Evaluation of Unit Cost of Time and Strain-and-Discomfort Cost of Non-Uniform Driving

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●THE EXCHANGE of ideas between Nathan Cherniack and myself left us both with the hope that a development more practicable and at the same time more rational in theory than either of us had contrived would point to a valid solution for the problem of time and impedance costs.

It should be made plain that here we are dealing with costs which are experienced primarily by the passenger-car driver; and which, although objective and tangible enough in their impact upon him, are subjective in the values that he puts on them. These subjectively-valued costs are experienced by the driver of a commercial vehicle—indeed by anyone who operates or occupies any kind of vehicle—but the costs of commercial-vehicle time, involving wages, labor relations, business opportunities afforded or denied, and like factors, are so differently oriented that it seems best to treat them as an entirely different problem.

There was an inclination in earlier years to regard uncompensated passenger-car-driver time as free time, and to take no account of it in the reckoning of user benefits. The experience of the toll facilities tends to contradict this line of reasoning and has demonstrated the fact that the desire of the motorist for time savings is the dominant element in the demand for high-speed limited-access service. But it became clear that time-savings alone do not fully account for this extraordinary demand—the motorist is buying a package of advantages in paying, by tolls or taxes, for high-speed expressways. Studies of expressway origins and destinations have shown that a substantial percentage of motorists will sacrifice time-savings to gain the free-flowing traffic that the high-grade facility affords (1, 2).

Cherniack's earlier scheme was to set up a cost item for each identifiable cost (or cost-savings) in the package of expressway benefits (running costs, running time, waiting time, the cost of right turns, left turns, passing maneuvers, etc.) and to rely on the accumulation of instances occurring in the presence of toll charges to provide a statistical solution for the unit values of this collection of unknowns. The general method still offers great promise; but it became evident fairly early that a greatly needed economy would be gained by reducing the number of variables, or unknowns to be solved for. One method of doing this is to solve for some of the cost elements independently, so that they will enter as known terms, rather than unknown, in the statistical equations. This can be done with running costs (gasoline, tire wear, etc.) and, somewhat less easily and directly, with accident costs.

There remain the cost of time, or value of time-savings (subjectively appriased by the motorist) and a collection of other subjective cost evaluations, all of them linked with the package of advantages provided by the free-riding expressway, or more generally, with the difference in riding advantages afforded by any two trips subject to comparison. It is obvious that if a single attribute could be found that would act as surrogate for all the blessings of the free-flowing expressway that are associated with the reduction of strain, annoyance, or discomfort, the unknowns in the equation would be reduced to two: the cost of time and the cost of this special attribute.

At the time the authors were fumbling with these ideas, others were experimenting with the significance and practicability of measuring speed changes. Greenshields (3) in developing an index of the quality of traffic flow used the summation of speed changes as the principal factor. Saal and others have used the measurement of speed changes as a tool in the study of vehicle performance under varying conditions of traffic and road.

Nearly all the factors that contribute to annoyance, discomfort, and nervous tension on a trip have their most direct and immediate effects in causing changes in speed (including reduction to zero speed). Sharp curves, steep grades, narrow roads, poor conditions of repair, left turns, right turns, stop signs and signals, passing maneuvers and many other items cause the motorist repeatedly to check his speed, to accelerate, to stop, to start, or, in other words, to depart from the condition of uniform speed which is the characteristic of a pleasant trip. The necessity for changing speed requires certain physical movements on the part of the driver and an increase of the attention he must pay to driving. On all occupants of the car acceleration or deceleration exerts forces that are proportional to the magnitudes of the speed changes. On this point Greenshields wrote as follows:

> ... It is not only slow speed but the range and the frequency of speed changes that annoy the driver and often cause him to seek a longer route that may take more time to travel .... It is reasonable to assume that the annoyance factor increases as the frequency and magnitude of speed changes increase. (3, p. 510)

Consideration of these facts led to the notion that the summation of speed changes on a trip might be used as a common denominator for the entire catalogue of impedances to uniform driving. This procedure would reduce the number of unknowns, or subjective factors, in the equation to two—the unit cost of time, to be measured in cents per minute, and the unit cost of the strain-discomfort factor, to be measured in cents per unit of speed change. The experimental work of Paul Claffey in measuring time and speed changes on toll roads and alternative routes, has been directed in part toward (a) testing the validity of the speed-change unit, and (b) determining reasonable average values of these two unknowns.

Claffey has stated informally that this summer's experience at 14 toll situations has caused him to question the adequacy of the speed-change unit as an index of the strain-discomfort factor, largely because it fails to take account of the annoyance caused by forced driving at reduced speeds on 2-lane highways, occasioned by slow-moving vehicles. There is also the case of prolonged stops, such as those at a red light, which involve a speed change at the beginning and at the end, but none during the duration. It is to be hoped the combination of time and speed change as subjective costs will be adequate to care for these two elements, but this cannot be counted on with assurance at this time.

THE EQUATION

Given a situation in which the number of motorists using a toll road, and the number using an alternative route for the same trip, are known or can be measured, the two requirements for a solution are: (a) that a valid equation of trip costs, including the two unknowns to be solved for, can be written; and (b) that a condition or situation can be found in which the trip costs on the toll road and its alternative route can be equated. If this can be done for a satisfactory number of cases, the group of equations can be subjected to a solution for the two unknowns by the method of least squares.

