Chapter 2

CAR FOLLOWING AND ACCELERATION NOISE

E. W. Montroll
International Business Machines Corporation

and

R. B. Potts
University of Adelaide, Australia

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2.1 INTRODUCTION

Traffic phenomena are more a part of the behavioral than the physical sciences, for they result from the response of humans to various stimuli. Certain stimulus response equations can be analyzed, however, in the same manner that physicists analyze dynamic equations of motion.

The average speed or travel time for smooth safe driving on a given road depends on many phenomena (weather, mechanical condition of vehicles, driver behavior patterns, curves, hills, pedestrians, etc.). Two factors determine the maintenance of a smooth safe trip—the motion of an isolated vehicle and the interference of vehicles with each other.

Theoretically, traffic can be considered as the behavior of an assembly of vehicles which are influenced by their environment and by each other. Each vehicle is capable of either acceleration or deceleration. The "traffic problem" concerns the large-scale motions of these vehicles at high density. In this state they are forced to follow each other in lanes and they have only occasional opportunities to pass. Traffic theory in this regard then is the study of the acceleration and decleration patterns of these vehicles and the flows resulting when they are regulated in various ways.

2.2 THE ISOLATED VEHICLE

When a car is driven on an open road in the absence of traffic, the driver generally attempts, consciously or unconsciously, to maintain a rather uniform velocity, but he never quite succeeds. His acceleration pattern, as a function of time, has a random appearance. An acceleration distribution function can be easily obtained from such a pattern. This distribution is essentially normal. The random component of the acceleration pattern is called "acceleration noise" (*4, 5, 7*).

A measure of the smoothness or jerkiness of the driving is then given by the dispersion σ of the acceleration noise. The mathematical definition of this quantity is

$$\sigma^2 = \frac{1}{T} \int_0^T [a\,(t)]^2\, dt \qquad (2.1)$$

in which $a(t)$ is the acceleration (positive or negative) at time t, and T is the total running time. Alternatively, if one considers that the acceleration is sampled at successive time intervals, Δt, then

$$\sigma^2 = \frac{1}{T} \sum [a\,(t)]^2\, \Delta t \qquad (2.2)$$

The dispersion, or standard deviation, σ, is simply the root-mean-square of the acceleration, and it has the dimensions of acceleration. Its values are usually quoted in ft/sec^2 or as a fraction or multiple of $g = 32$ ft/sec^2.

Runs made on a section of the General Motors test track (an almost perfect roadbed) by four operators while driving in the range of 20 to 60 mph yielded normal acceleration noise distributions with standard deviates of $0.01g \pm 0.002g$, which are about 0.32 ft/sec^2. This dispersion increases at extreme speeds greater than 50 mph or less than 20 mph.

The acceleration noise of a driver will vary considerably as he drives on different roads or under different physiological or psychological conditions. The acceleration noise observed in a run in the Holland Tunnel of the New York Port Authority (with no traffic interference in the lane in which the run was made) was 0.73 ft/sec^2. Although the roadbed of the Holland Tunnel is quite good, the narrow lanes, artificial lighting and confined conditions induce a tension in a driver which is reflected in the doubling of his acceleration noise dispersion from its perfect road value. Preliminary studies of the acceleration noise associated with runs on poorly surfaced, winding country roads indicate that dispersions of 1.5 to 2 ft/sec^2 are not unusual.

Both transverse and longitudinal acceleration noises exist, but no measurement of the transverse (left-right) noise has been made. The latter would be large on winding roads and in the pattern of drivers who change lanes frequently while driving in heavy traffic. Both components of the noise would be large in the case of an intoxicated or fatigued driver or in situations in which the attention of the driver is shared between the road and his traveling companions. Noise measurements have not yet been made in these situations.

