grained material is a better thermal insulator than the coarse-grained. Thus, a

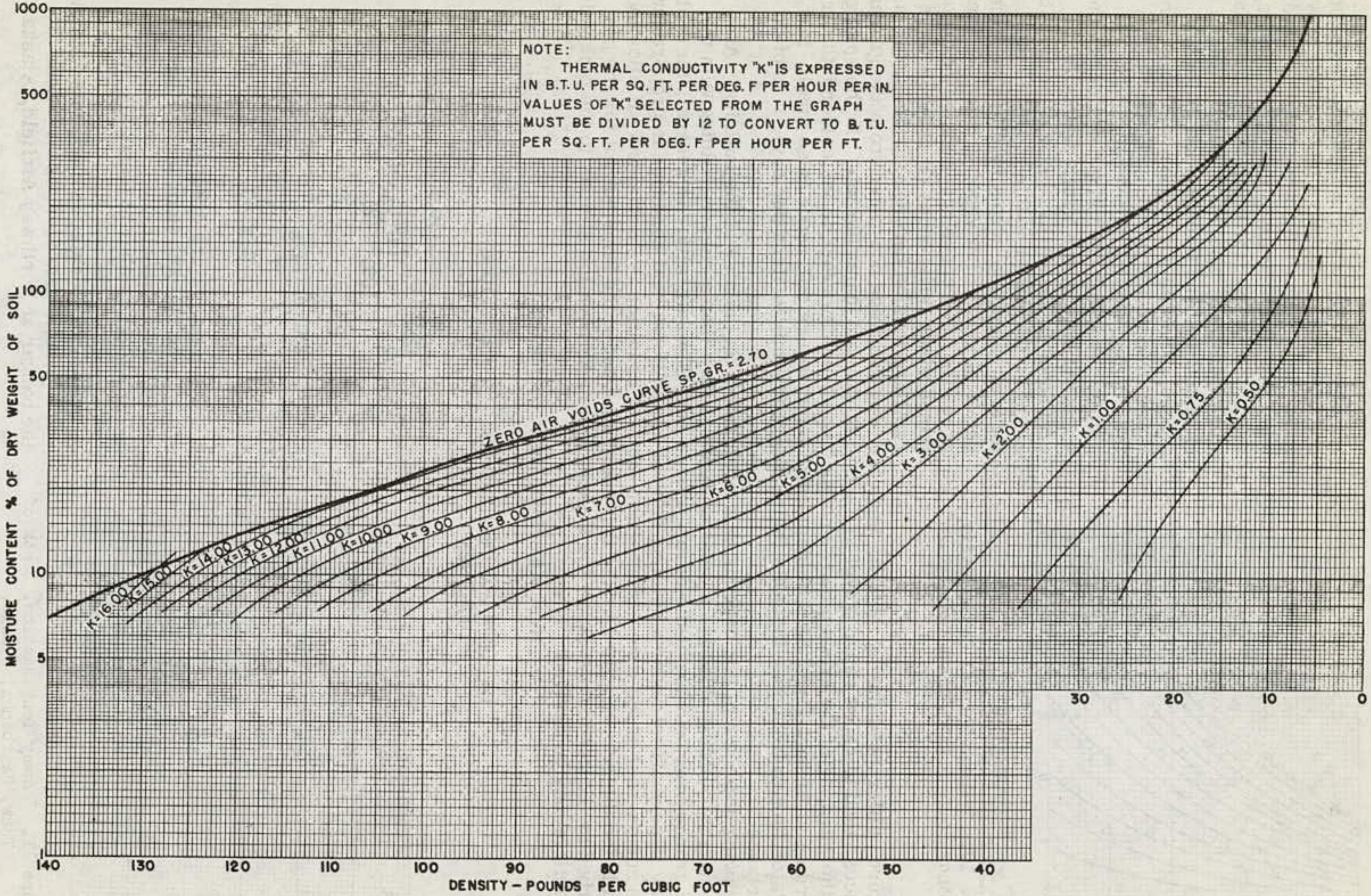


Figure 9. Average Thermal Conductivity For Silt, Clay and Peat Frozen, Mean Temperature 25 Deg. F.