The equation of trip costs is as follows:

$$C = O + A + P + Tx + Dz \tag{1}$$

in which

C = Total trip cost in cents;
O = Trip operating costs (mileage or running costs);
A = Accident costs (an expectancy term, based on the accident experience of the two classes of road);
P = Toll charge, if any (P for "Pay");
T = Time of trip in minutes;

x = Unit cost of time in cents per minute;
D = Number of speed-change units developed on the trip (the strain-discomfort term); and
z = Unit cost of speed changes in cents per unit of speed change.

Speed-change units have, of course, the dimensions of acceleration. In the 1958 summer tests in Maine and Pennsylvania speed changes were measured in miles per hour per 6-second interval.

Objections have been made to the procedure of setting up the unit value of time as applicable to the total time of trip, because of the logical conflict involved in equating the sum of a great many minute time-savings (such as a few seconds each) with the arithmetically equal sum of a much smaller number of much greater time-savings. It has been suggested that the conflict can be avoided by defining the quantity sought as the unit value of time-savings. This does not seem to avoid the conflict, which perhaps can only be evaded by a gentlemen's agreement not to accumulate huge sums of minute time-savings. In the actual working out of Eq. 1, however, the solution for the unit cost of time, x, will be dependent on observed values of time-savings on one alternative route as compared with another; and in actual applications to economic analysis this unit cost would be applied to time-savings rather than to total values of time of trip.

THE CONCEPT OF VARIABLE SUBJECTIVE APPRAISALS

Nearly all would agree that there is variation in the subjective appraisals that motorists put on the value or cost of time and of the strains and discomforts of non-uniform driving. For example, if the traffic between two points breaks 80/20 between a turnpike and an alternative route it is clear that those who pay the toll place a higher value on time-savings and on uninterrupted driving than do those who drive the less free-flowing alternative route. The toll charge tends to sort drivers into two classes; but it is reasonable to suppose that these appraisals range from very low to very high and group themselves about a mean value according to some statistical distribution. It is natural to assume that they are distributed normally, although there is something to be said for the assumption of a skewed distribution.

A normal distribution is characterized by two parameters, the mean $\bar{x}$, and the standard deviation σ . Since, in the trip-cost equation there are two subjective cost factors to evaluate, x and z, the number of unknowns to solve for increases to four, if one is to follow through with the assumption of a normal distribution of subjective appraisals.

Thus,

Time involves $\bar{x}$ and σ_x; strain-discomfort involves $\bar{z}$ and σ_z.

EQUATING TRIP COSTS AT THE MARGIN

The assumption of variable subjective costs makes it possible to equate trip costs for the two alternative routes. For, if the split is 80/20 in favor of the turnpike, and the toll is \$1.00, that means that to 80 percent of those making the choice the advantages of the toll road are worth \$1.00 or more; and to 20 percent these advantages are worth \$1.00 or less. At a considerably higher toll, presumably, the traffic would break 50/50, and this toll would correspond with mean values of the subjective cost appraisals. Still higher tolls would provide 40/60, 30/70, and 20/80 distributions of the traffic. But note that, in the actual case, the trip-costs on the two routes are equal in the estimation of that group of motorists to whom the turnpike advantages are worth just one dollar. This group will break 50/50, the most trifling preference turning them to one or the other route.

Let us forget strain-discomfort for the moment and take the case where the only subjective cost item is the value of time. Let us denote as x_0 the value of time at the breakover point, and characterize by the subscripts 1 and 2, the costs on the toll-road and the alternative route. Equating costs,

$$O_1 + A_1 + P + T_1x_o = O_2 + A_2 + T_2x_o$$

$$x_o = \frac{(O_1 - O_2) + (A_1 - A_2) + P}{T_2 - T_1} \tag{2}$$

$$= \frac{P - (O_2 - O_1) - (A_2 - A_1)}{T_2 - T_1}$$

Since the values of O and A are objective and therefore given, and the toll is known, the value of x_o is given by the data of the example. One more definition is necessary:

p_1 = The proportion of travelers using the preferred route.

For this single-variate case the equation relating to the mean value, $\bar{x}$, and the standard deviation, σ_x, can be readily derived by reference to the normal probability curve (Fig. 1). The proportion of motorists, p_1, using the preferred route is represented by the area under the curve to the right of the plotted value of x_o.

For any value, x_o, the area to the right of it can be expressed, as a proportion, by the integral—

$$p_1 = \int_{t = t_1}^{t = a} e^{-\frac{t^2}{2}} \, dt$$

in which

$$t = \frac{x}{\sigma_x}; t_1 = \frac{x_o}{\sigma_x}$$

This formula for t is written for the case where $\bar{x}$ is put equal to zero. For the case where $\bar{x}$ has a positive finite value, the x-coordinate is shifted by replacing x with the expression, $x - \bar{x}$. For the value of t corresponding to the marginal or breakover values, p_1 and x_o, —

$$t_1 = \frac{x_o - \bar{x}}{\sigma_x}$$

The equation is then developed as follows:

$$x_o - \bar{x} = \sigma_x t_1$$

$$x_o = \bar{x} + \sigma_x t_1 \tag{3}$$

The value of t_1 in any given case can be obtained by finding in the Table of Areas under the normal probability curve (see any standard text (5) in statistics) the value of t corresponding to the area value given by $p_1 - \frac{1}{2}$.