The dispersion of the acceleration noise of a vehicle was first measured by Herman *et al.* (*4*) by using an accelerometer to record on photographic film the car's acceleration as a function of time. From an analysis of the curve, the value of the dispersion σ was determined. Although preliminary results were obtained by this method, the reduction of the data was rather tedious. Apparatus for automatically recording the acceleration in a form which can be converted to digital data suitable for computer input has been developed by Herman and his group. This apparatus enables accurate

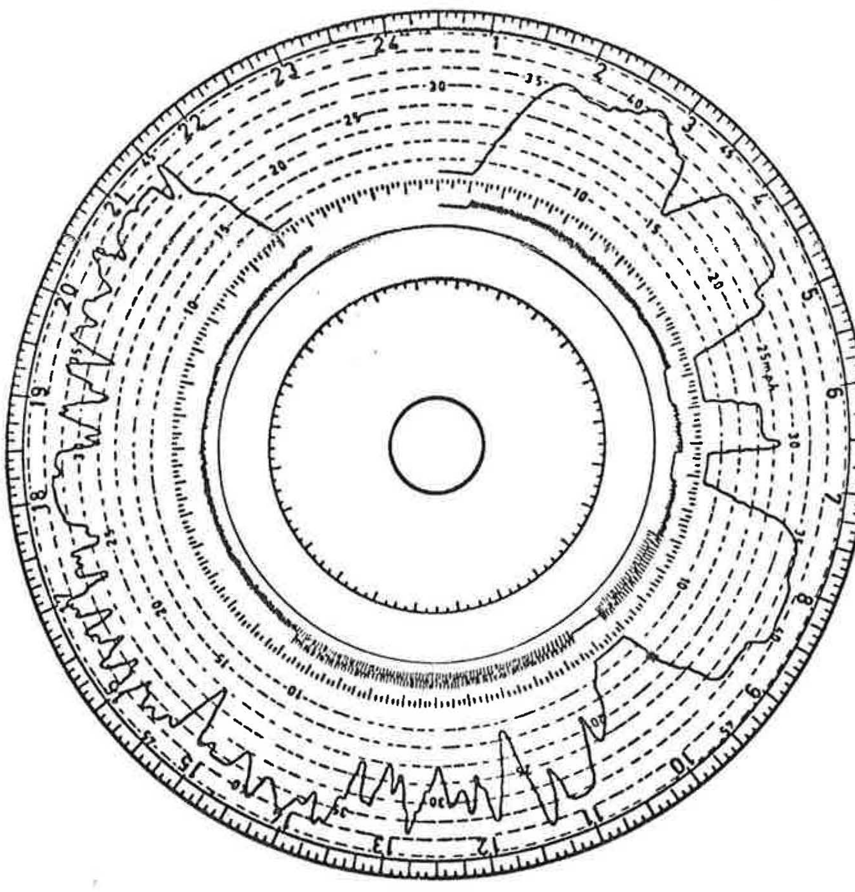

Figure 2.1. Sketch of a recording obtained on the circular chart of Kienzle TCO 8F tachograph. The concentric circles give the speed in mph; the scale on the outer circumference is in minutes. The inner trace is formed by an additional stylus whose mode of vibration is chosen by the driver by operating a key on the tachograph. The record illustrates a period of comparatively smooth driving with some stops (medium acceleration dispersion) followed by frequent accelerations and brakings (large acceleration dispersion).

Table 2.1 Value of $n^2/\Delta t$

Δt	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$
1	1.00	4.00	9.00	16.00	25.00	36.00	49.00	64.00
2	0.50	2.00	4.50	8.00	12.50	18.00	24.50	32.00
3	0.33	1.33	3.00	5.33	8.33	12.00	16.33	21.33
4	0.25	1.00	2.25	4.00	6.25	9.00	12.25	16.00
5	0.20	0.80	1.80	3.20	5.00	7.20	9.80	12.80
6	0.17	0.67	1.50	2.67	4.17	6.00	8.17	10.67
7	0.14	0.57	1.29	2.29	3.57	5.14	7.00	9.15
8	0.13	0.50	1.13	2.00	3.13	4.50	6.13	8.00
9	0.11	0.44	1.00	1.78	2.78	4.00	5.44	7.11
10	0.10	0.40	0.90	1.50	2.50	3.60	4.90	6.40
11	0.09	0.36	0.82	1.45	2.27	3.27	4.45	5.82
12	0.08	0.33	0.75	1.33	2.08	3.00	4.08	5.33
13	0.08	0.31	0.69	1.23	1.92	2.77	3.77	4.92
14	0.07	0.29	0.64	1.14	1.79	2.57	3.50	4.57
15	0.07	0.27	0.60	1.07	1.67	2.40	3.27	4.27
16	0.06	0.25	0.56	1.00	1.56	2.25	3.06	4.00
17	0.06	0.24	0.53	0.94	1.47	2.12	2.88	3.76
18	0.06	0.22	0.50	0.89	1.39	2.00	2.72	3.56
19	0.05	0.21	0.47	0.84	1.32	1.89	2.58	3.37
20	0.05	0.20	0.45	0.80	1.25	1.80	2.45	3.20

estimations of the acceleration dispersion.

