Water Movement in Soils Under Pressure Potentials

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Introductory Remarks by the Chairman

During recent years a great deal of emphasis has been given to moisture conduction in soils under suction, thermal and electric potentials. From this, the impression might be gained that either all there is to know on water movement under pressure potentials, or that such movement is of lesser importance in soil engineering than movement caused by other potentials. Nothing could be farther from the truth on both counts. In the over-all picture, pressure potentials are still the most important water-moving agents in soil engineering and certainly there is still much to learn about the physics of pressure flow in porous media. Professor Schmid has prepared a brief and precise account of the historic development and present state of knowledge in this all-important area. While he traces with sure hand the essential role played by mathematics in the quantitative evaluation of flow conditions, he gives due consideration to the material-physical aspect of flow through porous hydrophilic systems as exemplified by soils.

● PRESSURE potentials were the first driving forces recognized to cause movement of water through soils, and originally they were believed to be the only driving forces for such movement. It is not surprising then that the problem of water movement under pressure potentials has received extensive treatment, and the existing literature is tremendous. An attempt will be made to review—as far as this is at all possible—some of the more important aspects of the problem in the following paragraphs and to summarize in a general way the present state of knowledge.

SINGLE PHASE FLOW

The first quantitative insight was achieved by Darcy experimenting with filter sands in the water works of Dijon (1), and his empirical law states that the filter velocity is proportional to the first power of the pressure gradient:

$$V = \frac{Q}{A} = -K\frac{\Delta p}{\Delta L} \tag{1}$$

Where V is the filter—or seepage velocity, Q the permeation discharge, A the permeated area, K the proportionality coefficient which can be resolved into permeability and viscosity: $K = \frac{k}{\mu}$, and $\Delta p = p_2 - p_1$, the pressure dissipated along the flow path ΔL. The minus sign indicates flow in the direction of decreasing pressures.

Equation 1 describes the movement of a liquid through an isotropic porous medium in the absence of an exterior force field or for horizontal flow. If the velocity V has a vertical component, the gravitational force must be included and Eq. 1 now reads:

$$V = -K\frac{\Delta p + \rho g \Delta Z}{\Delta L} \tag{2}$$

where ρ is the density of the liquid, g the gravitational acceleration and ΔZ the difference in vertical coordinates. Darcy's law in the form of Eq. 1 or 2 is of very restricted use since it describes only uniform conditions over a finite length. A more general expression is the differential form of Eq. 2:

$$\bar{V} = -\frac{k}{\mu}(\text{grad } p + \rho g) \tag{3a}$$

Here $\bar{V}$ is the local filter velocity vector. Eq. 3a can be simplified by the introduction of a force potential ϕ where

$$\phi = g \cdot Z + \int_{p_0}^{p} \frac{dp}{\rho(p)} \tag{3b}$$

and then

$$\bar{V} = -\frac{k}{\mu} \text{ grad } \phi \tag{3c}$$

Since any solution of a flow problem under pressure potentials requires the determination of three unknowns, namely $\bar{V}$, p, and ρ, Darcy's law alone is not yet sufficient for complete specification of the problem. The additional conditions are supplied by the relation between fluid density and pressure:

$$\rho = \rho\,(p) \tag{4}$$

and by the continuity condition:

$$n\frac{\partial \rho}{\partial t} = \text{div }(\rho\bar{V}) \tag{5}$$

where n is the porosity and t the time.

Combination of Eqs. 3, 4 and 5 yields:

$$n\frac{\partial \rho}{\partial t} = \text{div}\left[\frac{\rho k}{\mu}\text{ (grad } \phi)\right] \tag{6}$$

Thus, if the water movement through the soil is described by Eq. 3c, the solution of Eq. 6 for the given boundary conditions constitutes the solution of the flow problem. The solution may be sought for the following conditions:

(a) Steady state flow with geometrically prescribed boundary conditions.
(b) Steady state flow with a free boundary (free surface gravity flow).
(c) Unsteady state flow.

For cases (a) and (b), the time derivative vanishes and Eq. 6 is reduced to

$$\text{div}\left[\frac{\rho k}{\mu}\text{ (grad } \phi)\right] = 0 \tag{7}$$

Assuming the water to be incompressible (ρ = const), and the soil to be homogeneous,

$$\nabla^2 \phi = 0$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial Z^2} \tag{8}$$

is the Laplace operator.

METHODS OF SOLUTION

A. Steady State Flow with Geometrically Prescribed Boundary Conditions

1. Analytical Solutions. In all cases where the flow is fully determined by the geometry of the boundaries, the boundary conditions can be defined by:

$$\bar{V}_n = 0 \tag{9}$$

along the boundary, where $\bar{V}_n$ is the component of the velocity vector normal to the boundary. Two dimensional solutions of Eq. 8 for the boundary conditions, Eq. 9, are relatively simple if the geometry of the boundary is such that it can be conveniently expressed analytically. This can be achieved by the use of complex variables and their conformal mapping, since it is known that solutions of the Cauchy-Riemann differential equations:

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}; \quad \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$
$$\frac{\partial x}{\partial \phi} = \frac{\partial y}{\partial \psi}; \quad \frac{\partial x}{\partial \psi} = -\frac{\partial y}{\partial \phi} \tag{10}$$

satisfy also the Laplace Eq. 8. This procedure applied to the two-dimensional problem of Figure 1 gives

$$\frac{x^2}{d^2 \cos^2 \phi} - \frac{y^2}{d^2 \sin^2 \phi} = 1 \tag{10a}$$

and

$$\frac{x^2}{d^2 \cos^2 h\psi} + \frac{y^2}{d^2 \sin^2 h\psi} = 1 \tag{10b}$$

For constant values of ϕ and ψ, Eqs. 10a and 10b give the equipotential and streamlines respectively for the flow around a sheet pile wall in a semi-infinite soil mass. These two families of curves are confocal ellipses and hyperbolas with their common focus at the tip C of the wall. The components of the velocity vector are given by:

$$\bar{V}_x = K \frac{H - h}{\pi} \frac{\partial \psi}{\partial y} \tag{10c}$$

and

$$\bar{V}_y = K \frac{H - h}{\pi} \frac{\partial \psi}{\partial x} \tag{10d}$$

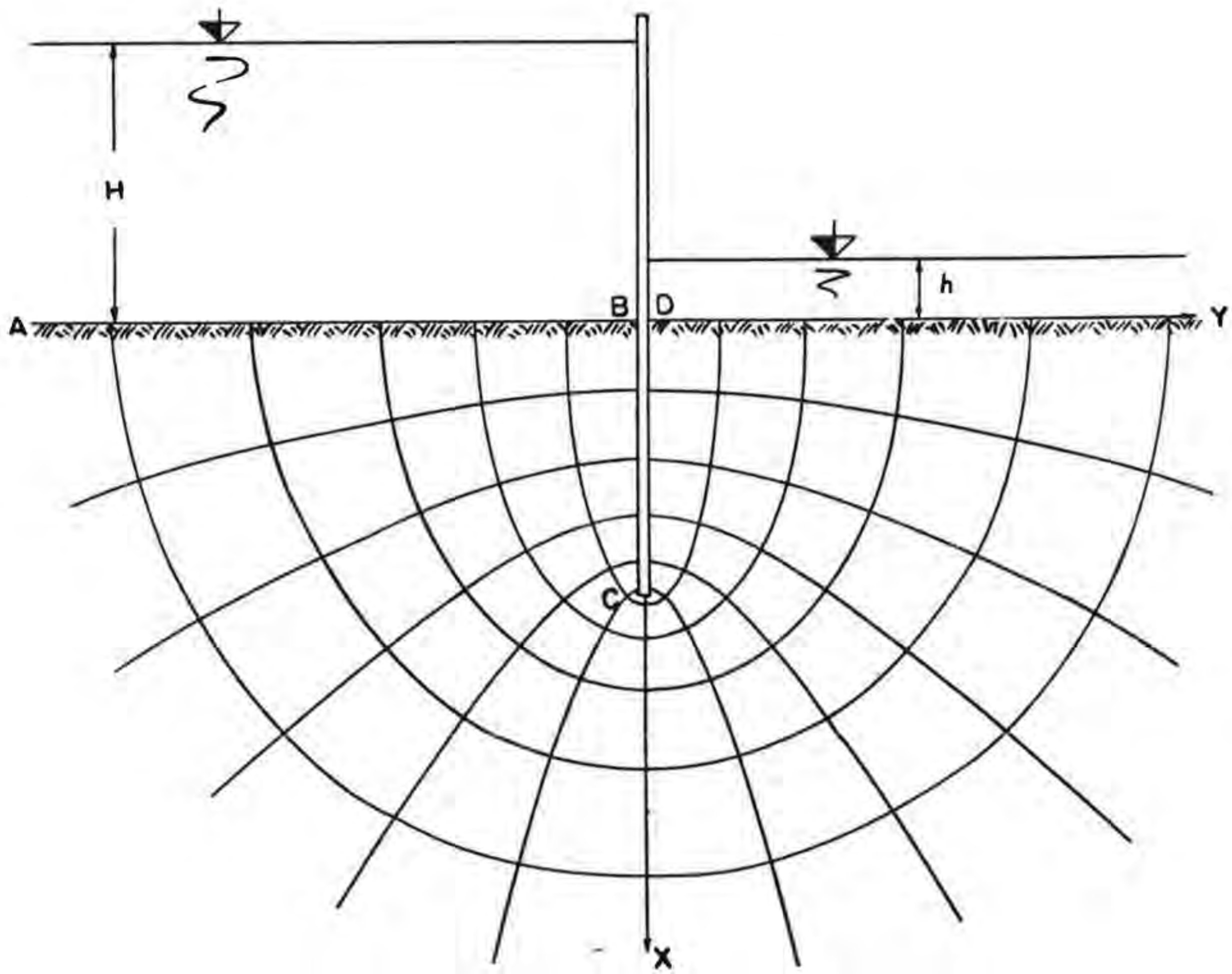


Figure 1. Flow around a sheet pile wall.

Of particular interest is the exit velocity at the down-stream surface of the soil, since it will indicate the safety against a quick condition. With

$$\frac{\partial \psi}{\partial y} = \frac{1}{d\sqrt{1 + (\frac{y}{d})^2}} \qquad \text{(for } x = 0\text{)}$$

gives

$$\bar{V}_x = K \frac{H - h}{\pi d \sqrt{1 + (\frac{y}{d})^2}} \tag{10e}$$

which has a maximum for $y = 0$: $\max \bar{V}_x = K \frac{H - h}{\pi d}$.

Since according to Eq. 3c grad $\phi = \frac{\bar{V}}{K}$ the maximum gradient thus will be

$$\frac{\max \bar{V}_x}{K} = \frac{H - h}{\pi d}$$

This maximum gradient must be smaller than the critical hydraulic gradient $S_{cr} = \frac{G_s - 1}{1 + e}$ to prevent the soil from "boiling" out. Here G_s is the specific gravity of the solids and e is the void ratio. Thus:

$$\frac{G_s - 1}{1 + e} > \frac{H - h}{\pi d} \tag{10f}$$

This condition determines the minimum depth of penetration which is possible. This type of problem is frequently encountered where an excavation pit has to be maintained in the dry. Hence, a condition such as Eq. 10f would give the height H to which the groundwater level has to be depressed by well points to safeguard against boils.

Of further interest is the distribution of the velocity $\bar{V}_x$ between point C and D as well as the horizontal velocity $\bar{V}_y$ below point C. Using Eqs. 10c and 10:

$$\frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x} = \frac{1}{d\sqrt{1 - (\frac{x}{d})^2}}$$

Thus

$$\bar{V}_x = K \frac{H - h}{\pi} \frac{1}{d\sqrt{1 - (\frac{x}{d})^2}} \tag{10g}$$

for $x = d$: $\bar{V}_x = \infty$

and

$$\bar{V}_y = K \frac{H - h}{\pi} \frac{1}{d\sqrt{(\frac{x}{d})^2 - 1}} \tag{10h}$$

for $x = d$: $\bar{V}_y = \infty$

The variations of $\bar{V}_x$ and $\bar{V}_y$ are shown in Figure 2.

