Fractional Factorial Analysis for Flexible Pavement Performance Data

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• The first objective of the AASHO Road Test was to find significant relationships between pavement performance and certain characteristics of pavement design and applied loads. The rationale for achieving this objective has been described in considerable detail by Irick *et al.* (*1*). An understanding of this reference would be helpful since essentially the same rationale was used in the analysis described in this paper. The pavement serviceability-performance concept which served to define a performance variable was described by Carey and Irick (*2*). Statistical concepts primarily oriented toward the Road Test were explained elsewhere (*3, 4, 5*).

Within each Test Road loop the main flexible experiment (Design 1) sections existed as a complete factorial experiment; that is, within a loop every design factor level occurred in combination with all other design factor levels. In addition, the factorial experiment was designed so that a properly selected subset of all the sections would be analyzed independently of the remaining sections. This subset was called a fractional replicate; at the Road Test it was also called a factorial block. It is the purpose of this paper to show that an analysis using a subset of the flexible sections would have produced essentially the same results as an analysis of the complete factorial experiment (*6*).

It should be emphasized that certain assumptions must be true before a subset or fractional replicate of a complete factorial experiment can give useful information. Therefore, it may be necessary to employ pilot experiments to determine if these assumptions are true or false. At the Road Test these assumptions turned out to be valid for certain variables including performance. It then follows that for other experiments where these assumptions are true the fractionally replicated experiment has an economic advantage over a complete factorial experiment since it is smaller and can give equivalent information.

A secondary purpose of this paper is to describe fractional replicate concepts. The conditions which must be met before fractions of complete factorial experiments can be used are explained. There are many books dealing with fractional replicate theory; for further reference (*7* through *12*).

A simple example will be used to explain the statistical and mathematical concepts of fractional replication. For this example, the design variables are asphalt surface thickness and crushed stone base thickness; the asphalt is 3 and 5 in. thick and the crushed stone 3 and 9 in. thick. The dependent variable Y can be any measured variable: performance (properly defined), deflection or rut depth, for example. For illustration, the dependent variable is the performance of a section under a specified load, the same load for all sections.

STATISTICAL CONCEPTS

The effect of a design variable, such as surface thickness, may be thought of as a change in the measured variable (performance) which is clearly attributable to a change in the design variable. If the principle of balance is used, the effect of surface thickness may be estimated independently of the effect of base thickness. Conventional application of this principle leads to a complete factorial experiment, a "strong" balanced experiment. The strength of a factorial experiment is due, in part, to the following:

1. The main effect of each design factor may be estimated independently of the main effect of every other design factor.
2. Interaction of the design factors can be examined. (Briefly an interaction occurs when the effect of one variable is different as the levels of another variable are changed.)
3. Effects can be determined with relatively high precision and efficiency since all of the measured Y's (dependent variables) are used to estimate each effect.
4. If certain effects are small or not significant they may be used to estimate error; that is, the effect of uncontrolled variation. This is sometimes referred to as the "hidden replication" present in a factorial experiment.

A factorial layout of the illustrative experiment is shown in Table 1. Balance is achieved by having 3 and 9 in. of base under both 3 and

TABLE 1

FACTORIAL LAYOUT OF THE ILLUSTRATIVE EXPERIMENT

Base Thickness (in.)	Surface Thickness (in.)	
	3	5
3	Y_{11}	Y_{21}
9	Y_{12}	Y_{22}

5 in. of surface. Thus, this experiment has $2 \times 2 = 4$ sections.

Let Y_{ij} be the performance of the section at the i^{th} level of surface thickness and j^{th} level of base thickness, where $i = 1, 2$ and $j = 1, 2$. For example, Y_{21} denotes the performance of the section with 5 in. of surface ($i = 2$) and 3 in. of base ($j = 1$).

The effect of base under 3 in. of surface can be measured by observing the difference between Y_{11} and Y_{12} ($Y_{12} - Y_{11}$); the effect of base can also be measured under 5 in. of surface by observing the difference between Y_{21} and Y_{22} ($Y_{22} - Y_{21}$). An interaction implies that these differences are not the same, *i.e.*, $Y_{12} - Y_{11}$ is not equal to $Y_{22} - Y_{21}$.

Before any statement can be made about the significance of a surface thickness effect, for example, it must be known how differently two sections with the same surface thickness perform. That is, how do uncontrolled variables affect performance. In general, the same designs must be repeated or replicated in space and/or time if any conclusion is to be made about the significance of the effect of a design variable. Replication makes it possible to estimate the effects of uncontrolled variables on the dependent variables. Replicates can be constructed in a variety of ways. The entire experiment can be constructed at another time or place, one or more of the original sections can be repeated, or replicate measurements can be made on the same section. Thus, at any level of observation replication must be used if there is any reason to draw conclusions about the effects of controlled variables at that level. Replication variation or replication error serves as a yardstick against which the apparent effects of controlled variables can be gaged.

At the Road Test certain sections were replicated in every loop and at times replicate measurements were made on every section. However, the lanes were not replicated nor were the loops. Therefore, there was no direct estimate of error for gaging the effect of lane or loop. The illustrative experiment could be replicated by taking repeated measurements (without changing the level of any other variable such as load applications), by building some or all of the sections more than once, or by repeating the entire experiment. The amount of replication would depend on whether conclusions would be made about measurement variables, section design variables or "loop" variables.

CONFOUNDING

Randomization reduces the risk that the effects of controlled variables will be confused with the effects of uncontrolled variables that vary systematically. For example, if the illustrative experiment was laid out so that both of the 3-in. surface sections were on relatively poor soil and both of the 5-in. surface sections were on relatively good soil then the effect of surface would be confused or confounded with the effect of soil. It would not be possible to determine how much increased performance was due to increasing surface thickness and how much was due to changing from poor to good soil. The randomized order of the illustrative sections is shown in Figure 1.

While it is never desirable that effects of uncontrolled variables be confused or confounded with effects of controlled variables, there may be advantages to purposeful confounding of certain effects of controlled variables with one another. In general, it is most useful to confound interacting effects of controlled variables. It will be shown that this type of confounding is most useful for reducing the size of an experiment or for gaining information about more variables.

One of the penalties of confounding "main effects", *e.g.*, the surface effect and the base effect, is that it is impossible to separate the effects of the different variables. For example, an experiment might be designed so that the thinnest section had 0 in. of surface and 0 in. of base and the next thickest section had 1 in. of surface and 3 in. of base with each thicker section having 1 in. more of surface and, at the same time, 3 in. more of base. It would be impossible to tell how much change in a dependent variable was associated with the 1-in. increase in surface and how much was associated with the 3-in. increase in base. There is, obviously, no information about interacting effects. If it can be assumed that an effect is negligible or is not present, this effect can be confounded with a second effect with no loss of information about the second effect. In summary, confounding, if it is properly done, may be beneficial; improperly done, confounding may make it impossible to discriminate between the effects of variables.

Confounding may be used for one of two purposes:

1. To include more variables without changing the size of an experiment; the number of sections will remain the same but the effects of more variables will be examined.
2. To reduce the size of an experiment

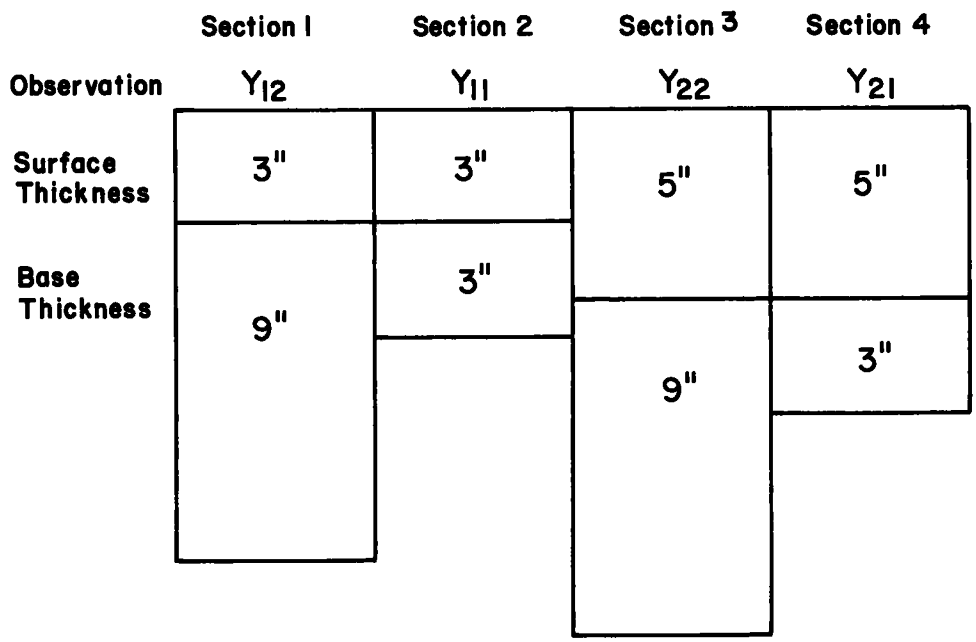


Figure 1. Randomized order of illustrative sections.

through the use of fractional replication; the number of factors remains the same but there are fewer sections than in a complete factorial experiment.