An inexpensive and simple method of estimating the dispersion employs the Kienzle TCO8F model tachograph* with a speed recording range of 0 to 45 mph. The speed is recorded by a stylus on a circular chart which revolves once in 24 min. A typical record is shown in Figure 2.1. The inner trace is formed by an additional stylus that can vibrate in any of three modes of vibration. The choice of the mode is decided by the position of a tachograph key which can be operated by the driver. It enables him to indicate when he passes selected points on the highway. A stylus for recording distance traveled was not used, as the mileometer on the tachograph was more suitable and accurate.

Inasmuch as times are proportional to angles on the circular chart, it is a simple matter to use a protractor to measure the travel time TT, the stopped time ST and the running time RT (= TT − ST). A special analyzer is available from the tachograph manufacturers which allows the record to be mounted on a protractor and viewed through a magnifying glass. The acceleration dispersion was estimated by approximating Eq. 2.1 by

$$\sigma^2 \approx \frac{1}{T} \sum \left(\frac{\Delta u}{\Delta t} \right)^2 \Delta t \tag{2.3}$$

or

$$\sigma^2 = \frac{(\Delta u)^2}{T} \sum \frac{n^2}{\Delta t} \tag{2.4}$$

in which Δt is the time taken for a change $n\,\Delta u$ in speed, n being an integer and Δu a small speed interval taken constant throughout the measurement of the record. The time T is taken as the running time RT and not the travel time. For a chart recording speeds in the 0- to 45-mph range, a value $\Delta u = 2.5$ mph proved to be convenient. The record is first marked at speed intervals $n\,\Delta u$, as indicated at the beginning of the record in Figure 2.1. The chart is then placed in an analyzer and successive values of Δt are measured. It is convenient to use a table of values of $n^2/\Delta t$ (such as Table 2.1) to enable the value of $n^2/\Delta t$ to be cal-

* The Kienzle tachograph is distributed under the name ARGO in the United States. Other tachographs are manufactured by VDO and Wagner. Various models are available. Some have circular charts; others use paper wound on spools. Models with slow-moving charts are used by trucking and bus companies. Those with fast-moving charts are ideal for many traffic engineering purposes.

culated progressively on a desk calculator. To illustrate the method, the values of n, Δt and $n^2/\Delta t$ for the beginning of the marked record in Figure 2.1 are:

n	Δt	$n^2/\Delta t$
8	20	3.20
4	15	1.07
2	10	0.40
0	—	0
1	16	0.06
1	6	0.17
0	—	0
2	14	0.29

If Δt is in seconds, the running time T in seconds, and $\Delta u = 2.5$ mph, then $(\Delta u)^2 = \left(2.5 \times \frac{22}{15}\right)^2 \approx 13.44$ ft²/sec², which when inserted in Eq. 2.4 gives σ in ft/sec².

Some of the advantages of this method of measuring the acceleration dispersion are:

1. The equipment is inexpensive.
2. The chart requires no processing.
3. The chart forms a convenient permanent record of the test run.
4. Travel times, stopped times and running times are easily measured from the chart.
5. Small speed fluctuations are ignored.

The main disadvantages are:

1. Each record takes up to 30 min to analyze.
2. The accuracy of the determination of σ is only about 10 percent.

It must be emphasized that the acceleration dispersion σ is suggested as a useful traffic parameter, enabling the comparison of different traffic situations. Although the error is about 10 percent, the estimated value is consistently smaller than the exact value because the subdivision of the record into speed intervals, which are multiples of Δu, essentially replaces the speed-time curve by a set of linear segments. In any case the many factors contributing to the acceleration noise denote that its dispersion varies from run to run, and the usual care must be taken to design a set of experiments with a sufficient number of runs so that significant statistical tests can be made on the results.

