A Study of the Relation Between Creep and the Gain of Strength of Concrete

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The relation between creep and stress-strength ratio at the time of loading is extended to include the influence of the strength of concrete at any time under load. In particular, creep is shown to be a function of the fractional increase in strength while the concrete is under load. The relative creep is shown to be a function of the relative strength increase, so that the rate at which the ultimate creep of any concrete is approached can be estimated from the strength-time curve.

Quantitative relations between creep and the strength-time characteristics of concrete are derived. These relations have been verified for tests in which the stress-strength ratio was maintained constant as well as for the conventional constant stress tests.

•EARLIER PAPERS (1, 2) have shown a relation between the strength of concrete at the time of application of load and creep after a given time under load. In particular, it was found that creep (after a given time) is directly proportional to the stress-strength ratio of the concrete (at the time of application of load), regardless of the water-cement ratio or ambient humidity conditions, provided no drying or swelling takes place concurrently with creep. The content of cement paste is, however, of considerable influence on the magnitude of creep, as shown in another paper (3).

In the discussion of the relation between creep and stress-strength ratio it was noted that the apparent discrepancies in the observed data can be related to the differences between the different test specimens in the gain of strength after the application of load. The relation between creep and the stress-strength ratio at the time of application of load was in fact considered to be a first, albeit fairly close, approximation: a more accurate relation would describe creep as an integral function of the strength of concrete from the time of application of load to the time when creep is determined (1).

TESTS

To study this relation creep tests were performed on specimens having various relations of strength and time since loading. This was achieved by: (a) varying the water-cement ratio of the mix, the content of cement paste remaining substantially constant; (b) varying the age at which the load is applied; and (c) varying the relative humidity of the curing medium.

In all cases the stress-strength ratio at the time of application of load had the same value, namely 0.4. In series A this value was maintained constant by increasing periodically the applied stress in proportion to the increase in strength. In series B the applied stress remained unaltered so that, as the strength of concrete increased, the effective stress-strength ratio decreased. Specimens in this series will be referred to as unadjusted while those in series A will be called adjusted.

The unadjusted tests represent the usual type of creep test. The adjusted tests were made in order to establish the influence on creep of the stress-strength ratio at any time; it is believed that tests of this type have not been made in the past.

The test layout was similar to that described in earlier papers (1, 4). As before, the specimens were 2 in. in diameter and 9¼ in. long, and had embedded lugs, so that the measured creep was that in the core rather than on the surface of the specimen.

A blended batch of Type III cement was used throughout; the composition and properties are summarized in Appendix A, which also gives the grading of the aggregate. Two relative humidities were used: 95 percent, referred to as wet storage, and, in the case of one mix, 50 percent, called dry storage. All specimens were stored from the time of demolding at 24 hours onwards at the same humidity at which they were subsequently loaded. Owing to this, shrinkage was small and its effects were minimized. Details of the mixes used are given in Table 1. Figure 1 shows the strength-age curves for the various mixes, the strength having been determined on specimens of the same shape as those used for the determination of creep. Table 2 summarizes the creep data.

GAIN OF STRENGTH

In the present tests all specimens were subjected to the same stress-strength ratio at the time of loading, and the variation in the strength of concrete should therefore be expressed not in absolute terms but should be referred to the strength at the time of loading. There are several possible ways of doing so.

Let u_0 be strength at the age of loading t_0, and u be strength at any time $t > t_0$. Then, we define the fractional strength increase as

$$f_u = \frac{u - u_0}{u_0} \tag{1}$$

Thus, even though a given mix has a characteristic strength-age curve, f_u depends on the age at loading; Table 3 gives a summary of the principal values, and Figure 2 shows the variation in f_u. It appears that:

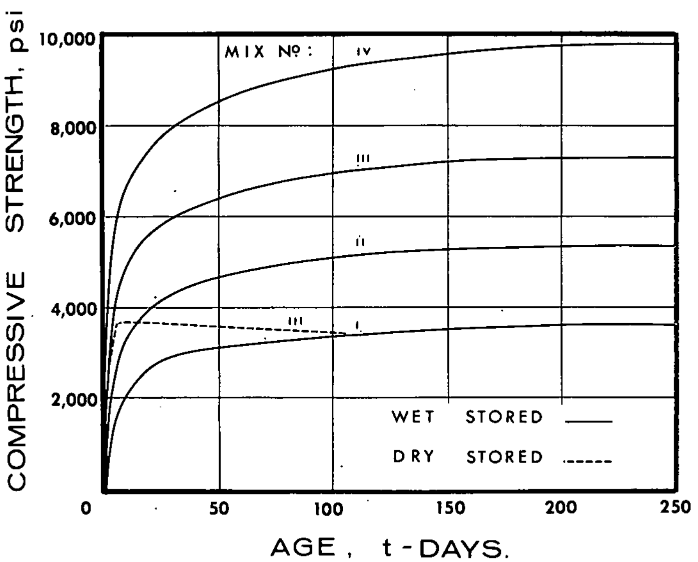

Figure 1. Relation between strength and age for various mixes.

