Mechanism of Soil Moisture Extraction from a Pressure-Membrane Apparatus

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Introductory Remarks by Chairman

Modern science is based on observations and measurements. To use the latter properly one must know what is measured, how reproducible the measurements are, what sources of deviations exist, and how to correct for known sources of deviations. This means that one must know as well as possible what is going on within the apparatus, whether the measurements really answer the requirements, what accuracy of measurement is obtained, and whether or not corrections of the raw data are called for. Professor Fukuda has made a significant contribution to the symposium by analyzing theoretically and experimentally what goes on when soil moisture is extracted in a pressure-membrane apparatus.

FOR ONE of the fundamental studies of irrigation, soil moisture tension can be measured by a pressure-membrane apparatus developed by Richards.[1]

In the apparatus the soil from which moisture is to be removed is placed in a chamber in which nitrogen gas pressure is increased above atmospheric pressure.

The side of the chamber which supports the soil consists of a cellophane membrane supported on a brass screen and a brass plate in such a way that any moisture passing through the membrane is conducted away. Thus it is considered, that the moisture content of the soil in contact with the membrane will be reduced by the amount that would be necessary under normal atmospheric conditions to make the pressure deficiency of the soil water equal to the excess gas pressure in the extracted chamber.

This paper deals with a mechanism by which soil moisture is to be extracted through the cellophane membrane.

Conditions of Soil Moisture in a Pressure-Membrane Apparatus

As a quantity to express soil moisture content, an equivalent height h is used here, which means the height above the ground water level. At this height the soil moisture in equilibrium is assumed to be constant, hence instead of moisture content the height h is used to express this factor.

Taking out a part of such soil (Fig. 1), and considering p_w, and p as a pressure in meniscus and out of it respectively, gives:

$$p - p_w = \sigma\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (1)$$

where R_1, R_2 are the principal radii of curvature, σ being surface tension of water.

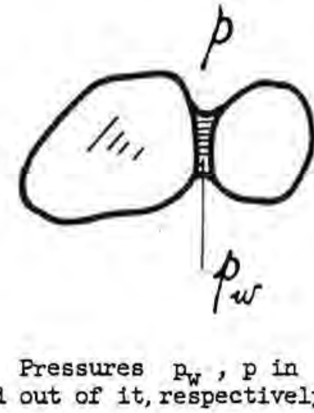

Figure 1. Pressures p_w, p in meniscus and out of it, respectively.

At the ground water surface, water and air pressures are both one atmosphere (p_0). In Figure 1, when soil is at the height h above the ground water:

[1] Richards, L. A., "A Pressure-Membrane Extraction Apparatus for Soil Solution." Soil Sci., 51:2, (1941).

$$p = p_o \, e^{-\frac{M_a g h}{R T}} \qquad (2)$$

$$p_w = p_o - \rho_w g h \qquad (3)$$

where M_a = molecular weight of air $\doteqdot$ 29, ρ_w = density of water = 1 gr/cm^3, R = gas constant = 8.34×10^7 erg/mol. deg, T = absolute temperature, g = acceleration of gravity = 980 cm/sec.^{2}, equation 2 is a so-called barometric formula and equation 3 a hydrostatic one. It could be considered for soil moisture in Figure 1 to have a direct contact with ground water through a small pipe.

Supposing the soil in Figure 1 put in a pressure-membrane apparatus, in which a pressure P is applied (Fig. 2), to keep the same moisture condition with that in Figure 1, a pressure in meniscus must be one atmosphere (p_o). It is now considered that the outside of the apparatus is saturated with vapor and kept in equilibrium with water under atmospheric pressure, and also at this condition water drops are expected to be on the outer surface of the membrane. As water can flow easily through the membrane, water pressure becomes p_o by the hydrostatic relation just like equation 3.

Because the geometrical form of soil moisture in Figure 2 is the same as that in Figure 1,

$$P - p_o = \sigma \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \qquad (4)$$

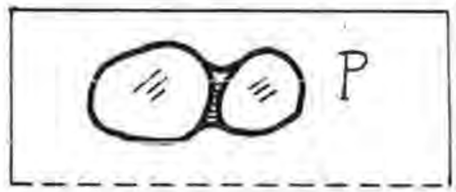

Figure 2. Pressure P in the chamber around soil particles.

