SESSION FOUR

Friday, September 18, at 9:00 A.M.

THE DETERMINATION AND MEASUREMENT OF USER BENEFITS

JAMES S. BURCH, North Carolina Highway Commission, Presiding

Effects of Travel Impedance Costs

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● BEGINNING in 1924, the author was involved in several types of traffic and revenue studies. One type had to do with the actual determination of the probable redistribution of vehicular traffic among existing ferries or free bridges and proposed toll crossings. Another type had to do with the determination of the probable traffic volumes that would be "generated" by the proposed crossing, in addition to the anticipated annual organic growth of vehicular traffic. A third type of study was to demonstrate through statistical research the fact that a proposed crossing, by reducing travel impedance costs and despite the levying of a toll or a higher toll than on existing crossings, would actually stimulate cross-river trips. A fourth type of study was to demonstrate through research that, despite differences in the toll cost between a cheaper competitive ferry or an alternate free bridge, the proposed toll crossing would actually divert sufficient traffic from cheaper ferries or free crossings to be competitive with them and thus prove to be economically feasible.

In a 1940 paper (1) several hypotheses on traffic distribution and generation were brought together. The author set forth equations and determinants of generation of vehicular trip volumes, distribution among alternate routes, and organic growth, on the basis of types of data then available for such trip determinations.

In 1945 the author set forth two hypotheses: one on traffic distribution and the other on traffic generation.

The traffic distribution hypothesis, expressed mathematically, stated that equal numerical differences in impedance costs between a selected standard and any other existing alternate route, are associated with equal percentage differences, in the opposite direction, in the quality ratings of the existing alternate routes.

The traffic generation hypothesis, expressed mathematically, stated that the differences in trips between any pair of zones (one a residence and the other a non-residence zone) and a standard pair of corresponding zones, are (a) proportional to the differences in auto registrations in the two resident zones, (b) proportional to the differences in the indexes of "attraction" in the non-resident or purpose zones, and (c) inversely proportional to a function of the differences in impedance costs between the given pair of residence and non-residence zones and the corresponding standard zones. The inverse mathematical functional relation between trip differences and impedance cost differences (between any pair of zones and a standard pair) was an exponential function and not a power function, like the so-called gravity model.

G. P. St. Clair put these two hypotheses and their corresponding series of equations through thorough and rigorous mathematical tests. In connection with the author's traffic distribution hypothesis, he suggested an alternate hypothesis which might briefly be stated as follows: "all alternate routes have equal impedance costs." In this connection, he also suggested that a relationship would have to be established between running and waiting time impedances and the traffic volume flowing along alternate routes. This would provide another basis for determining redistributions of traffic among existing and proposed alternate routes.

In connection with the author's traffic generation hypothesis, St. Clair agreed that the mass product of (a) auto registrations in the resident zones and (b) indicators of attraction in the non-residence zones, were partial determinants of inter-zonal trips. He also agreed to the idea that impedances and impedance costs presented real possibilities for the relative economic evaluation of proposed highways.

In addition, St. Clair derived from the author's exponential functions, by the method of the calculus, an exponential-type-skewed function and normal-type-skewed function. The designations of functions sound like formidable mathematical challenges. Though they may appear complex, they seem to be realistic reflections of both traffic generation and the types of linkages that appear to exist between homes and sites of economic activities. The author has discovered data which not only points to the realism of these mathematical functions but also to the fact that some of the corollary "constants", which they would yield, would be quite meaningful and valuable in real life if determined from certain types of data.

These analyses and the extended reciprocal correspondence between the author and St. Clair were not published at the time (in 1946). The author felt that the data that would be forthcoming from the "home interview" O-D studies, then newly developed by the Bureau of Public Roads, might yield the very types of trip volume data, in sufficient quantities, together with correlative determinant factors that might be assembled simultaneously, to test scientifically, the validity of the author's trip-impedance cost hypotheses. These data might also provide the data necessary to test the validity of the mathematical functions suggested by the author as well as those derived by St. Clair.