The example will be used to show how confounding may be applied in a statistical sense to satisfy either of the objectives. First of all for the illustrative experiment, let D_1 stand for the surface variable and let D_{1i} stand for the i^{th} level of this variable where $i = 1, 2$. Similarly let D_{2j} stand for the j^{th} level of the base variable where $j = 1, 2$. For convenience D_1 and D_2 are coded so that the thinner level is coded to -1 and the thicker level is coded to $+1$. The coded variables are called A and B. For example,

$$A_1 = \frac{D_{11} - 4}{1} = \frac{3 - 4}{1} = -1$$

and

$$A_2 = \frac{D_{12} - 4}{1} = \frac{5 - 4}{1} = +1$$

Thus, Section 2 with an observed performance equal to Y_{11} is coded as $(-1, -1)$. The interaction between surface and base is called AB and is the product of A_i times B_j. The coding is given in Table 2.

Before the experiment is performed however, another variable, subbase thickness, is included. This variable is denoted by D_3, its levels are denoted by D_{3k} where $k = 1, 2$. It must be assumed that there are no interactions between either surface and subbase or base and subbase. Furthermore, it is assumed that there is no interaction between surface and base, *i.e.*, that the AB interaction is negligible. AB can be equated to or confounded with the new variable subbase in the following manner. Subbase

TABLE 2

ILLUSTRATIVE EXPERIMENT, BLOCK 1

Section	Observation	Surface Thickness (in.) D_{1i}	Base Thickness (in.) D_{2j}	Coded Levels* Surface A_i	Coded Levels* Base B_j	Coded Levels* Subbase $A_iB_j(=C_k)$	Subbase Thickness (in.) D_{3k}
2	Y_{11}	3	3	−1	−1	+1	8
1	Y_{12}	3	9	−1	+1	−1	4
4	Y_{21}	5	3	+1	−1	−1	4
3	Y_{22}	5	9	+1	+1	+1	8

$^*A_i = \dfrac{D_{1i} - 4}{1}$, $B_j = \dfrac{D_{2j} - 6}{3}$, $C_k = \dfrac{D_{3k} - 6}{2}$

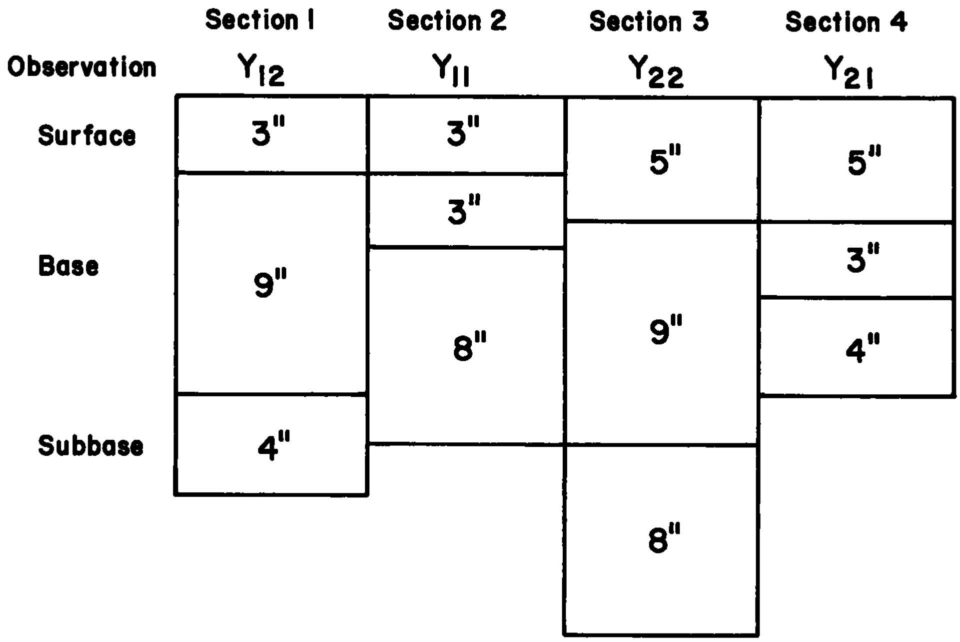


Figure 2. Randomized order of illustrative sections including subbase thickness, Block 1.

TABLE 3

ILLUSTRATIVE EXPERIMENT, BLOCK 2

Section	Observation	Surface Thickness (in.) D_{1i}	Surface Thickness (in.) D_{2j}	Coded Levels* Surface A_i	Base B_j	Subbase $-A_iB_j(=C_k)$	Subbase Thickness (in.) D_{3k}
8	Y'_{11}	3	3	−1	−1	−1	4
6	Y'_{12}	3	9	−1	+1	+1	8
5	Y'_{21}	5	3	+1	−1	+1	8
7	Y'_{22}	5	9	+1	+1	−1	4

$^*A_i = \dfrac{D_{1i} - 4}{1}$, $B_j = \dfrac{D_{2j} - 6}{3}$, $C_k = \dfrac{D_{3k} - 6}{2}$

thickness (D_3) is coded to −1 for the thinner level and to +1 for the thicker level. The coded levels are denoted by C_k. For example,

$$C_1 = \frac{D_{31} - 6}{2} = \frac{4 - 6}{2} = -1.$$

Then $C = -1$ can be equated to $AB = -1$ and $C = +1$ can be equated to $AB = +1$. This technique results in 8 in. of subbase being confounded with $AB = +1$ while 4 in. of subbase is confounded with $AB = -1$ (Table 2). In other words, subbase thickness has been confounded with the surface-base (AB) interaction. The result is that another factor has been included without increasing the size of the original experiment. It must be emphasized again that any interacting effects between the variables must be negligible.

The foregoing explanation showed that with proper design an extra factor could be included in an experiment without increasing the size of the experiment. Using the same assumptions of no interactions a subset or a fractional replicate of a complete factorial experiment could also be constructed. It will be shown that the experimental design in Table 2 can also be viewed as a one-half replicate of a complete factorial experiment. The complete factorial experiment would have three factors at two levels and contain eight sections. Let the experiment shown in Table 2 be called Block 1 to imply that it is a fractional replicate. The randomized layout of this experiment (Block 1) is shown in Figure 2.

Table 3 shows the design of an experiment much like that in Table 2. However, there are four new sections called 5, 6, 7 and 8 with observations on the dependent variable (performance) denoted by Y'. As before, the 3-in.

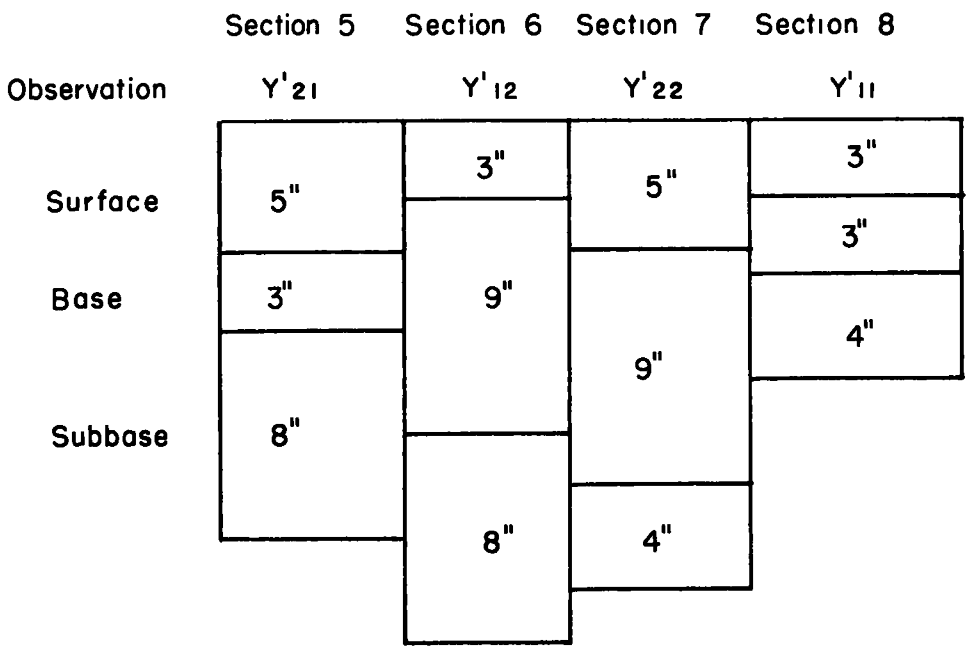


Figure 3. Randomized order of illustrative sections including subbase thickness, Block 2.