For any given toll situation the value of x_o would be obtained by solving Eq. 2. The value of t_1 would be obtained from the given value of p_1 by reference to the Table of Areas under the normal probability curve. From a series of toll situations a sufficient number of pairs of values of x_o and t_1 would be obtained to make possible a solution, by the method of least squares, for the value of $\bar{x}$, the average unit cost of time, and that of σ_x, the standard deviation of subjective appraisals about the mean value. The difficulty is that time is not the only thing that is paid for at the toll booths; and therefore such a solution is necessarily defective.

Figure 1 illustrates the solution for $\bar{x}$, the average subjective unit cost of time,

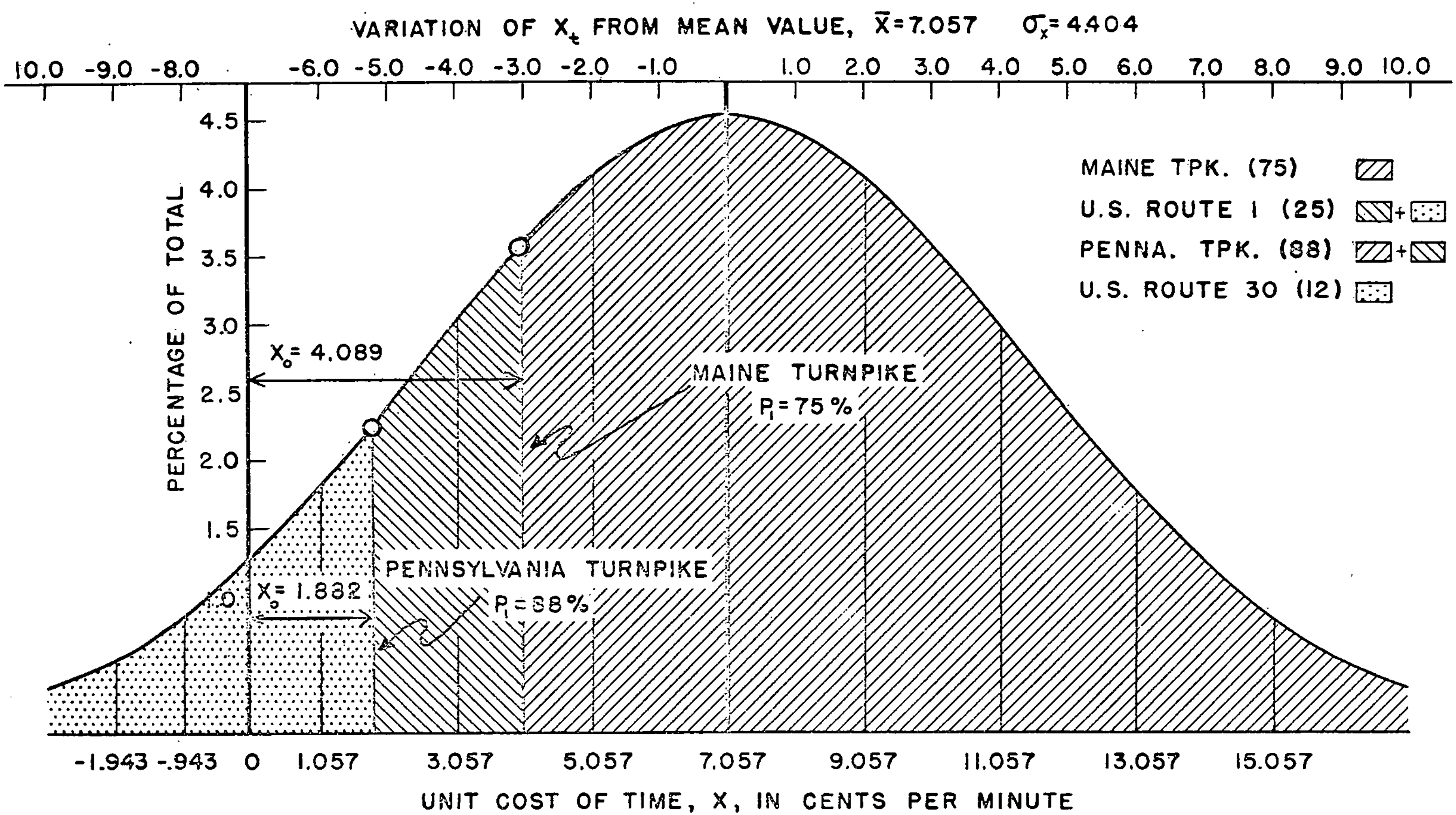

Figure 1. Solution for unit cost of time, assuming normal distribution of travelers' subjective evaluations and using results of Maine and Pennsylvania tests, 1958.

derived from the data obtained by Paul Claffey in the summer of 1958 from test runs on the Pennsylvania Turnpike and U S 30, and on the Maine Turnpike and U S 1. In this calculation the assumption was made that trip costs on the turnpike and the alternative route were equal except for the toll charge, P, and Tx, the cost of time. The percentages of motorists using the alternative routes were taken from the results of previous studies (4) which gave the ratios 75/25 for the Maine Turnpike and U S 1 (between Kittery and Portland) and 88/12 for the Pennsylvania Turnpike and U S 30 (between Philadelphia and Breezewood).