The author has since found that the data summarized from the "home interview" method of assembling O-D data had serious defects for use in research, particularly for testing the validity of these hypotheses. For one thing, the data on trips to and from given O-D zones were usually mixtures of primary trips, some originating or destined for residences in the zones, and other trips originating or destined for non-residence sites in the same zones. As a result of these trip mixtures, correlations between trips into and out of zones, with autos domiciled in these zones, could not possibly yield satisfactory correlations. It was essential therefore that this "chemical mixture" of trip data be broken down first. By going back to the original schedules and obtaining pure "elemental" trips to and from zones originating and destined for exclusively in residences in those zones, meaningful correlations could then be established.

Every O-D zone is not only a residence zone but also a non-residence zone. The same type of "chemical breakdown" is required to obtain trips to and from every O-D zone as a non-residence zone. If such breakdowns could be made, then such data could be correlated with data on gainfully employed for journey-to-work trips, with floor space in commercial buildings for business trips, floor space in retail establishments for shipping trips, and floor space in other buildings for amusement and recreation trips, etc. These types of land use data for non-residence zones have since become available, but only very recently.

Bringing together some 35 years of continuing studies, bearing on generation of traffic, distribution among alternate routes and modes of travel, and organic annual growth in vehicular traffic volumes, present knowledge and understanding suggests these types of future studies.

To arrive at the minimum number of fundamental determinants of vehicular and person trip generation as well as their distribution among routes and modes of travel, the first essentials of course, are data on vehicle and person trips between as large a number of pairs of zones as possible in study areas. In one of the pairs of zones, the trip ends must be exclusively at residences or "residence zones." In the other of the pairs of zones, the trip ends must be exclusively at non-residences where the purpose of the trip was satisfied or in "zones of purpose." Such zones of purpose are: employment zones, retail shopping zones, commercial or amusement and recreation zones. Vehicle and person trip data should therefore be assembled on the basis of several significantly different purposes but also by modes of travel such as by auto, bus, rapid transit and via commuter rails.

However, supplemental data on fundamental traffic determinants are equally essential. Such data must be assembled for small areas—like census tracts, postal zones or O-D zones. Such data consist of population, households, auto ownership, numbers of gainfully employed at sites of zones of employment, net residential acreages, floor space at retail establishments, at all other commercial establishments and at amusement and recreation sites.

Equally essential data that must be assembled, if possible, simultaneously with trip and land use data, are those obtained through test runs between every pair of zones in the study area and along various routes made with autos, by riding buses, and railroads. These test run data consist of the following: distances, travel time, both running, stopping and waiting times; auto operating costs; tolls at bridges, tunnels and highways; parking fees in non-residence zones; vehicle volumes along all major arterials and well-traveled routes, and notations of annoying and irritating potential hazardous aspects of routes, like left turns, parked cars, pedestrian crossings and so forth.

With such trip volume and supplemental fundamental data at hand, trip data may be correlated with the above fundamental determinants, and excellent results anticipated. Trips for any given period of time and for one or more purposes, to and from zones with trip ends exclusively at residences would yield excellent correlations with auto ownerships in those zones. In turn, densities of auto ownerships (expressed in cars per acre) would be correlated quite closely with household densities (expressed in households per acre).

Thus, auto ownership densities appear to increase in proportion to household densities up to about 10 to 15 households per acre; after that, auto densities do not increase as fast as household densities. Auto ownerships per household, on the other hand, decrease as household densities increase. Thus, there are more cars per acre in Manhattan (about 21 cars per acre) than in Westchester County (about 5 cars per acre). On the other hand, on Manhattan there are only about 40 cars per 100 households compared to about 120 cars per 100 households in Westchester County (in 1955). In the author's opinion, household densities (households per acre), are far more stable indicators of auto ownerships in small areas than are, for example, such indicators as distances of zones from CBD's, or income per household.

Similarly, trips for any given period of time and for any given purpose—such as journey to work, to and from the same zones but with trip ends now exclusively at sites which satisfy the purpose (such as at sites of employment or retail establishments)—will correlate with indicators of the purpose, such as number of gainfully employed or sales volume or floor space, in these non-residence zones.

Trips between pairs of zones, one a residence and the other a non-residence zone, made for one or more purposes, will thus correlate closely with the product of (a) auto ownership in the residence zone and (b) indicators of the purposes satisfied in the non-residence or purpose zone. The mass product influence on trips between pairs of zones may thus be "filtered out" by dividing trips by the product of auto ownership and the purpose indicator, to yield a very significant series of auto or person trip ratios, for each of the zones in the study area. It is these auto or person trip ratios, for each of the zones, which are inversely related to travel impedance costs, between pairs of zones.

