Chapter Nine

Stability Analyses and Design of Control Methods

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Discussions throughout this text have emphasized the fact that more than one method can be used to prevent or correct a given landslide problem. In earlier chapters the various corrective and preventive measures have been described and general recommendations for the use of each have been made. Experience has been the basis for the recommendations. It has been shown, however, that the unfavorable experience record of certain treatments has been in part the result of failure to understand the magnitude of the forces involved. The extrapolation of experience with one type of slide in one particular region and type of material to other slide types in other regions and other materials is obviously difficult, if not dangerous. Moreover, where performance records for two different corrective treatments show equal success, some basis is needed for deciding which will be most economical in a new situation. Obviously, some quantitative means of evaluation is needed.

Even though some landslides do not lend themselves entirely to the assumptions commonly used in soil engineering, stability analyses made according to the classic theories of soil mechanics still present the best hope for a quantitative means of evaluating experience and provide a rational basis for extending experience for the purpose of prediction. The analyses cannot be made for every type of landslide and for any type a number of assumptions based on idealized conditions and materials will be required. It is impossible to treat mathematically all of the variables imposed by nature. Further mathematical simplification is required to prevent the analysis from becoming unwieldy. In application, then, the results are always dependent on the validity of the assumptions and simplifications. The results should not be considered as exact solutions of the problem and the possible variance between real and assumed conditions should always be kept in mind. Even with its limitations, applied theoretical analysis has advantages that are useful and it can, on occasion, be of considerable value.

The principal use of a mathematical approach may lie in making it possible to weigh the cost of the treatment against the value received, rather than in the actual quantitative answer. For example, if the use of a given corrective measure is questioned on the basis of experience, the cost of the treatment can be estimated, the before and after safety factors computed, and an evaluation made of how much relative stability is produced for the given amount of money. Even though one may question the accuracy of the mathematics with the attendant assumptions and, therefore, the exact values of safety factors derived, there is less question in considering relative stabilities; that is, in ranking the before and after safety factors.

This chapter is not intended to be a critical review of the methods and theories of soil mechanics as applied to landslides, and space cannot be given to a detailed discussion of all of the ramifications of stability analyses. The reader is referred to texts on soil mechanics for detailed treatments of the problem and for discussions of the variables and simplifying assumptions that are required in any of the mathematical treatments.

Background material is provided to acquaint management, the field engineer, and the geologist with analytical methods and to permit an understanding of the major part of the discussion, which consists of examples of analyses involving the major methods of landslide control. Principal emphasis is placed on a single method of attack, the Swedish slice method, and on its application to the economics of various treatments. Numerous other methods can be used and are preferred by some workers. Most standard texts on soil mechanics may be consulted for other methods of analysis; the Corps of Engineers Manual (1952) provides a compact discussion with working examples of several of them.

The examples given here are for situations where slides have occurred. With some necessary modifications, however, the same methods are applicable to the analysis of slope stability where construction may create an unstable condition in previously stable slopes. Moreover, the analysis of an existing slide often provides the easiest and perhaps most accurate method of arriving at an estimate of stability for slopes in similar materials in adjacent areas. The discussion and the examples are included in order to demonstrate the method and principles that are involved so that the reader can make similar applications if the principles are applicable. Attention is again called to the need for understanding of the variables and assumptions involved.

Method and Principles

Several methods are available for quantitative study of the stability of slopes. Each varies to a slight degree, and each requires certain assumptions, including one as to the form of the surface of sliding. The real surface of sliding is often a composite surface having a section made up of two or more arcs of circles or approximated by an arc of an ellipse.[6] However, an exact duplication of the potential sliding surface is seldom warranted.

Most methods of analysis, therefore, replace the real surface of sliding with one having a section of either an arc of a circle or of a logarithmic spiral (Rendulic, 1935). The use of the circular arc assumption is based on studies of actual failure surfaces by the Swedish Geotechnical Commission and is fundamental to a method of analysis developed by W. Fellenius (1927, 1936). The general approach of this method has been widely adopted by soils engineers to estimate the factor of safety of slopes against failure.

In addition to an assumption as to the form of the failure surface, conventional stability analyses require certain other facts and assumptions as follows:

1. A shear failure must have occurred or must be a threat. This assumption will be true for slides, but not for falls and some flows. Flow materials will not have significant shearing resistance, so that stability analyses will not generally be made.
2. The average shearing resistance along the slip-surface at the time of failure must be known, as must any major variation from the average. The shearing resistance is at once the most critical value and the most difficult one to obtain unless a failure has occurred.
3. An assumption must be made that the conditions that exist along a narrow slice or cross-section of the slide can be used to design against movement in the remainder of the area. A related assumption (common to all stability analyses) is that no lateral shearing resistance exists along the sides of the slice. It is

[6] Kjellman (1955), indeed, has raised the question of the actual existence of a "surface" of sliding, pointing out that perhaps no true surface exists although all mathematical stability analyses assume such a surface.

believed that this assumption affects the quantitative answer in a minor way. Three-dimensional analyses can be used, but considerably more work is required and an assumption of increased accuracy may not be warranted.

4. An assumption must be made as to the location of the piezometric or the ground water surface at the instant of failure. This will apply to those movements where hydrostatic pressures could have played a significant part. One of two assumptions will be necessary, for rarely will it be possible or practical to obtain the necessary hydrostatic or ground water data. The first is to assume a reasonable location for the piezometric surface based on subsurface water conditions. If the shearing resistance is known, the location can be checked against the fact that a failure developed (or has not yet developed); that is, if a failure has developed, then certain hydrostatic pressure conditions could have produced the failure (higher pressures would have brought failure sooner, and lower pressures would have produced no failure). The other approach is to use a value of shearing resistance which incorporates the effect of hydrostatic pressure. This approach is more useful if a correction other than drainage is to be analyzed, and if the value of the shearing resistance is based on the developed slide.

5. The value of the safety factor to apply must be established. This facet can be a very difficult one to handle, for a relatively minor change in safety factor may more than double the cost of the treatment. Also of some importance is the selection of the type of safety factor to be used. Safety factors can be expressed in terms of the ratio of slide resisting forces to slide-inducing forces, or they may be expressed in terms of the relationship between soil strength factors (for example, in terms of the developed unit cohesion as compared to the unit cohesion adopted for design). The definition selected will vary with the method of analysis and the conditions of the individual situation. For an excellent discussion of safety factors as related to slope stability analyses, see Corps of Engineers (1952).

In the discussion and examples of this chapter the most frequently used expression of the safety factor will be as a ratio between total shearing resistance and total shearing force. In analyses of failed slopes the concept will be used that failure occurred when total shearing force just exceeded total shearing resistance. Thus, for the analysis of the failed slope a factor of safety of one is assumed. This assumption is fundamentally sound and, it is felt, allows the best estimate of the values of cohesion, c, and angle of internal friction, ϕ, as they existed in the ground prior to movement.

Swedish Method of Slices

The Swedish Method of Slices was developed to a relatively high degree by W. Fellenius (1927, 1936). This method applies to most cohesive soils above the water table which have a shearing resistance, s, approximately equal to

$$s = c + \sigma \tan \phi \qquad (1)$$

in which

c = cohesion;
σ = stress normal to the slip-surface; and
ϕ = angle of internal friction.

Difficulty is generally encountered in establishing accurate values of cohesion and angle of internal friction, due to inadequate sampling and testing techniques. However, the method can be applied to materials that are non-uniform in character and is most useful in estimating factors of safety against failure.

In the analysis, the assumption is made that the surface of failure of a slope can be defined as having a section represented by the arc of a circle, and that the soil within the circle rotates about point 0, the center of the circle (Fig. 116). The arc along which the soil may be assumed to move will be deter-

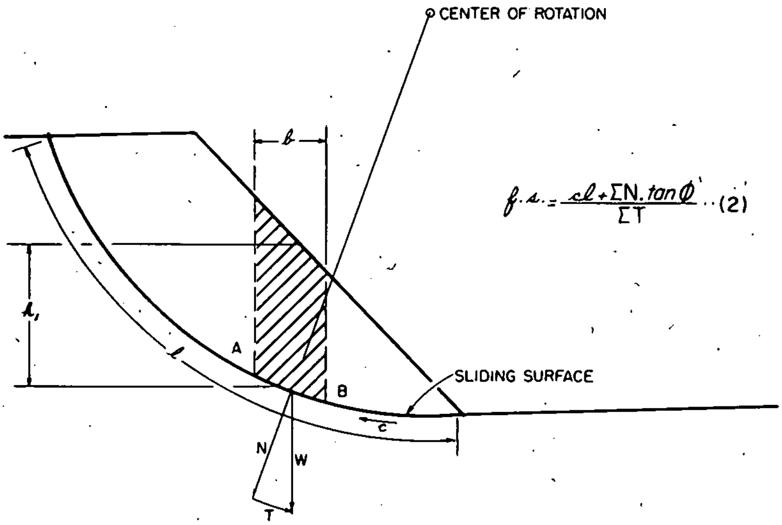


Figure 116. Forces acting on a slide wedge.

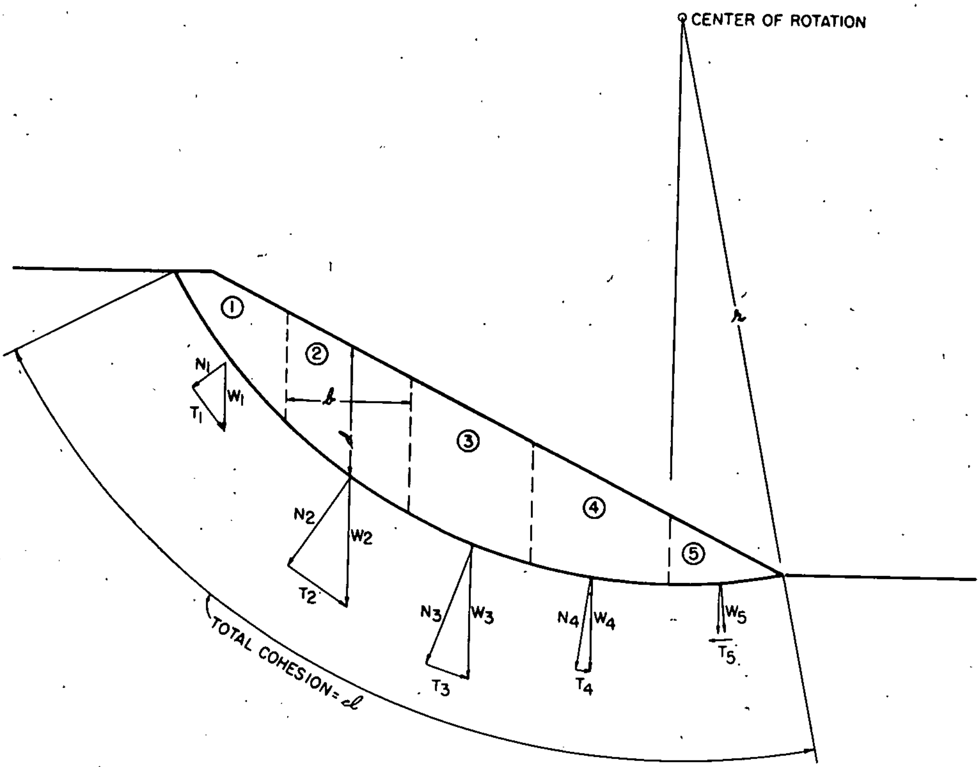


Figure 117. Graphical solution of forces for the method of slices.

mined by stratification within the sliding mass, depth to a firm material, and several other factors. In many cases the sliding surface will not approximate that of an arc of a single circle, but will be made up of composite arcs.