At point C, $\bar{V}_x$ as well as $\bar{V}_y$ become infinite. However, because of Eq. 3c this would require grad ϕ to become infinite. Since the upper value of ϕ is finite and more or less determined by H. The condition grad $\phi = \infty$ would require an infinite negative pressure in the vicinity of C. This is, of course, physically not possible. Point C thus is a singular point where the solution of the potential theory is impossible. It must be concluded that there the boundary streamline separates from the

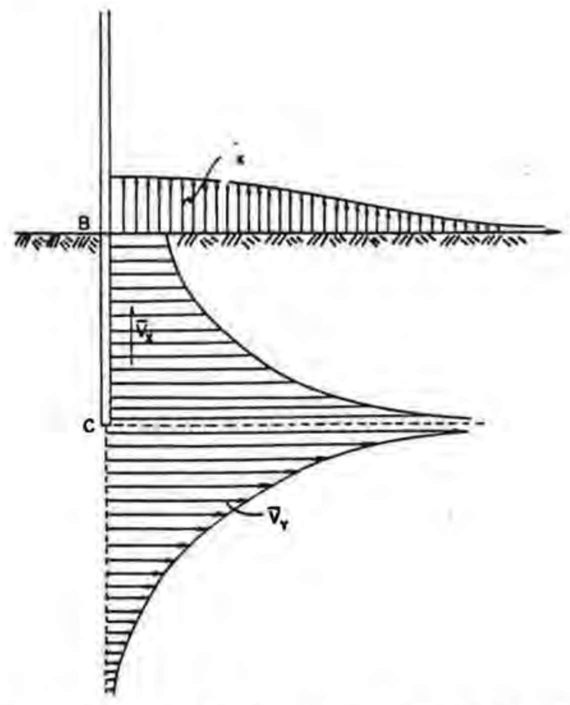


Figure 2. Velocity distribution along sheet pile wall.

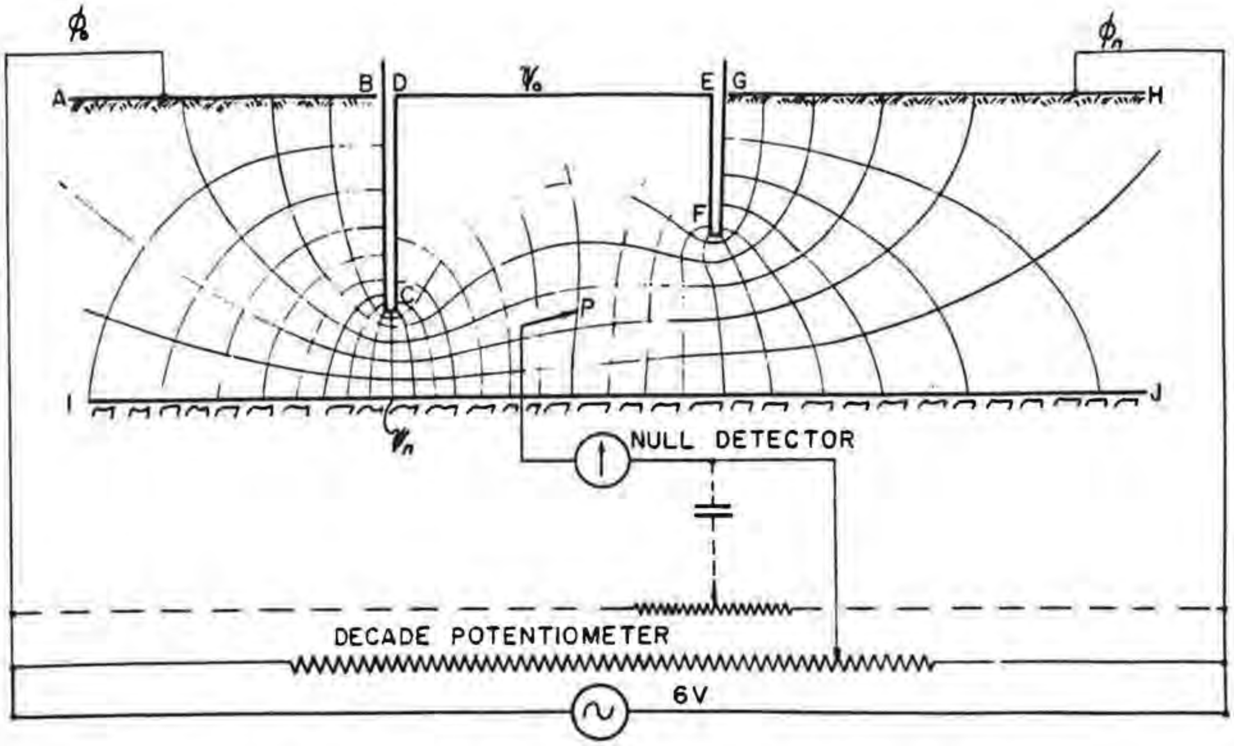


Figure 3. Solution of flow problems by electrical analog.

sheet pile wall and a small pocket is formed in which the absolute pressure is zero (vacuum). The situation at C is similar to the problem of laminar flow over a sharp crested weir.

Besides using the method of conformal transformation of complex variables, solutions for Eq. 8 may be obtained by using Green's function or by relaxation techniques. Many problems will require a reasonable amount of mathematical sophistication and for general methods appropriate mathematical texts should be consulted since the solutions are essentially a problem of mathematics.

A large number of frequently encountered problems have been treated in the foregoing fashion and their solutions are given in the literature notably by Muskat (2) and Polubarinova-Kochina (3). Scheidegger in his excellent recent book (4) gives a comprehensive bibliography.

2. Analog Solutions Whenever the boundary values are too complicated for obtaining solutions by the above mentioned methods, the use of analog models and analog computors may prove to be advantageous. The method is based on the principle that the differential Eq. 6 describing the force potential for water seepage through soil is analogous to the differential equations defining a number of other physical processes or phenomena. Among those are the electrical potential field, the magnetic potential field, the thermal potential field, as well as the trace of a two-dimensional stress tensor. The idea is by no means new and was used probably for the first time by G. Kirchhoff in 1845 (5). In contrast to analytical solutions which manipulate with numerical quantities, the analog solutions operate with physical quantities. These methods use the experimental determination of such physical quantities as magnetic flux, electric current and potential, temperature and stress which by their analog interpretation then give the velocity and force potentials for the seepage flow problem. Since the accuracy of such solutions depends on the precision of the measurement, the electric potential analog is most widely used, because electrical quantities such as resistance, potential and current can be measured easily with high precision by simple equipment. The discussion, therefore, will be confined to electrical analogs.