TABLE 4

LEVELS FOR A 2×2×2 COMPLETE FACTORIAL EXPERIMENT

Section	Observation	Design Levels: Surface Thickness (in.) D_{1i}	Design Levels: Base Thickness (in.) D_{2j}	Design Levels: Subbase Thickness (in.) D_{3k}	Coded Levels*: Surface A_i	Coded Levels*: Base B_j	Coded Levels*: Subbase C_k	Three Factor Interaction $A_iB_jC_k$	Block
8	Y'_{11}	3	3	4	−1	−1	−1	−1	2
2	Y_{11}	3	3	8	−1	−1	+1	+1	1
1	Y_{12}	3	9	4	−1	+1	−1	+1	1
6	Y'_{12}	3	9	8	−1	+1	+1	−1	2
4	Y_{21}	5	3	4	+1	−1	−1	+1	1
5	Y'_{21}	5	3	8	+1	−1	+1	−1	2
7	Y'_{22}	5	9	4	+1	+1	−1	−1	2
3	Y_{22}	5	9	8	+1	+1	+1	+1	1

$*A_i = \dfrac{D_{1i} - 4}{1}$, $B_j = \dfrac{D_{2j} - 6}{3}$, $C_k = \dfrac{D_{3k} - 6}{2}$

level of surface has been coded to −1 and the 5-in. level has been coded to +1; the 3-in. level of base has been coded to −1 while the 9-in. level has been coded to +1. Again, with the assumption of no interaction, a third variable, subbase, has been introduced. However, this time coded subbase (C_k) has been equated to or confounded with $-AB$. Table 3 indicates that 8 in. of subbase has been confounded with $AB = +1$, whereas 4 in. of subbase has been confounded with $AB = -1$. Subbase thickness has now been confounded in a slightly different manner than in Block 1 with the surface-base ($-AB$) interaction. This is another one-half replicate of an experiment with three factors at two levels. This one-half replicate will be called Block 2 (Fig. 3).

Blocks 1 and 2 are in a sense complementary. This can be seen by observing Section 2 in Block 1 (Table 2) and Section 8 in Block 2 (Table 3). Both sections have 3 in. of surface and 3 in. of base. However, under Section 2 the subbase is 8 in. thick, whereas under Section 8 it is 4 in. thick. Essentially the same is true for each pair of sections with equal surface and subbase levels; a section with 4 in. of subbase in one block will have 8 in. of subbase in the other block or vice-versa.

Table 4 shows the design level layout for an experiment with three factors, surface thickness, base thickness, and subbase thickness at two levels. There are $2 \times 2 \times 2 = 2^3 = 8$ experimental sections. The coded levels A_i, B_j and C_k are calculated using exactly the same method described previously. There are also three 2-factor interactions denoted by AB, AC and BC and a 3-factor interaction denoted by ABC. Only the levels of the 3-factor interaction are shown. Four of the sections have exactly the same levels as the sections in Block 1 (Table 2); they are labeled in the last column of Table 4 as Block 1. In addition, each of these sections labeled Block 1 has +1 for ABC. The remaining sections have exactly the same levels as the sections in Block 2 (Table 3); they are labeled Block 2. Each of these sections has a —1 for ABC. Thus, Block 1 and Block 2 correspond to four sections selected from eight sections of a 2^3 complete factorial experiment. Therefore, the blocks are complementary to each other. This is the reason why each block is called a one-half replicate of 2^3 complete factorial experiment. Each block can be utilized as an independent experiment and analyzed for the effects of surface, base and subbase (provided that there are no interactions). The complete factorial experiment would, of course, give complete information about the effects of the design variables and their interactions.

In summary, it has been demonstrated that with four sections and negligible interacting effects it is possible to test three factors at two levels. The first purpose was satisfied since it was desired to include an additional variable subbase without changing the size of the original 2×2 experiment. A complete factorial experiment containing three factors at two levels would have eight sections. Full information on the effects of the three factors and their interactions would have been available. However, with the assumption that interactions were negligible it was possible to obtain information about the main effects of the three factors using only four sections. Obviously no information is available about interactions. This would satisfy the second purpose since the size of the experiment was halved and, in general, four sections would cost less to test than eight sections.

It should be added that the illustrative experiment is impractical for a normal experiment for the following reasons:

1. There is no provision for estimating replication error—this could be remedied by replicating.
2. With only two levels for each variable there is no information available about curvature or trends; thus, it might be rash to fit an empirical model to the data.
3. With only four observations one extreme value or outlying point would have a drastic effect on the analysis.

This experiment might be of some practical value if there is a prior estimate of error or if it is only necessary to estimate the magnitude of an effect.

The preceding discussion about confounding was intended to show that if there is information about interactions it is possible to design an experiment which makes information available more economically than a complete factorial experiment. This might be done in one of two ways:

1. Retaining the same number of sections but putting additional factors into the experiment.
2. Studying the same number of factors but constructing fewer sections.

The same objective, that of studying the effects of variables and their interactions, could still be reached.

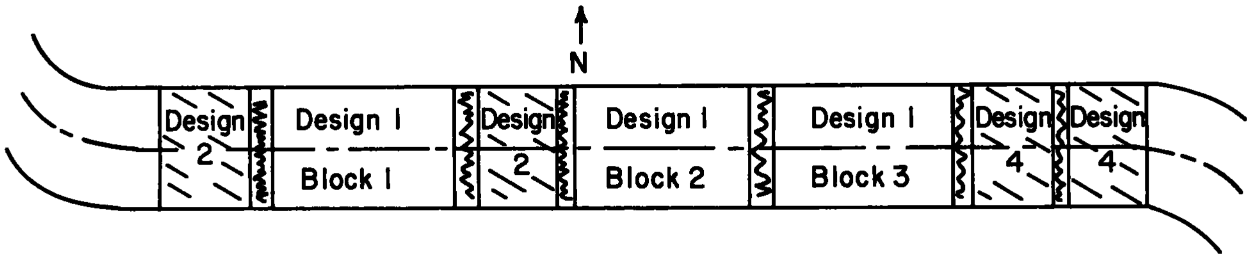
In addition, the fractional replicate has other advantages:

1. If the number of factors and/or levels of an experiment is large the number of sections needed for a complete factorial experiment may become prohibitive. There may not be enough time or money available to perform the complete factorial experiment.
2. Using a fraction of a complete factorial it is less disheartening and costly to include design variables which show up to have no effect on the dependent variable. Furthermore, if an experiment is part of a series of experiments the next experiment, where the nonsignificant factor will be dropped, will occur sooner.
3. If it is necessary to obtain more information about a set of effects and their interactions more of the sets of fractional replicates or even the complete factorial experiment can be performed.

CONFOUNDING AT THE AASHO ROAD TEST

The flexible tangent of each loop at the AASHO Road Test was designed as a complete factorial experiment. There were three pavement design factors, asphalt surface thickness convention (usually called surfacing thickness), crushed limestone base thickness and sand-gravel subbase thickness. Surface thickness convention implies that somewhat different construction procedures and materials were used for different thicknesses of surface.

The pavement design factors in Loops 3 to 6 (called the main traffic loops) had three levels. For example, in Loop 3 there were 2, 3 and 4 in. of surface, 0, 3 and 6 in. of base and 0, 4 and 8 in. of subbase. Thus, there were $3 \times 3 \times 3 = 3^3 = 27$ structural sections in the flexible tangent of each main traffic loop.