The procedure requires a cross-section, plotted to scale, of the slope being analyzed. The circular arc that represents the failure surface is then drawn on the cross-section, forming a circular segment representing the sliding mass. The segment is then divided into several slices of equal width, as shown in Figure 117. The shaded area of Figure 116 represents a single slice.

The forces acting on this slice are indicated at the sliding surface. Neglecting the forces acting on the sides, the forces acting on the slice are the normal and tangential components, N and T, of the weight W; the unit cohesion per unit of slice width, c, acting along the arc, BA, and the frictional force induced by N. The tangential vector, T, represents the slide-inducing force, whereas the resisting forces are the cohesion plus the normal force, N times the tangent of the angle of internal friction. Thus, the factor of safety (f.s.) against sliding along an arc of length l can be written as

$$\text{f.s.} = \frac{\text{shearing resistance}}{\text{shearing force}}$$

$$= \frac{c\,l + \Sigma N \tan\phi}{\Sigma T} \qquad (2)$$

In which ΣT and ΣN represent the sum of values of T and N for all the slices. ΣT is the total slide-inducing force; $cl + \Sigma N \tan\phi$ represents the total resisting force, l being the length of the sliding surface.

This method of analysis lends itself readily to the design of corrective measures. If the critical slide surface of a slope can be established, and the shearing forces evaluated, the increase in factor of safety realized by placing an additional resisting force at the toe can be calculated readily. The expression then becomes

$$\text{f.s.} = \frac{c\,l + \Sigma N \tan\phi + P}{\Sigma T} \qquad (3)$$

in which P is the additional resisting force per unit of width.

Neutral Pressures

In the foregoing equations the weight of the soil mass is equal to the volume of soil times the soil's unit weight. Where the ground water table is below the failure surface (thus no seepage forces are encountered) the unit weight used in the calculations is the weight of a unit volume of the soil and its included water. However, should the ground water table be at some point above the failure surface, the resisting force is reduced due to the neutral pressure, μ, of the soil water. In this case the factor of safety against sliding is given by

$$\text{f.s.} = \frac{c\,l + \Sigma (N - \mu) \tan\phi}{\Sigma T} \qquad (4)$$

in which μ represents the total force of the soil water exerted on the bottom of the soil slice (Fig. 116). For example, if the water table is at the ground surface in Figure 116 and no flow of water exists, the neutral pressure acting on a slice is given by

$$\mu = h_1 \gamma_\omega \text{BA} \qquad (5)$$

in which γ_ω is the unit weight of water.

Expressed in another way, the slide-inducing forces are determined by using the weight of the soil plus water; the resisting forces are determined using the submerged unit weight of the soil

$$\gamma'_m = \gamma_m - \gamma_\omega \qquad (6)$$

in which γ'_m is the effective or submerged unit weight of the soil, γ_m is the mass unit weight of soil plus water, and γ_ω is the unit weight of the water.

Method of Estimating Stability

In the ideal case of relatively homogeneous soil, the factor of safety of a slope against sliding can be determined conveniently by graphical procedures, as illustrated in Figure 117.

The sliding elements of equal width are obtained. The weights (W_1, W_2, W_3 . . . W_n) or areas of each slice are laid out, respectively, as a vertical vector to any convenient scale at the center of each slice at the sliding arc. If the slices are of equal width, this may be done by making the vector distance numerically equal to the average depth of the slice. Lines are then drawn through the center of the circle and through the origin of each W vector at the sliding surface; this locates the line of action of the normal forces. The tangential forces are next drawn at right angles to these lines and to the lower end of the W vectors. The T and N forces may then be determined by use of an engineer's scale. As an aid in the solution it is best to set up the problem in the form of a table (see Table A, Fig. 118).

Table A, Figure 118, applies to a slump or rotational type of failure. Where the slip-surface is nearly a straight line in a planar failure, the same approach may be used (see Fig. 119). In this case the resisting force is again made up of the unit cohesion times the length of sliding plane plus the product of the normal force times the tangent of the angle of internal friction, or $c\,l + \Sigma N \tan\phi$, and the sliding force is equal to T.

Location of Sliding Surface

The success of this method of analysis, and of any mathematical treatment of slides, is contingent upon adequate boring and strength data. A sound field exploration program is essential before any type of theoretical analysis is made. Moreover, many landslides are not adapted to mathematical analysis; among these are rockfalls of all types. For the purpose of analysis, failures for artificial embankments are generally broken down into (a) slope failures, (b) toe failures, and (c) base failures. The first two of these are perhaps self-explanatory. The last, base failure, denotes a deep circle that intersects the ground line well below the toe of the slope. This type of failure, if influenced entirely by soil, is generally a midpoint failure; that is, one where the center of the circle exists at some point on a vertical line drawn midway between the toe and the top of the original slope. The location of the center on this line must be found, however, by trial and error.

The location of the circle must be compatible with the known conditions. If a layer of weak, soft material exists at some depth, the circle will be so situated that its major portion lies within this layer. If materials of different shearing resistance are present, such as soil overburden on rock, or on a firm base such as gravel, the circle will generally be tangent to the firm base. Seepage planes may likewise influence the location of the circle.

Methods are available for mathematically estimating the potential sliding surface of unfailed artificial slopes in homogeneous soils (see Taylor, 1948). After an estimate is made of the potential failure surface, taking into account the natural soil conditions, calculations are made as illustrated in previous paragraphs. The factor of safety is then computed and a new trial is made by shifting the center of rotation to both the left and the right. By repeating the process after the center of rotation is moved vertically, one can determine the critical center which is the one giving the least factor of safety.

If, after a slide occurs, the positions of at least two points on the slide can be fixed in relation to the positions which they had on the original ground, the sliding surface may be determined by simple geometry. This is done as illustrated in Figure 120. Straight lines are drawn from the original to the final location of the known points. Perpendicular bisectors of these lines will intersect at the center of rotation of the mass. In practice it is best to utilize at least three points, more if possible. In this way any error which arises from inaccurate measurements in the field or which arises from the fact that the sliding plane is not the arc of a circle will be averaged. The lines will be found to intersect at several points and the true center can then be taken as the average of these. A slightly different empirical method of determining the location of the slip plane is described in the section on "Estimating Depth of Slump Slides: Slip Circle Method" (Chapter Six) and illustrated in Figure 62.

Determination of Strength Factors

It is extremely important that proper estimates be made of the values of cohesion and internal friction. In Chapter Three under "Factors that Contribute to Low Shear Strength," several items that contribute to shear failures of earth and rock masses are listed. Among these are neutral pressures and pressures caused by percolating water, sensitivity of clays, inherently weak materials, and others. If the rational approach to slope design is to be adequate, each of these must be evaluated.

In the special case of saturated natural clay deposits the shearing resistance can be approximated by

$$s = c \qquad (7)$$

This is true because the permeability of clay is very low; therefore, if a shearing force is applied rapidly before drainage can take place, the load is taken in large part by interstitial water and the apparent angle of internal friction is equal to zero. Thus, when the analysis is made for saturated clays, the shearing resistance is made up of cohesion alone. For this case, the value of cohesion is determined by performing the unconfined compression test. Cohesion is then equal to one-half the ultimate strength in compression. Rewriting Eq. 2 with $\phi = 0$ gives

$$\text{f.s.} = \frac{c\,l}{\Sigma T} \qquad (8)$$

In another special case, that of clean uncemented sand, essentially no cohesion exists and Eq. 1 becomes

$$s = \sigma \tan\phi \qquad (9)$$

For these cohesionless sands the angle of repose, the natural slope assumed by sand when poured loosely on a flat surface, although not in exact agreement with the angle of internal friction, will give results which are sufficiently accurate.

For the more general case where both cohesion and internal friction must be considered, the laboratory determination of these factors is more complicated. The shear resistance may be determined by direct shear tests or, preferably, by triaxial shear tests. However, many analyses of failed slopes which have been based on laboratory values for c and ϕ have given safety factors greater than 1.0. In other examples, natural slopes of known stability have given computed safety factors as low as 0.75. This failure of laboratory-derived values to give the expected results in computations is no doubt caused by irregularities in the soil, difficulties in obtaining undisturbed samples, problems in laboratory technique, and the effect of seepage forces.

In the redesign of failed slopes an estimate of the average shearing resistance which is safe and which lessens the effects of the troublesome variables previously listed can be arrived at by basing the computations on the conditions in the failed slope.

The average shearing resistance along the failed surface can be computed by balancing forces around the center of rotation (see Fig. 121). For the redesign of slopes it is not advantageous to divide the resisting forces into their components (cohesion and friction) unless a drainage solution is involved; instead, a composite figure acting along the sliding surface should give the desired accuracy. If the latter method is used (see Fig.

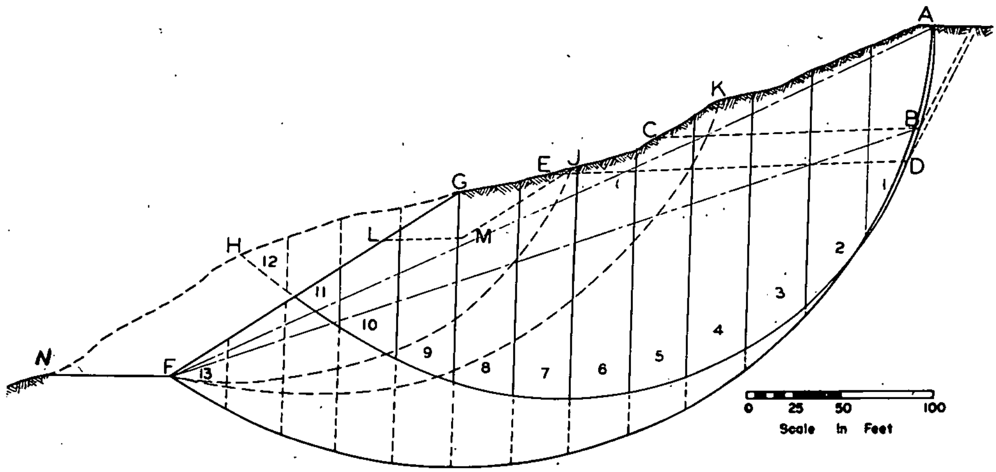


Figure 118. Determination of shearing resistance and design for excavation methods.

Table A. For use in determining original values of ϕ and c.

Segments Within Arc AH

	1	2	3	4	5	6	7	8	9	10	11	12	Total
Area (A), sq ft	2580	3730	4120	4450	4160	4140	3600	3430	2860	2230	1260	320	36 880
Normal (N_A)	1000	2500	3350	4000	4000	4100	3600	3400	2750	2050	1050	250	32 050
Tangential (T_A)	2400	2800	2400	1900	1150	600	0	–400	–800	–1000	–700	–200	8 150

Table B. For use in determining stability if toe of slide (FNG) is removed.

Segments Within Arc AF and Below Excavation Line GF

	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
Area (A), sq ft	2350	3780	4460	4980	4780	4750	4730	4740	4110	3340	2290	1570	740	46,620
Normal (N_A)	650	1800	3200	3950	4650	4500	4600	4700	4100	3300	2200	1500	650	39,800
Tangential (T_A)	2250	2870	3050	2850	1350	1500	1000	400	–200	–500	–600	–500	–350	13,120

Table C. For use in determining stability if head (ABC) and toe (FNG) of slide are removed.

Segments Within Arc AF and Below Excavation Lines ABC and GF

	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
Area (A), sq ft	610	2750	3400	4240	4620	4750	4730	4740	4110	3340	2290	1570	740	41,890
Normal (N_A)	250	1450	2400	3400	4100	4500	4600	4700	4100	3300	2200	1500	650	37,150
Tangential (T_A)	550	2350	2400	2500	2100	1500	1000	400	–200	–500	–600	–500	–350	10,650

Table D. For use in determining stability if larger head (ADE) and toe (FNG) of slide are removed.