2.3 LAW OF CAR FOLLOWING AND VARIATION OF FLOW WITH DENSITY

In this section the effect of the road is neglected, and consideration is given only to the interaction between cars. Consider a line of traffic so dense that passing is impossible and the driver of each vehicle is forced to drive slower than he would on his own volition. Also suppose that the road is excellent, so that the acceleration pattern of each vehicle depends more on the behavior of its predecessor than on its own natural acceleration noise.

The acceleration of the nth vehicle at time t can be expected to depend on various relative characteristics of the $(n-1)$st and the nth vehicles. Some of these characteristics are relative velocity and separation distance. The manner in which one vehicle follows another is referred to as the *law of following* (*1, 7*).

Several qualitative features of the law are self-evident. First, *a moving line of traffic must not amplify small disturbances.* That is, if the first vehicle in the line slows down slightly and then speeds up to his old rate, this slight perturbation must not be amplified as it is transmitted down the line to the extent that a collision occurs far behind the point of perturbation or that the cars sufficiently far back must stop to avoid collisions. Secondly, the law of following must not be such that a strong perturbation such as a sudden stop cannot sometime cause a rear-end collision, for such collisions occur rather frequently. *Responses are never instantaneous.* A certain time t_1 is required for a driver to notice that his relative speed and separation distance with his predecessor have changed. A time t_2 is required to decide on the proper response to a variation. A time t_3 is required for the vehicle to act on the response. In practice, $t_1 + t_2 + t_3$ is about 1.5 sec.

As a standard from which perturbations are to be measured, consider a hypothetical line of traffic moving at constant velocity u with all cars separated by a distance s (= distance from the front bumper of one car to the front bumper of the car behind it). The traffic in Figure 2.2 is postulated to be moving to the right and $X_n(t)$ is the position of the nth car at the time t. Then, if the origin is chosen as the location of the front bumper of the first car at time $t = 0$,

$$X_n(t) = u\,t - (n-1)s \qquad (2.5)$$

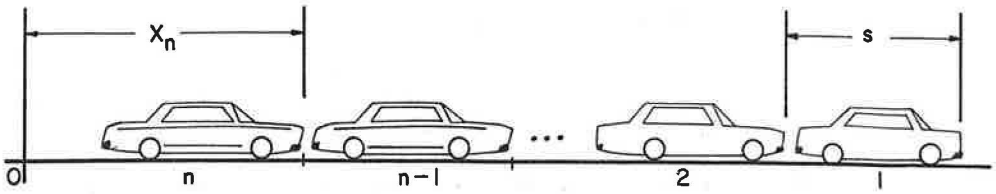

Figure 2.2. Postulation of moving vehicles.

Of course, cars in a real stream of traffic do not move with constant velocity, nor is the separation distance fixed. Let $x_n(t)$ be the deviation from $X_n(t)$ of the location of the nth car at time t and let $y_n(t)$ be the actual location. Then

$$y_n(t) = x_n(t) + X_n(t) \qquad (2.6)$$

and the velocity of the nth car is

$$u_n(t) = \dot{x}_n(t) + u \qquad (2.7)$$

while the acceleration is

$$a_n(t) = \ddot{y}_n = \ddot{x}_n \qquad (2.8)$$

Suppose that the line of traffic flows almost in the described manner so that $x_n(t)$ and $\dot{x}_n(t)$ are very small for all n. By accelerating and decelerating, each driver makes small compensations to arrive at the steady-stream velocity and spacing. Now examine several possible laws of following to see which might be realistic and then try to compare with experimental results. In the limit of very small $x_j(t)$ and $\dot{x}_j(t)$, three possibilities might exist:

(a) The nth driver accelerates or decelerates by an amount proportional to the deviation in relative separation from the desired amount s. That is,

$$a_n(t) = \ddot{x}_n(t) = \mu\,[x_{n-1}(t) - x_n(t)] \qquad (2.9)$$

in which the parameter μ would be determined from observations on relative motions of cars in the traffic stream.