It is usually exceedingly difficult to establish, from non-physical statistical data, the precise mathematical functions connecting trips with their correlative determinants, because any one of several families of inverse functions will usually yield equally good statistical correlations, and closeness of fit of any curve, is the only statistical test for choice of the precise function. However, in the opinion of this researcher, the so-called gravity formula, which states that trips vary inversely as some power of distance or time, does not appear to be the mathematical function best suited to express the inverse relationship between trips and impedance costs. The gravity formula being a power function states, in effect, that a one percent increase in distance or time produces an x percent decrease in trips. The type of mathematical function that best expresses the inverse relationship between trips and impedance costs is an inverse exponential type function. An inverse exponential trip-versus-impedance-cost function postulates that a numerical difference in determinants like distance (miles), running or waiting time (minutes), tolls (in cents) or parking fees and one or more forms of usually unmeasurable types of impedance which produce irritations, annoyances and potential hazards to travel, or numerical differences in unit costs of these impedances will produce percentage differences in trips, in the opposite direction.

These mathematical functions also postulate that motorists in the study area, place average unit values on the differences in each type of impedance, between any given pair and that for the standard pair of zones. Thus, motorists place an average unit cost on every mile more or less than the distance between the standard pair of zones or on the differnce of every minute of running or waiting time, as well as on differences in the not directly measurable impedances that produce annoyances and irritations to motorists. In order to avoid these impedances, the motorist is thus willing to pay so many cents either per identifiable impedance difference or for all residual not directly measurable impedance differences.

If these impedances have been measured along routes that connect a large number of pairs of zones in the study area which have widely varying impedance characteristics and widely varying volume of trips, it is possible, by the method of least squares, to determine the most probable average unit impedance costs of the limited number of measurable impedance differences.

The end product of the above procedure yields quite a simple formula. It states that between any pair of zones and any other pair in the study area numerical differences in aggregate impedance cost differences are associated with percentage differences, in the opposite direction, in trips.

There is one more philosophical step required before practical application can be made of such formulas to estimate probable future changes in trips resulting from changes in travel impedances. It is this: a numerical change of one cent in impedance cost during a given interval of time, would be equivalent to the effect of a difference of one cent, at the time of the study. In defense of this philosophical step, the author has this experience to offer.

In 1940, the author produced this "rule of thumb": that a one cent difference in impedance cost would produce a one percent difference in the opposite direction in trips. Recently, Westchester County Toll Parkways increased tolls from 10 cents to 25 cents or a difference of 15 cents. According to the above rule, assuming a change and a difference to be equivalent, there should have been a reduction in traffic volume of 15 percent. The reduction in traffic volume turned out to be 16 percent. But, with tolls 250 percent and traffic 84 percent before the toll increase, revenues turned out to be 210 percent of those before the toll increase. This revenue increase clearly indicates what the motorist is willing to pay for avoiding the not directly measurable impedances on alternate routes, of which there are a number.

The author has worked consistently with a mathematical function that relates percentage differences in trip volumes with numerical differences in impedance costs. St. Clair, some twelve years ago, as mentioned above, derived several other distinct types of integral functions. Although they are more complex than the author's original difference functions, they are nvertheless highly useful. Data could be collected to implement them. These functions would yield measures for a number of highly interesting characteristics of trip linkages between sites of concentrated economic activities and surrounding tributary residence areas. They would reveal relationships between increasing travel impedance costs and decreasing rentals in residential areas located at various travel impedance costs from concentrated economic areas, like CBD's. These functions also indicate that person and vehicle trip data should be collected at concentrated sites and areas of economic and recreational activities, rather than in the homes through the "home interview" method.

The derivation of the formula for redistribution of traffic between any pair of zones among alternate routes connecting them, follows similar lines of reasoning and results in a hypothesis similar to the traffic generation theory. This hypothesis states that the percentage ratio of any given route to that of a standard route (which ratio the author terms the route's "quality rating") is inversely proportional to the numerical difference in the impedance cost between the given route and the standard route. Thus, on the Hudson River where between some pairs of zones, there were as many as 18 alternate ferry, tunnel and bridge routes, carrying significant portions of trans-Hudson trips between given pairs of zones, it was essential to reflect in the formula the number of significant alternate routes competing for revenue traffic.