From equations 1 and 4

$$P - p_o = p - p_w$$

Substituting equations 2 and 3,

$$P - p_o = p_o e^{-\frac{M_a g h}{RT}} - (p_o - \rho_w g h)$$

$$= \rho_w g h - p_o \left(1 - e^{-\frac{M_a g h}{RT}} \right)$$

Dividing both sides of this equation by p_o, and putting $p_o / \rho_w g$ = 1,034 cm = h_o (water head equivalent of one atmosphere of pressure), gives

$$\frac{P - p_o}{p_o} = \frac{h}{h_o} - \left(1 - e^{-\frac{M_a g h}{R T}}\right) \qquad (5)$$

in which the left side shows an excess pressure in the apparatus expressed in units of atmospheric pressure, the first term and the right is an equivalent height based on h_o, and the second right hand term is a small correction and almost negligible in the ordinary conditions as shown in Table 1.

TABLE 1

RELATION BETWEEN h/h_o AND $(P - p_o)/p_o$

h/h_o	0	1	10	100	1,000	10,000
$(P - p_o)/p_o$	0	0.9988	9.988	99.89	999.3	9,999

Water Extraction Into Saturated Air Out of the Apparatus

When soil moisture is so plentiful as to make all capillary water connect with each other and keep good contact with the other side (pressure = p_o) of the apparatus through cellophane membrane, it is easily understood that, by increasing the pressure in the apparatus, the pressure in the capillary water exceeds p_o, the pressure outside the apparatus; hence, soil water flows out through the membrane.

Even when soil moisture decreases, and the connection between capillary water disappears, the fact of water flowing out through the membrane under a higher pressure may be explained as follows: By vapor pressure is meant physically that pressure condition at which a surface of water is in equilibrium with its own vapor; the vapor pressure of water will tend to increase in case of the vapor coexisting with another gas (for example air).

Now, when air is pushed into the apparatus containing wet soil, the vapor pressure of water increases; since the water holding capacity of the air in the chamber increases with increasing air pressure soil water begins to evaporate. When the vapor pressure in the apparatus exceeds that on the outside, vapor goes out through the membrane and condenses outside if the air outside the apparatus is saturated with water vapor. As water goes out in that way, soil moisture decreases and the radii of curvature in the capillary water become smaller, also the vapor pressure decreases until it becomes equal to that outside. At that point water extraction stops. Thus the moisture inside and outside of the apparatus remains in equilibrium as a result of the exchange of water vapor. Starting from this concept, equation 5 can be obtained purely from thermodynamical considerations.

Water Extraction Into Unsaturated Air Out of the Apparatus

In this case equation 4 must be slightly modified. Supposing a small tube in the membrane shown in Figure 3, which has a water pressure p'. As this water is in equilibrium with capillary water, the pressure in the capillary water must be p' too.

Thus, instead of equation 4:

$$P - p' = \sigma\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (6)$$

and instead of equation 5 there is obtained:

$$\frac{P - p'}{p_o} = \frac{h}{h_o} - \left(1 - e^{-\frac{M_a g h}{RT}}\right) \qquad (7)$$

On the other hand, there is a relation shown by

$$p_o - p' = \sigma\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad (8)$$

between pressure p_o outside of the membrane and p' in a small tube, where r_1, r_2 are radii of curvature of the water surface in this small tube.

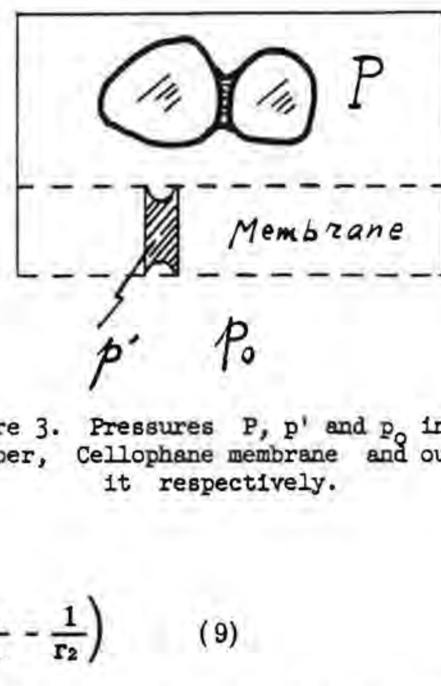

Figure 3. Pressures P, p' and p_o in the chamber, Cellophane membrane and out of it respectively.