The differential equation for the electric potential field is given by:

$$\operatorname{div}\ (g \cdot \operatorname{grad} E) = 0 \tag{11}$$

where g is the conductivity and E the electrical potential. Substituting $E = A \cdot \phi$; $g = B \cdot K$ into Eq. 11, the equation becomes identical to Eq. 7. A and B are scaling constants. Thus, the drop in electric potential between two electrodes in a conducting medium corresponds to the drop of the force potential between the extreme equipotential lines of the flow problem. Figure 3 demonstrates the solution of the two-dimensional flow below a dam with cut-off walls by an analog model. The flow field ABC . . . J is cut out from a sheet of conducting paper and the boundaries $AB = \phi_o$, and $GH = \phi_n$, which are the extreme equipotential lines are drawn with a conducting paint as well as the extreme flow lines $B\,C\,D\,E\,F\,G = \psi_o$ and $I\,J = \psi_n$. By applying an ac potential difference between ϕ_o and ϕ_n as shown, the equipotential lines $\phi_1\ \phi_2\ \phi_3$ . . . can be traced using the probe P and the null detector. Similarly, by impressing the potential along ψ_o and ψ_n, the streamlines $\psi_1\ \psi_2$ . . . can be found. The conducting paper technique is simple and cheap, but is only suitable for two-dimensional problems. Three-dimensional problems can be solved by applying the same principle to a model in an electrolytic tank, where an electrolytic solution serves as the conducting medium. The shape of this tank is modeled after the boundaries of the flow problem.

A third type of electric analog model is the resistance network. Here the continuous flow field is approximated by a grid system of resistors. This is permissible if the grid distance "h" is made sufficiently small. The method corresponds to the numerical solution of Laplace's Eq. 8 by a finite difference relaxation method where the field is divided into small square lattices. In this case, Eq. 8 can be written:

$$\nabla^2 \phi = \frac{\phi_1 + \phi_2 + \phi_3 + \phi_4 - 4\phi_o}{h^2} + \frac{h^2}{12}\left[\frac{d^4\phi}{\partial x^4} + \frac{\partial^4\phi}{\partial y^4}\right] \tag{12}$$

The term in the bracket is small of higher order and may be neglected. In terms of the electrical analog Eq. 12 corresponds to

$$\frac{\phi_1 + \phi_2 + \phi_3 + \phi_4 - 4\phi_o}{R} = 0$$ where R is the resistance between the node

points of the grid. A detailed description of such an analog computer is given by S. C. Ling (6).

3. Graphical Solution. An alternate procedure for solving Eq. 8 is due to Forchheimer. Here the flow pattern is obtained by a trial and error method. The derivation is given by Tschebotarioff (7), see also Taylor (8). The stream- and equipotential lines are sketched by watching the boundary conditions as well as the Cauchy-Riemann conditions, and the pattern is subsequently corrected such as to satisfy these conditions at every point. These conditions will be satisfied if the following rules are observed:

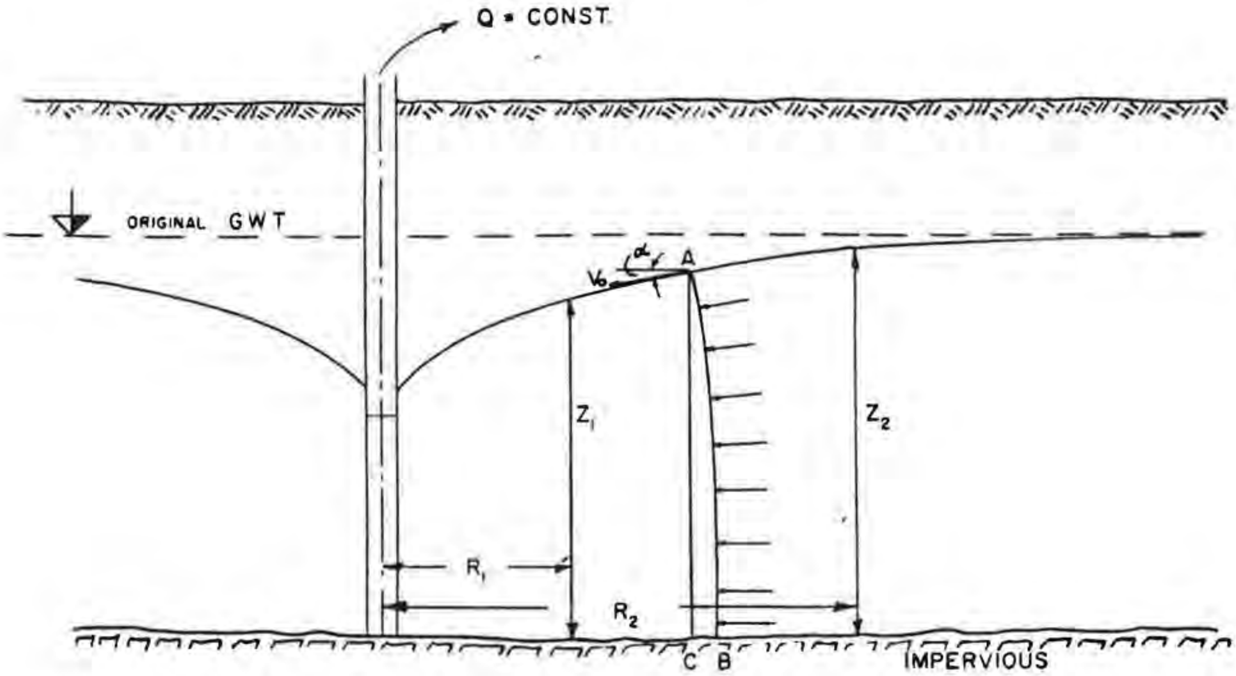
(a) The flow boundaries are at the same time the extreme streamlines.
(b) Lines of equal head are the equipotential lines.
(c) Streamlines and equipotential lines intersect at right angles. (This applies also to the boundaries.)
(d) All figures should approximate squares.

This procedure often gives a quick and sufficiently accurate solution which would be tedious to obtain analytically. It becomes impracticable, however, for the solution of three-dimensional problems unless there is radial symmetry.

4. Mechanical Models. Since for homogeneous isotropic coarse grained soils, the force and velocity potentials are independent of the permeability k, the problem could also be solved by direct model tests as long as the geometric relations are maintained. This method, however, is subject to relatively large experimental errors caused by capillary tension, air bubbles, etc. and will yield only qualitative results.