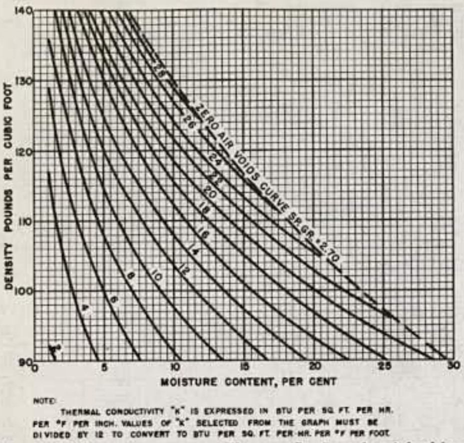


Figure 10. Average Thermal Conductivity for Sandy Soils Frozen, Mean Temperature 25 F.

combination of layers of fine and coarse materials may, under certain circumstances and with proper drainage techniques, prove to be the most satisfactory solution. Medium sand has a capillarity of only a few inches; therefore, a one foot layer of this material, if well drained, would normally be ample to insure that the fine-grained material placed above did not receive sufficient water to form ice lenses and cause heaving. Frost heaving is not caused in any soil by the water normally contained in the voids but only by the water drawn from groundwater reservoirs by capillarity. It is known that water exists in soil in the form of vapor and moves by diffusion from points of high to low vapor pressure. Such pressure differences occur as the result of temperature differences or differences in capillary tension between surfaces. Temperature differences during the fall, when freezing proceeds downward from the ground surfaces, tend to cause vapor travel in the soil to the cold upper surface. In the spring with the warming of the ground, surface vapor travel is downward to the colder interior. According to Beskow (1), "the diffusion due to temperature difference is so small, that for water flow in frost-heaving soils (being only a few millimeters per day), it is of no importance."

Thawing under a building depends to a large extent on the size of the building. A long, narrow building will have a smaller depth of thaw than a square building with the same ground area because of the differences in heat flow laterally. In general, the depth of thaw in permafrost under a building subject to a uniform heat condition is proportional to the square root of the time during which the heat condition is maintained. If allowance is made for the heat absorbed by the thawing ground under the floor but not for the heat absorbed by the permafrost below the thawing layer, an equation similar to the others in this article may be derived:

Heat absorbed = heat conducted

$$Lx + \left(\frac{w}{100}\right)(d)\left(\frac{I}{2}\right)(x) + (d)(c)\left(\frac{I}{2}\right)(x) = \frac{(k)(T)(24t)}{x/2}$$

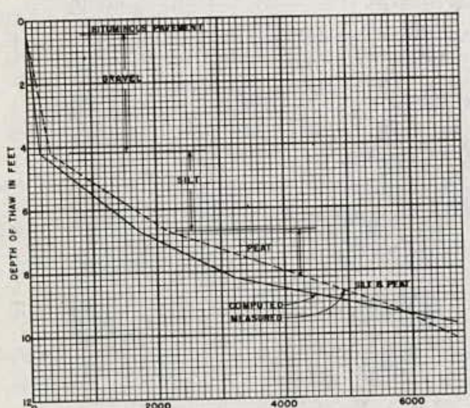


Figure 11. Runway Test Section, Depth of Thaw, Fairbanks, Alaska.