Now, generally there is a relation[2]:

$$-\frac{\rho_w RT}{M_w}\ln\frac{p_v}{p_{v_o}} = \sigma\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad (9)$$

between vapor pressure p_v on a curved water surface and p_{v_o} on a plane one, where M_w = molecular weight of water, ρ_w = density of water.

Substituting equation 9 into equation 8, and setting $p_v/p_{v_o} = \phi$, the relative humidity outside of the apparatus, gives

$$p_o - p' = -\frac{\rho_w RT}{M_w}\ln\phi$$

[2] Edlefsen, N. E. and Anderson, A. B. C., "Thermodynamics of Soil Moisture." Hilgardia, 15:2, 145 (Eq. 272) (1943).

$$\frac{p_o - p'}{p_o} = -\frac{\rho_w RT}{M_w p_o}\ln\phi$$
$$= -\frac{\rho_w}{M_w}\cdot\frac{M_a}{\rho_a}\ln\phi$$
$$\frac{p_o - p'}{p_o} = \frac{-22400}{18}\cdot\frac{T}{T_o}\ln\phi \qquad (10)$$

where ρ_a is the air density under atmospheric pressure and temperature T, and T_o = 273.

From equations 10 and 7, neglecting the second right hand term in equation 7, gives

$$\frac{P - p_o}{p_o} = \frac{h}{h_o} + 1250\frac{T}{T_o}\ln\phi \qquad (11)$$

in which the second term right is a correction for relative humidity and temperature. Because of the large numerical coefficient, this correction becomes exceedingly large; for example, when $\phi = 0.9$, the correction is 130. Thus, it can be understood that the outside of the membrane should be surrounded by saturated air to get the accurate amount of soil moisture to remain in equilibrium with the applied pressure in the apparatus.

In equation 11, when $P = p_o$, gives

$$h = -\left(1250\frac{T}{T_o}\ln\phi\right)h_o \qquad (12)$$

The effects of humidity and temperature expressed by this equation are shown graphically in Figure 4. This figure shows that humidity has a much greater influence than temperature.

In the apparatus commonly used, it can be expected that the space between the cellophane membrane and a brass screen under it, shown in Figure 5, to be always saturated; if this is the case, then $\ln\phi = 0$ in equation 11 and there will be no effect on the soil moisture remaining in the chamber, of any changes in humidity and temperature outside of the apparatus.

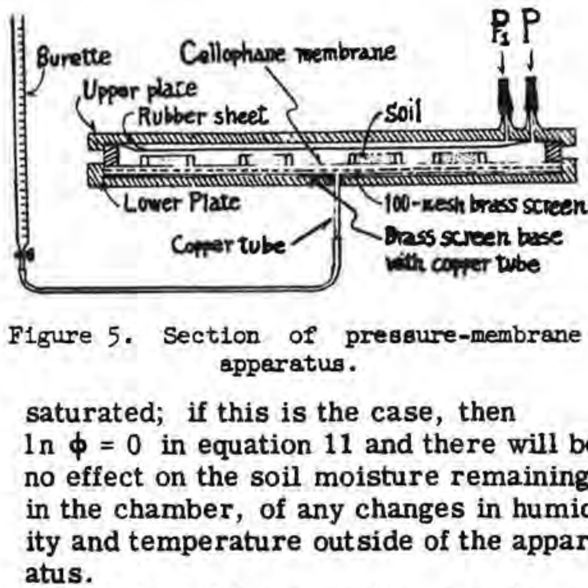

Figure 5. Section of pressure-membrane apparatus.