(b) The nth driver accelerates by an amount proportional to the difference in relative velocity of nth and $(n{-}1)$st cars, giving

$$a_n(t) = \dot{u}_n(t) = \alpha\,[u_{n-1}(t) - u_n(t)] \qquad (2.10)$$

If the $(n{-}1)$st car is moving faster than the nth, the nth driver accelerates to compensate and reduce velocity differences and vice versa when $u_{n-1}(t) < u_n(t)$, the parameter α being chosen to be positive.

(c) A linear combination of the previous two laws:

$$a_n(t) = \mu\,[x_{n-1}(t) - x_n(t)] + \alpha\,[u_{n-1}(t) - u_n(t)] \qquad (2.11)$$

All these laws are *linear* laws, which might be appropriate only for *small* deviations from the desired state of traffic. The response of the nth driver is proportional to a deviation for which he wishes to compensate. The parameters α and μ are called sensitivities of the response to the deviations. Large values of α and μ correspond to strong compensation, and small values correspond to weak compensations. Experiments have been performed to determine whether these possibilities are sensible. Before resorting to experimental evidence, however, determine if any of these laws can be ruled out on the basis that they violate the requirement that a line of traffic must not be an amplifier of small disturbances.

A standard way to investigate the effect of disturbances and of stability of linear systems is to make a harmonic (frequency) analysis of the disturbance to see how individual frequency components are propagated through the system. Assuming that the deviation of the motion of the lead car in a platoon is the source of the disturbance, its motion can then be harmonically analyzed. When this is done in law (a), it turns out that a resonance exists at frequency $\omega = \mu^{1/2}$. That is, any frequency components at frequencies near $\mu^{1/2}$ are amplified strongly by the traffic, the law of amplification of the amplitude of the ω component being $[1 - \omega^2/\mu]^{-n}$. On the other hand, law (b) damps out a disturbance as

$$[1 + \omega^2/\alpha^2]^{-n} \tag{2.12}$$

for the nth car behind the source of the disturbance. Hence, law (b) is a reasonable one to investigate further while law (a) is not. If one investigates mixed laws such as (c) or any other law in which the acceleration is proportional to the difference in ith derivatives of the separation distance between two successive vehicles, he finds resonances (instabilities) in those laws which contain terms with even values of i. Inasmuch as it is doubtful that a driver could be sensitive to third derivatives, one is left with only law (b) as a possible one for investigation.

Before law (b) is compared with experimental data, additional features of the law must be examined. It will be recalled from the discussion at the beginning of this section that responses are never instantaneous. Even though law (b) may be suggestive for further consideration, it should be amended to take into account the time lag between the time of the actual development of a disturbance and the moment of effective response; therefore, law (b) should now read

$$a_n (t + \Delta) = \dot{u}_n (t + \Delta) = \alpha [u_{n-1} (t) - u_n (t)] \tag{2.13}$$

in which the velocities on the right side are to be taken at time t to influence the acceleration of the left at time $t + \Delta$. When time lags are incorporated into linear systems, instabilities may result. If one reacts too strongly (large α) to an event which occurred too far in the distant past (large response lag Δ), the situation at the moment of response may have changed to the point where the response is actually in the wrong direction. Hence, when lags are long there should be weak responses to insure stability. In fact, a line of traffic following Eq. 2.13 is stable, not amplifying small disturbances, only when

$$2 \alpha \Delta < 1 \tag{2.14}$$

A disturbance of unit amplitude is propagated back to the nth car so that its amplitude at arrival is equal to or less than

$$[1 + (\omega^2/\alpha^2) (1 - 2 \alpha \Delta)]^{-n} \tag{2.15}$$

It should be noted that a resonance appears when Eq. 2.14 is violated.

Before comparing Eq. 2.13 with experimental data, it is worth trying to extend the formula slightly so that it is applicable to cases in which, for some reason, rather large gaps have formed between cars. Clearly, when the separation distance is large one will not drive as sensitively as he would in a bumper-to-bumper situation. Hence α should depend on the separation distance in such a way that when two successive vehicles are separated by an enormous distance no interaction exists between them at all. One possible law is that the sensitivity α_0 should be inversely proportional to the car spacing (distance between cars plus car length) so that

$$a_n (t + \Delta) = \ddot{y}_n (t + \Delta) = \alpha_0 \left\{ \frac{\dot{y}_{n-1} (t) - \dot{y}_n (t)}{y_{n-1} (t) - y_n (t)} \right\} \tag{2.16}$$

in which α_0 is a measure of sensitivity.