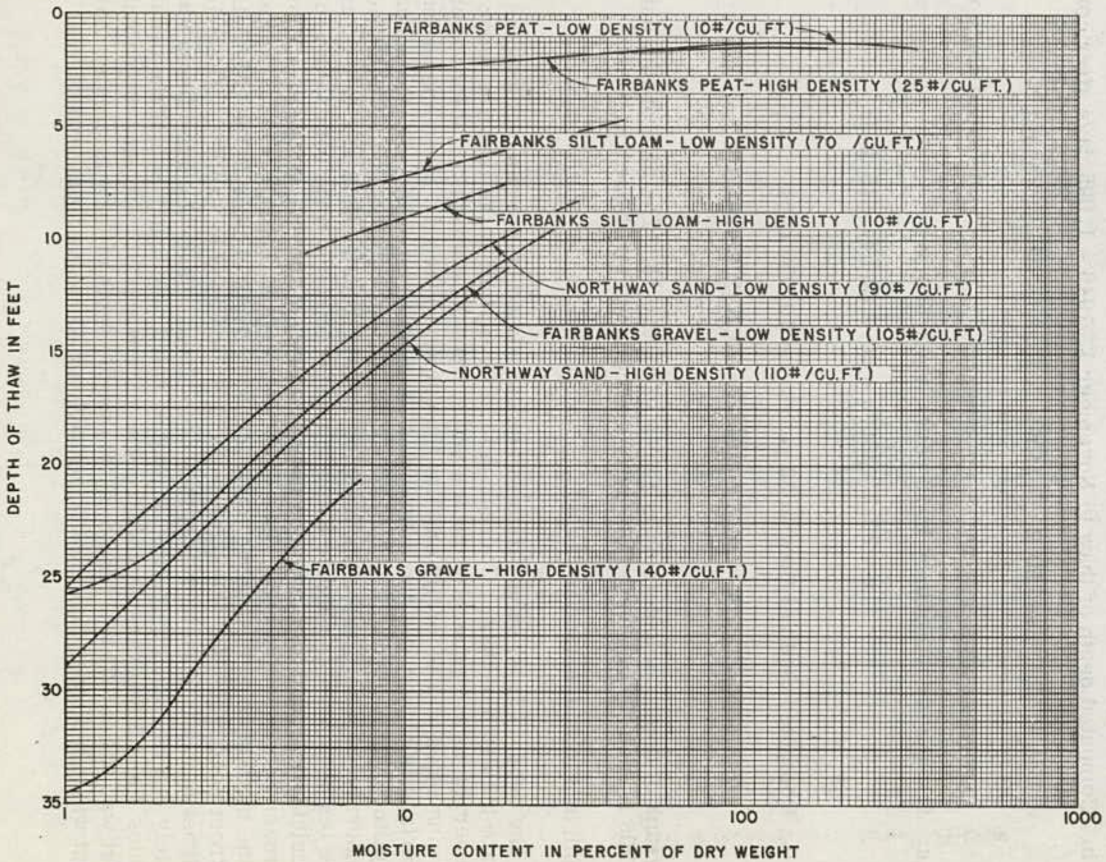
$$x = \sqrt{\frac{48kTt}{L + \left(\frac{w}{100}\right)(d)\left(\frac{T}{2}\right) + (d)(c)\left(\frac{T}{2}\right)}}$$

$$x = \sqrt{\frac{48kTt}{L + \frac{Td}{2}\left(\frac{w}{100} + c\right)}} \qquad (22)$$

T = temperature difference between the building floor and the permafrost surface

t = total number of days

A sample calculation using this formula and available data from observations of the hangar at Northway Airfield, Alaska,



NOTE:
FORMULA FOR DEPTH OF THAW IN SINGLE LAYER OF UNIFORM SOIL IS

$$h = \sqrt{\frac{24\,CI \times 2k}{L}} \quad \text{WHERE}$$

h = DEPTH OF THAW IN FEET.
I = THAWING INDEX IN DEGREE DAYS.
C = CONSTANT TO CORRECT THAWING INDEX FOR SURFACE CONDITIONS.
k = COEFFICIENT OF THERMAL CONDUCTIVITY.
L = 1.434 x DENSITY x MOISTURE CONTENT.

FOR THIS GRAPH, THAWING INDEX TAKEN EQUAL TO 3000 AND FACTOR "C" AS FOLLOWS.

C	SOIL
0.37	PEAT
1.22	FAIRBANKS SILT LOAM
2.00	FAIRBANKS GRAVEL
2.00	NORTHWAY SAND

Figure 12. Computed Depth of Thaw in Peat, Silt Loam, Sand, and Gravel, Fairbanks, Alaska.

is as follows, based on uniform soil conditions:

Sandy soil $w = 25$ percent $d = 93$ lb. per cu. ft. $k = \frac{19.5}{12} = 1.62$ $c = 0.17$

$T = 60$ deg. (at floor surface) - 32 deg. = 28 deg.

$L = 143.4\,(w)(d) = 143.4\left(\frac{25}{100}\right)(93) = 3334$ Btu per cu. ft.

The hangar was constructed in 1943 and heating started on November 1, 1943.

a. Computed depth of thaw to 1 November 1945 = 730 days after construction.

$$x = \sqrt{\frac{48\,(1.62)(28)(730)}{3334 + \frac{28}{2}(93)(.25 + .17)}} = \sqrt{\frac{1,590,000}{3334 + 560}} = \sqrt{408} = 20.2 \text{ ft.}$$

Observed depth was 20 ft.

b. Computed depth of thaw to November 1, 1946 = 1,095 days after construction.

$$x = 20.2 \cdot \sqrt{\frac{1095}{730}} = 20.2\,(1.225) = 24.8 \text{ ft.}$$

Observed depth was 22.5 ft.

c. Computed depth of thaw in 10 years $t = 3,650$ days

$$x = 20.2\sqrt{\frac{3650}{730}} = 20.2\,(2.24) = 45.2$$

d. Computed depth of thaw in 30 years $t = 10,950$ days

$$x = 20.2\sqrt{\frac{10950}{730}} = 20.2\,(3.88) = 78.4$$

A graph of these computations is shown in Figure 13. The size of this building is 162 ft. by 208 ft. and the concrete floor is in direct contact with the ground.