The terms of the solution gave values of x_o (the appraised cost of time at the break-over point) of 4.089 cents per minute for the Maine Turnpike and 1.882 cents per minute for the Pennsylvania Turnpike. These points are plotted (Fig. 1) on the normal probability curve. The ordinates represent the proportions of motorists whose subjective appraisal of the value of time is equal to x as it takes different values along the abscissa of the chart, the class interval in the illustration being ½ cent. The cross-hatched area extending to the right of the point $x_o = 4.089$ (Maine Turnpike) is 75 percent of the total area under the curve. The area in reverse cross-hatch extending from this point to the point $x_o = 1.882$ (Pennsylvania Turnpike) is 13 percent of the area under the curve; so that the two areas combined equal 88 percent—the percentage of motorists using the Pennsylvania Turnpike. The remaining 12 percent of area, representing motorists preferring U S 30 is shown in dotted hatching. The two left-hand areas combined represent the 25 percent of motorists preferring U S 1 to the Maine Turnpike.

If 88 percent of motorists in the one case value time at 1.9 cents per minute or more, and 75 percent in the other case value time at 4.1 cents per minute or more, a solution satisfying both cases and postulating a 50/50 distribution of preferences will give a unit value of time considerably greater. When the normal probability curve is used the value is $x = 7.057$, with a standard deviation σ_x, equal to 4.404.

This very high value of the unit cost of time has to be deflated. If we assume that the effect of other cost items, such as the strain-discomfort factor, the motorists' appraisal of reduction in accident-cost hazard, and some slight difference in running costs, would have equal weight with time-savings, we may reduce the value by one-half to 3½ cents per minute, which is within shooting distance of the probabilities. The real point is that we have achieved a solution, such as it is, for the single-variate case of an assumed normal distribution of motorists' subjective appraisals of the unit cost of time.

BIVARIATE SOLUTION

No equation has yet been developed for the bivariate solution, where, in addition to the time-values, $\bar{x}$ and σ_x, we must solve for $\bar{z}$ and σ_z, the mean and standard-deviation values for the unit cost of speed change, the strain-discomfort factor. We have, however, experimented with a solution by trial-and-error or successive approximations, using the Maine-Pennsylvania data.

The principal instrument of this trial-and-error solution was the setting up of a table giving the ordinates of a bivariate distribution, assuming the two attributes concerned, x and z, to be independent of one another. For the purpose of generalization the class limits for both attributes were expressed as multiples of the standard deviation. An interval of 0.5σ was used in the calculation. Table 1 gives the bivariate distribution in abbreviated form, with a class interval of 1.0σ. The values tabulated are, in effect, the ordinates of a 3-dimensional surface of which values in the other two dimensions are the x-values (subjective unit cost of time) and the z-values (subjective cost of speed-change).

The Assumption of Independence

Since the two favorable attributes of expressways—time-savings and reduction of strain-discomfort—tend to go together it may be held that they are not independent, that is, that one is correlated with the other. If this were the case, or assumed to be the case, the equation for the solution would be complicated by the necessity to include

TABLE 1

BIVARIATE NORMAL DISTRIBUTION OF 1,000 INDIVIDUALS ACCORDING TO TWO INDEPENDENT ATTRIBUTES

Class Limits in Multiples of Standard Deviation, σ	$-\alpha$ to -3.0	-3.0 to -2.0	-2.0 to -1.0	-1.0 to 0.0	0.0 to 1.0	1.0 to 2.0	2.0 to 3.0	3.0 to α	Rim Totals
$-\alpha$ to -3.0	-	-	-	1	1	-	-	-	2
-3.0 to -2.0	-	1	3	7	7	3	1	-	22
-2.0 to -1.0	-	3	19	46	46	19	3	-	136
-1.0 to 0.0	1	7	46	116	116	46	7	1	340
0.0 to 1.0	1	7	46	116	116	46	7	1	340
1.0 to 2.0	-	3	19	46	46	19	3	-	136
2.0 to 3.0	-	1	3	7	7	3	1	-	22
3.0 to α	-	-	-	1	1	-	-	-	2
Rim Totals	2	22	136	340	340	136	22	2	1,000

in it an expression of the correlative relationship between the two attributes, which might, for example, be inverse. The assumption of independence of the two attributes has the advantage of relative simplicity.

There is, moreover, support for this assumption in both evidence and logic. Partial verification is found in the fact that a significant proportion of expressway users travel them at a sacrifice in loss of time. Conceptually, the independence of these two subjective attributes is attested to by the fact that they are psychologically very different. The value of time, stripped of irrelevant considerations, inheres only in the desirability of getting from point A to point B in a minimum of time. A motorist's subjective appraisal of the value of time is probably affected by (a) his economic and occupational status, (b) his personal characteristics, and (c) the nature of the particular trip, for example, vacation, home-to-work, or driving his wife to the maternity hospital. The reduction of strain, discomfort, and impatience, achieved by making it possible to drive at a uniform and uninterrupted speed, is an entirely different thing, both in motivation and in the nature of the satisfaction received, even though the same road characteristics produce both results. Thus there is no obvious reason why motorists' subjective evaluations of the strain-discomfort factor should vary either as a direct or as an inverse function of their subjective evaluations of time.