PAVEMENT DESIGNS FOR FRACTIONAL REPLICATES

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Block 1	Block 2	Block 3
3-0-4	3-0-8	5-0-8
5-3-8	5-0-4	3-6-4
5-6-4 *	4-0-12	4-0-4
4-3-4	3-6-12	5-6-12
4-6-12	3-3-4	5-3-4
3-6-8	4-3-8 *	4-6-8
5-0-12	4-6-4	4-3-12
4-0-8	5-3-12	3-3-8
3-3-12	5-6-8	3-0-12 *
5-6-4 *	4-3-8 *	3-0-12 *
Avg. 4-3-8	Avg. 4-3-8	Avg. 4-3-8

* Replicates

Figure 4. Layout for main loop flexible pavement tangent.

Loop 2 had only two levels of subbase thickness, 0 and 4 in., so there were $3 \times 3 \times 2 = 18$ structural sections on the flexible side of Loop 2.

In addition, there was a load (or lane) factor in each of the traffic loops (2 to 6). For example, Loop 3 had 12-kip single-axle trucks in the inner lane and 24-kip tandem-axle trucks in the outer lane. In the main traffic loops all single-axle vehicles were in the inner lane and all tandem-axle vehicles were in the outer lane so that load type was confounded with lane. Loop 2 had single-axle vehicles with different loads in each lane.

The 27 structural sections in each of the main traffic loops were partitioned into three blocks. Each block was a one-third fractional replicate of the complete factorial experiment in that loop. (The fractional replicates were designed according to the layout in Table 9.62 (*7*).) This was not done in Loop 2. The fractional replicates were commonly called factorial blocks. (For brevity a block in this paper will denote a one-third fractional replicate of the complete factorial experiment in a loop.) Each block was assigned at random to one portion of the flexible side of a loop. The same randomization was used in all four loops. The randomized blocks were designated as the west factorial block, the center factorial block and the east factorial block; or alternatively as Block 1, Block 2 and Block 3. A typical layout of the randomized factorial blocks is shown in Figure 4. Structural sections were arranged in random order from west to east within their respective blocks. Figure 4 also shows a typical arrangement of structural sections in blocks. The section-in-block randomization was different for each loop and block. The remaining flexible sections in each loop were used in special studies of base type and shoulder paving. The special studies were also laid out in randomized blocks, there being four special study blocks in each main loop tangent.

The blocks or fractional replicates had some important characteristics designed into them; some of these characteristics which were useful for analyzing the results of the experiment were as follows:

1. All three levels of each of the pavement design factors occurred in each block.
2. The average total pavement thickness was the same for each block in a loop.
3. Three sections were replicated or constructed twice in each loop; one of these sections was assigned to each block.
4. The total thickness of these replicate sec-

tions was equal to the average total thickness of the blocks, providing an arbitrary rule for deciding which designs to replicate.

It might be profitable to discuss the possibility of having confounded effects of variables at the Road Test. As has been pointed out confounding could have been used to advantage in one of two ways:

1. The size of the experiment could have been reduced by including only one or two of the three fractional replicates (blocks).
2. More variables could have been introduced with the same number of sections.

The possibility of confounding and thereby reducing the size of the experiment or including more variables was carefully considered prior to designing the Test Road. However, as was shown in the illustrative example, certain interactions between surface, base and subbase had to be negligible before a confounded design could provide useful information. Because it was felt that there was insufficient information about interactions to warrant confounding at the Road Test, the complete factorial experiment was constructed in each loop. To provide additional useful information about future possibilities for using confounding, the sections within the complete factorial experiment were partitioned into fractional replicates or blocks. It is the purpose of this paper to show that since the complete factorial experiment was used and interactions were virtually negligible, any fractional replicate should give essentially the same information as the complete factorial experiment. Inasmuch as the complete factorials were available, it was possible to compare the results from each fractional replicate with each other and with the complete factorial experiment.

PAVEMENT PERFORMANCE

The same pavement performance data were used as were previously reported (*6*). In addition, the following definitions have been given (*1*):

Serviceability trend is the plot of smoothed serviceability history values p on an accumulated axle applications scale W where axle applications may be weighted or unweighted; and

Performance of a pavement is given by its serviceability trend.

For the purposes of this paper flexible weighted applications only were used. It was supposed that unweighted applications would result in essentially the same conclusions. Every section was represented by five pairs of coordinates from its serviceability trend. Sections whose serviceability trend had fallen to 1.5 were represented by five values of W noted when p reached 3.5, 3.0, 2.5, 2.0 and 1.5. Sections whose trend had not reached 1.5 were represented by five values of p recorded on 11, 22, 33, 44 and 55 index days. For analytical purposes it was more convenient to use log W instead of W; therefore, a section which reached 1.5 after one million weighted applications of load would have as one of its coordinates 6.0 and 1.5 for log W and p, respectively.

ANALYSIS

The procedures used in the analysis described in this paper were essentially the same as those previously described (*1*). The procedures were used to fit an equation to the observed performance data so that the equation related performance to pavement design and load variables. Certain constants were specified; others were estimated using statistical and mathematical techniques. The analysis described in this paper differed from previous analyses in that each factorial block was treated as a separate entity. The results from the blocks were compared among themselves and with the results from an analysis of the complete factorial experiment.

The results of this analysis cannot prove that the procedures used will always result in unbiased estimates of the coefficients; the results can only show that when the procedures are used with slightly different sets of data the results do not change very much.

The performance equation used at the AASHO Road Test was of the form:

$$p = c_0 - (c_0 - c_1)\left(\frac{W}{\rho}\right)^{\beta} \qquad (1)$$

in which $c_1 \leq p \leq c_0$.

This equation was transformed so that the resulting equation was linear in β and log ρ

$$\log (c_0 - p) = \log (c_0 - c_1) + \beta (\log W - \log \rho) \qquad (2)$$

or

$$\log\left(\frac{c_0 - p}{c_0 - c_1}\right) = \beta (\log W - \log \rho) \qquad (3)$$

For convenience the left side of Eq. 3 was called G, so that

$$G = \log\left(\frac{c_0 - p}{c_0 - c_1}\right) \qquad (4)$$

in which G is undefined unless $p < c_0$. Substituting into Eq. 3 from Eq. 4

$$G = \beta (\log W - \log \rho) \qquad (5)$$

For the flexible pavement analysis c_0 was 4.2, the average initial serviceability of all flexible factorial sections, while c_1 was 1.5. The average initial serviceability of all the sections in a block