B. Steady State Flow with a Free Boundary

1. Analytical Solutions. Rigorous analytical solutions are very difficult and tedious to obtain and are based on the assumption of a sharply defined free surface along which the fluid pressure is constant and equal to the pressure in the gas above it. The most



(STEADY FLOW WITH A FREE SURFACE)

Figure 4. Flow towards a well.

promising method is one using a velocity potential ψ such that with the previous notation

$$\psi = K(p + \rho g z)$$

$$\bar{V} = -\text{grad}\ \psi$$

and

$$\nabla 2 \psi = 0$$

using again a complex number representation and the hodograph transformation $u = \frac{\partial \psi}{\delta x}$, $V = \frac{\partial \psi}{\partial y}$ the free surface is determined in the hodograph (u, V) plane. Once the free surface is known, it can be treated like a geometric boundary. The method was pioneered by Hamel (9) and a series of solutions are given by Muskat (2) and especially by Polubarinova-Kochina (3).

A much simpler and, for many engineering purposes, sufficiently accurate, approximate solution can be obtained using Dupuit's assumption (Fig. 4).

Let V_0 be the tangential filter velocity at the point A of the free surface which has an inclination of α with the horizontal. Then Dupuit's assumption is that the horizontal component of the filter velocity along the line AC is uniform and equal to $K \frac{dZ}{dY}$. Although this assumption has been questioned (2), it seems to give results in good agreement with rigorous solutions whenever the slope α of the free surface is not too large. Applying this assumption then to the problem of flow towards a well

$$Q = V \cdot A = K \frac{dz}{dr} \cdot 2 r \pi \cdot z \tag{13}$$

After integrating this separable differential equation and solving for the constant of integration

$$Q = \pi K (Z_1^2 - Z_2^2) \cdot \ln \frac{r_2}{r_1} \tag{14}$$

This equation now gives the yield of a well on a horizontal impervious layer under steady flow conditions.

2. Analog Solutions. In view of the difficulty of obtaining rigorous analytical solutions, the electrical analog method may be particularly useful whenever high accuracy is required. Because the free surface boundary is not known, it must be determined first by trial and error. With the conducting paper technique, it amounts to using a trial shape and check whether the electric potential at the free surface is equal to zero. The correct shape can be found by successive approximations. The resistance analog computer can also be used in a similar manner to great advantage.

3. Graphical Solutions. The graphical method described under A could only be used if a guess is made with respect to the free surface boundary. The quality of the solution depends on the accuracy of this guess. There is unfortunately no direct way of checking the results.

C. Unsteady State Flow

In the general case, the continuity Eq. 5 together with Darcy's law, Eqs. 3c and 4, leads to a non-linear partial differential equation which makes an analytical treatment rather difficult. However, Shchelkachev (10, 11, 12) by using the assumptions of fluid, as well as soil being compressible, and following Hook's law succeeded in developing the linearized equation:

$$\nabla^2 \phi = \frac{\mu}{K} (\alpha n + \beta) \frac{\partial \phi}{\partial t} \tag{15}$$

where $\phi = \int_{p_0}^{p} \frac{dp}{\rho(p)}$; and $\alpha = \frac{1}{\rho} \frac{d\rho}{dp}$, $\beta = \frac{dn}{dp}$ are the compressibility coefficients of liquid and soil which are assumed constant. It should be noted that in the force potential ϕ the gravity term is neglected. Eq. 13 is of the same type as the transient heat flow equation and methods for its solution are given by Carslaw and Jaeger (13) and Courant and Hilbert (14).

D. Anisotropy

In 1948, Ferrandon (15) derived formulas for the flow through anisotropic porous media which led to the notion of permeability as a tensor quantity. Ferrandon's theory has been substantiated by Scheidegger who checked the permeability tensor concept, comparing directional measurements of permeability. As a consequence, it can be stated that in the general case, the direction of the filter velocity $\bar{V}$ and the force potential gradient grad ϕ do not coincide except in three orthogonal directions in space. These three directions may be called the principal axes of the permeability tensor. A flow problem in anisotropic soil can be reduced to the case of isotropic flow by a geometric distortion with respect to the principal axes of the permeability tensor. In the case of two-dimensional flow, for example, by a reduction in scale in the direction "2" by the amount $\sqrt{\frac{k_1}{k_2}}$ where k_1 and k_2 are the permeabilities in the principal directions.

E. Permeability

The discussion above might suggest the idea that the water movement in soils under pressure potentials is a solved problem from the scientific point of view except for possibly some isolated questions at the periphery. Nothing could be farther from the truth. All the considerations so far were made under the tacit assumption that the permeability of the soil was a known and constant material property. While it is not too difficult to measure the permeability in the laboratory under a given set of conditions, the problem remains how a variation of these conditions will influence the permeability and also, how to appraise the conditions likely to be encountered in the field.

Of course, the problem of changing soil properties in stratified deposits as well as the lack of uniformity within an individual layer itself always will be with the soil engineer. He is often dealing with natural soil deposits which have been formed in quite an erratic way. Yet, even if he should fine—in a rare case—a completely uniform, homogeneous soil, it often will be practically impossible to duplicate the field conditions in a laboratory permeability test. This holds particularly for sands, because undisturbed sampling of sandy and gravely soils is not possible without excessive costs. Disturbed samples, however, have a different structure, aggregation and porosity and hence a different permeability. Therefore, the influence of these factors on the permeability must be known.

It seems logical that some relationship should exist between the geometry of the pore space in the soil and the permeability. Because of the irregularity of the particle shapes, the lack of uniformity of the grain size in most soils and because of the inumerable possibilities for particle aggregation into some more or less pronounced structure, no parameter defining the geometry of the voids in a unique way has as yet been discovered.

Attempts to correlate soil permeability with such other physical quantities as porosity n, grain diameter d, capillary diameter D, surface area S, are almost as old as Darcy's law.

Hazen, for example, related permeability to an effective grain size diameter: $k \approx C\, d_{10}^2$ where d_{10} is the sieve opening passed by 10 percent of the sample. It is frequently overlooked that the formula is valid only for clean, uniform filter sands.

The relationship between porosity and permeability in particular has been discussed and investigated extensively in the past 60 years. Today there are at least ten conflicting formulae (16) giving a relationship between permeability and porosity. Some of them were derived from theoretical considerations, others from experimental results or a combination of both.

The porosity alone can never completely specify the geometry of the voids. It is obviously possible for any one particular soil to have quite different structural arrangements and thus to have a different pore geometry at the same porosity. The most promising approach appears to be the use of an equivalent or effective capillary diameter D_e. This diameter is introduced if the permeate (soil) is replaced by an idealized capillaric model consisting of a system of capillary tubes through which flow occurs according to the Hagen-Poiseulle equation. Unfortunately, this effective capillary diameter D_e cannot be measured directly by independent means, but it can be determined from permeability tests. The author has shown recently (17) that above a critical porosity D_e may be considered constant.