Segments Within Arc AF and Below Excavation Lines ADE and GF.

	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
Area (A), sq ft	350	2030	2960	3780	4220	4510	4630	4740	4110	3340	2290	1570	740	39,270
Normal (N_A)	150	1100	2100	3100	3750	4250	4400	4700	4100	3300	2200	1500	650	35,400
Tangential (T_A)	300	1700	2100	2150	1900	1450	1000	400	–200	–500	–600	–500	–350	8,850

Table E. For use in determining stability if a straight 2:1 slope is excavated.

Segments Within Arc AF and Below 2:1 Excavation Line AF

	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
Area (A), sq ft	2130	3360	3870	4450	4480	4680	4210	3930	3410	2640	1750	1350	500	40,760
Normal (N_A)	650	1850	2850	3600	4000	4400	4100	3900	3400	2600	1700	1300	400	34,750
Tangential (T_A)	1700	2800	2600	2600	2000	1550	900	400	−200	−350	−400	−550	−300	12,750

Table F. For use in determining stability if a straight 3:1 slope is excavated.

Segments Within Arc AF and Below 3:1 Excavation Line BF

	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
Area (A), sq ft	670	2050	2820	3270	3480	3480	3380	3170	2770	2520	1650	1120	470	30,850
Normal (N_A)	400	1100	2000	2650	3100	3300	3300	3150	2750	2450	1600	1050	450	27,300
Tangential (T_A)	550	1700	1950	1900	1550	1100	750	300	−150	−350	−400	−400	−250	8,450

Table G. For use in determining stability if toe of slide (FNG) is removed (assuming arc JF is potential sliding surface).

Segments Within Arc JF and Below Excavation Line GF

	7	8	9	10	11	12	13	Total
Area (A), sq ft	765	1880	2160	1840	1550	910	465	9,570
Normal (N_A)	400	1400	1850	1700	1500	900	450	8,200
Tangential (T_A)	650	1250	1100	700	350	100	−100	4,050

Table H. For use in determining stability if a bench (JGLM) is cut.

Segments Within Arc JF and Beneath Bench JGLM

	7	8	9	10	11	12	13	Total
Area (A), sq ft	470	1100	1525	1800	1550	910	465	7,820
Normal (N_A)	100	850	1200	1700	1500	900	450	6,700
Tangential (T_A)	450	700	900	700	350	100	−100	3,050

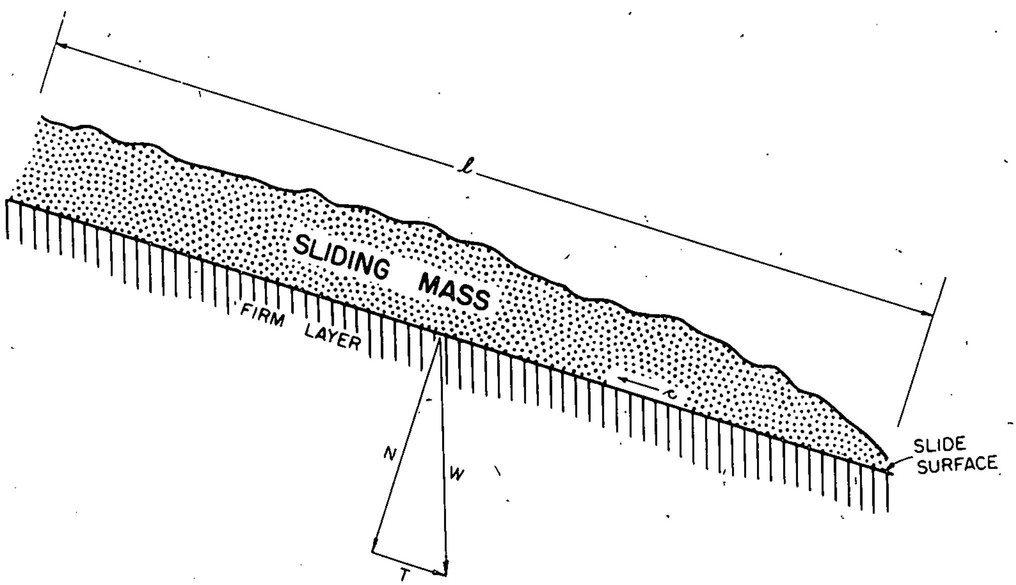


Figure 119. Forces acting on sliding mass, planar slide surface.

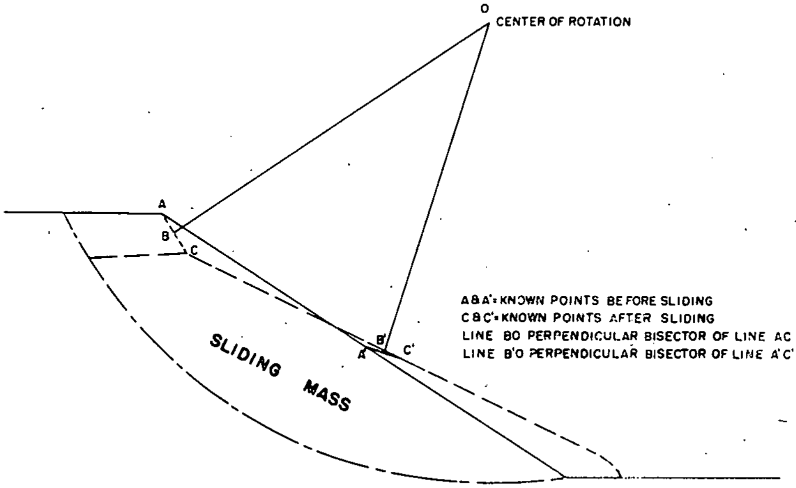


Figure 120. Method of locating center of rotation of slide mass.

121), moments can be balanced about the center of rotation and the average shearing resistance at failure is calculated from

$$s = \frac{W_1 d_1 - W_2 d_2}{l r} \qquad (10)$$

in which s is the average shearing resistance, composed of either, or both, cohesion, c, and $N \tan\phi$.

Use of Slide Data for Determining Shearing Resistance

As has been pointed out, the values for ϕ and c to be used in Eq. 2 can be obtained by laboratory shear tests on undisturbed samples taken from the zone of the shear surface (slip-plane). For computations where the pore pressures are to be ignored, however, technique of the type described in the preceding paragraph is recommended.

The method of slices (see Fig. 117 and discussion) can also be used in estimating the shearing resistance at the time of failure. A condition of safety factor = 1.0 is assumed. In the example that follows, such pairs of values for c and ϕ are used as to permit computation of either c or ϕ if the value of the other quantity is assumed. The average unit weight of the soil mass must be determined by sampling and measuring. An estimate of the location of the slip-surface is required, and the cross-section must be divided into increments as shown in Figure 118. In this and the following figures in this chapter the areas and the normal and tangential forces of each segment of the diagram are shown in tables. The components for normal (N_A) and tangential (T_A) forces are expressed in terms of area so as to simplify computations. Assuming that the average unit weight of the soil has been determined to be 125 lb per cu ft, that a 1-ft slice is used, and that prior to the excavation and the subsequent slide the arc AH represents a first estimate of the slip-surface, then the total normal force, in pounds, is

$$\sum_1^{12} N = \sum_1^{12} N_A \, (\gamma_m) \qquad (11a)$$

and the total tangential force, in pounds, is

$$\sum_1^{12} T = \sum_1^{12} T_A \, (\gamma_m) \qquad (12a)$$

in which

$\sum_{1}^{12} N_A =$ summation of the normal forces for increments 1-12, inclusive, in sq ft of area;

$\sum_{1}^{12} T_A =$ summation of tangential forces for increments 1-12, inclusive, in sq ft of area; and

$\gamma_m =$ unit weight of the landslide mass, in lb per cu ft.

Thus, the following values are determined:

$$\sum_{1}^{12} N = 32{,}050 \times 125 = 4{,}010{,}000 \text{ lb}$$

$$\sum_{1}^{12} T = 8{,}150 \times 125 = 1{,}020{,}000 \text{ lb}$$

$$l = 505 \text{ ft (scaled)}$$

$$\text{f.s.} = 1.0 \text{ (assumed)}$$

With these four factors known, Eq. 2 now contains only two unknowns, ϕ and c. By assuming a value for one of these factors, the other may be computed. For example, assuming that $\phi = 5°$, and expressing Eq. 2 as

$$c = \frac{\text{f.s. } \Sigma T - \Sigma N \tan\phi}{l} \qquad (13)$$

c has a value of

$$\frac{1{,}020{,}000 - (4{,}010{,}000 \times 0.0875)}{505 \times 1}$$

or 1,320 lb per sq ft.

Assuming that $\phi = 10°$, then c has a value of

$$\frac{1{,}020{,}000 - (4{,}010{,}000 \times 0.1763)}{505}$$

or 619 lb per sq ft.

Therefore, for future computations, the pairs of values to be used together are: $\phi = 5°$ and $c = 1{,}320$ lb per sq ft; $\phi = 10°$ and $c = 619$ lb per sq ft.

The shearing resistance thus computed indicates the strength needed to maintain equilibrium prior to any recent movement. Any other stability analyses made on the basis of these shear values will represent a stability with reference to that before recent movement. For example, a safety factor of 1.0 after treatment will mean that a condition will exist that is approximately as stable as the original hillside. For very stable slopes that may be encountered, this approach may represent an overdesign. For very unstable slopes, a greater relative safety factor may be desired for the corrective treatment.

The preceding method for estimating shearing resistance is of primary use for analysis of landslides that have occurred. The technique has been used on relatively stable slopes, but more danger of overdesign exists. The cross-section of the ground surface after movement (or the original ground surface for a potential slide area) is used, which represents an assumption that little or no change in shearing resistance has taken place. This will be true except where extensive movement has taken place, such as when sensitive clays are encountered. For these materials, the movement will resemble a flow rather than a slide and a stability analysis probably will not be attempted. Where sensitive clays are suspected (potential slide case), laboratory tests will disclose the truth very quickly.

The fact that the shearing resistance does not change radically in a slump-type failure (except where sensitive clays are involved) may not be readily acceptable. However, if shear resistance computations are made for the slip-surface and ground line before movement and the results are compared with those for the ground line after movement, only a slight difference will be developed. In fact, minor changes in driving and resisting forces make such a result self-evident.

For relatively stable hillsides, the technique is slightly more involved. The probable slip-surface location must be considered in light of subsurface data on bedrock location, weak strata, etc. In general, the circle that progresses farthest uphill will produce the highest shearing resistance, which is the value desired; that is, if the resistance to shear was lower, a failure would have occurred.

Another means of estimating the shearing resistance is to compute the value along a failed surface after excavation (arc AF in Fig. 118 after the slope FG has been cut). Possibilities exist for errors, however, if pore pressures can be expected to increase at a later date. The advantage to considering the slope as it existed before failure is that, within the lifetime of the slope, any pore pressures that have existed will be reflected in the stability of the natural slope.

Examples of the Method Applied to Specific Control Measures

Excavation

If the approximate location of the surface of rupture and the average shear strength characteristics are known, and if the influences of hydrostatic pressure are neglected, Eq. 2 can be used to estimate the effect of excavation anywhere on the slope.