A number of car-following experiments were performed on the General Motors test track, as well as in the Holland and Lincoln Tunnels in New York. Each of a number of drivers using an instrumented car was told to follow a lead car as he would in normal city driving. In each case a continuous record was taken of the acceleration of the second car $a(t)$, as well as the relative velocities $u_r (t)$ and spacing $s(t)$ of the two cars. For each driver a best value of α and Δ was obtained in

$$a(t + \Delta) = \alpha_0 [u_r (t)/s(t)] \tag{2.17}$$

which is equivalent to Eq. 2.16 so that

$$\sum_t [a(t + \Delta) - \alpha_0 u_r(t)/s(t)]^2 = \min \tag{2.18}$$

The results (5) of the car-following experiments are summarized in Table 2.2. The correlation coefficients for the best values of α_0 and Δ were usually greater than 0.9, and for some drivers as high as 0.97. If Eq. 2.16 were exact and no experimental error existed in the data, the correlation coefficients would be 1. Some deviation from 1 must be expected because the acceleration noise contribution to $a(t)$ has been omitted. There is some variation in the values of α_0

Table 2.2 Summary of Car-Following Experiments

Locality	Number of Drivers	α_0 (mph)	Δ (sec)
General Motors test track	8	27.4	1.5
Holland Tunnel	10	18.2	1.4
Lincoln Tunnel	16	20.3	1.2

and Δ for different drivers. For example, in the General Motors test track experiments, Δ varied from 1.0 to 2.2 sec, with one-half the drivers having Δ values between 1.4 and 1.7. It would be interesting to find these constants on a given road for a large number of drivers. This would enable one to obtain reliable statistics on personal variations between drivers. In applying Eq. 2.16 to a line of traffic, it is assumed that all drivers have the same characteristics; namely, the average ones.

An interesting consequence of the law of following (Eq. 2.13) is that the formula for the rate of propagation of a disturbance down a line of traffic (in cars per second) is $n/t = \alpha$.

Although a line of traffic is stable to small perturbations, it is well known that most rear-end collisions are due to local instabilities in which one or more cars are unable to compensate for large disturbances ahead of them. It can be shown that no such local instabilities would occur in the law of following if the inequality $\alpha e \Delta < 1$ were satisfied, a condition rarely exhibited in follow-the-leader experiments.

Although Eq. 2.17 was derived to form a basis for the law of following of one vehicle by another, it can also be employed to relate the flow rate of single-lane traffic to the traffic density (*3*). The flow rate q (say in vehicles per hour) is the product of the density k (cars per mile) and the velocity u (miles per hour). Thus, $q = u k$. Qualitatively the equation of state of the traffic, the name given to the flow-versus-density relation, can be expected to have the form given in Figure 2.3. When there are no cars on the road ($k = 0$) the flow rate is zero. At close packing (bumper to bumper) where $k = k_j$, the density is greatest, but no cars

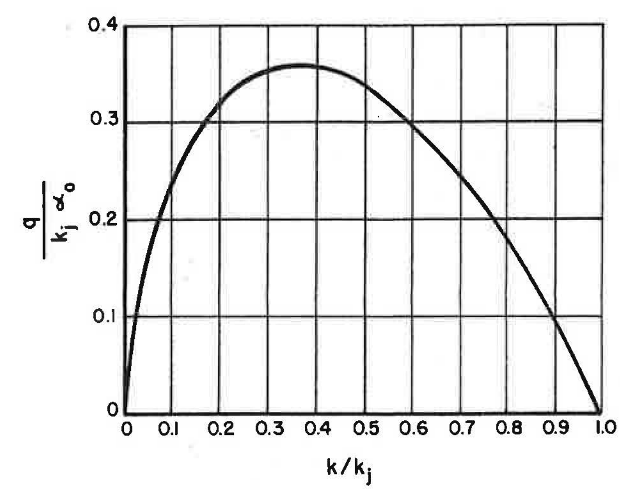

Figure 2.3. Normalized traffic flow versus density as obtained from Eq. 2.21. Curve compares with data obtained by Greenberg from experiments in the Lincoln Tunnel.