A modification of the capillaric model theory was made by Kozeny who introduced the notion of the hydraulic radius. His formula

$$k = \frac{C\, n^3}{S^2} \tag{16}$$

as well as several modifications of it so, for example, the Kozeny-Carman equation:

$$k = \frac{n^3}{5\, S_0^2\, (1 - n)^2} \tag{17}$$

still enjoy wide popularity at present, but they have been severely criticized recently (4, 17) and their validity is questionable. In these formulae, C is the Kozeny constant varying between 0.5 and 0.667, S is the surface area per unit volume, S_0 is Carman's "specific" surface, that is, surface per unit volume of solids and n is the porosity.

One reason for the discrepancies and contradictions between the various proposed permeability relations may be derived from the fact that with the exception of Baver (18) and Winterkorn (19), all investigators are concerned with the full porosity of the soil. This is unrealistic. It has been known for some time that water is adsorbed at the particle surfaces by surface forces resulting in a complete fixation of several layers of water molecules and, some distance farther away, these surface forces cause an increase in the viscosity of the water.

Also, in the complex system of the inter-connected pore space there will be dead ends, side pockets, and cross capillaries without an appreciable pressure gradient. None of these will contribute to the flow, yet all of them do contribute to the porosity. Although several investigators mention these facts, they fail to consider them in the derivation of their formulae.

On the basis of these arguments, the author has derived the following permeability relationship (17):

$$K = \frac{\gamma}{32\,\mu} D_e^2 (n - n_0) \qquad (18)$$

where γ is the unit weight of the water, μ the viscosity, D_e the effective capillary diameter, n and n_0 is the total and the ineffective porosity, respectively. Experimental results are in good agreement with this equation. Figures 6 and 7 show permeability data giving values for D_e and n_0 computed from Eq. 18. D_e is constant above a certain porosity and thus may be a function of the granulometry of the particles, whereas n_0 appears to be a function of the structure or aggregation of the soil. Winterkorn (19) under somewhat different conditions also found a limiting or specific porosity analogous to Eq. 18.

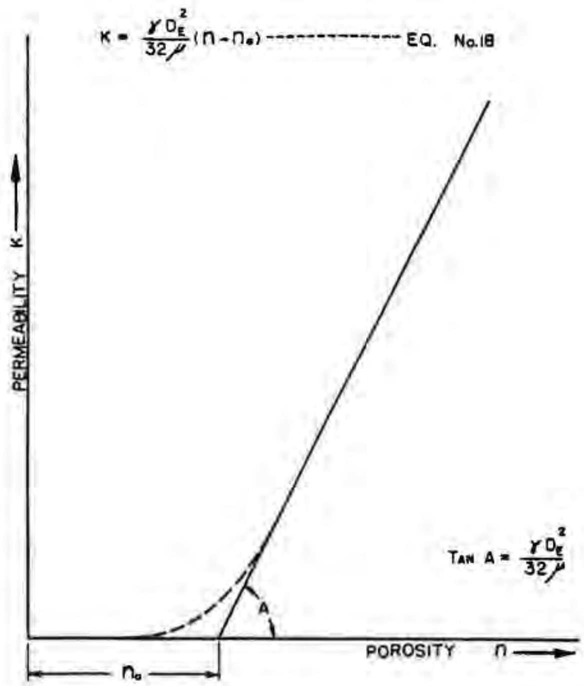


Figure 5. Porosity vs permeability after Equation 18.

The fact that the permeability values when plotted against the porosity "n" show a deviation from the straight line as n approaches n_0, suggests that in this range D_e is no longer constant but becomes a function of the pressure. A functional relationship between permeability and pressure was found indeed empirically by Tiller (21, 22) who established that

$$k = k_0 (\delta - p)^{-m} \qquad (19)$$

where δ is the total—or overburden pressure on the soil and p is the porewater pressure. This relationship holds above some experimentally determined value for $(\delta - p)$. A similar relationship between permeability and pressure was found by Fatt and Davis (23). Another interesting phenomenon in this connection is the existence of an initial pressure gradient in clays demonstrated by Derjaguin and Krylov (24) and supported by recent measurements of residual pore water pressures in consolidated clays (25). Since Eqs. 16, 17, 18, and 19 give the soil permeability as a function of the porosity n or as a function of the total pressure δ on the soil, permeability values determined in the laboratory can be reduced to the conditions of the soil in situ, provided the structure of the test sample has been relatively undisturbed. It is always necessary, however, to determine the permeability at least at two different porosities, and it is advisable to determine k at three or more different values for n and plot the diagram k vs n.

Permeabilities can be measured in the laboratory by either the constant head test, the falling head test, or indirectly from consolidation test data. In the former two tests, particular care is required to avoid the formation of air bubbles which would influence the results. This may be achieved by either assembling the test setup under vacuum and using de-aired water or by running the permeability tests under sufficient back pressure that any air present remains in solution. The latter can be done most conveniently in a triaxial test chamber. The indirect determination of the permeability from consolidation test data is open to some criticism because of the simplifying assumptions of the consolidation theory. In some cases, it may be advantageous to determine the permeability directly in the field, namely, whenever a high reliability of the results is required, or, when the integrity of the samples will be questionable, or, when a laboratory determination would be of little value because of the non-uniformity of a particular deposit. In all these cases the permeability may be determined by a pumping test. After the subsoil conditions have been ascertained by borings, the appropriate well equation can be used to compute the permeability. For the ground water