Solution Procedure Illustrated

The guessing procedure in this operation consisted of selecting a given set of values of the four unknowns, $\bar{x}$, σ_x, $\bar{z}$, and σ_z; and then computing the trip costs on the two turnpikes and their alternative routes for the values of x and z given by each cell in the bivariate distribution, or by a sufficient number of cells so that the percentage choosing each route was definitely determined. The computation for a single cell is illustrated in Table 2. The motorists comprising this cell, who value time at 2.375 cents per minute and relief from strain-discomfort at 0.015 cents per speed-change unit, would choose U S 1 in preference to the Maine Turnpike, but they would choose the Pennsylvania Turnpike in preference to U S 30.

After a considerable number of trials it was found that the values $\bar{x} = 3.0$ and $\bar{z} = 0.06$ gave a range of values in percentage turnpike choice close to the 75 percent for the Maine Turnpike and 88 percent for the Pennsylvania Turnpike that was the object of the quest. By varying the values σ_x and σ_z an area was delineated in which the percentages varied from 74/87 to 77/89. At this point it was decided that a qualified success had been achieved.

Figure 2 shows, in the form of a 3-dimensional surface in isometric projection, the results of the calculation for the following values of the four variables, or unknowns: $\bar{x} = 3.0$; $\sigma_x = 0.5$ (cents per minute); and $\bar{z} = 0.06$; $\sigma_z = 0.06$ (cents per speed-change unit).

TABLE 2

EXAMPLE OF CALCULATION OF ROUTE CHOICES

1. Values of mean and standard deviation
   $\bar{x} = 3.0$ $\sigma_x = 0.5$ (cents per minute)
   $\bar{z} = 0.06$ $\sigma_z = 0.06$ (cents per speed-change unit)
2. Coordinates of cell chosen for calculation
   Class interval of σ_x: -1.5 to -1.0
   x-values:
   Class interval: 2.25 to 2.50
   Midpoint: 2.375
   Class interval of σ_z: -1.0 to -0.5
   z-values:
   Class interval: 0.000 to 0.030
   Midpoint: 0.015
3. Calculation

Item	Maine US 1	Maine Turnpike	Pennsylvania US 30	Pennsylvania Turnpike
a. Time of trip, minutes	69.3	46.4	260.4	169.8
b. Total speed-change units	1,519	140	6,968	448
c. Time cost in cents (a x 2.375)	165	110	618	403
d. Speed-change costs, cents (b x 0.015)	22	2	105	7
e. Toll charge on turnpike, cents	-	90	-	170
f. Total cost of trip, cents	187	202	723	580
g. Choice of motorists in this cell	X	-	-	X

This particular solution gave turnpike ratios of 77/89 rather than the desired value, 75/88. It was chosen because the values of σ were such that multiples thereof could be plotted conveniently on coordinate lines of the isometric chart from which this drawing was traced. The vertical ordinates represent the number of motorists (out of 10,000) within a given cell of x, z values. The class intervals are 0.25 on the x-scale (time in cents per minute) and 0.03 on the z-scale (strain-discomfort factor in cents per speed-change unit). The surface is outlined by a sort of basket of profile curves, which are normal probability curves plotted in isometric projection.

The volume outlined in heavy full lines (Fig. 2) includes 77 percent of the volume, or number of motorists; and represents the number who would choose the Maine Turnpike in preference to US 1. The segment in broken line represents the additional 12 percent of motorists who would perfer the Pennsylvania Turnpike to US 30. Because of the discrete character of calculations using class intervals it was found difficult to make a satisfactory portrayal of this middle segment. The segment in light full line represents the 11 percent of motorists who would prefer US 30 to the Pennsylvania Turnpike. The light-line and broken-line segments combined represent the 23 percent who would choose US 1 in preference to the Maine Turnpike.

The values of x, the subjective appraisal of unit cost of time, are all positive and contained neatly between the values 1.00 and 5.00 cents per minute. The values of z, on the other hand, spill over rather badly into the negative quadrant. This fact signalizes our partial failure in this experiment; and at the same time is perhaps indicative of the relative accuracy with which the value of time and the value of that other factor can be measured at this stage. A skewed distribution, defined so as to exclude negative values, might prove successful. We are in hopes, however, that with a larger number of observations, an acceptable solution using the bivariate normal distribution will emerge.

In summary, we have a notion here that does not quite prove out. The essential germ of the idea is that any solution for the unit value of time and/or the strain-discomfort factor should take account, mathematically, of the variation in individual subjective appraisals of those values. It is not an easy thing to do, and this experiment is a first faltering step in that direction.