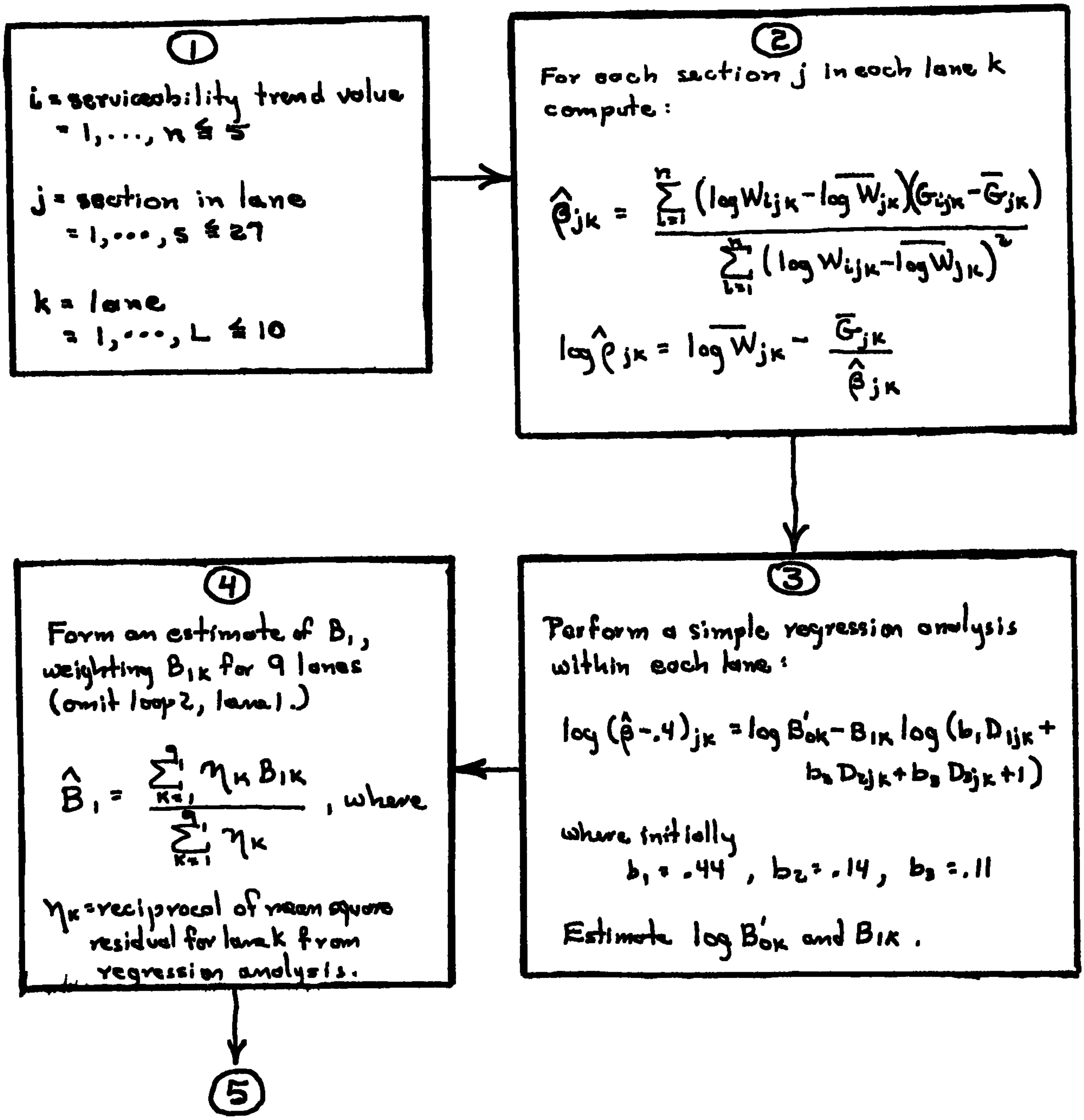

Figure 5. Flow chart for performance data analysis.

was also computed for each block. These values either rounded to 4.2 or were not significantly different from 4.2; therefore, $c_0 = 4.2$ was used for all blocks. The p and W coordinates for each section were converted to G and log W coordinates. In general, there were five new coordinates, however, if any p's were greater than 4.2 there were correspondingly fewer points. A least squares linear regression analysis was used to fit a straight line to these points. The slope of the line, β, was estimated by $\hat{\beta}$; the intercept on the abscissa, log ρ, was estimated by log $\hat{\rho}$. The equations for $\hat{\beta}$ and log $\hat{\rho}$ are shown in Figure 5, box 2.

It was not possible in this analysis to use Loop 2 as it was designed since there were only two levels of subbase (0 and 4 in.) and no factorial blocks as such. However, for the block analysis hypothetical values of $\hat{\beta}$ and log $\hat{\rho}$ were inserted for fictitious sections with 8 in. of subbase. These hypothetical values were based on experience at the Road Test indicating that 4 in. of subbase were approximately equal to 3 in. of base. This assumed, for example, an

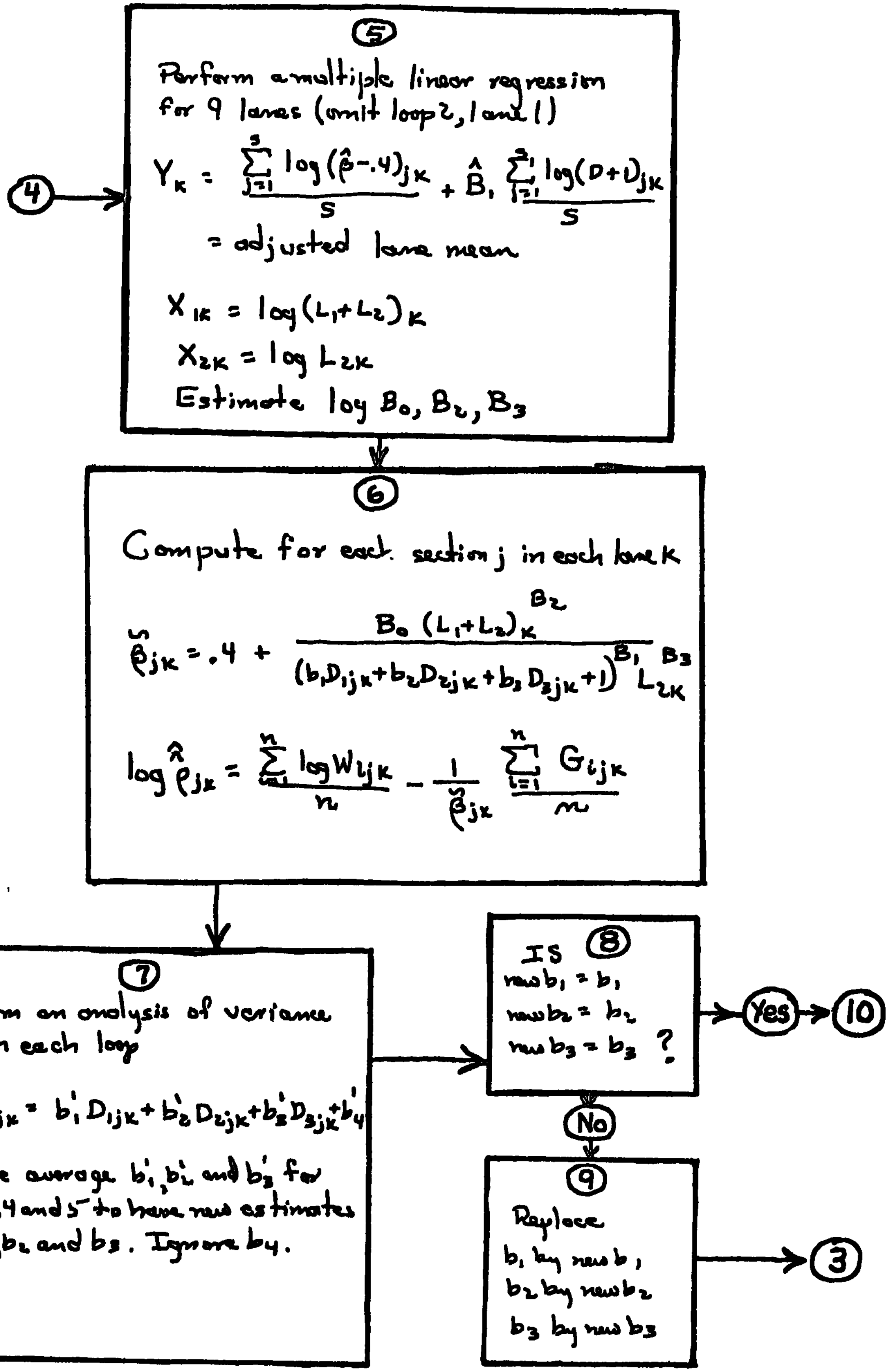

Figure 5. Flow chart (continued).