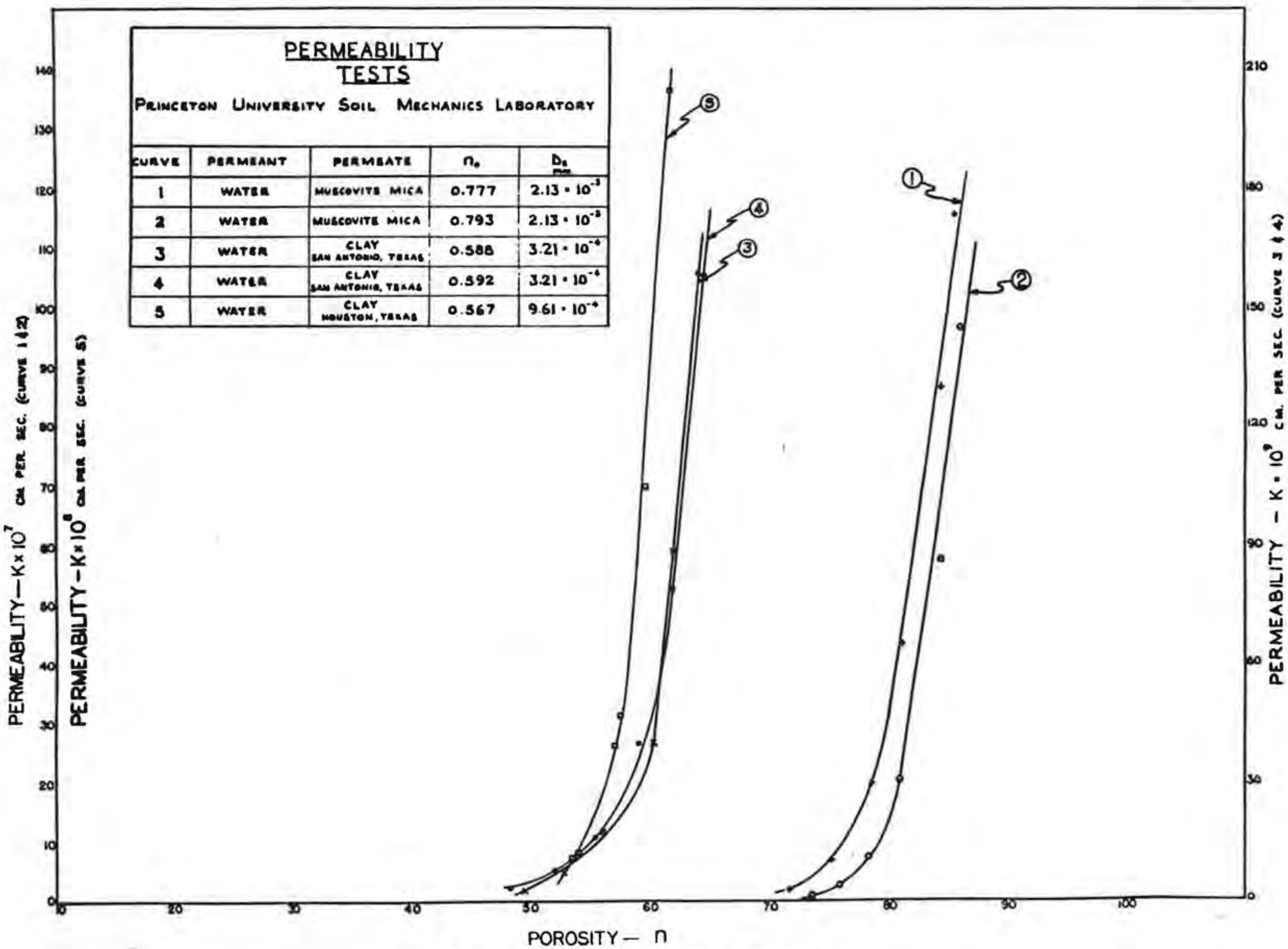
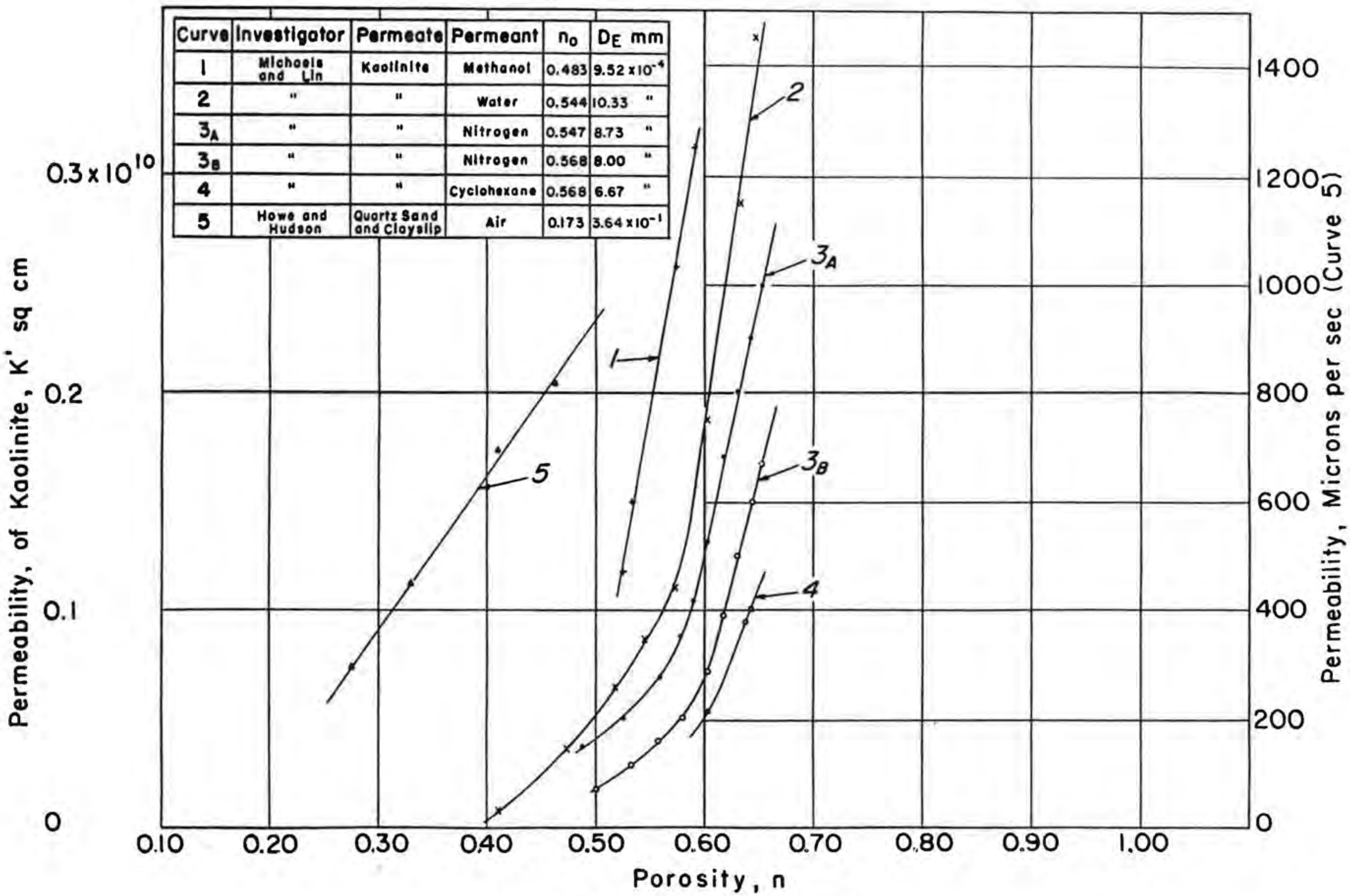


Figure 6. Permeability vs porosity.