Removal of Material at Head of Slide

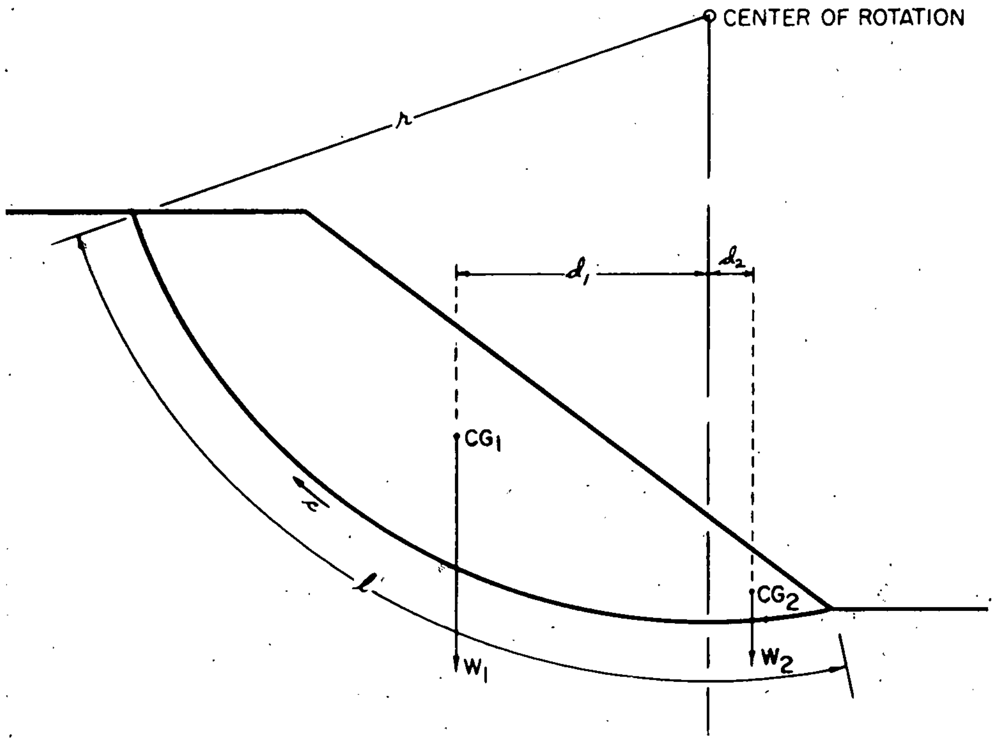
Considering first the removal of material from the head, one can use a technique consisting of a trial-and-error method to develop the desired safety factor. From Figure 118, an area (ABC) approximately 10 to 25 percent of the moving mass is selected. Eq. 2 can then be used to determine the safety factor after area ABC is removed and the lower slope is excavated to line FG. The stability will be improved due to the decrease of ΣT, but it will be lessened by decrease in the length of the slip-plane and the loss of forces normal to the slip-plane. However, as the major portion of the shearing force comes from the head, the net result of such excavations is an improvement of stability conditions. As shown in Figure 117, the ratio of T to N is relatively larger in the head region of a slide than it is in the middle and toe regions.

If the area selected (ABC, Fig. 118) does not produce a sufficient increase in the safety factor, a larger area is then tried (ADE). Conversely, if the increase is too great, a smaller area is considered for economic reasons.

The following is an example of the computations required, neglecting the effect of pore pressure. Referring to Figure 118 for a slump failure, and to Eq. 2, assume that undisturbed samples have indicated an average unit weight of 125 lb per cu ft. Also assume that laboratory tests or slide analyses indicate that $\phi = 5°$ and $c = 1{,}320$ lb per sq ft. For the first computations, the arc AF will be used as the slip-surface, and the effect on the stability by an excavation along FG will be determined. By graphical methods described previously, the values in Table B, Figure 118, are computed. Using the method of computation given in the previous section, $\Sigma N = 4{,}970{,}000$ lb; $\Sigma T = 1{,}640{,}000$ lb; $l = 550$ ft; and f.s. $= \dfrac{(4{,}970{,}000 \times 0.0875) + (1{,}320 \times 550 \times 1)}{1{,}640{,}000} = 0.700$. To estimate the influence of removing the upper portion of the slide (area ABC), the following factors are determined for the slip-surface AF, with the upper and lower areas (ABC and FNG) excavated (Table C, Fig. 118): $\Sigma N = 4{,}650{,}000$ lb; $\Sigma T = 1{,}330{,}000$ lb; $l = 495$ ft; and f.s. $= \dfrac{(4{,}650{,}000 \times 0.0875) + (1{,}320 \times 495 \times 1)}{1{,}330{,}000} = 0.795$. The larger area at the head (ADE), together with the same toe removal (FNG), are then assumed to be removed (Table D, Fig. 118), the values becoming: $\Sigma N = 4{,}430{,}000$ lb; $\Sigma T = 1{,}100{,}000$ lb; $l = 475$ feet; and f.s. $= \dfrac{(4{,}430{,}000 \times 0.0875) + (1{,}320 \times 475 \times 1)}{1{,}100{,}000} = 0.923$.

Flattening the Slope

For a comparison of the foregoing removals of head and toe with the stability for straight slopes, assume that the 2:1 (horizontal:vertical) slope, AF, is cut (Table E, Fig. 118). The values then be-



CG_1, CG_2 = Centers of gravity, respectively, of driving and resisting masses;
W_1 = Weight of driving mass;
W_2 = Weight of resisting mass;
d_1, d_2 = Lever arms, respectively, of W_1, W_2;
c = Cohesion per unit of length and width of slice; and
l = Length of slip surface.

Figure 121. Determination of average shearing resistance by balancing of forces.

come: ΣN = 4,350,000 lb; ΣT = 1,591,000 lb; l = 550 ft; and f.s. =

$$\frac{(4,350,000 \times 0.0875) + (1,320 \times 550 \times 1)}{1,591,000} = 0.697.$$

The stability of the flatter 3:1 slope, BF (Table F, Fig. 118), would be: ΣN = 3,410,000 lb; ΣT = 1,055,000 lb; l = 495 ft; and f.s. =

$$\frac{(3,410,000 \times 0.0875) + (1,320 \times 495 \times 1)}{1,055,000} = 0.902.$$

Thus, removal of material near the top of the slide produces a greater influence on the stability than do the other corrective measures for which calculations were made. An economic comparison of excavation at the head of the slide over slope-flattening can also be made from the examples. The removal of area ADE requires only 785 cu yd of excavation per yard of slide length measured normal to direction of movement. The excavation of a 3:1 slope gives nearly the same safety factor, but requires removal of nearly 2½ times as much material, or 1,720 cu yd, for the same length of slide.

The effect of excess hydrostatic pressures (seepage forces) and the reduction in shearing resistance due to removal of the load were not considered in the foregoing. In the following section on drainage methods, a theoretical approach is suggested for considering the hydrostatic forces when sufficient data are available.

For a slide having a planar sliding surface (Fig. 119) it is obvious that removing the head has no more effect on the measure of stability as obtained from Eq. 2 than the same removal from any other place in the moving mass. This is true because the relation of N to T is the same at any point on the slide. One exception would be at the toe of the slide. If the cut slope were not sufficiently flat, a failure could develop on that slope and progress uphill, successively undermining the upper areas. There is another factor to be considered for toe excavation. If the slip-plane is curved at the toe but elsewhere straight, toe removal would be more severe in terms of undermining. The increased detrimental effect is caused by (a) a decrease in shearing resistance resulting from the removal at the toe of a mass which contributes to the frictional part of the shearing resistance; and (b) an increase in the tangential component (shearing force), as the values of T would be negative at the toe (see Fig. 117).

Benching of Slopes

Computations relative to the benching of slopes are essentially the same as described previously for other excavation methods. Because slopes containing cohesive materials are limited to a "critical height" (above which failure occurs) for a given angle, many too-steep slopes that are within the limits of their individual critical heights can be separated by a bench and thus be made stable.

The following is an example of designing benches in cohesive soil slopes. Referring to Figure 118, Table G, and using Eq. 2, assume that the slope FG has been excavated, that $\phi = 5°$ and $c = 1{,}320$ lb per sq ft[7], and that arc JF represents a potential slip-surface (Table G, Fig. 118). The values then are: $\Sigma N = 1{,}025{,}000$ lb; $\Sigma T = 506{,}000$ lb; $l = 260$ ft; and f.s. =

$$\frac{(1{,}025{,}000 \times 0.0875) + (1{,}320 \times 260)}{506{,}000} = 0.855.$$

In order to determine the effect on the stability, assume that the bench, JGLM, is excavated (Table H, Fig. 118), the resulting values being: $\Sigma N = 837{,}000$ lb; $\Sigma T = 371{,}000$ lb; $l = 260$ ft; and f.s. =

$$\frac{(790{,}000 \times 0.0875) + (1{,}320 \times 260)}{371{,}000} = 1.12.$$

The same slip-surface, JF, was used in both computations. More dangerous slip-surfaces (KF, for example), farther up the slope, also should be checked.

[7] It must, of course, be assumed also that the danger of sliding along arc AF and similar planes has been removed by excavation at the head of the slide or by other means.

Drainage

The summary of "Drainage Methods" in Chapter Eight indicates five possible detrimental influences of water in a slide area. The factor of reduction in weight, the change in shearing resistance of the material at the slip-surface, and the effect on the shearing resistance due to geochemical and physical changes are difficult to evaluate quantitatively. Necessarily, a stability analysis based on these three factors will lie in the realm of conjecture until better techniques have been developed. In particular, the decrease in weight by the installation of drains is likely to show little influence on the stability as pictured by the safety factor.

However, the lowering of the ground water table or the elimination of excess hydrostatic pressures (or seepage forces) can materially influence the value of the safety factor. In this respect, subdrainage measures can be evaluated analytically.

Two factors that are difficult to determine are: (a) drain spacing, and (b) the prediction of the water table or piezometric surface after the drains have become effective. At the moment, trial-and-error methods must be used and field observations are needed. For example, in clay soils the drain spacing might need to be as close as 10 to 25 ft (perpendicular to direction of landslide movement).

$\mu 1_A$ = Neutral pressures before drainage (area);
$\mu 2_A$ = Neutral pressures after drainage (area);
h_1 = Height of piezometric surface above slip-plane before drainage; and
h_2 = Height of piezometric surface above slip-plane after drainage (bottom of pipe).

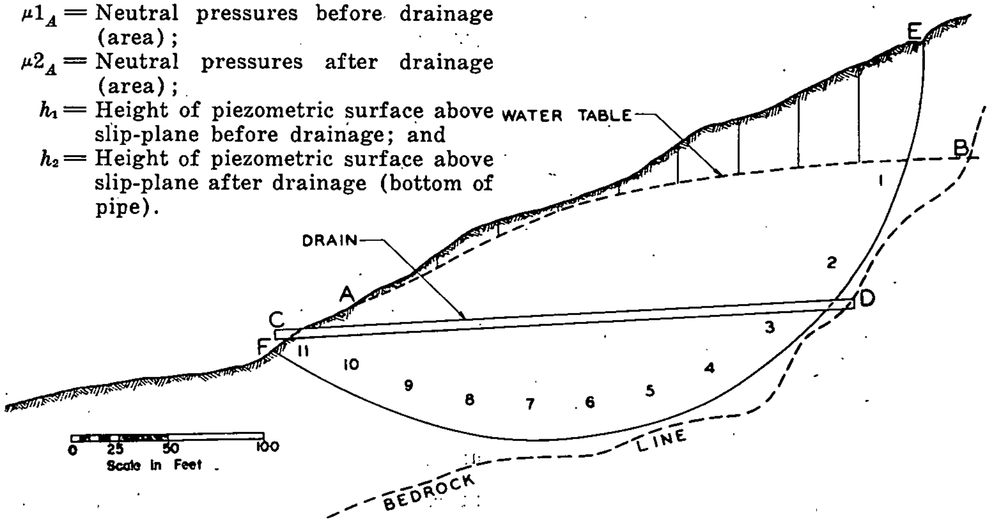


Figure 122. Analysis of horizontal drainage installation.