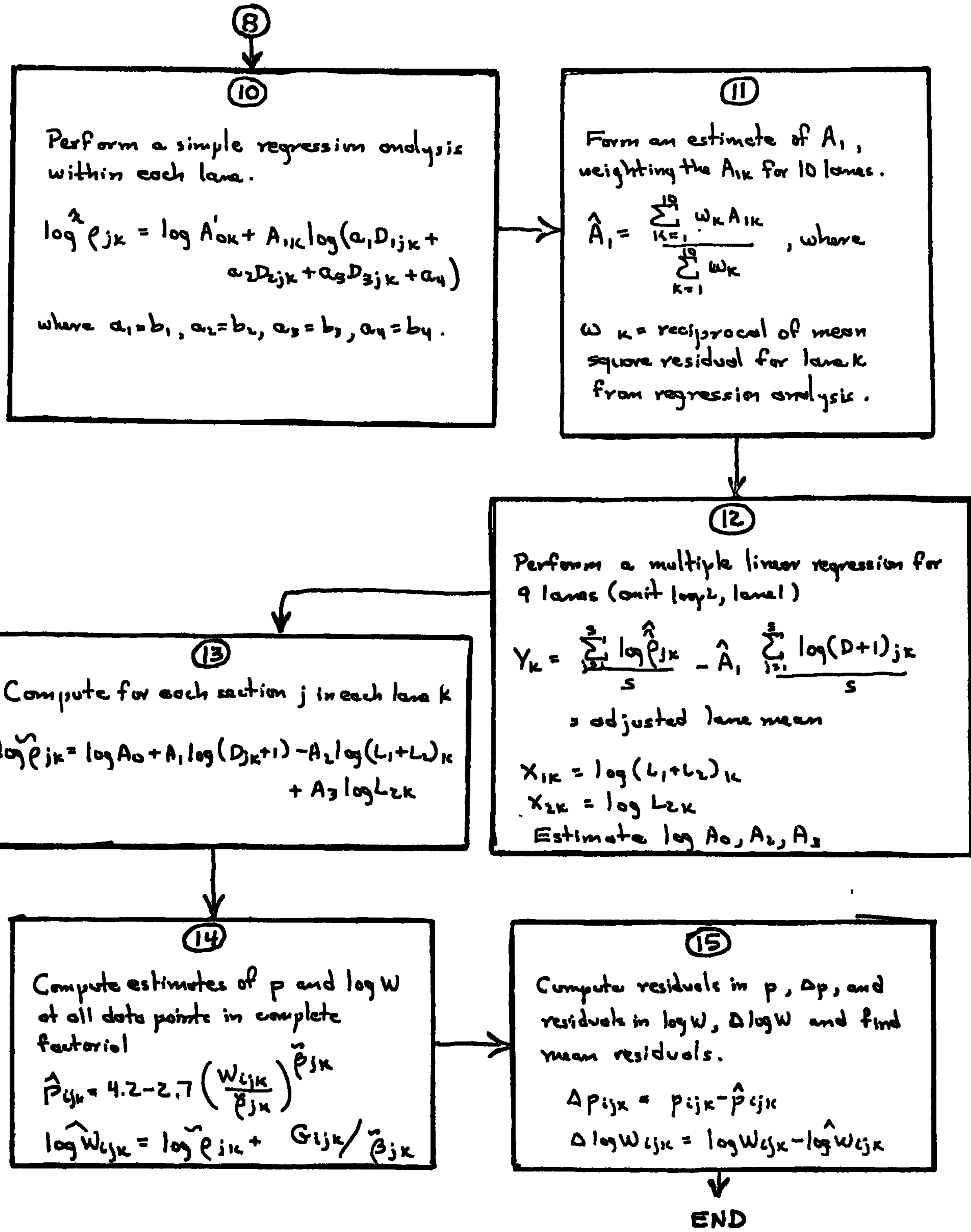

Figure 5. Flow chart (continued).

approximately equivalent relationship between a real section of 2-3-4 design for surface, base and subbase, respectively, and a section of 2-0-8 design. Removing 3 in. of base could be compensated for by adding 4 in. of subbase. This does not imply that this relationship holds outside of the Road Test, for other combinations of base and subbase material or for analyses of other data. It should be noted that the effect of this assumption on the analysis was relatively small since, in general, Loop 2, lane 1 was not included in the performance analysis and the nine fictitious sections in Loop 2, lane 2 were among 234 observed sections.

It was assumed that β was related to design and load in the following manner,

$$\beta = \beta_0 + \frac{B_0 (L_1 + L_2)^{B_2}}{(b_1D_1 + b_2D_2 + b_3D_3 + b_4)^{B_1} L_2^{B_3}} \tag{6}$$

in which

β_0 = an assumed minimum value for β;
L_1 = the nominal axle load, in kips;
L_2 = 1 for single-axle vehicles, 2 for tandem-axle vehicles;
D_1 = surface thickness in inches;
D_2 = base thickness, in inches;
D_3 = subbase thickness, in inches; and
b_4 = a positive constant with an assumed value.

The remaining letters denote positive constants which were estimated from the $\hat{\beta}$ values. Rearranging and taking logarithms, Eq. 6 reduced to

$$\log (\beta - \beta_0) = \log B_0 - B_1 \log (b_1D_1 + b_2D_2 + b_3D_3 + b_4) + B_2 \log (L_1 + L_2) - B_3 \log L_2 \tag{7}$$

At the Road Test it was assumed that $\beta_0 = 0.4$ and $b_4 = 1.0$. Eq. 7 reduced to

$$\log (\beta - 0.4) = \log B_0 - B_1 \log (b_1D_1 + b_2D_2 + b_3D_3 + 1) + B_2 \log (L_1 + L_2) - B_3 \log L_2 \tag{8}$$

This equation was a linear regression equation with log $(\beta - 0.4)$ as the dependent variable and three independent variables: (a) log $(b_1D_1 + b_2D_2 + b_3D_3 + 1)$; (b) log $(L_1 + L_2)$; and (c) log L_2.

The parameters were estimated by an iterative process. The flow chart for this process is shown in Figure 5. Each of the three blocks were analyzed independently using this iterative process. In other words, three parallel block analyses were conducted simultaneously following the procedure in the flow chart.

First of all (as shown in Box 3), regression analyses were made for each of nine lanes using log $(\hat{\beta} - 0.4)$ as the dependent variable and log $(0.44D_1 + 0.14D_2 + 0.11D_3 + 1)$ as the independent variable. Loop 2, lane 1 was omitted since very few of the $\hat{\beta}$ values were greater than the minimum of 0.4, *i.e.*, not enough sections showed a decline in serviceability. The estimates of log B_0 were ignored; the nine lane estimates of B_1 were formed into a weighted average (Box 4) to give an estimate of B_1 called $\hat{B}_1$. Table 5 shows the estimate of B_1 for the necessary iterations as well as for the estimates derived from the complete factorial experiment (*6*).

For simplicity let $D = b_1D_1 + b_2D_2 + b_3D_3$. A transposition of B_1 log $(D + 1)$ in Eq. 8 yielded

$$\log (\beta - 0.4) + B_1 \log (D + 1) = \log B_0 + B_2 \log (L_1 + L_2) - B_3 \log L_2 \tag{9}$$

The left side of Eq. 9 was called an adjusted log $(\beta - 0.4)$. The lane average of the left

TABLE 5

ESTIMATES OF PARAMETERS FOR $\tilde{\beta}$ AND LOG $\tilde{\rho}$ EQUATIONS

(*a*) ESTIMATES OF PARAMETERS FOR $\tilde{\beta}$

		β_0	B_0	B_1	b_1	b_2	b_3	b_4	B_2	B_3
Reference (*6*)	Over-all Equation	0 4	0 081	5 19	0 44	0 14	0 11	1	3 23	3 23
Initial Values First Iteration	Block 1	0 4	0 068	5 97	0 44	0 14	0 11	1	3 67	3 67
	Block 2	0 4	0 138	4 15	0 44	0 14	0 11	1	2 54	2 54
	Block 3	0 4	0 083	5 80	0 44	0 14	0 11	1	3 56	3 56
Second Iteration	Block 1	0 4	0 065	5 91	0 446	0 125	0 108	1	3 62	3 62
	Block 2	0 4	0 151	4 15	0 464	0 147	0 105	1	2 54	2 54
	Block 3	0 4	0 085	5 82	0 423	0 156	0 120	1	3 60	3 60
Final Values Third Iteration	Block 1	0 4	0 063	5 90	0 45	0 12	0 11	1	3 63	3 63
	Block 2	0 4	0 151	4 18	0 46	0 15	0 10	1	2 54	2 54
	Block 3	0 4	0 085	5 83	0 42	0 16	0 12	1	3 61	3 61

(*b*) ESTIMATES OF PARAMETERS FOR LOG $\tilde{\rho}$

		log A_0	A_1	a_1	a_2	a_3	a_4	A_2	A_3
Reference (*6*)	Over-all Equation	5 93	9 36	0 44	0 14	0 11	1	4 79	4 33
Block Analysis	Block 1	5 96	9 30	0 45	0 12	0 11	1	4 81	4 48
	Block 2	5 72	9 12	0 46	0 15	0 10	1	4 50	4 00
	Block 3	6 06	9 40	0 42	0 16	0 11	1	4 99	4 43

side of Eq. 9 was called an adjusted lane mean; it was also a linear function of log $(L_1 + L_2)$ and log L_2. Next in the procedure, average [log $(\beta - 0.4)$] + $\hat{B}_1$ average [log $(D + 1)$] was regressed on log $(L_1 + L_2)$ and log L_2 to obtain estimates of log B_0, B_2 and B_3 (Box 5, Fig. 5). The estimates for B_0, B_2 and B_3 are shown in Table 5. For each block there was an equation which related β to the specified design and load variables. The equation derived at the Road Test from the complete factorial experiment was the following (*6*):

$$\log(\tilde{\beta} - 0.4) = -1.09 - 5.19 \log(0.44D_1 + 0.14D_2 + 0.11D_3 + 1) + 3.23 \log(L_1 + L_2) - 3.23 \log L_2 \qquad (10)$$

or

$$\tilde{\beta} = 0.4 + \frac{0.081\ (L_1 + L_2)^{3.23}}{(0.44D_1 + 0.14D_2 + 0.11D_3 + 1)^{5.19}\ L_2^{3.23}} \qquad (11)$$

where $\tilde{\beta}$ was a final estimate of β.