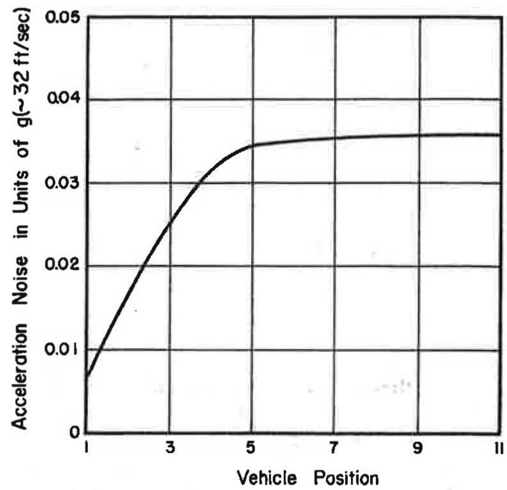

Figure 2.4. Acceleration noise of vehicles at different locations in a platoon.

can move ($u = 0$). At some intermediate density, a maximum flow rate exists.

Eq. 2.16 can be integrated to yield

$$u_n(t+\Delta) - u'_n = \alpha_0 \log [y_{n-1}(t) - y_n(t)]/s'_n \quad (2.19)$$

in which $s'_n = y_{n-1} - y_n$ at time when velocities are u'_n. Now choose s'_n to be the close packing bumper-to-bumper distance. Because there is no motion under this condition, $u'_n = 0$ and

$$u_n(t+\Delta) = \alpha_0 \log [y_{n-1}(t) - y_n(t)]/s'_n \quad (2.20)$$

Now suppose that the traffic flow has become steady. Then its average velocity at time t is about the same as that at $t + \Delta$ (Δ being about 1.5 sec). Therefore, $u_n(t+\Delta)$ can be replaced by the average velocity u, and $[y_{n-1}(t) - y_n(t)]$ can be replaced by the average separation distance, which is the reciprocal of the average density, k^{-1}. Actually u is the arithmetic mean velocity and k the geometric mean density. Hence,

$$u = \alpha_0 \log_e k_j/k \quad (2.21)$$

in which k_j is the density at close packing ($k_j = 1/s'_n$).

The flow rate q is then given by

$$q = u\,k = k\,\alpha_0 \log_e k_j/k \quad (2.22)$$

This function, plotted in Figure 2.3 compares with experimental data taken in the Lincoln Tunnel in New York. From a large sample of more than 24,000 vehicles (*7*) in the Holland Tunnel, the best fit value of α_0 was found to be 18.95 mph, which is to be compared with $\alpha_0 = 18.2$ mph obtained in car-following experiments in the same tunnel (Table 2.2). This provides a good check for the theory.

Notice that α_0 is the velocity which gives a maximum flow rate. It has been observed that α_0 is small under hazardous driving conditions, such as poor lighting or narrow roadway with two lanes in tunnels, whereas it is large on good roads such as freeways with no turns. Because the expensive parts of a highway system, such as bridges and tunnels, are frequently its bottlenecks, the traffic engineer should make α_0 as large as possible to increase the maximum possible flow rate and to regulate traffic so that for a given α_0 this maximum is achieved.

2.4 ACCELERATION NOISE OF A VEHICLE IN TRAFFIC

In Section 2.2, the acceleration noise of an isolated vehicle was discussed. In Section 2.3, several simple car-following laws for traffic in the absence of acceleration noise were exhibited. Clearly, the total acceleration noise of a vehicle in traffic is a superposition of its natural noise and its response to that of its predecessors through the law of following. In stable, smooth-flowing traffic the effect of the natural noise of a given vehicle dies out as it is propagated down the line. The total acceleration noise of vehicles at different locations in a platoon has been measured by Herman and Rothery (*6*) (see Fig. 2.4). It is noted that traffic has broadened the acceleration distribution function so that the dispersion far down the platoon is about three times that of the lead car, which is effectively moving freely on the road. Figure 2.4 also shows that in the absence of any violent disturbances the influence of the noise of a single vehicle is dampened out by the time the signal of its motion has propagated down to the fifth or sixth car behind it. Traffic broadens the acceleration distribution, the broadening being smaller for the conservative driver who is satisfied to follow the stream than for the "cowboy" who by weaving attempts to drive 5 to 10 mph faster than the stream. This is shown in Figure 2.5 for traffic on Woodward Avenue in Detroit (*4*).