Table I. For use in determining stability if a drain CD is installed.

	Segments Within Arc EF											
	1	2	3	4	5	6	7	8	9	10	11	Total
Area (A), sq ft	2460	3730	4120	4450	4150	4100	3650	3280	2300	1340	390	33,970
Normal (N_A)	900	2500	3300	4000	3900	3950	3600	3200	2150	1150	300	28,950
Tangential (T_A)	2300	2800	2450	1850	1100	500	–200	–700	–800	–700	–250	8,350
Average h_1	30	80	98	119	125	123	115	96	70	45	18	
Average l	65	50	40	38	32	32	32	32	35	35	30	
$\mu 1_A$, sq ft	1950	4000	3920	4520	4000	3930	3680	3070	2450	1575	540	33,635
Average h_2	0	2	23	44	55	61	60	56	45	30	13	
$\mu 2_A$, sq ft	0	100	920	1670	1760	1950	1920	1790	1575	1050	390	13,125

For the drainage of permeable layers, a 25- to 50-ft spacing might be adequate. Complete interception of all water, of course, calls for a system of drains that is fitted to the local geologic conditions, rather than one that follows a set geometric pattern.

In the following example, four simplifying assumptions are involved (Fig. 122), as follows:

1. The new water table will lie at the elevation of the drain.
2. No change in ϕ and c occurs with the lowering of the water table.
3. There is no reduction in the average unit weight of the soil.
4. A static condition (no seepage) exists for the conventional flow line.

The first three do not represent a conservative approach, and estimated adjustments can be made if desired. The fourth eliminates the development of a flow net, and a conservative answer is obtained unless deep flow exists. As the water table is drawn in Figure 122 it is obvious that some seepage will actually take place at A, where the water table intersects the surface. The assumtion that no seepage exists is made for problem simplification, but is justifiable. The quantity of flow is not important and static hydrostatic pressure conditions will normally be more severe than those for seepage.

Figure 122 and its table are used for the following examples and the following conditions are known or assumed. The average unit weight of the soil, γ_s, is 125 lb per cu ft, the drain CD has been installed, the line AB represents the highest original water table or piezometric surface to be anticipated, and $\phi = 10°$.

$$\sum_1^{11} N = \sum_1^{11} N_A \gamma_m \qquad (11b)$$

$$= 28{,}950 \times 125 = 3{,}620{,}000 \text{ lb}$$

$$\sum_1^{11} T = \sum_1^{11} T_A \gamma_m \qquad (12b)$$

$$= 8{,}350 \times 125 = 1{,}040{,}000 \text{ lb}$$

$$\mu_1 = \mu_{1A} \gamma_\omega \qquad (14a)$$

$$= 33{,}635 \times 62.4$$
$$= 2{,}100{,}000 \text{ lb (before drainage)}$$

$$\mu_2 = \mu_{2A} \gamma_\omega \qquad (14b)$$

$$= 13{,}125 \times 62.4$$
$$= 819{,}000 \text{ lb (after drainage)}$$

Also, $l = 465$ ft, $\Sigma(N - \mu_1) = 1{,}520{,}000$ lb, and $\Sigma(N - \mu_2) = 2{,}801{,}000$ lb.

In order to estimate shear characteristics, a safety factor of 1.0 is assumed for the original hillside and Eq. 13 gives

$$c = \frac{1{,}040{,}000 - (1{,}520{,}000 \times 0.1763)}{465}$$

$= 1{,}660$ lb per sq ft. For the influence on stability produced by the drain, Eq. 4 can be used to find the factor of safety, or f.s. =

$$\frac{(2{,}801{,}000 \times 0.1763) + (1{,}660 \times 465)}{1{,}040{,}000}$$

$= 1.22$.

Thus, lowering the water table under the stated conditions produces a safety factor of 1.22 as compared to 1.0 without drainage.

The preceding example illustrates the importance of ϕ with regard to drainage solutions. Unless ϕ is at least 10° to 20° the influence of drainage may not be great. The safety factor for a cohesive material is given by Eq. 8. For such materials, the benefits from drainage must result from loss of weight and increase in shearing resistance. Once more using Figure 122, but this time assuming that $\phi = 0°$ and that the unit weight can be reduced from 125 to 115 lb per cu ft, the value for cohesion can be obtained by expressing Eq. 8 in terms of c for a safety factor of 1.0, giving $c = \frac{1{,}040{,}000}{465}$ $= 2{,}240$ lb per sq ft. For the assumed loss of weight accomplished by drainage, and assuming no increase in shearing resistance, f.s. $= \frac{1{,}040{,}000}{960{,}000} = 1.09$. Furthermore, in order to improve the safety factor to a value of 1.25, the increase in shearing resistance required would be $c = \frac{960{,}000 \times 1.25}{465} = 2{,}580$ lb per sq. ft. Laboratory testing could be used to estimate whether the assumed decrease in unit weight and increase in shearing resistance are reasonable for the type of soil and for the drainage characteristics of the landslide.

Restraining Structures

For a quantitative approach to determination of the size of a restraining structure, an estimate is needed of the force against the restraining device, the shearing resistance along the slip-surface, and the resistance afforded by the retainer. For flow conditions, there will be very little resistance along any surface of separation that develops and the forces against a structure will be relatively large. For slides, if an estimate can be obtained of the sizable resistance along the slip-surface at the time of movement, the factors which must be known are the thrust against the retaining device and the point and direction of its application.

The degree of stability (or relative safety factor) should be at least 1.5 for restraining structures. However, values as low as 1.25 may be required due to economic considerations.

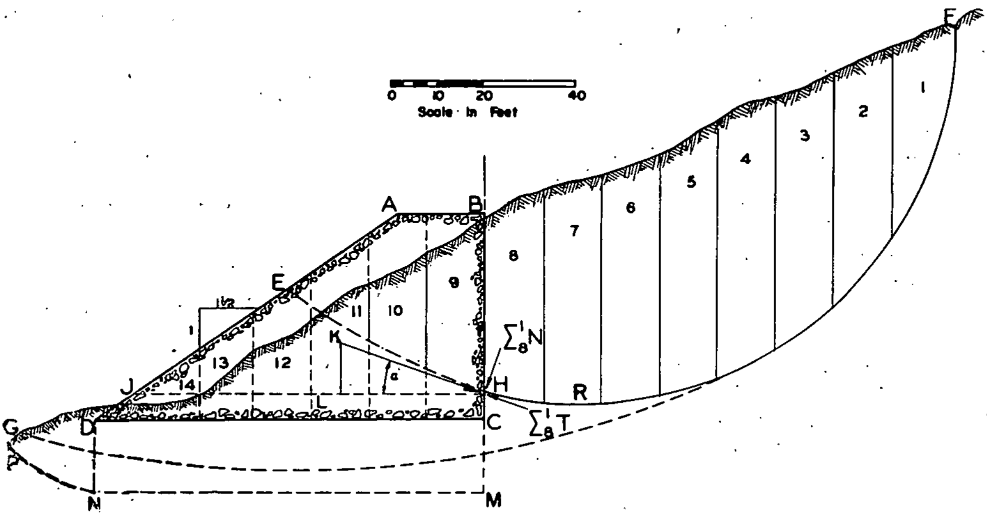


Figure 123. Design of a rock buttress.

Table K. For use in determining the shearing resistance, assuming a slip plane EF in the original hillside.

Segments Within Arc EF

	1	2	3	4	5	6	7	8	9	10	11	Total
Area (A), sq ft	385	465	640	680	650	620	550	545	335	195	60	5,125
Normal (N_A)	160	400	540	620	630	615	550	540	330	180	55	4,620
Tangential (T_A)	350	420	350	280	155	75	0	–35	–70	–70	–20	1,435

Table L. For use in determining stability of a rock buttress if shear failure develops along arc EH through the buttress.

Segments Within Arc EH

	9	10	11	12	Total
Area (A), sq ft	435	420	200	60	1,115
Normal (N_A)	425	390	185	30	1,030
Tangential (T_A)	–115	–170	–80	–45	–410

Construction of this type will frequently involve special procedures in order to permit the near-vertical, BH. This will be particularly true if the slope is relatively unstable.

Buttresses

For buttresses, failure may develop in one of three ways (see Fig. 123), as follows:

1. Shear through the buttress (along line HE or HJ).
2. Foundation failure beneath the buttress (along arc FG).
3. Friction or shear failure between the buttress and the foundation (along line CD).

Rock buttresses can be constructed on either a solid rock foundation or on soil. Where it is possible or practicable, a solid rock foundation should be obtained, as foundation failures through rock are unlikely. Thus, the possibilities of failure at the contact surface between buttress and foundation are minimized, and the costs of adequately providing against a foundation failure beneath the buttress are eliminated.

For a rock buttress with foundations on soil, it will be necessary to determine the value of the shearing resistance of the underlying soil. This can be done by laboratory testing or by the evaluation of performance in a manner similar to the determination of the shearing resistance elsewhere in the slide area. The shearing resistance of the soil in the foundation material is needed both for a check on stability against a foundation failure and for the estimate of the stability along the contact surface between the buttress and the foundation.

The primary difference between a rock buttress and an earth buttress lies in the distinction between a granular, noncohesive material (rock buttress) and a fine-grained, cohesive soil (earth buttress). The granular material develops shearing resistance, due to friction, which is proportioned to the weight of the material above the shear plane. A cohesive material develops a shearing resistance from cohesion (which is not materially affected by weight above the shear plane) and from friction. The shearing resistance of a clay soil develops primarily from cohesion, with little or no friction benefit. This fact has led to the $\phi = 0°$ approach, a simplification that may be warranted in many instances. However, the frictional component may be tangible, with values ranging from 5° to 15° for clays and silty clays.

Rock Buttress. — Consider first a rock buttress. The slip-surface can be extended through an assumed buttress (line HE in Fig. 123) and a stability analysis used to determine the degree of stability. Such an analysis is difficult because trial-and-error methods are involved and because curved slip-surfaces must be faced. The curved slip-surface complication can be avoided without serious error by assuming a straight line extension of the slip-surface through the buttress. Normally, it will be easier to determine the size of the buttress on a preliminary basis by checking the stability against a friction or shear failure at the base of the buttress (line JH). Stability with reference to a shear failure through the buttress and the relative stability against a foundation failure beneath the buttress can then be checked.

The location of the buttress with reference to the toe of the movement is related to the position and shape of the slip-surface. An effective location for the back of the buttress is near that part of the slip-surface that is tangent to the horizontal (point R). It is recommended that for the first computations one edge of the buttress (line BC) be placed so that the tangent to the slip-plane at point H makes an angle of less than 10° with the horizontal.

Using the principles explained previously for excavation methods, one can estimate the summation and direction of application of the tangential and normal stresses at any point along the slip-surface. In order to obtain a preliminary estimate of the size of the buttress, the summations are made for the upper portion of the mass between the top of the slide and the upper edge of the buttress (increments 1-8, inclusive, in Fig. 123). To determine the amount of resistance required from the buttress, a safety factor for design is established, and the following form of Eq. 3 is used:

$$P_R = \text{f.s.} \left(\sum_a^b T + \sum_b^c T \right) - \sum_a^b N \tan\phi - c_s l_{a-b} \quad (15a)$$

or

$$P_R = \text{f.s.} \sum_a^b T - \sum_a^b N \tan\phi - c_s l_{a-b} \quad (15b)$$

in which

c_s = cohesion in the natural soil;
P_R = resistance required from the buttress, in pounds.