The next step in the procedure was to rewrite Eq. 5 so that log ρ became a function of β.

$$\log \rho = \log W - \frac{G}{\beta} \qquad (12)$$

$\tilde{\beta}$ was introduced as an estimate of β and a new, second estimate of log ρ was computed from

$$\log \hat{\rho} = \overline{\log W} - \frac{\overline{G}}{\tilde{\beta}} \qquad (13)$$

Box 6, Figure 5 shows the computations.

New estimates of b_1, b_2 and b_3 were then found by making an analysis of variance in each loop using log $\hat{\rho}$ as the dependent variable and D_1, D_2 and D_3 as the independent variables (Box 7, Fig. 5). This analysis assumed a linear relationship between log ρ and D_1, D_2 and D_3. This was done only to provide estimates of b_1, b_2 and b_3 and was not hypothesized as a model. The new estimates of b_1, b_2 and b_3 were averages of the coefficients computed from the Loop 3, 4 and 5 analyses. This was done because the estimates from Loops 3, 4 and 5 always tended to be homogeneous, whereas those from Loop 2 were high and those from Loop 6 were low.

The new estimates of b_1, b_2 and b_3 were compared to those found in the previous iteration (Table 5). If they were equal the iteration was stopped. If the coefficients were different the new estimates of b_1, b_2 and b_3 were used in log $(D + 1)$ and the complete procedure of estimating β was repeated starting with Eq. 8 (Box 3). The estimates of the various parameters computed during successive iterations are also shown in Table 5.

By the third iteration the input values of b_1, b_2 and b_3 (2nd iteration) and the output of the analysis of variance (3rd iteration) were equal when rounded to two decimal places. Iteration was stopped and the final procedure for estimating log ρ was started.

The equation relating log ρ to design and load variables was assumed to have the form

$$\rho = \frac{A_0\ (a_1D_1 + a_2D_2 + a_3D_3 + a_4)^{A_1}\ L_2^{A_3}}{(L_1 + L_2)^{A_2}} \qquad (14)$$

in which the variables and parameters had the same properties as specified for Eq. 6. In addition, it was assumed that $b_1 = a_1$, $b_2 = a_2$ and $b_3 = a_3$. This was the reason for the iterative procedure.

Logarithms of both sides of Eq. 14 were taken to produce the following multiple regression equation:

$$\log \rho = \log A_0 + A_1 \log(D + a_4) - A_2 \log(L_1 + L_2) + A_3 \log L_2 \qquad (15)$$

in which $D = a_1D_1 + a_2D_2 + a_3D_3 = b_1D_1 + b_2D_2 + b_3D_3$, and $a_4 = 1$.

The log $\hat{\rho}$ data obtained in Eq. 13 (Box 6, Fig. 5) were regressed on log $(D + 1)$ to obtain estimates of A_1 for each lane (Box 10). Loop 2, lane 1 was included since the log $\hat{\rho}$ data for this lane were quite reasonable. The regression slopes A_1 were averaged (Box 11) for the ten lanes to give an estimate, $\hat{A}_1$, for each block. Table 5 shows these estimates.

A transposition of A_1 log $(D + 1)$ in Eq. 15 then yielded

$$\log \rho - A_1 \log(D + 1) = \log A_0 - A_2 \log(L_1 + L_2) + A_3 \log L_2 \qquad (16)$$

The left side of Eq. 16 was called an adjusted log ρ. The lane average of the left side of Eq. 16 was called an adjusted lane mean; as for log $(\beta - 0.4)$ it was a linear function of log $(L_1 + L_2)$ and log L_2. The lane average was denoted by average log ρ. Next, average [log ρ] $- \hat{A}_1$ average [log $(D + 1)$] was regressed on log $(L_1 + L_2)$ and log L_2 (Box 12), yielding estimates of log A_0, A_2 and A_3. For each block there was now an equation which related log ρ to the specified design and load variables. The equation derived from the complete factorial analysis at the Road Test was (*6*)

$$\log \tilde{\rho} = 5.93 + 9.36 \log(0.44D_1 + 0.14D_2 + 0.11D_3 + 1) - 4.79 \log(L_1 + L_2) + 4.33 \log L_2 \qquad (17)$$

or

$$\tilde{\rho} = \frac{10^{5.93}\ (0.44D_1 + 0.14D_2 + 0.11D_3 + 1)^{9.36}\ L_2^{4.33}}{(L_1 + L_2)^{4.79}} \qquad (18)$$

in which $\log \bar{\rho}$ was the final estimate of $\log \rho$. The corresponding parameter estimates for each block are shown in Table 5.

COMPARISON OF THE ANALYSES

It would be extremely difficult to construct a test which could tell if a_1, a_2 or a_3 were significantly different between blocks. However, a mere scanning of these coefficients seemed to indicate that they were homogeneous. In addition, the average of a_1, a_2 and a_3 across blocks was 0.44, 0.14 and 0.11, respectively, the same as the values from the analysis of the complete factorial experiment. This was an indication that this procedure would give consistent results when applied to data that were somewhat different.

Tests were made to determine if the other coefficients, such as $\log B_0$, B_1 and B_2 were significantly different between blocks. An exact significance test would necessitate an explicit formula for the variance of the estimated coefficients, an extremely difficult task for this procedure. Statistical "t" tests were used which indicated that the coefficients B_2, B_1, A_2 and A_3 were significantly different for Block 2 compared to Blocks 1 or 3. Because of the strong relationship between the procedures for estimating $\log (\beta - \beta_0)$ and $\log \rho$ it was difficult to reduce what these significant differences implied. However, these differences had little effect on the practical outcome of the experiment.

In HRB Special Report 61E, data from the complete factorial experiment were substituted into the performance equation in order to determine residuals from the over-all equation. The summary of these residuals is given in Table 6. In order to compare the goodness of fit of the block equations with the over-all equation, it was decided to compute residuals for all factorial section data, using first the performance equation derived only from Block 1 sections, then Block 2 sections, and finally from Block 3 sections. The following equations were used to obtain estimates for log W and for p.

$$\hat{p} = 4.2 - 2.7\left(\frac{W}{\bar{\rho}}\right)^{\bar{\beta}} \tag{19}$$

$$\widehat{\log W} = \overline{\log}\, \bar{\rho} + \frac{G}{\bar{\beta}} \tag{20}$$

Residuals were computed according to the equations:

$$\Delta p = p - \hat{p} \tag{21}$$

$$\Delta \log W = \log W - \widehat{\log W}. \tag{22}$$

The mean absolute residuals for all sections are shown in Table 6 for both the equations derived from the complete factorial experiment and each of the three factorial blocks. Correlation indexes are also shown for the log W estimates in each of the four cases.

Generally the loop-lane-block residuals were of the same size as the loop-lane residuals for the complete factorial experiment. There was no apparent trend for any one block to have smaller residuals than those for the complete factorial experiment.

If residuals from block equations are computed for only those sections that were in the respective blocks, then the residuals average to be somewhat smaller than for the over-all equations. However, all sections have been included in the residual summary of Table 6 to show that the equation from any one block can be used to predict the performance of sections whose designs did not appear in the block at all—and that the precision of prediction is essentially as though complete factorial data had been used to determine the equation.