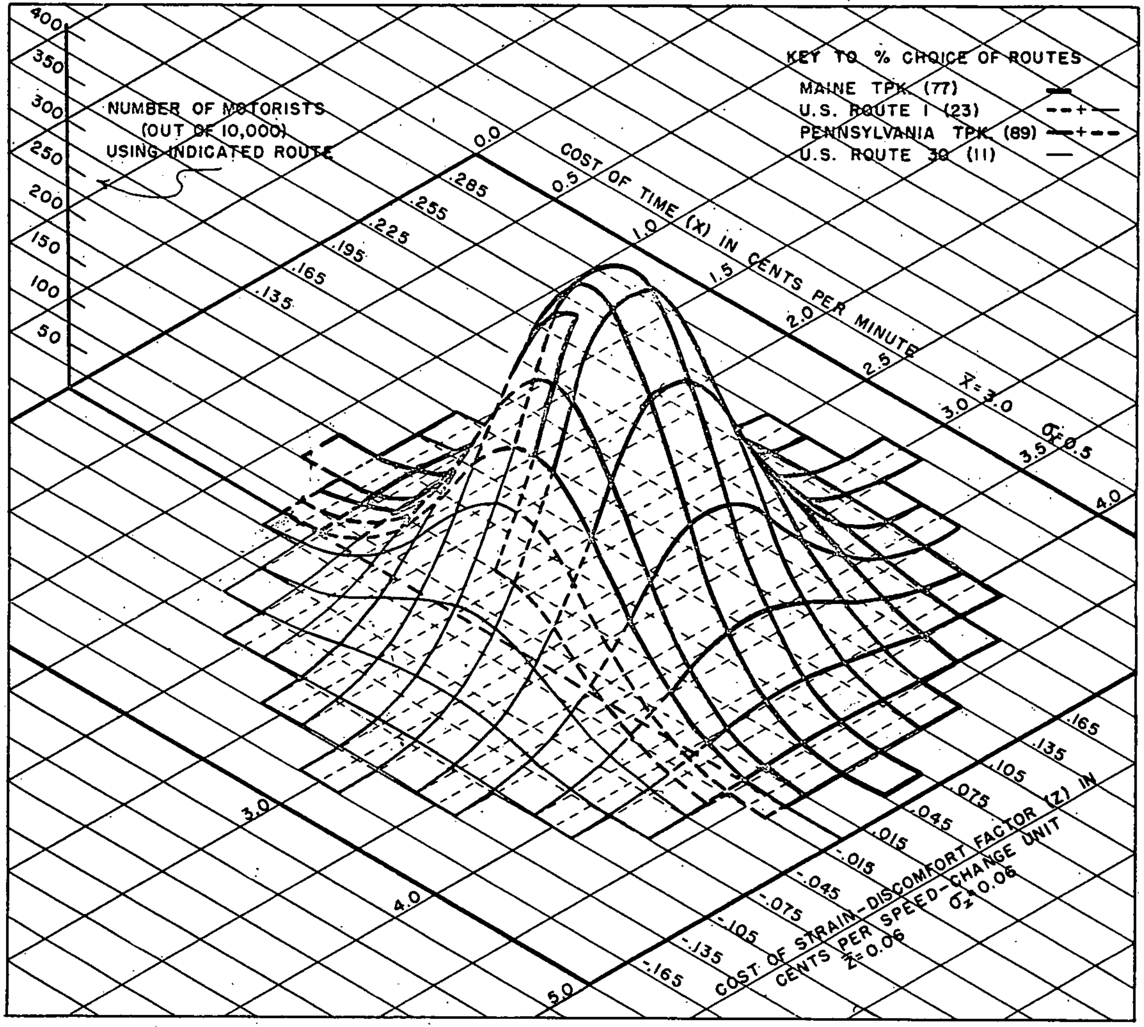

Figure 2. Illustrating bivariate normal distribution of subjective cost appraisals.

SINGLE-VARIATE SOLUTION

For comparative purposes the single-variate solution for mean value, $\bar{x}$, and standard deviation, σ_x, of motorists' subjective appraisals of unit cost of time, based on observations made in 1958, is as follows:

The experimental data on which this trial solution is based were, as previously mentioned, collected by Paul J. Claffey in the summer of 1958. Test runs were made on the Maine Turnpike and US 1 between common terminal points in Kittery and in Portland. Similar runs were made on the Pennsylvania Turnpike and US 30, between common terminal points in Philadelphia and in Breezewood. In each of the states, four pairs of trips were made (two in each direction), during which, in addition to the trip-time record, speedometer readings were made at 6-sec intervals. The speed changes thus recorded were tied in with their causes, such as red lights, restricted speed zones, turns, and passing maneuvers, by means of a log record. The speed-change data are not pertinent to this illustrative solution and are therefore not shown.

No O-D studies were made in connection with these test runs to determine the percentages of total travel between the termini that used the turnpike and the alternative routes. Instead, the reported results of origin-destination surveys made a few years previous were used (4). These studies indicated a 75/25 distribution of choice between the Maine Turnpike and US 1 on the trip between Kittery and Portland; and an 88/12 distribution between the Pennsylvania Turnpike and US 30 on the trip between Philadelphia and Breezewood.

The data used in the statistical solution for $\bar{x}$ and σ_x are given in Table 3. The eight observations are not entirely independent, since the proportion using the turnpike, p_1, varies only as between the Pennsylvania and the Maine situations. If it were practicable to have test runs on a turnpike and its alternative made at different times of day, days of the week, and seasons of the year, and to make O-D surveys that would supply values of p_1 applicable to each particular pair of trips, then the individual observations for a single trunpike situation would have greater independence, which would enhance the validity of the statistical solution. The test runs made under Claffey's direction in the summer of 1959 were accompanied by approximately simultaneous O-D studies conducted by the state highway departments. Values of p_1 (proportion using turnpike) will thus be closely tied in with the time of conduct of the test runs.