Eqs. 15*a* and 15*b* represent the general equation where *a*, *b*, and *c* are any three increments between which summations are desired. For example, in Figure 123

$a = 1$, $b = 8$, and $c = 14$. For preliminary estimates, the value of $\sum_b^c T$ can be assumed to equal zero (Eq. 15*a*), which will give a conservative result (algebraic value is normally negative).

Given the summation of the shearing resistance required from the buttress along the extension of the slip-surface (EH in Fig. 123) and the direction of its application (tangent to the slip-plane at point H), the value for P_R represents the summation of the resistance within the buttress (increments 9 to 14, inclusive). The source of the shearing resistance will be the frictional component of the weight of the mass for (a) shear surfaces within the buttress, and (b) at the contact between the rock buttress and bedrock. Most of the available resistance will be friction. The resistance offered by the tangential component for a nonhorizontal shear plane is included in $\sum_b^c T$. By graphics, the horizontal thrust against the buttress (required frictional resistance) can be determined (LH) so as to resist a shear failure through the buttress (along line HJ), or can be obtained by multiplying P_R by $\cos\alpha$. Therefore, the following equations can be used to express the resistance required from the buttress per unit of width.

For horizontal shear through the buttress:

$$P_R \cos\alpha = \gamma_B A_B \tan\phi_B \qquad (16a)$$

or

$$A_B = \frac{P_R \cos\alpha}{\gamma_B \times \tan\phi_B} \qquad (16b)$$

in which

α = angle formed by the tangent to the slip-surface and the horizontal at back of buttress;

γ_B = unit weight of the buttress, in lb per cu ft;

A_B = area of the buttress, in sq ft; and

ϕ_B = angle of internal friction for the rock in the buttress.

For shear failure at contact — soil foundations:

$$P_R \cos\alpha = \gamma_B A_B \tan\phi_s + c_s\left(\frac{A_B}{h} + \frac{1.5h}{2}\right) \qquad (17a)$$

$$A_B = \frac{P_R \cos\alpha - \frac{1.5h}{2}c_s}{\gamma_B \tan\phi_s + \frac{c_s}{h}} \qquad (17b)$$

in which

ϕ_s = angle of internal friction for the foundation soil;

c_s = unit cohesion of the natural soil; and

h = height of buttress, in ft.

Assuming that the buttress is to be constructed with one vertical face (BC) and the other (AD) on a 1.5:1 (horizontal to vertical) slope, the length of the bases can be expressed as

$$\text{Length of bases} = \frac{A_B}{h} \pm \frac{1.5h}{2} \qquad (18)$$

After obtaining a preliminary estimate of the dimensions of the buttress, the stability with reference to the other conditions of failure should be determined. Assuming that Eqs. 15*a* and 16*b* have been used for the previously determined values, one needs to check for the degree of stability with reference to a shear failure through the buttress (line HE). The values for $\sum_a^b T$ and $\sum_a^b N$ between the top of the slide and the edge of the buttress are obtained and the $\sum_b^c T$ and $\sum_b^c N$ within the buttress and above the slip-plane are then determined. From these values, the safety factor can be determined by

$$\text{f.s.} = \frac{\sum_a^b N \tan\phi_s + c_s l_{a-b} + \sum_b^c N \tan\phi_B}{\sum_a^b T + \Sigma \sum_b^c T} \qquad (19)$$

If the safety factor is too low, additional material will be needed above the shear plane. The needed amount can be estimated from Eq. 3 by trial and error. An alternate solution would be a new location for the buttress.

To determine the safety factor relative to a foundation failure (soil foundations), a slip-plane extending below the buttress should be studied (FG in Fig. 123). Unless drilling has indicated the presence of a very weak stratum, the same values for shearing resistance can be used as for the upper portions of the slide, and a circular slip-plane assumed.

The following is an example of the computations (Table K, Fig. 123) for a rock buttress with the unit weight of the soil and buttress material assumed equal to 125 and 100 lb per cu ft, respectively. If one assumes no hydrostatic pressures present, a value of $\phi = 10°$, and a safety factor of 1.0 for the original hillside, the cohesion can be determined as follows: $\sum_1^{11} N = 577{,}000$ lb; $\sum_1^{11} T = 179{,}000$ lb; $l_{1-11} = 188$ ft; and

$$\text{(Eq. 2)}\ c = \frac{179{,}000 - (577{,}000 \times 0.1763)}{188} = 410 \text{ lb per sq ft.}$$

To obtain an estimate of resistance required from the buttress for a safety factor of 1.5, $\sum_1^{8} N = 507{,}000$ lb; $\sum_1^{8} T = 199{,}000$ lb; $l_{1-8} = 149$ ft; and (Eq. 15*b*) $P_R = 1.5\ (199{,}000) - (507{,}000 \times 0.1763) - (410 \times 149) = 148{,}000$ lb.

For preliminary estimates of the size of the buttress, and assuming that the buttress will be founded on bedrock, Eqs. 16*b* and 18 can be applied with the following data: $\alpha = 20°$; $\phi_B = 35°$; $h = 40$ ft; $P_R \cos\alpha = 148{,}000 \times 0.9397 = 139{,}000$; and (Eq. 16 *b*) $A_B = \dfrac{139{,}000}{100 \times 0.700} = 1{,}985$ sq ft.

To determine the length of the bases for a buttress with a rear vertical face and a front slope of 1.5:1 (horizontal: vertical) (Eq. 18), bases $= \dfrac{1{,}985}{40} \pm \dfrac{60}{2}$; therefore, upper base = 20 ft, and lower base = 80 ft.

The stability with reference to a shear failure through the buttress (EH), can be checked by Eq. 19 (Table L, Fig 123): $\sum_9^{12} N = 103{,}000$ lb; $\sum_9^{12} T = -41{,}000$ lb; and (Eq. 19) f.s. = 1.41.

To increase this value, another location of the buttress can be selected or the height can be increased, and the stability estimates recomputed.

For conditions where bedrock is not encountered, Eq. 17*b* can be used for obtaining the dimensions required for the buttress: $A_B = \dfrac{139{,}000 - \dfrac{1.5 \times 40 \times 410}{2}}{100 \times 0.1763 + \dfrac{410}{40}} = 4{,}545$ sq ft; length of lower base $= \dfrac{4{,}545}{40} + \dfrac{1.5 \times 40}{2} = 144$ ft; and length of upper base $= \dfrac{4{,}545}{40} - \dfrac{1.5 \times 40}{2} = 84$ ft.

An alternate to the use of the larger buttress that is required with soil foundations would be one with deeper foundations (MN). Although drainage would be required, additional resistance would be afforded along NP, and the slip-plane FG (failure benenath buttress) would be lowered.

Earth Buttress. — The procedure for designing an earth buttress is quite similar to that described for one composed of rock. Eq. 15*a* can be used to determine the needed resistance. The general procedure described for rock buttresses can be followed, or a method based on assumed dimensions is quite useful. A buttress of approximately one-third to one-half the volume of the mass to be retained is selected. Laboratory tests on remolded samples of the earth buttress material will produce the necessary values for ϕ and c. The resistance produced by the buttress is then checked against the required resistance by extending the slip-plane through the buttress and by use of the following equation (see Fig. 123):

$$P_R = \text{f.s.} \sum_b^c N \tan\phi_b + c_b l_{b-c}) \quad (20)$$

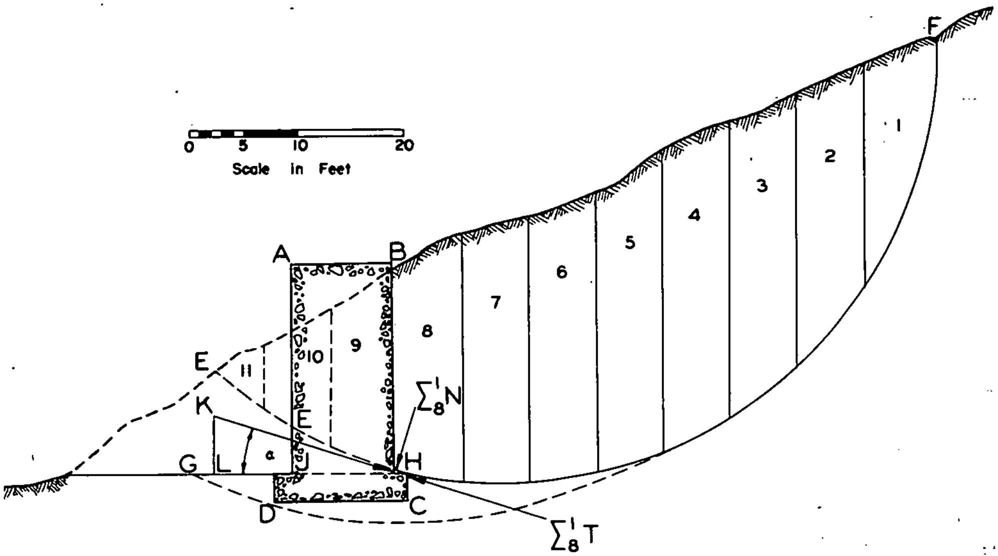


Figure 124. Design of a wall.

Table M. For use in determining stability of a crib wall ABCD. Construction of this type will frequently involve special procedures in order to permit the near-vertical cut, BH. This will be particularly true if the slope is relatively unstable.

	Segments Within Arc EF											
	1	2	3	4	5	6	7	8	9	10	11	Total
Area (A), sq ft	100	120	160	170	165	160	140	140	85	50	20	1,310
Normal (N_A)	40	100	135	155	160	155	140	135	80	45	15	1,160
Tangential (T_A)	90	105	90	70	40	20	0	–10	–20	–20	–5	360

in which

c_b = cohesion for soil in buttress, in lb per sq ft;

ϕ_b = angle of internal friction for soil in the buttress; and

l_{b-c} = length of slip-plane within the buttress, in ft.

If the buttress selected does not produce sufficient shearing resistance, a larger buttress should be considered; if too much resistance is developed, estimates for a smaller one can be made.

In order to determine the adequacy of the buttress with regard to a failure at the contact with the foundation soil, Eq. 17a can be used. The shear characteristics of the foundation material can be assumed to be equal to those for the natural hillside soil, unless obvious differences exist. In the latter event, previously described methods for determining shearing resistance will be required. The failure beneath the earth buttress is checked in the same manner as for a rock buttress (FG of Fig. 123).

Cribs and Retaining Walls

In determining the size or adequacy of a crib or retaining wall, the same basic equations are employed as for the buttresses. The required resistance is obtained by using Eq. 15a. This type of structure must be checked for the following types of failures (Fig. 124):

1. Shear failure through the wall (arc EH or line JH).
2. Foundation failure beneath the wall (arc FG).

3. Friction or shear failure at contact between the wall footer and the foundation (line CD).
4. Overturning.

The possibilities of a foundation failure are checked by the same technique as explained in the foregoing for a rock buttress, and the procedure will not be discussed further. It should be remembered that if the wall is founded in or on bedrock, the possibilities of a foundation failure are remote.

For shear failure of the wall and overturning, the reader is referred to structural texts for the design of a retaining wall. For determining the force required due to inherent instability of the slide area, Eq. 15*a* is used. Knowing the direction and magnitude of the force, KH, that is acting at the slip-plane, normal design methods can be used. The resisting force can be assumed to be applied at the one-third point between the slip-surface and the top of the wall.