Figures 22 and 23 (*6*) show graphically the results of fitting Eq. 1 to the observed data. The relationships between design and axle load applications for various loads are illustrated at $p = 2.5$ and 1.5. As was stated in AASHO Road Test Report 5, "*These relationships are not intended to be design equations.* However, they can serve as a basis for design procedures in which variables not included in the Road Test, such as soil type, are considered." The corresponding relationships can be compared for a one-third fractional replicate and the complete factorial experiment. Figure 6 shows a comparison for an 18-kip single-axle load.

There is obviously little practical difference between the results from a one-third fractional replicate and the complete factorial experiment. In the range of weighted axle-load applications shown, the maximum discrepancy between the complete factorial analysis (*6*) and a factorial block analysis for a constant number of axle-load applications corresponds to an approximate thickness index of 0.2. This roughly corresponds to ½ in. of surface, 1½ in. of base or 2 in. of subbase. However, this variation is within the plausible amount of variation when other factors such as soil, construction, climate, methodology for analysis, and location are varied. In other words, there are many more variables which would affect the final estimates if they were included. There is no reason to believe that the variation between block estimates is any greater than the variation attributable to other omitted factors. From this reasoning the block analyses can be said to give acceptable and valid performance equations relative to the equations determined by data from the complete factorial experiment.

CONCLUSIONS

1. The performance analysis could have been made with a properly selected one-third fractional replicate of the flexible pavement sec-

TABLE 6

SUMMARY OF PERFORMANCE EQUATION RESIDUALS FOR FLEXIBLE PAVEMENTS[1]

Loop	Lane	Load L_1	Load L_2	Source of Data for Equation[2]	p Residuals: No. of Residuals	p Residuals: Mean Absolute Residual			Log W Residuals: No. of Residuals	Log W Residuals: Mean Absolute Residual		
2	1	2	1	FR	97	0 39	0 41	0 43	42	0 87	0 87	0 97
				CF			0 41				0 92	
2	2	6	1	FR	55	0 67	0 46	0 48	83	0 36	0 35	0 37
				CF			0 51				0 36	
3	1	12	1	FR	37	0 55	0 44	0 48	138	0 27	0 30	0 28
				CF			0 46				0 28	
3	2	24	2	FR	31	0 47	0 45	0 48	146	0 32	0 30	0 30
				CF			0 44				0 29	
4	1	18	1	FR	62	0 66	0 50	0 78	136	0 19	0 20	0 21
				CF			0 53				0 19	
4	2	32	2	FR	66	0 33	0 41	0 65	138	0 16	0 17	0 22
				CF			0 41				0 16	
5	1	22.4	1	FR	56	0 64	0 45	0 54	137	0 18	0 25	0 19
				CF			0 46				0 20	
5	2	40	2	FR	63	0 40	0 40	0 49	138	0 25	0 25	0 18
				CF			0 39				0 21	
6	1	30	1	FR	77	0 78	0 46	0 81	133	0 17	0 20	0 19
				CF			0 56				0 18	
6	2	48	2	FR	94	0 42	0 43	0 60	122	0 22	0 23	0 21
				CF			0 46				0 21	
Over-all mean residuals				FR	638	0 54	0 44	0 61	1,171	0 22	0 25	0 23
				CF			0 46				0 23	
Over-all root mean square residuals				FR					1,171	0 31	0 32	0 32
				CF							0 31	
Correlation indexes				FR					1,171	0 69	0 66	0 68
				CF							0 70	

[1] Weighted applications.
[2] FR = fractional replicate, values given in the order Block 1, Block 2, Block 3; CF = complete factorial.

tions. However, this must be qualified in two ways: (a) The fractional factorial analysis depended on negligible interaction effects between pavement design variables*; and (b) it may be unwarranted to assume in advance of an experiment that interacting effects are negligible.

2. Experience has shown that at the Road Test certain interactions may be considered negligible.

3. Interactions may be neglected for two other reasons: (a) Although an interaction effect may be significant statistically it may have no practical significance; and (b) interaction effects can be difficult to interpret and in the interests of simplicity and convenience they may be omitted.

4. Other Road Test data can be analyzed using only a fractional replicate of the complete factorial experiment. Experience with Road Test data has shown which dependent variables were affected by interactions and which were not. An analysis of part of the data could save considerable time and money, particularly if there were no facilities for rapid calculations with large amounts of data.

5. Regional road tests could be conducted on a block basis with the differences between blocks confounded with the differences between regions. In this way a complete factorial ex-

* It was learned only during the course of many analyses that this was a valid assumption.

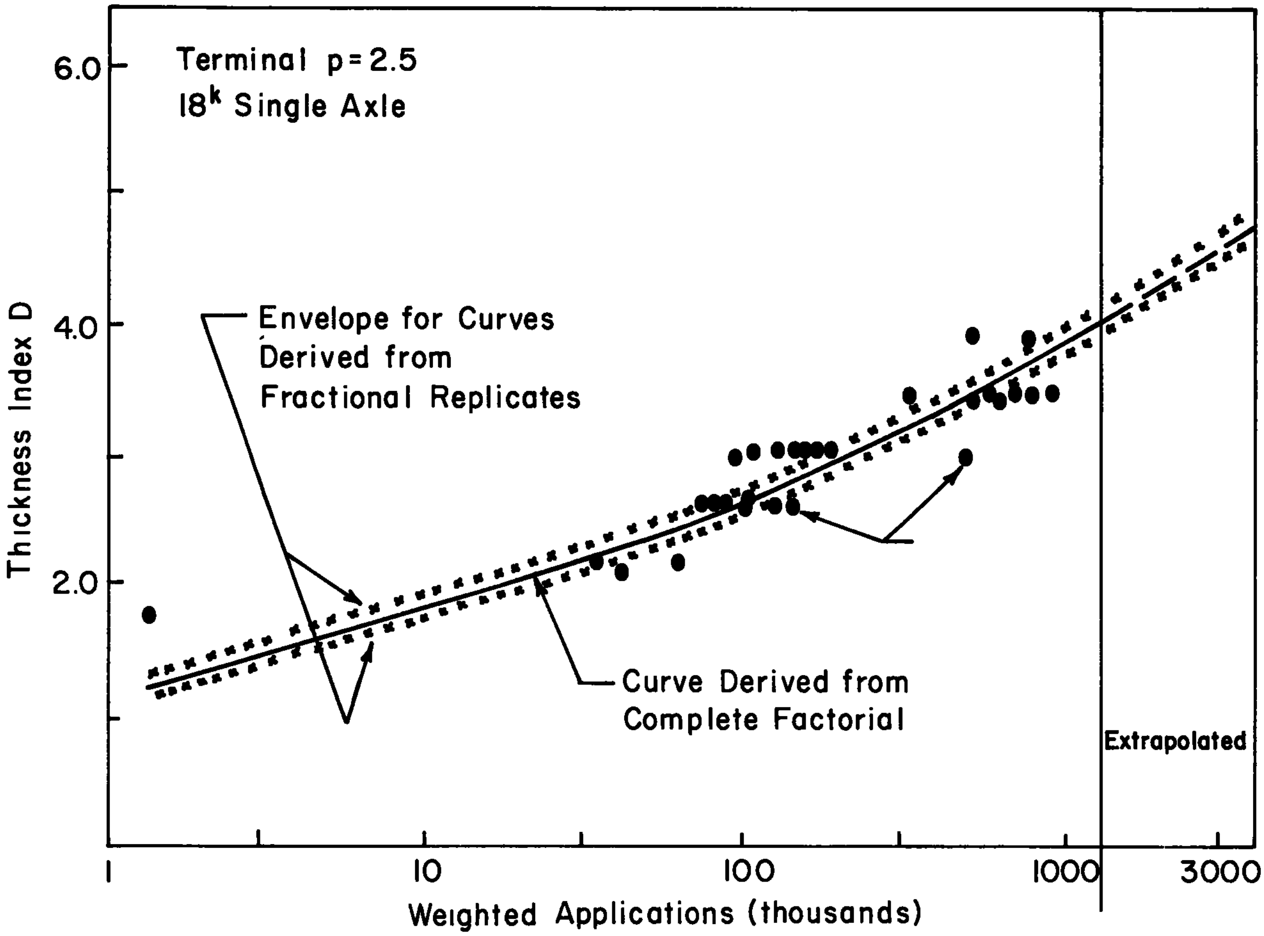

Figure 6. Flexible pavement performance equation curves.

periment could be designed to cover several regions and at the same time yield estimates of main effects, interacting effects and block effects with the precision of a factorial experiment. Differences between regional effects could be estimated.

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