The traffic broadening is not large for smoothly flowing traffic, but the dispersion increases rapidly at the onset of congestion. For stop-and-go traffic the dispersion is small because cars are unable to accelerate to appreciable speeds.

The broadening of the acceleration distribution by traffic depends on the parameters of the law of following. The acceleration of the *n*th car at time t is a superposition of its natural acceleration noise and its response to the motion of its predecessor. In smoothly moving traffic the separation distance varies only slightly from the equilibrium distance s. Hence, Eq. 2.16 can be linearized so that addition of the natural acceleration $\beta(t)$ gives

$$\dot{u}_n(t+\Delta) = \alpha\,[u_{n-1}(t) - u_n(t)] + \beta(t) \qquad (2.23)$$

in which

$$\alpha = \alpha_0/s \qquad (2.24)$$

The $\beta(t)$ is a random function whose value at time t is not specified. It is determined by its distribution function $f(a)$ so that $f(a)\,da$ is the probability that $\beta(t)$ has a value between a and $a + da$ at time t. For simplicity, assume that $\beta(t)$ has the same distribution for all drivers on the road of interest. One can use the standard methods of the theory of Brownian motion to determine the statistical differences of properties of $a_n(t) = \dot{u}_n(t)$ from those of $\beta(t)$ in terms of α and Δ. If the acceleration noise is peaked in the low frequency range, one finds that the dispersion σ of the distribution function of $a_n(t)$ (as $n \to \infty$; *i.e.*, for cars far from the beginning of a platoon) is related to the dispersion σ_0 of $\beta(t)$ by

$$\sigma = \sigma_0/(1-2\,\alpha\,\Delta)^{1/2} \quad \text{if } 2\,\alpha\,\Delta < 1 \qquad (2.25)$$

The stability condition (Eq. 2.14) again makes its appearance. The closer the traffic reaches the limit of stability ($2\,\alpha\,\Delta \to 1$) the larger the traffic broadening of the acceleration noise.

If Eq. 2.24 is substituted in Eq. 2.25, the average spacing may be expressed as

$$s = 2\,\alpha_0\,\Delta/[1 - (\sigma_0/\sigma)^2] \qquad (2.26)$$

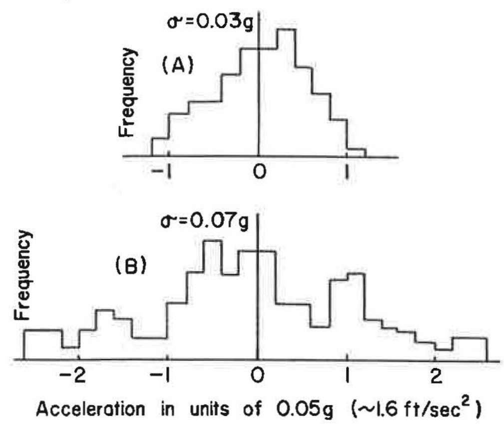

Figure 2.5. Acceleration distribution functions for a driver (A) moving with a traffic stream at approximately 35 mph and (B) attempting to drive 5 to 10 mph faster than the stream average.

This equation was checked with the Holland Tunnel observations of Herman, Potts and Rothery. The traffic broadening of the acceleration noise dispersions σ/σ_0 in the tunnel varied from about 1.50 to 1.75, depending on the density during the experiment. The value of α_0 was determined by fitting Eq. 2.15 to the observed flow-versus-density curve for the tunnel. The average time lag of 1.5 sec, which was observed in car-following experiments, was substituted in Eq. 2.26, as was the observed ratio σ/σ_0. The computed values of s were then converted into appropriate densities ($s = 1/k$), which were compared with the observed densities made at the same time as σ/σ_0 was determined. These calculated values generally did not deviate from the measured ones by more than 10 or 15 percent.

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