Depth of Frost Penetration

The depth of frost penetration can be calculated by the same principles used to calculate the depth of thaw. Correction factors applied to the freezing index based on air temperatures are generally much smaller than the correction factors applied to the thawing index and generally smaller than unity. When the mean annual temperature of a surface is higher than the mean annual temperature of the air, the annual temperature amplitude would have to be increased more than the difference between the mean annual temperatures just to make the freezing indexes equal. However, on the average, surface temperatures are higher than air temperatures during both winter and summer. In calculating depth of frost penetration, values of thermal conductivity for the soil in a frozen condition are used. The example shown in Table 3 illustrates the application of the method described above for calculating depth of thaw to the calculation of depth of frost penetration for the same soil conditions used in Table 2. The calculated depth of frost is 8.4 ft., while the depth of frost from temperature gradients was approximately 8.5 ft. See Figure 14. It may be noted on comparison of the results obtained in Tables 2 and 3 that the calculated depth of thaw was 9.25 ft. and the calculated depth of frost was 8.4 ft. These results show that the depth of thaw is greater than the depth of frost which indicates that thaw is probably progressing from year to year and that perma-

TABLE 3

COMPUTED DEPTH OF FROST

RUNWAY TEST SECTION RN-4 FAIRBANKS RESEARCH AREA

Layer	Material	"b" Thickness of Layer in ft.	"d" Dry Density of Soil lb. per cu. ft.	"w" Water Content of Soil in Percent of Dry Wt.	"L" 1.434 wd Volumetric Latent Heat of Fusion	"k" Thermal Conductivity Btu/sq. ft./hr/°F/ft.	$R = \frac{b}{k}$ Thermal Resistance	ΣR	$\Sigma R + \frac{R_n}{2}$	F = Freezing Index: Increment*	F = Freezing Index: Summation of Increments
1	Asphalt	0.4	150	0	0	$\frac{10.3}{12} = 0.86$	0.47	0	0.23	0	0
2	Gravel (GW)	3.8	143	3.7	759	$\frac{20}{12} = 1.67$	2.28	0.47	1.61	193	193
3	Silt (MH)	2.5	99	27.7	3932	$\frac{16}{12} = 1.33$	1.88	2.75	3.69	1511	1704
4	Peat	1.5	25	81.9	2936	$\frac{2.3}{12} = 0.19$	7.89	4.63	8.57	1572	3276
5	Silt & Peat (1.0)	x = 0.2	62	50.0	4445	$\frac{12.8}{12} = 1.07$	0.93	12.52	$12.52 + \frac{x}{2(0.93)}$	354	3630**
6	Silt & Peat		78.2	39.5	4429	$\frac{15}{12} = 1.25$	$\frac{x}{1.25}$				

Total = 8.4'

* $F_1 = \frac{L_1 b_1}{24} \cdot \frac{R_1}{2}$ $\quad F_2 = \frac{L_2 b_2}{24}\left(R_1 + \frac{R_2}{2}\right)$ $\quad F_3 = \frac{L_3 b_3}{24}\left(R_1 + R_2 + \frac{R_3}{2}\right)$ $\quad F_n = \frac{L_n b_n}{24}\left(\Sigma R + \frac{R_n}{2}\right)$

** Corrected freezing index = 5042 · 0.72 = 3630

Solving for depth x in layer 5 $\quad F_n = \frac{L_n b_n}{24}\left(\Sigma R + \frac{b_n}{2k}\right)$

$$354 = \frac{4445}{24} x \left(12.52 + \frac{x}{2(1.07)}\right)$$

$$x = 0.2'$$

Computed Depth of Frost = 8.4'

Depth of Frost from Temperature (Fig. 14) = 8.5'