For crib walls, shearing resistance is primarily developed in the material used to backfill the crib. The shear values, either for rock or soil, can be determined by shear tests in the laboratory. Soil is rarely, if ever, desirable for the backfill. Although some resistance can be attributed to the interlocking members of the crib, such resistance will be relatively small and can, for safe and conservative design, be disregarded.

Overturning of a crib wall is not a problem if standard recommendations for batter are followed. A combination shear failure and overturning may develop, but the lack of tensile strength precludes normal overturning. The resistance to a friction or shear failure at the contact between the footer and the foundation can be determined as for a buttress, using a form of Eq. 15*a*.

For bedrock foundations

$$P_R \cos\alpha = W \tan\phi_F \qquad (21)$$

and for soil foundations

$$P_R \cos\alpha = W \tan\phi_s + c_s l_w \qquad (22)$$

in which

W = weight per foot of wall length, in lb;
$\tan\phi_F$ = coefficient of friction between wall and foundation material; and
l_w = length of slip-surface beneath the wall, in ft.

Another estimate of the stability of a retainer can be made if the device is to replace an excavation in a reasonably stable slope. By comparing the natural resistance of the soil removed to that afforded by the retaining device, relative stability can be determined. If a slight movement has developed but halted, or if a stable slope is excavated, the technique may be useful. The approach consists of determining the toe portion of the shearing resistance along a surface of rupture by the previously described methods. The shearing resistance of the excavated soil and, consequently, that required from the retainer is

$$P_R = \text{f.s.}\left(\sum_b^c N \tan\phi_s + c_s l_{b-c} - \sum_b^c T\right) \qquad (23)$$

Thus, the stability with reference to the original soils is equal to the ratio of the retainer resistance to the soil resistance before excavation; that is, if a safety factor of 1.5 is used, the retainer will produce a state of stability (as reflected in the safety factor) 1.5 times that developed by the soil before excavation. The over-all safety factor of the hillside may be less than 1.5 in such cases, however. Obviously, this method of computing stability is useful only if the natural hillside appears relatively stable.

The following example of the design of a crib wall refers to Figure 124. Although in practice the wall will be constructed on a batter (approximately 1:6), computations are somewhat simplified and the results are conservative if one assumes a vertical wall. The unit weight of the soil is assumed to be 125 lb per cu ft and the unit weight of the rock backfill for the crib wall to be 100 lb per cu ft. The length of the slip-surface EF shown in Figure 124 is 94 ft. To determine the shearing resistance under an assumed safety factor of 1.0, and a value of $\phi = 10°$ (Table M); $\sum_1^{11} N = 145{,}000$ lb; $\sum_1^{11} T = 45{,}000$ lb; and $c_s = \frac{45{,}000 - (145{,}000 \times 0.1763)}{94} = 207$ lb per sq ft.

To determine P_R (the resistance required from the wall) for a safety factor of 1.5 (relative to the original stability), $\sum_1^{8} N = 127{,}000$ lb; $\sum_1^{8} T = 51{,}000$ lb; $l_{1-8} = 75$ ft; and (Eq. 15b) $P_R = 1.5(51{,}000) - (127{,}000 \times 0.1763) - (207 \times 75) = 38{,}600$ lb.

The weight required from the wall if the angle of internal friction of the backfill is 35° and α (Fig. 124) is 22° is:

$$W = \frac{-P_R \cos\alpha}{\tan\phi_w} \qquad (24)$$

$$= \frac{38{,}600 \times 0.927}{0.700} = 51{,}100 \text{ lb}$$

in which

ϕ_w = angle of internal friction on the backfill material of the crib wall.

The weight available per foot of height can be estimated for a single-cell crib, closed face, with standard 6-in. by 8-in. by 6-ft concrete stretchers. Each of the stretchers weighs 300 lb. If one assumes eight per foot of height, the total weight available is 2,400 lb, or 400 lb per foot of length. The weight of the rock backfill is approximately 600 lb per foot of length for each foot of height. Therefore, the crib wall will weigh 1,000 lb per foot of height per foot of length.

If a double cell is used for 12 ft and the remaining 8 ft of wall height is a single cell, the weight of the double-cell portion is 12 x 2 x 1,000 = 24,000 lb, that of the single-cell portion is 8 x 1,000 = 8,000 lb, and the total is 32,000 lb. Thus, an additional 19,000 lb of weight is required to produce a safety factor of 1.5. To determine the safety factor of the preceding design (Eq. 19), f.s. = 1.18.

To use a crib wall in this instance, a backfill between the wall and the slope would be desirable. Furthermore, a factor of safety as high as 1.5 may be impractical in this case.

Piling

For analyzing the benefits derived from piling, consideration is given to the two basic types of piling installations that are currently employed: one type anchors the piling to an unyielding foundation, whereas the other drives to refusal, and may or may not be properly fixed at the surface of rupture. The use of the latter type will not be generally acceptable except as an expedient. The resistance developed at the foot of such a pile cannot be great, but in cases involving small quantities of material, nonfixed piles have proven adequate for an extended period of time.

For a piling installation considered as fixed, the piles should penetrate one-third their total length into a stable foundation material. Where the foundation is bedrock and the piling is grouted at the toe, the depth of anchorage can be reduced to one-fourth the total pile length. Fixed piling fails in one of the following ways:

1. Shear through the pile.
2. Flexure through bending by cantilever action.
3. Soil shear around and past the piles.
4. Foundation failure beneath the piles.

Foundation failures of the type that would follow the arc FG beneath the pile (Fig. 125), have been discussed by Krynine (1931) and Hennes (1936). The increase in safety factor for conditions of a foundation failure produced by piling can be checked in the same manner as for failures beneath a buttress or a wall by the use of Eq. 2 or Eq. 4.

The preliminary spacing of the piling can be determined on the basis of the bending moment developed. The thrust

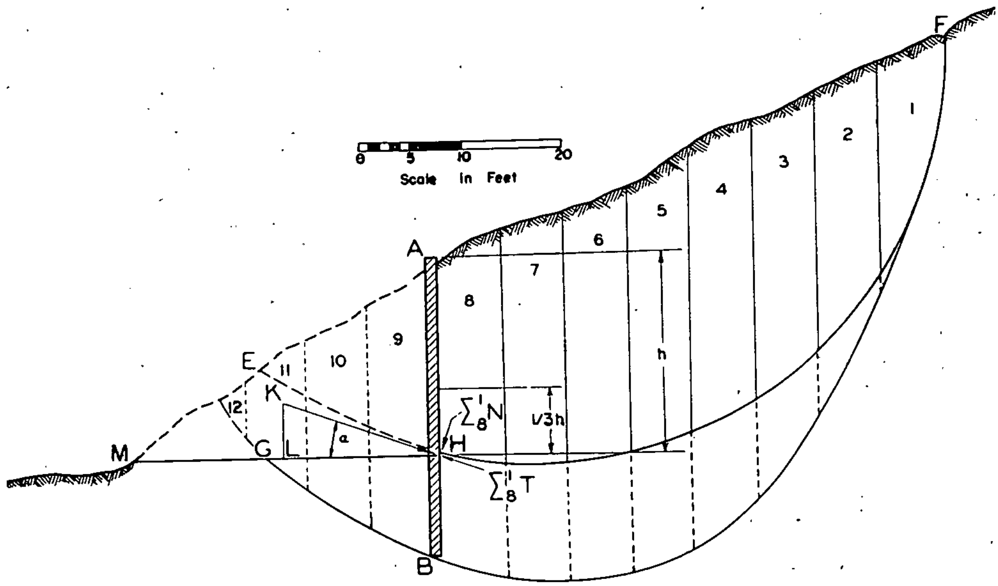


Figure 125. Design of piling.

Table N. For use in determining stability of a piling installation if foundation failure should occur along arc FG. The original slip-plane is arc EF.

	Segments Within Arc FG												
	1	2	3	4	5	6	7	8	9	10	11	12	Totals
Area (A), sq ft	115	160	230	260	260	230	210	195	135	90	65	5	1,965
Normal (N_A)	45	80	150	230	240	225	200	180	120	80	50	–	1,500
Tangential (I_A)	110	140	170	140	80	10	–30	–65	–55	–45	–45	5	405

(P_R) against the piling is determined by Eq. 15a for the slip-surface FE, and the horizontal force HL is obtained graphically or from the expression P_R cosα. The slip-surface FE is determined to be the most critical slip-surface — the one for which the piling is being installed. Furthermore, P_R is the shearing force exerted on the piling at the slip-surface. If the piling is assumed fixed in the area BH, and if it is further assumed that the loading diagram on the pile is triangular, a cantilever exists with the total load equal to the shearing force. This load is then assumed to be acting one-third the distance between points H and A. The moment can be expressed as

$$M = P_R \cos\alpha \, D\frac{h}{3}12 \qquad (25)$$

in which

M = maximum bending moment per foot of width, in in.-lb;
h = length of pile above surface of rupture, in ft; and
D = center-to-center spacing of piling divided by the number of lines of piles, in ft per pile.

Having obtained the bending moment, the size of the pile can be determined by the method appropriate to the type of material being used. Eq. 25 assumes no resistance from the surrounding soil, so that the size of the pile obtained will be conservative in some instances. However, for the piling to be effective, full bending moment is very likely to develop.

For preliminary design purposes, an estimate of the pile spacing should be made in order to determine the total thrust against a pile. If too large a pile is required, the spacing should be reduced by trial and error.

To determine the resistance to shear developed by the piles at the surface of rupture for a unit slice, the following form of an equation applied by Hennes (1936) to this problem may be used:

$$v_p = \frac{A_p\, f_v}{D} \qquad (26)$$

in which

v_p = shearing resistance of the pile installation per foot of slide width, in lb per ft;

A_p = cross-section area of the pile, in sq in.; and

f_v = allowable shearing stress for the piles, in lb per sq in.

The ratio of v_p to P_R cosα is the stability of the pile with reference to shear.

Hennes (1936) has also suggested use of an equation for determining the stability with reference to a shear failure of the soil around and past the pile. A form of that equation is

$$S_s = \frac{2\ c\ h\ d}{D} \qquad (27)$$

in which

S_s = shearing resistance of the soil per foot of slide width, in lb per ft;

c = cohesion of the soil, in lb per sq ft;

h = height from surface of rupture to ground surface, in ft; and

d = diameter of pile, in ft.

The ratio of S_s to P_R cosα (total force per foot against pile, from Eq. 15a) is the relative stability with reference to soil shear around the pile.

The following is a typical example for steel piling (Fig. 125) assuming, (a) that P_R, c, and ϕ have been determined in the same manner as for the example for a crib wall, and (b) that the shearing resistance of the material downslope from the pile is neglected:

P_R = 38,600 lb; ϕ = 10°; α = 22°; c = 207 lb per sq ft; f_s = 40,000 lb per sq in.; f_v = 25,000 lb per sq in.; h = 20 ft; and l_{1-8} = 75 ft. Assume that four lines of piling are driven with an 18-in. center-to-center spacing in each line, determine the size of beam necessary to resist a bending moment (Eq. 25, in which $D = \frac{1.5}{4} = 0.375$ ft) of 38,600 x 0.927 x 0.375 x 80 = 1,074,000 in.-lb.

For steel I-beams and from the AISC Handbook,

$f_s = \frac{M\ c}{I} = \frac{M}{S}$, and in this case $S = \frac{1{,}074{,}000}{40{,}000} = 26.8$.

For a 10-in. beam weighing 25 lb per ft, $A_p = 7.35$, $S = 26.4$, and $f_s = \frac{1{,}074{,}000}{26.4}$ = 40,680 lb per sq in.