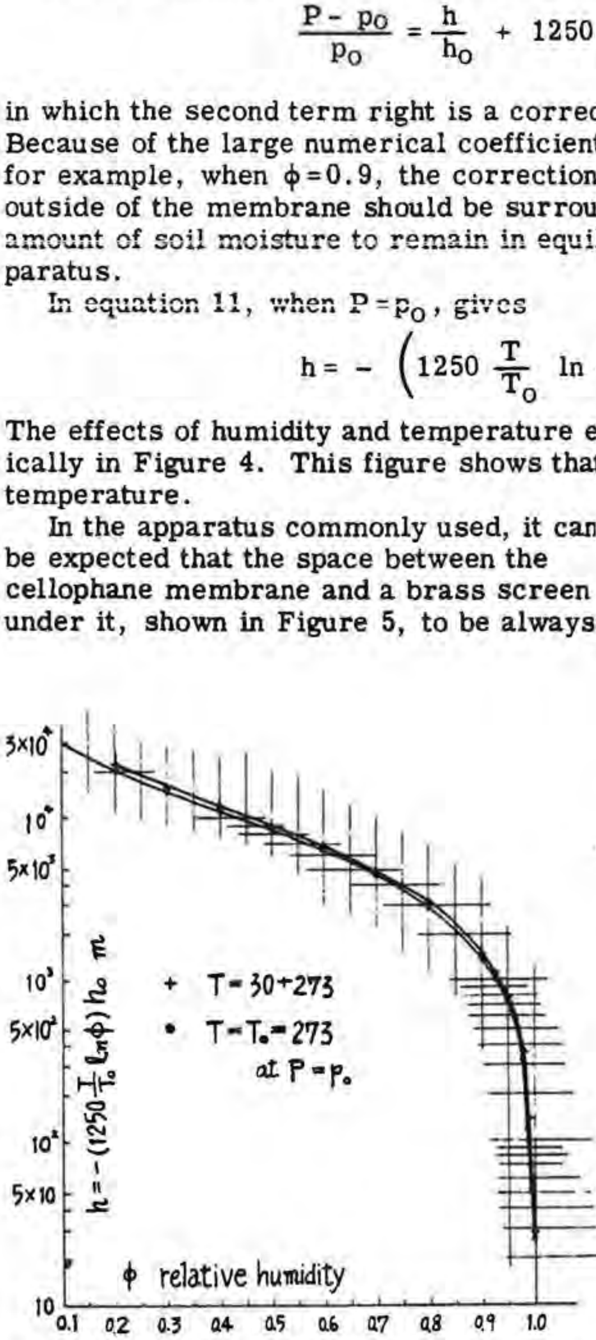

Figure 4. Relation between equivalent height h m. and relative humidity.

To prove this assumption experimentally, humidity changes at a pressure of ten atmospheres and temperature change at a pressure of 15 atmospheres were considered. The amounts of soil moisture remaining in equilibrium are shown in Table 2 and 3 respectively.

To maintain a saturated condition outside of the apparatus (100 percent in

Table 2), the extracted water passed through a copper tube into a burette filled with water, thus the effect of humidity change can be eliminated. For the relative humidity of 70 percent the copper tube was connected with air possessing a relative humidity of 70 percent.

Table 3 gives the measured amounts of soil moisture remaining in the apparatus, which was kept at various air temperatures; the extracted water always passed into a burette filled with water.

Thus, from the results shown in Table 2 and 3 we can see no practical effects of humidity and temperature on soil moisture in equilibrium with the gas pressure inside the apparatus; but it would increase the stability of the experiment to connect a copper tube outlet with a burette with water for keeping a saturated condition outside of the pressure membrane.

TABLE 2

EFFECT OF AIR HUMIDITY OUTSIDE OF THE APPARATUS ON SOIL MOISTURE IN EQUILIBRIUM WITH PRESSURE, 10 ATMOSPHERES

Soil	Relative Humidity out of the Apparatus, %		Room Temperature
	100	70	C
Volcanic ashy loam	25.3	25.4	21.0 — 22.5
Sandy loam	5.54	5.52	22.0 — 22.5

TABLE 3

EFFECT OF AIR TEMPERATURE ON SOIL MOISTURE EQUILIBRIUM WITH PRESSURE, 15 ATMOSPHERES

Soil	Structure	Diameter mm.	Room Temperature C			
			10.5-14.0	13-14	30-32	40-41
Clay	Single	< 0.25	22.0	22.8	20.9	20.6
	Granular	< 2.0	20.0	20.1	19.5	19.1
Volcanic ashy loam	Single	< 0.5	25.9	26.2	25.3	24.9
	Granular	< 0.5	31.8	31.6	30.2	29.3
Sandy loam	Single	< 0.5	5.0	4.9	4.6	4.6
	Granular	< 0.5	5.2	5.2	5.2	4.9

SUMMARY

In this paper a mechanism by which soil moisture is extracted in the case of the pressure-membrane apparatus developed by Richards was considered theoretically and the effects of air humidity and room temperature outside the apparatus on soil moisture in equilibrium with various pressures inside the apparatus were proved experimentally to be of little importance.