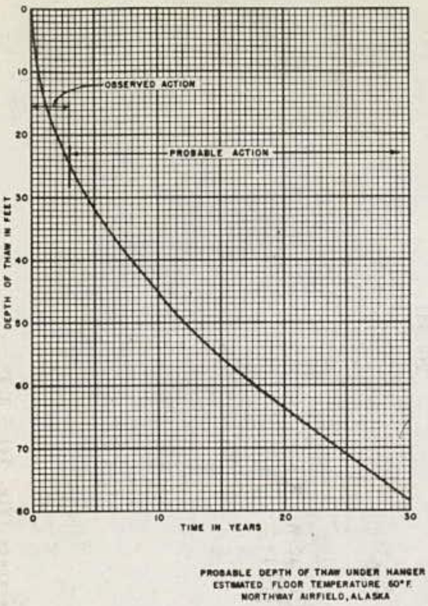


Figure 13. Probable Depth of Thaw Under Hangar Estimated Floor Temperature 60 F., Northway Airfield, Alaska.

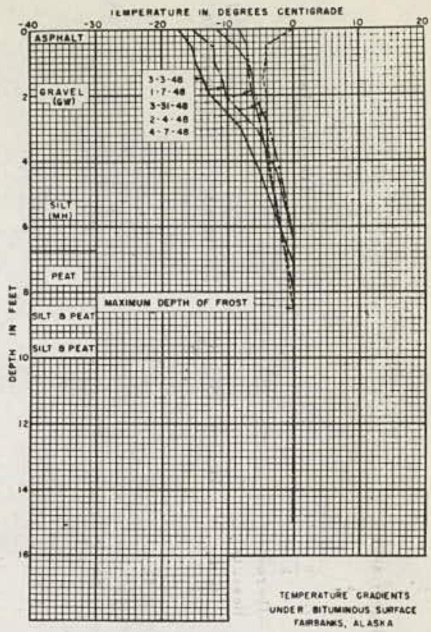


Figure 14. Temperature Gradients Under Bituminous Surface, Fairbanks, Alaska.

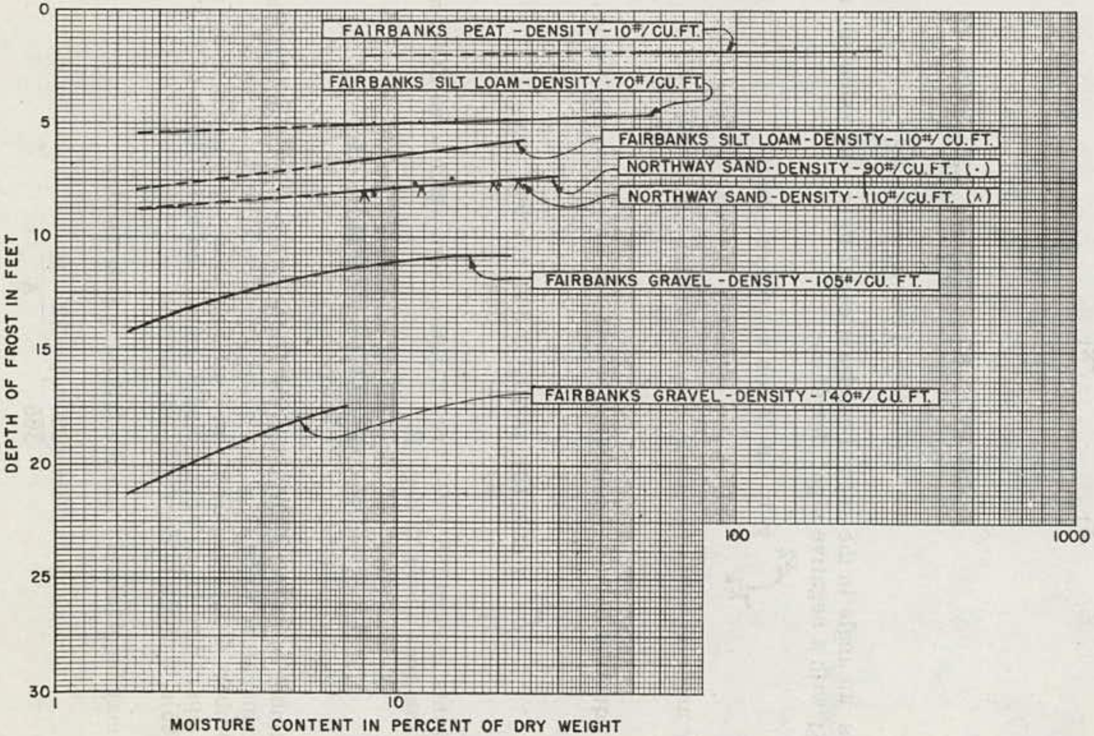
frost is degrading. It accounts theoretically for the discrepancy between the computed depth of thaw, 9.25 ft., and the observed depth, 10.2 ft. See Table 2. The rate of degradation of permafrost might be calculated in this case by subtracting the number of degree days necessary to thaw 8.4 ft. of seasonal frost from the corrected thawing index, 6,690 deg.-days, and using the difference in the equation for depth of thaw to compute the increments of thaw gained below 8.4 ft. each year. Figure 15 illustrates the computed relative depths of frost penetration for typical soil types in the vicinity of Fairbanks, Alaska, where the average freezing index is 5,220 deg.-days based on a 45-year record of air temperatures through 1948.