The share of total trips, between any given pair of zones that any given existing or proposed alternate route would handle, was equal to the ratio which its "quality rating" (relative to a standard route) bore to the sum of the quality ratings of all alternate routes. The quality rating of any existing route was obtained from the trip data by dividing the trips between a given pair of zones via any given route, by those via the standard route. Correlations of quality ratings between alternate routes and impedance costs via alternate routes indicated that a numerical difference of one cent in impedance costs was associated with about a one percent difference, in the opposite direction, in the quality rating between alternate routes.

Today there is a wealth of trip volume data between small areas in more than 100 cities for which millions of dollars have been spent to assemble, and more millions for analysis. Far greater understanding of the basic factors which determine generation of traffic volumes, distributions among alternate routes and traffic expansion over time, could be derived from the voluminous original household interview data that have been assembled. Some of these original data should therefore be "exhumed" and re-punched on new cards. Some supplemental data should be punched into those cards. They should be tabulated and examined, at first by hand, in the light of the equations discussed in this paper. Electronic machines or computers could then be used to determine the most probable impedance costs. At the same time, the best types of mathematical functions could be firmly established. A great wealth of understanding would flow from such re-analyses.

Also serious consideration should be given to assemble future O-D data at sites and areas of concentrated economic, social and recreational activities in urban areas. In the opinion of the author a much richer body of data would thus become available for research on the underlying economic determinants of urban transportation.

FORMULAS FOR TRAFFIC DISTRIBUTION AND RE-DISTRIBUTION AMONG ALTERNATE ROUTES

$$J_{HP} = J_1 + J_2 + \ldots + J_s + J_m + = S_1^m J \quad (1)$$

in which

$J_{HP} = S_1^m J$ = all journeys between a residence zone H and a purpose zone P, via existing alternate routes 1 to m inclusive.

J_1, J_2, J_s, J_m, = journeys between the residence zone H and purpose zone P, via individual alternate routes 1, 2, any other existing route m, and the most traveled route between zones H and P, adopted as standard route s.

$$s_m = \frac{J_m}{J_{HP}} \quad \frac{J_m}{J_1 + J_2 + \ldots J_m \; J_s} \quad (2)$$

in which

s_m = share which any existing alternate route m handles of all journeys between residence zone R and purpose zone P, via all existing alternate routes, and other terms as above.

$$Q_m = \frac{J_m}{J_s} \; ; \; Q_s = \frac{J_s}{J_s} = 1 \tag{3}$$

in which

Q_m = "quality rating" of route m, relative to standard route s and other terms as above.

$$sm = \frac{J_m}{S_1^m J} = \frac{Q_m}{S_1^m Q} \tag{4}$$

in which

$S_1^m Q$ = sum of the quality ratings of all alternate routes, 1 to m between residence zone H and purpose zone P, and all other terms as above.

$$Q_m = (1-d)^{\Delta c_m}; \tag{5}$$

in which

d = discount from unity in "quality rating" of any alternate route, for every impedance cost difference of one cent via route m, compared with standard route s. Numerical value of discount (d) measures the "keenness of competition" among alternate routes, between zones H and P.

Δc_m or Δc_p = algebraic sum of the impedance cost differentials between either existing alternate route m or proposed alternate route p and the standard existing alternate route s and other terms as above.

$$\Delta c_m = \Delta m c_{\Delta m} + \Delta r c_{\Delta r} + \Delta w c_{\Delta w} + c_{\Delta t} + \Delta i c_{\Delta i} \tag{6}$$

in which

For journeys between zones H and P, the impedance differences between alternate route m and standard route s are:

- Δm = distance difference (in miles)
- Δr = running time difference (in minutes)
- Δw = waiting time differences while stopping for traffic signals and other delays (in minutes)
- Δi = impedance differences like left turns, pedestrian crossing, unparking of cars, etc. (in numbers)
- $c_{\Delta m}$ = unit cost of mileage differences (in cent/min)
- $c_{\Delta r}$ = unit cost of running time differences (in cent/min)
- $c_{\Delta w}$ = unit cost of waiting time differences (in cent/min)
- $c_{\Delta t}$ = cost of toll differences (in cents)
- $c_{\Delta i}$ = unit cost of all other identifiable or residual impedance (in cent per impedance)

$$Q_m = (1-d)^{\Delta c_m} \tag{5}$$

$$\log Q_m = \Delta c_m \log (1-d) \tag{7}$$

let $\log Q_m = q_m$ and $\log (1-d) = d'$

then $q_m = d' \Delta cm$ (8)

but $\Delta c_m = \Delta m c_{\Delta m} + \Delta r\, c_{\Delta r} + \Delta w c_{\Delta w} + c_{\Delta t} + \Delta i c_{\Delta i}$ (6)

therefore $q_m = d'\, \Delta m c_{\Delta m} + d'\, \Delta r c_{\Delta r} + d'\, \Delta w c_{\Delta w} + d'\, c_{\Delta t} + d' \Delta i c_{\Delta i}$ (9)

Eq. 9 may be used to determine, by the method of least squares, motorist's evaluations of unit costs of mileage differentials, of running and waiting time differentials, and of other travel impedance differentials where differences in mileages, running and waiting times, and tolls have been determined from standard routes, for a large number of pairs of residence and purpose zones.

$$s_p = \frac{Q_p}{S_1^m Q + Q_p} \tag{10a}$$

$$s_{1-n} = \frac{S_1^n Q}{S_1^n Q + Q_p} \tag{10b}$$

in which

s_p = share which any proposed alternate route p.

s_{1-n} = share which all other existing routes would handle of all journeys between residence zone H and purpose zone P, via all existing alternate routes plus the proposed alternate route.

Q_p = "quality rating" of the proposed route p, obtained from Eq. 5,

$$Q_p = (1-d)^{\Delta c}p.$$

Eqs. 10a and 10b are used to compute shares of total journeys between a residence and purpose zone which a proposed route would divert from existing alternate routes. And the share remianing on all individual existing routes with the proposed route in operation.

FORMULAS FOR TRAFFIC GENERATION

$$J_{HP} = K\,A_H\,I_p\,F(c_{HP}) \tag{11}$$

in which

J_{HP} = all journeys between a residence zone H and a purpose zone P.

A_H = autos domiciled in residence zone H.

I_p = Index of purpose satisfaction, in purpose zone P.

$F(c_{HP})$ = Some inverse mathematical function of aggregate costs of all travel impedances between residence zone H and purpose zone P.

K = "dimensionality constant" which converts product of the numbers representing the terms of A, I, and c into J_{HP}, representing the journeys between residence zone H and purpose zone P.

$$K = \frac{S_J}{S_A} = j_a \tag{12}$$

in which

S_J = sum of all primary journeys from all residences in the study area.

S_A = sum of autos domiciled in all residences in the study area.

j_a = average number of journeys made to all purpose zones by each auto domiciled in the study area.

$$I_w = \frac{E_w}{E_s} = e_w;\; I_b = \frac{F_b}{F_s} = f_b \tag{13}$$

in which

I_w = Index of work purpose.

E_w = Employment in the work zone.

E_s = Total employment in the study area.

e_w = Percent of total employment of study area in work zone.

I_b = Index of Business.

F_b = Floor space in business area.

F_s = Total floor space devoted to business in the study area.
f_b = Percent of total floor space of study area, in business zone.

$$F(c_{HP}) = F(mc_m + rc_r + wc_w + T + P + Ic_i) \qquad (14)$$

Impedances between residence zone H and purpose zone P are:

m = distance (in miles)
r = running time (in minutes)
w = waiting time (in minutes)
I = residual impedances (like left turns, unparking of cars, pedestrian crossings, etc.)
c_m = unit cost per mile.
c_r = unit cost per minute of running time.
c_w = unit cost per minute of waiting time.
T = tolls at bridges, tunnels or toll highways.
P = parking fees in the purpose zone.
c_i = unit cost per identifiable or residual impedance.

$$J_H = j_a A_H F(c_H) \qquad (15)$$

in which

J_H = journeys from the residence zone to all purpose zones in the study area.
j_a = average number of journeys per auto (domiciled in study area) between residence and purpose zones in the study area.
A_H = autos domiciled in residence zone H.
$F(c_H)$ = some inverse mathematical function of aggregate impedance costs between residence and purpose zones.