The time differences do show considerable variation, from 83 to 96 minutes on the Pennsylvania Turnpike compared with US 30, and 16 to 29 minutes on the Maine Turnpike compared with US 1. No objective or measured costs other than the toll charge enter into the solution; and the z term, the strain-discomfort factor, was neglected in order to reduce the problem to that of a single-variate solution.

The normal equations and the results of the solution by the method of least squares are given in the lower part of Table 3. For $\bar{x}$, the mean value of motorists' subjective appraisals of the unit cost of time, the solution gives the value, 7.057 cents per minute. The standard deviation, σ_x, of subjective appraisals about their mean value turns out to be 4.404 cents per minute. Since two standard deviations embrace only 95 percent of the observations, it is evident that the extreme low values of x in this solution spill over into the negative quadrant, as was the case with the bivariate solution. The possibility of substituting a skewed distribution (such as the logarithmic normal) was investigated briefly in connection with the bivariate solution. This procedure gave reasonable values of $\bar{x}$ and $\bar{z}$, but produced inordinately large values of the standard deviations, when expressed in real rather than logarithmic figures.

A similar single-variate solution was calculated for $\bar{z}$ and σ_z, the mean value and standard deviation of the strain-discomfort factor, on the assumption that this factor, rather than the value of time, was the quantity to be solved for. This solution gave the value of $\bar{z}$ as 0.3097 cents per speed-change unit, and the value of σ_z as 0.2304.

These solutions, based on somewhat disparate data from only two turnpike situations, are not of great significance in themselves. They indicate however, as does the bivariate trial-and-error solution, that, with a respectable body of data taken at turnpike situations, reasonable solutions for the unit value of time and of the strain-discomfort factor, based on the assumption of a normal (or perhaps skewed normal) distribution of subjective appraisals of these values, may be forthcoming.

TABLE 3

SINGLE-VARIATE SOLUTION ALTERNATIVE ROUTES IN 1958

State and Observation No.	Difference in Objective costs[1] P (cents)	Time Difference, $T_2 - T_1$ (min)	$\frac{P}{T_2 - T_1} = x_o$ (cents min)	Proportion Using Turnpike, p_1	$p_1 - \frac{1}{2}$	$t = \frac{x_o - \bar{x}}{\sigma_x}$
Pa. 1	170	95.7	1.776	0.88	0.38	1.175
2	170	93.9	1.810	0.88	0.38	1.175
3	170	83.0	2.048	0.88	0.38	1.175
4	170	89.5	1.899	0.88	0.38	1.175
Me. 5	90	29.0	3.103	0.75	0.25	0.674
6	90	16.4	5.488	0.75	0.25	0.674
7	90	23.5	3.830	0.75	0.25	0.674
8	90	22.9	3.930	0.75	0.25	0.674
Total	-	-	23.884	-	-	7.396

Equation:

$$x_o = \bar{x} + \sigma_x t$$

Normal equations:

$$\Sigma x_o = N\bar{x} + \sigma_x \Sigma t$$

$$\Sigma x_o t = \bar{x}\Sigma t + \sigma_x \Sigma t^2$$

$$23.884 = 8.000\bar{x} + 7.396\,\sigma_x$$

$$19.871 = 7.396\bar{x} + 7.340\,\sigma_x$$

Solution: $\bar{x} = 7.057$ $\sigma_x = 4.404$ (cents per minute)

Summations:

$N = 8$

$\Sigma x_o = 23.884$ $\Sigma t = 7.396$

$\Sigma x_o t = 19.871$

$\Sigma t^2 = 7.340$

[1]Toll charge only.

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Discussion

Burch. —Mr. St. Clair, it is apparent that you and your associates have done a lot of work to assign values to something which has heretofore been an intangible, and that you have made considerable progess in doing so.

Hoch. —I would like to say that I think this is a giant step, rather than a faltering step. As a further step, I think that you might well try a logarithmic normal distribution.

St. Clair. —We tried one and it looked like a promising thing because with the same parameters, you get a skewed distribution but it did not, in the brief trial we made, produce a satisfactory solution.

Hoch. —The reasons for it are that you would get negative values, and it might approximate the income distribution.

St. Clair. — You mean the income distribution of the respondents, of the people.

Hoch. —Yes, or just in the United States economy.

Grant. —I have a technical statistical comment and one question. There seems to be here need for another sigma, another standard deviation, and this is the standard deviation of vehicle running cost. I mean, some people have large old cars that make only 10 mi per gal, and some have small foreign cars that get 35.

I am not quite clear what this would do if it were included. One thing, of course, it would do, would be to give you a four-dimensional picture that you could not flash on a screen. This seems to me to be an element that belongs in the equation. There is that variability along with these other variabilities. The fellow that has the low mileage car has a great advantage in saving distance, let's say, on the Turnpike, if it does save distance, that is miles per gallon.

My question is, you are rather out on the extremes of preference here in the Maine and Pennsylvania cases. Do you have any unpopular turnpikes like West Virginia, or something like that, that you can operate on?