Suggested Research

Thawing and freezing indexes for use in the equations described in this paper have been determined by measuring the area between the time temperature curve and the freezing temperature line for the thawing and freezing seasons. Air and surface temperatures have been obtained periodically by observations of potentiometer or wheatstone bridge type temperature indicators. The ratio of the surface index to the air index is the correction factor used in Equation 14. Studies are now being made of temperatures of the air and various surfaces which are measured many times each day by automatic temperature recorders.

Data concerning air temperatures are available for weather stations all over the world. From these data the mean annual temperature and annual temperature amplitude of the air at any locality can be determined. It appears that relations may exist between the mean annual temperatures of the air and ground surface and the annual temperature amplitudes of the air and ground surface. Additional research will be required to validate this assumption.

For locations where the mean annual temperature and annual temperature amplitude of the air are known it is possible to compute the thawing index or freezing index from the sine law. The annual temperature amplitude may be determined from a graph of monthly mean temperatures or by calculation using the following Equation 23:



NOTE:

FORMULA FOR DEPTH OF FROST IN SINGLE LAYER OF UNIFORM SOIL IS

$$X=\sqrt{\frac{24\,CF\,(2k)}{L}} \quad \text{WHERE}$$

X = DEPTH OF FROST IN FEET.
F = FREEZING INDEX IN DEGREE DAYS.
C = CONSTANT TO CORRECT FREEZING INDEX.
k = COEFFICIENT OF THERMAL CONDUCTIVITIY OF FROZEN SOIL.
L = 1.434 x DENSITY x MOISTURE CONTENT.

FOR THIS GRAPH, THE FREEZING INDEX IS TAKEN EQUAL TO 5220 DEGREE DAYS AND FACTOR "C" AS FOLLOWS.

C	SOIL
.25	PEAT *
.33	FAIRBANKS SILT LOAM *
.70	FAIRBANKS GRAVEL
.70	NORTHWAY SAND (ESTIMATED)

* SNOW COVER NOT REMOVED

Figure 15. Computed Depth of Frost in Peat, Silt Loam, Sand, and Gravel, Fairbanks, Alaska.

$$A = 1.414 \sqrt{\frac{\Sigma (M - m)^2}{12}} \tag{23}$$

A = amplitude in degrees
1.414 = factor to change the root mean square value to amplitude
Σ = summational symbol
M = mean annual temperature
m = mean monthly temperature

The equation for a sine curve may be integrated to obtain equations for the thawing index as follows:

$y = A \sin x$
A = amplitude
x = angle in radians
$y\,dx = A \sin x\,dx$

$$\int_{x_1}^{x_2} y\,dx = A \int_{x_1}^{x_2} \sin x\,dx$$
$$= A \Big[-\cos x\Big]_{x_1}^{x_2}$$
$$= A \Big[-\cos x_2 - (-\cos x_1)\Big]$$
$$= A \Big[\cos x_1 - \cos x_2\Big]$$

and since x_2 is an angle in the second quadrant equal to ($\pi - x_1$ radians), $\cos x_2$ is the same as $\cos x_1$ with a negative sign; therefore;

$$\int_{x_1}^{x_2} y\,dx = A\,(2 \cos x_1) \text{ degree radii}$$

To convert the angle x_1 into degrees x_1 may be written $x_1 = \frac{360}{365} t_1$

and the whole equation must be multiplied by $\frac{365}{2\pi}$ to convert to deg. -days.