$$\frac{J_H}{A_H} = j_H \qquad (16)$$

in which

j_H = average number of journeys to all purpose zones in study area per auto domiciled in residence zone H.

$$\frac{j_H}{j_a} = g_H \qquad (17)$$

in which

g_H = ratio of frequency of journeys per domiciled auto from residence zone H, to that of average frequency of journeys from all residence zones in the study area. (This becomes an indicator of "generation" of journeys, with reduction in impedance costs.)

$$g_H = (1 + p)^{-\Delta c_H} \qquad (18)$$

in which

p is the percent difference in journeys, for every difference of one cent in impedance cost differentials between journeys from residence zone H to the centroid of purpose zones, compared with journeys from the centroid of residences to the centroid of purpose zones.
Δc is the aggregate difference in impedance cost differentials between the residence zone H and the centroid of purpose zones compared with those between the centroid of all residence zones and the centroid of all purpose zones in the study area.

$$\Delta c_H = \Delta m c_{\Delta m} + \Delta r c_{\Delta r} + \Delta W c_{\Delta W} + c_{\Delta T} + c_{\Delta P} + \Delta I c_{\Delta I} \qquad (19)$$

Impedances between the centroid of residence zones and centroid of purpose zones are compared with those between the residence zone H and the centroid of purpose zones, and the resulting impedance differences are as follows:

Δm is distance difference in miles.
Δr is running time difference in minutes.
ΔW is waiting time difference in minutes.
ΔI is residual impedance difference, in numbers.
$c_{\Delta m}$ is unit cost of mileage difference.
$c_{\Delta R}$ is unit cost of running time difference.
$c_{\Delta w}$ is unit cost of waiting time difference.
$c_{\Delta T}$ is toll cost difference.
$c_{\Delta P}$ is parking cost difference.
$c_{\Delta I}$ is differential cost of residual impedances.

$$g_H = (1 + p)^{-\Delta c_H} \qquad (18)$$

$$\log g_H = -\Delta c_H \log (1 + p) \qquad (20)$$

let $\log g_H = g'_H$ and $\log (1 + p) = p'$

$$g'_H = p' \Delta c_H \qquad (21)$$

$$g'_H = -p' \Delta m c_{\Delta m} - p' \Delta r c_{\Delta r} - p' \Delta w c_{\Delta w} - p' c_{\Delta T} - p' c_{\Delta P} - p' \Delta I c_{\Delta I} \qquad (22)$$

Eq. 22 may be used to determine, by the method of least squares, the generation factor "p" and the unit costs of impedance differences, in the study area.

REFERENCE

1. Cherniack, Nathan, "Measuring the Potential Traffic of a Proposed Vehicular Crossing." ASCE Trans., Vol. 106, p. 520 (1941).

Discussion

Burch. —I know that everyone who has worked in O-D surveys and their analysis recognizes the great variety and great complexity of traffic movement. It often seems impossible to unravel the motivations and the decisions that are made by drivers as they choose their routes of travel.

Pendleton. —One of the frequent recommendations that economists make, particularly when urban facilities become very congested, is that a price of some sort, or a higher price at least than has been charged, be placed upon that facility as the most efficient way (at least in the short run) of meeting the problem.

I was wondering, in connection with the Westchester County freeways, what the motivation was for jumping the toll from ten cents to a quarter, and how you could justify to the public the collection of all this additional revenue which was presumably not matched immediately by increased costs.

Cherniack. —I can't speak, of course, for the Westchester Park County Commission, but I would guess that they were faced with a problem of building expanded highways and parkways; and under the present toll rates, they were unable to do that, while with the added toll they might be able to do it.

I can give you an example of one effect of inflation. The Port Authority built the first two tubes for about $80 million. When they came to build the third tube, one tube cost $100 million. In effect, you are really trying to retrieve the present 1959 costs with this added toll, and also to have the funds to expand when necessary.

Now, I can give you perhaps another illustration but in reverse. The floating bridge in Seattle was originally a toll bridge. It had paid itself off, amortized its debt sooner than expected, and so the bridge was made free of toll charges—at present it is operating at capacity, and the public wants to expand it. Now the problem is how to build a toll bridge under the costs of 1959, and to operate it in competition with a free bridge.