Curve	Investigator	Permeate	Permeant	n_0	D_E mm
1	Michaels and Lin	Kaolinite	Methanol	0.483	9.52×10^{-4}
2	"	"	Water	0.544	10.33 "
3_A	"	"	Nitrogen	0.547	8.73 "
3_B	"	"	Nitrogen	0.568	8.00 "
4	"	"	Cyclohexane	0.568	6.67 "
5	Howe and Hudson	Quartz Sand and Clayslip	Air	0.173	3.64×10^{-1}

Figure 7. Permeability vs porosity.

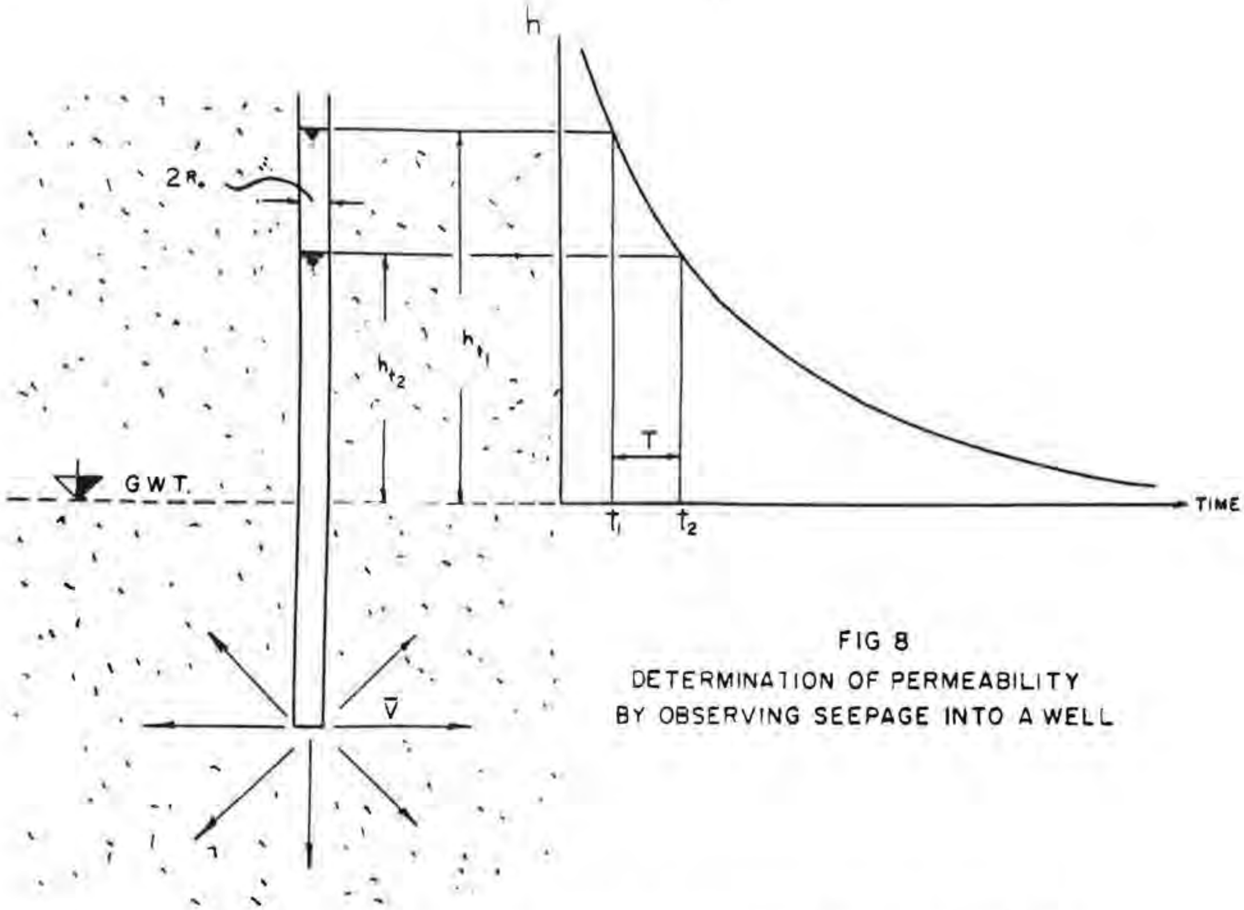


Figure 8. Determination of permeability by observing seepage into a well.

conditions of Figure 4, for example, Eq. 14 may be rewritten as follows:

$$k = \frac{Q}{\pi} \frac{\ln \frac{r_1}{r_2}}{(Z_1^2 - Z_2^2)} \tag{20}$$

Thus, a well can be driven into a pervious layer and pumped at a constant rate Q. After steady conditions have been reached, the water levels Z_1 and Z_2 in two observation wells a distance r_1 and r_2 from the pumped well can be measured and Eq. 20 yields the over-all permeability. It should be noted that the distances r_1 and r_2 should be sufficiently large as to be outside the immediate cone of depression in the vicinity of the pumped well. Otherwise, the Dupuit assumption under which Eq. 14 was derived, namely that the slope of the water table is small, would no longer be a good approximation. This method eliminates most of the uncertaintities discussed earlier, but it is relatively expensive.

A somewhat simpler method which requires only one well and also does not require steady conditions is possible by the use of Maag's equation:

$$k = \frac{R_0}{4T} \ln \frac{ht_1}{ht_2} \tag{21}$$

which gives the permeability of a soil in terms of the drop of the water level in a well projecting into the ground water table. The meaning of the symbols is shown in Figure 8.

While this method is cheaper and quicker than the previous one, it naturally allows the determination of the permeability only in the vicinity of the well. In contrast, the previous method allowed the determination of the over-all permeability of a relatively large area.

In summary it may be stated that much is still to be learned regarding the factors which influence the permeability of soils for water transport under pressure potentials. Particularly, the influence of soil aggregation and structure and the influence of the surface forces are yet little understood. Because of the large numbers of variables, the problem is a rather complex one. In this area there still is a wide field for future investigations and it is hoped that these questions will be resolved in due time.

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