$$\int_{t_1}^{t_2} y\,dx = \frac{365}{2\pi} (A)(2 \cos \frac{360}{365} t_1) \text{ deg. -days}$$

For Figure 16 this integration gives area EOQHI, from which area EOHI must be subtracted to get the area above 32 deg. F.

$$I = \frac{365}{2\pi} (A)(2 \cos \frac{360}{365} t_1) - (32 - M)(t_2 - t_1) \tag{24}$$

t_1 = number of days required for the temperature to change from the mean annual temperature to 32 F.
t_2 = number of days required for the temperature to rise from the mean annual temperature to maximum displacement and to fall back to 32 F.
$(360/365)t_1$ = angle in degrees, not radians

The factor t_1 may be obtained from the following relationship:

$$\sin \left(\frac{360}{365} t_1\right) = \frac{32 - M}{A} \tag{25}$$

and t_2 from the following:

$$t_2 = 182.5 - t_1 \tag{26}$$

By a similar integration, the equation for the freezing index may be obtained.

$$F = 2\left[(32 - M)t_1 - \frac{365}{2\pi}(A)\left(1 - \cos\frac{360}{365}t_1\right)\right] + \frac{365(32 - M)}{2} + \frac{365}{\pi}A \tag{27}$$

The preceding equations hold for I and F when the mean annual temperature is below 32 F., but the following equations are true when the mean annual temperature is above 32 F.

$$I = 2\left[(M - 32)\,t_1 - \frac{365}{2\pi}(A)\left(1 - \cos\frac{360}{365}t_1\right)\right] \tag{28}$$

$$+ \frac{365}{2}(M - 32) + \frac{365}{\pi}(A)$$

$$\sin\left(\frac{360}{365}t_1\right) = \frac{M - 32}{A} \tag{29}$$

$$F = \frac{365}{2\pi}(A)\left(2\cos\frac{360}{365}t_1\right) - (M - 32)(t_2 - t_1) \tag{30}$$

Data from Fairbanks, Alaska and Minneapolis, Minnesota for air temperatures have been applied to these equations and the resulting indexes have been very close to those determined by other methods.

The advantage of the equations above is that only two factors, namely, mean annual temperature and annual temperature amplitude are required for calculation of indexes at a given locality.

It appears possible to determine the mean annual temperature and the annual temperature amplitude of a given surface from similar properties of the air by adding or subtracting a temperature difference. The indexes for the surface may then be approximated by means of the equations above.

Further research is necessary and appears desirable to validate these assumptions and to determine the temperature difference between air and surface mean annual temperature and annual temperature amplitude for various localities.

Conclusions

The results indicate a practical method of calculating the depth of thaw and the depth of frost below plane surfaces. The procedures outlined herein can be applied to conditions in the temperate zone providing information is available concerning the relation between air temperatures and the temperature of the particular surface being investigated. Information must also be available concerning soil density, moisture, and thermal conductivity. It is believed that information obtained from such studies will be of considerable value in indicating the relative merits of various types of materials in preventing frost penetration and associated problems.

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METHOD I

EMPIRICAL DATA FOR INDEXES

MONTH	MEAN MONTHLY T	DEGREE DAYS OF FREEZE	DEGREE DAYS OF THAW
J	-16.7	1509.7	
F	-16.9	1369.2	
M	-14.8	1450.8	
A	- 0.2	966.0	
M.	19.5	387.5	
J	34.7	AOB=16.0	OCD 97.0
J	40.0		248.0
A	38.5		201.5
S	31.0	HLM 46.5	FHG=16.5
O	16.6	477.4	
N	0.0	960.0	
D	-11.7	1354.7	
	TOTAL	F = 8537.8° DAYS	I = 563.0° DAYS

METHOD II

THEORETICAL CALCULATION OF INDEXES

$$\text{AMPLITUDE (A)} = 1.414\sqrt{\frac{\Sigma(M-m)^2}{12}} = 30.6\ \text{F}^\circ$$

$$\text{Sin}\left(\frac{360}{365}t_1\right)^\circ = \frac{32-M}{A}$$

t_1 = 46.6 DAYS

$t_2 = \frac{365}{2} - 46.6 = 135.9$ DAYS

I = AREA EOQHI − AREA EOHI

$$\text{AREA EOQHI} = \frac{365}{2\pi}(A)\left(2\ \text{Cos}\ \frac{360}{365}t_1\right)$$