To determine the shearing resistance of the pile (Eq. 26), $v_p = \frac{7.35 \times 25{,}000}{0.375}$ = 490,000 lb per ft, and the factor of safety is

$$\text{f.s.} = \frac{v_p}{P_R \cos\alpha} = \frac{490{,}000}{35{,}700} = 13.7 \qquad (28)$$

To determine the stability with reference to shearing of the soil around the pile, laboratory tests on undisturbed samples from the piling area should be conducted. Assuming such tests produced a value of 750 lb per sq ft for c, by Eq. 27

$S_s = \frac{2 \times 750 \times 20 \times 0.833}{0.375}$ = 66,700 lb.

The safety factor is

$$\text{f.s.} = \frac{S_s}{P_R \cos\alpha} = \frac{66{,}700}{35{,}400} = 1.88 \qquad (29)$$

To determine the stability with reference to a foundation failure (FG): $\sum_{1}^{11} N = 187{,}000$ lb; $\sum_{1}^{11} T = 50{,}600$ lb; $l = 114$ ft; and (Eq. 2) f.s. $= \dfrac{(187{,}000 \times 0.1763) + (207 \times 114)}{50{,}600} = 1.12.$

This safety factor is too low and a longer pile would be required, thus forcing the slip-plane to a lower elevation. Other reasonable positions of the slip-surface should also be checked. If differences appear to exist between the shearing resistance of the strata, adjustments based on laboratory, field, or analytical studies will be necessary.

If neglecting the resistance that is offered by the mass on the downslope side of the piles is considered unduly conservative, the estimated force against the piles can be adjusted.

Assuming a safety factor of 1.0, the resistance needed from the piling is equal to the increase in safety factor multiplied by the original shearing resistance (or shearing force). With reference to Figure 125, P_R will be equal to $\dfrac{\sum_{9}^{11} T}{2}$ for a desired safety factor of 1.5 and for slip-surface EH. From the example for a crib wall (Fig. 124) this value for P_R can be obtained, and is equal to 22,500 lb. Assuming three lines of piling at 18-in. center-to-center spacing, $M = 22{,}500 \times 0.927 \times 0.5 \times 80 = 832{,}000$ in.-lb.

The required section modulus is $S = \dfrac{832{,}000}{40{,}000} = 20.8.$

For a 10-in. beam weighing 21 lb per ft, $A = 6.19$ sq in; $S = 21.5$; and $f_s = \dfrac{832{,}000}{21.5} = 38{,}700$ lb per sq in.

For the shearing resistance of the pile (Eq. 26), $v_p = \dfrac{6.19 \times 25{,}000}{0.5} = 309{,}500$ lb per ft, and the safety factor (Eq. 28) is f.s. $= \dfrac{309{,}500}{20{,}800} = 14.8.$

For stability with reference to the shearing of the soil past the piling (Eq. 27), $S_s = \dfrac{2 \times 750 \times 20 \times 0.833}{0.5} = 50{,}000$ lb, and the safety factor (Eq. 29) is f.s. $= \dfrac{50{,}000}{20{,}800} = 2.40.$

Miscellaneous Methods

From the viewpoint of stability analyses, the miscellaneous methods referred to in Chapter Eight do not lend themselves to a theoretical investigation. The changes produced by the hardening of the soil mass can be estimated by laboratory methods. In turn, the slope can be analyzed in the same fashion and with the same equations as used for excavation methods. However, incorporation of admixtures is so difficult to predict or to measure that little good can be accomplished by stability analyses.

Blasting

The value obtained by blasting is more amenable to prediction by a stability analysis. However, since an element of drainage is involved, a prediction of a change in ground water or piezometric surface is necessary. The other benefit of blasting is the relocation of the slip-surface. Here again, there will be considerable conjecture as to the effect of the blasting, but if the minimum slip-surface displacement is assumed, the stability as indicated by the safety factor will be conservative. Unless the drainage factor is included, no great change in the safety factor is to be anticipated by the minor displacement of the slip-surface.

For a stability analysis of a blasting operation, the shearing resistance should be obtained by methods described previously for existing slope failures. Then, assuming the displaced location of the slip-surface, Eq. 2 (for no drainage value) or Eq. 4 (including drainage values) can be used to estimate the new safety factor.

For the following example, the data of Figure 126 and Table O apply. The unit weight of the soil is 125 lb per cu ft and

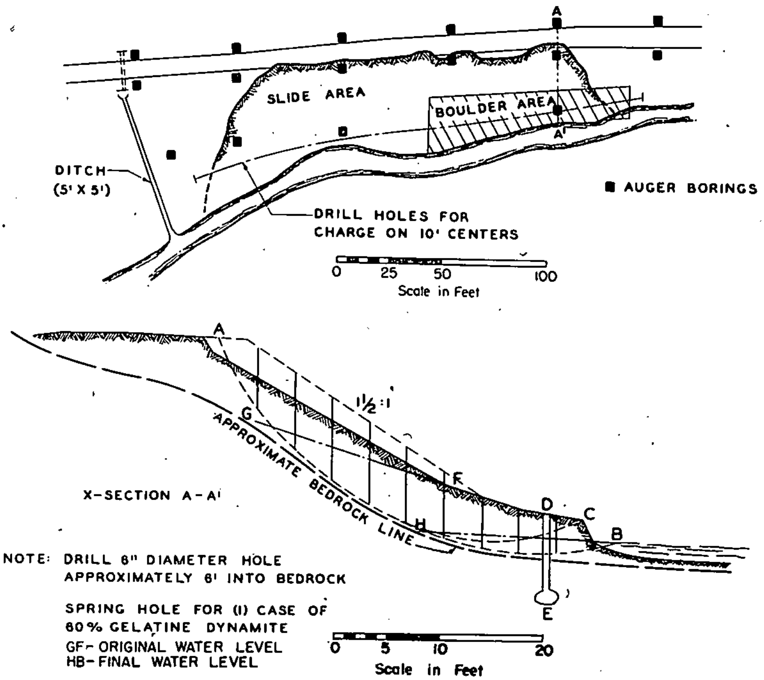


Figure 126. Design of blasting installation.

Table O. For use in determining stability along a slip-surface AB.

Segments Within Arc AB

	1	2	3	4	5	6	7	8	9	10	Total
Area (A), sq ft	12	23	28	29	29	29	20	14	13	9	206
Normal (N_A)	7.00	16	22	25	24	28	19	13	12	8	174
Tangential (T_A)	10.0	16	16	14	15	9	4	1	–1	–1	83
Average h	0	0.5	2.5	4.4	5.0	5.5	5.0	4.5	1.5	1.0	
	–	3.0	5.0	5.0	4.5	4.5	4.5	4.5	4.5	4.0	
(sq ft)		1.5	12.0	20.0	22.5	24.5	22.5	20.0	6.5	4.0	133.5

Table P. For use in determining stability after blasting has shifted the slip-surface to AC and has lowered water table from FG to BH.

Segments Within Arc AC

	1	2	3	4	5	6	7	8	9	10	Total
Area (A), sq ft	12	23	28	29	29	28	17	10	7	2	185
Normal (N_A)	7	16	22	25	24	26	16	10	6	1	153
Tangential (T_A)	10	16	16	14	15	9	1	0	–2	–1	79
Average h	0	0	0	0	0	0	0.5	1.0	0	0	
	–	–	–	–	–	–	4.0	4.0	–	–	
(sq ft)	0	0	0	0	0	0	2.0	4.0	0	0	6

a ϕ value of 10° is assumed. Considering first the slip-surface, AB, a value for c, corresponding to $\phi = 10°$, is determined. Also, $\Sigma N = 21{,}800$ lb; $\Sigma T = 10{,}400$ lb; $\Sigma\mu = 8{,}300$ lb; $l = 48$ ft; and (Eq. 4)

$$c = \frac{10{,}400 - (21{,}800 - 8{,}300)\ 0.1763}{48}$$

$= 167$ lb per sq ft.

If blasting is to be accomplished as indicated on Figure 126, one must assume in the design stage that the slip-surface will take a position such as AB, and that the water table will drop from FG to approximately BH. After blasting has been accomplished, the assumptions can be checked and the stability recomputed. The effect on the stability can be estimated as follows, using Table P, Figure 126: $\Sigma N = 19{,}200$ lb; $\Sigma T = 9{,}870$ lb; $\Sigma\mu = 375$ lb; $l = 47$ ft; and (Eq. 4)

$$\text{f.s.} = \frac{(19{,}200 - 375)\ 0.1763 + (167 \times 47)}{9{,}870}$$

$= 1.13$.

Thus, the blasting creates a maximum increase to 1.13 for the safety factor, as compared to a condition of f.s. $= 1.0$ at the time of failure. Additional blasting, or the replacement of the upper part of the fill with lightweight material (such as cinders) would tend to further increase the relative stability.

The fact that blasting did not make a significant change in the safety factor is of interest. It could mean either that the stability analyses do not measure adequately the degree of stability, or that this type of blasting is not very effective. The quantitative approach used in this preceding solution required several major assumptions with regard to the lowering of the water table and the displacement of the slip-surface. Even with most favorable assumptions the safety factor was not materially affected. One is reminded, however, that with such a quantitative approach, comparisons of the stability produced by various techniques are possible. Empirical methods do not provide adequate bases for such comparisons.

References

Allen, Harold, *et al.*, "Report of Committee on Classification of Materials for Subgrade and Granular Type Roads." *Highway Research Board Proceedings*, v. 25, 1945.

American Association of State Highway Officials, "Standard Methods of Determining the Liquid Limit of Soils." Am. Assoc. of State Highway Officials Designation T89-42; and "Standard Methods of Determining the Plastic Limit of Soils." Am. Assoc. of State Highway Officials Designation T90-42.

American Society for Testing Materials, "Procedures for Testing Soils. Nomenclature, Standard Methods and Suggested Methods." 1950.

American Society for Testing Materials, "Triaxial Testing of Soils and Bituminous Mixtures." John Wiley & Sons, New York, N. Y., 1950.

Corps of Engineers, U.S. Army, "The Unified Soil Classification System." Tech. Memo. No. 3-357, Waterways Experiment Station, Vicksburg, Miss., 1953.

Corps of Engineers, U.S. Army, "Soil Mechanics Design; Stability of Slopes and Foundations." U.S. Corps of Engineers, Eng. Man., Part CXIX, Chapter 2, 1952.

Fellenius, W., "Calculations of the Stability of Earth Dams." Transactions, 2nd Congress on Large Dams, v. 4, Washington, D.C., 1936.

Fellenius, W., "Erdstatische Berechnungen, etc.," W. Ernst u. Sohn, Berlin, 1927 (revised edition, 1939).

Hennes, R. G., "Analysis and Control of Landslides." Bull. No. 91, Univ. of Washington Eng. Exp. Sta., Seattle, Wash., 1936.

Kjellman, W., "Do Slip Surfaces Exist?" *Geotechnique*, v. 5, No. 1, p. 18-22, 1955.

Krynine, D. P., "Landslides and Pile Action." *Engineering News-Record*, v. 107, No. 122, Nov. 26, 1931.

Lambe, T. William, "Soil Testing for Engineers." John Wiley & Sons, New York, N.Y., 1951.

Rendulic, L., "Ein Beitrag zur Bestimmung der Gleitsicherheit." *Der Bauingenieur*, v. 16, p. 230-233, 1935.

Taylor, Donald W., "Fundamentals of Soil Mechanics." John Wiley & Sons, New York, N.Y., 1948.

Terzaghi, Karl, "Theoretical Soil Mechanics." John Wiley & Sons, New York, N.Y., 1943.

Terzaghi, Karl, and Peck, Ralph B., "Soil Mechanics in Engineering Practice." John Wiley & Sons, New York, N.Y., 1948.