Claffey. —We studied all of them, except the West Virginia and the Virginia Turnpike, Massachusetts Turnpike and those in Connecticut. We studied all the others and I do not know which of them might be considered poor or rich. We have not analyzed the data.

Lindman. —With regard to turnpikes, you might say that the West Virginia Turnpike would not be comparable to the others. With regard to the tax implications of this (and I assume that this work is being done with the objective of getting at the problem of financing highways) I am wondering if it leads to the conclusion that a tax for highways related to income, rather than a tax related to benefits of fuel consumed or some other such measure, is indicated.

St. Clair. —The primary objective in this study was to find a means of obtaining average values of motorists' subjective appraisals of cost of time and of non-uniform driving. This trail led us to the concept of statistical variations of such appraisals about their mean values. This conceptis a necessary tool in the tentative solution we have developed. To us it is solely a matter of evaluating these benefits with reasonable accuracy. The idea that motorists' subjective appraisals might be correlated with income status may have long-run implications, but there is no present plan to use this idea in our tax-allocation analysis.

Jorgensen. —With reference to the Maine and Pennsylvania Turnpike data, I would like to know if account is taken of the fact that we probably have a new generation of drivers that don't even know there is a US 1 or US 30. I don't understand this phase of the analysis. I assume that it is based on the assumption that people do know there is some other way to go from Portland to Boston than by the Turnpike, for example, I think there are a lot of people who start out for Portland, Maine, and assume that there is just one way to go, that is via the Turnpike.

St. Clair. —Mr. Claffey, I believe you more or less take the attitude that your testing should be done with the local people?

Claffey. —This last summer we made a more extensive study than we made last year. Not only did we use the vehicle data from the electronic device, but we also had O-D studies made. These O-D studies were made to pick up the people from an origin city at the beginning of each of the study locations, so that the people we are concerned with are people that live at one end of a toll road and presumably would be able to make a rational decision as to which route they would prefer. For example, one location in Kansas was between Topeka and Wichita. The people of Topeka would be the ones that were interviewed. Everyone who passed was interviewed, but they were asked to give their addresses. I plan to use only those people with residences in that initial point.

Jorgensen. —It strikes me that in most places you do have a situation where people, even residents in Boston, are accustomed to the idea of only one way to get to Maine, and that is on the turnpike.

Saal. —Mr. Chairman, just to clarify the discussion, the first two studies we made in 1958 were the turnpikes in Pennsylvania and Maine. We found we had to revise our procedures. We were using O-D data of 8 or 10 years ago. The sections selected were too long, and could not tie down the trip data, from Breezewood to Philadelphia, for example, and there was too much variation. In Maine it was somewhat better.

So we realized that we did not have very good cases. That is the reason we studied the 14 cases this year, very carefully selecting the sections so we could get the data we wanted, and so we could get fresh O-D information.

I think the data we got in 1959 should begin to answer the question.

Pendleton. —I have a comment concerning Mr. Claffey's misgivings about the speed-change unit with respect to sustained slow speeds and long stops at traffic signals. Both of these would be picked up in your time variable, so it does not make much difference, if you get the whole picture.

On Mr. Lindman's question, it seems to me that this sort of analysis and evaluation is fully as valuable, if not more so, for highway planning than it is for the financing function, and I think it should stand or fall on that, even more than on the question of whether it contributes to how much we should charge.

Gardner. —Did this O-D study get data on the type of travel? Isn't the type of travel significant in the picture? I am thinking of the business trip versus a vacation trip.

Claffey. —Yes, we asked everyone the question of the purpose of their trip. I do not know just how we will treat the data, but we have it classed in six different purposes.

Moskowitz. —Have you tried to see what this would do to the problem of variable time values that must be placed on people where they fall in the 30 percent portion of the assignment (diversion) curve?

St. Clair. —We have these two variables, and we assume that there is not one value but a distribution of values.

Moskowitz. —Then the thing to do would be to use the distribution of values, if you used a percent assignment curve wouldn't it?

St. Clair. —I believe that would be the rigorous method, although it would be more simple to use the mean value as determined. We have not tried any applications because we regard it as dubious right now.

Moskowitz. —I agree with Mr. Hoch that this is a terrific step forward in determining objectively what value auto drivers place on time.

California has been forced to accept but one value instead of a distribution of values. The value we have been using is not arbitrary. It is the amount actually paid by motorists who drove at the average "free choice" speeds prevailing at the time the value was established. It is well known that the cost per mile of operating a car rises as the speed increases. By equating the excess cost of driving at 53 mph against the time saved by driving at that speed instead of 53 less a differential, a value of 2.6 cents per minute was calculated (a curve showing cost per mile as ordinate and minutes per mile as abscissa was drawn. The slope of this curve at any point is the cost of time saved by driving at that speed. At 53 mph, the slope is 2.6 cents per minute, using unit costs in the 1952 AASHO Informational Report). At present speeds and present mileage costs, the slope might come out so much higher that we would be afraid to use it.

While it may be true that the motorist did not know how much he was paying for each minute, and that this kind of time may be different from the time lost sitting in a traffic jam, this approach at least is based on some fact instead of all opinion, and was resorted to in face of the necessity to assign a dollar value to the determinable benefits of time savings.