$$\text{AREA EOHI} = (32-M)(t_2-t_1)$$

I = 506.9° DAYS

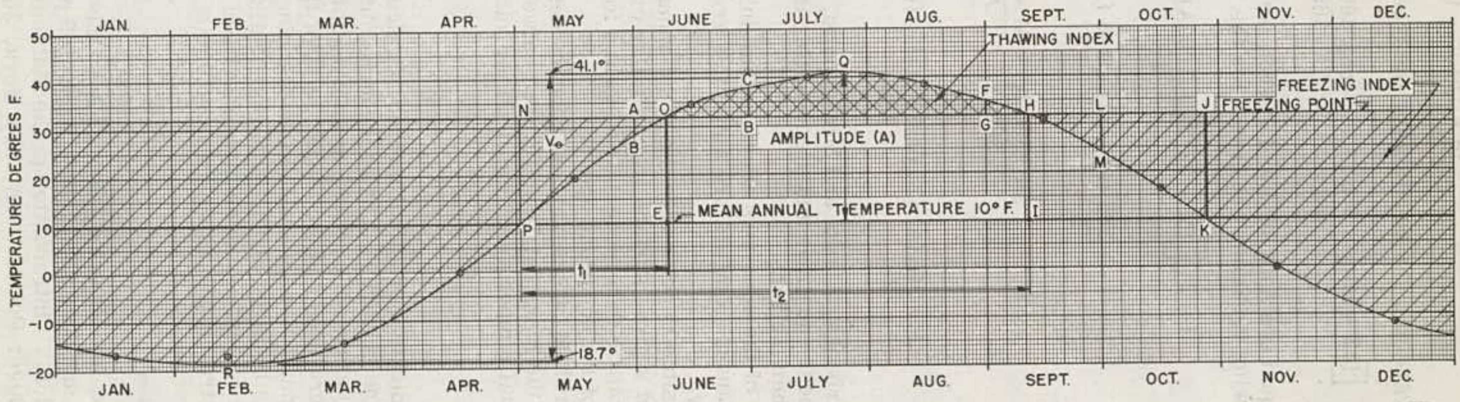
F = AREA NOP + AREA HJK + AREA KJNP + AREA KRP

$$\text{AREA NOP} + \text{AREA HJK} = 2\left[t_1(32-M) - \frac{365}{2\pi}(A)\left(1-\text{Cos}\ \frac{360}{365}t_1\right)\right] = 966.2^\circ\ \text{DAYS}$$

$$\text{AREA KJNP} = \frac{365}{2}(32-M) = 4015^\circ\ \text{DAYS}$$

$$\text{AREA KRP} = \frac{365}{\pi}A = 3555.7^\circ\ \text{DAYS}$$

F = 8536.9° DAYS



t_1 = TIME FOR AIR TEMPERATURE TO RISE FROM MEAN ANNUAL TO 32° F.
t_2 = TIME FOR AIR TEMPERATURE TO CHANGE FROM MEAN ANNUAL THROUGH THE MAXIMUM AND BACK TO 32° F.

M = MEAN ANNUAL TEMPERATURE
A = AMPLITUDE OF AIR TEMPERATURE VARIATION
m = MEAN MONTHLY TEMPERATURE.
⊙ = MEAN MONTHLY TEMPERATURE.

F = FREEZING INDEX.
I = THAWING INDEX.
V_o = MAXIMUM TEMPERATURE VARIATION.

TWO METHODS OF DETERMINING THE THAWING AND FREEZING INDEXES FROM MEAN MONTHLY AIR TEMPERATURES AT BARROW, ALASKA

Figure 16. Two Methods of Determining the Thawing and Freezing Indexes from Mean Monthly Air Temperatures at Barrow, Alaska.

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INTERPRETATION OF PERMAFROST FEATURES FROM AIRPHOTOS

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Synopsis

This report describes the use of aerial photography for predicting the presence of permafrost and certain permafrost features in arctic and subarctic regions. The techniques of airphoto interpretation for use in identifying engineering soil conditions and drainage conditions as developed in temperate climates has been expanded to include the application of the principles to the Arctic. These techniques have been used in the far northern regions in connection with highway and airfield location and general soil survey, particularly in Alaska.

Permafrost research at Purdue is being done as a part of the general permafrost program by the St. Paul District under the direct supervision of the Office, Chief of Engineers, Department of the Army. Five field trips to Alaska by members of the Purdue Staff have provided an opportunity to