TABLE 1

DETAILS OF MIXES

Mix No.	Water-Cement Ratio	Aggregate-Cement Ratio (by weight)	Cement Paste Content (% by volume)
I	0.86	6.73	31.4
II	0.73	6.00	31.4
III	0.60	5.25	31.4
IV	0.47	4.52	31.4

TABLE 2

SUMMARY OF CREEP DATA FOR SERIES B (UNADJUSTED TESTS)

Relative Humidity (%)	Mix No.	Water-Cement Ratio	Age at Loading t_0 (days)	Creep (10^{-6}) After Time Under Load T Days					
				3	7	14	28	56	150
95	I	0.86	3	170	210	250	270	300	375
			7	240	315	370	440	530	(740)
			(56)	190	285	370	470	575	(790)
	II	0.73	(56)	320	420	495	610	730	970
	III	0.60	3	380	450	520	610	690	750
			14	390	460	550	655	765	885
			28	380	490	580	700	840	1000
			(56)	220	400	520	655	815	1030
	IV	0.47	3	440	530	610	700	775	870
			7	400	510	600	685	770	925
			(56)	490	605	700	815	960	(1160)
50	III		3	670	920	1090	1220	1350	
			7	520	670	830	970	1110	
			28	290	410	520	640	770	

Note: The values given are averages for three specimens. Ages of loading in parentheses are for adjusted tests. Values of creep in parentheses are estimated.

TABLE 3

FRACTIONAL INCREASE IN STRENGTH AND CREEP FOR SERIES B (UNADJUSTED TESTS)

Mix No.	Age at Loading t_0 (days)	Fractional Increase in Strength f_u	Creep (10^{-6}) After 150 Days Under Load c
I	3	2.22	375
	7	1.02	(740)
	(56)	0.149	(790)
II	(56)	0.140	970
III	3	1.26	750
	14	0.368	885
	28	0.237	1000
	(56)	0.135	1030
IV	3	0.920	870
	7	0.480	925
	(56)	0.133	(1160)

Note: All data for concrete stored at a relative humidity of 95 percent. Values of creep in parentheses are extrapolated from creep at 130 days. Ages of loading in parentheses are for adjusted tests.

1. For a given mix, f_u is larger the earlier the age at loading, t_o;
2. For a given t_o, f_u is larger the higher the water-cement ratio of the mix; and
3. The influence of the water-cement ratio is greater the lower the value of t_o.

As will be shown later, the magnitude of creep at time t depends not only on the magnitude of the fractional strength increase f_u at time t, but also on during which part of the time interval $(t-t_o)$ this increase took place. We should therefore consider the progress of the increase until time t_m when the strength is u_m. The total increase in strength from the time of loading to time t_m, as a proportion of strength at time t_m, is then $\frac{u_m - u_o}{u_m}$. The increase up to time t, as a proportion of the strength at that time, is $\frac{u - u_o}{u}$. Hence the ratio of the latter to the former is

$$r_u = \frac{\frac{u - u_o}{u}}{\frac{u_m - u_o}{u_m}}$$

or

$$r_u = \frac{u_m}{u}\left(\frac{u - u_o}{u_m - u_o}\right) \tag{2}$$

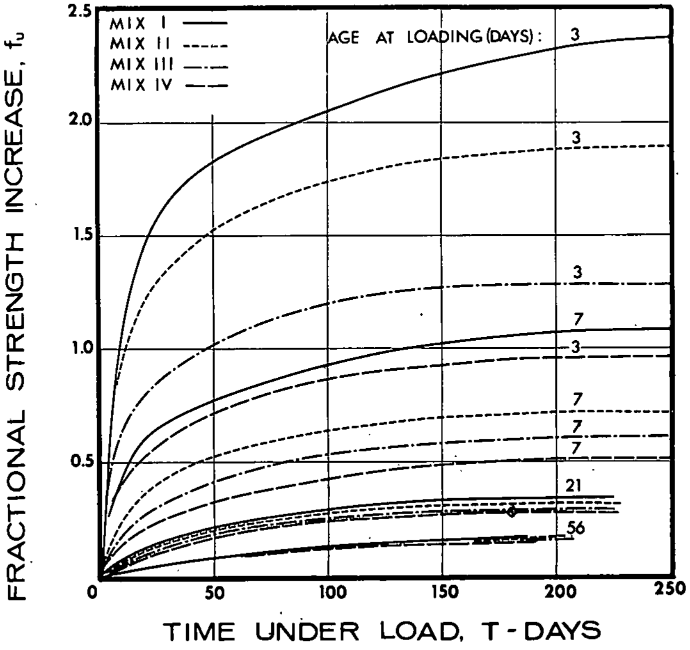

Figure 2. Fractional strength increase, f_u, for different mixes and ages at loading.

We call r_u the relative strength increase by analogy with the relative creep introduced in an earlier paper (3).

In Eq. 2 the numerator of the term in parentheses represents the increase in strength during an interval $T = t - t_0$, such that $t_0 < t < t_m$, and the denominator of the same term gives the increase in strength during the interval $t_m - t_0$.

Taking $t_m = 150$ days, the relation between r_u and T for the four mixes and for $t_0 =$ 3, 7, and 56 days is as shown in Figure 3. We can observe that r_u is higher the earlier the age at loading. Furthermore, r_u is higher the higher the water-cement ratio of the mix; for $t_0 = 3$ and $t_0 = 7$ days the influence of the water-cement ratio on r_u is greatest during the first 50 days or so after loading.

Another approach is to consider the strength of concrete as a function of the degree of hydration of the cement paste. We can express strength at any age t as a fraction of the final strength u'_m, and call this fractional strength

$$f_m = \frac{u}{u'_m} \tag{3}$$

This differs significantly from f_u and r_u in that it reflects the degree of gain of strength from the time of casting and not from the time of loading. Thus f_m is characteristic of the degree of hydration of the cement paste in the concrete, and can be related to Powers' (5) gel-space ratio which is used in Ali and Kesler's (8) work on basic creep. Powers found the relation between strength u and gel-space ratio x to be of the form $u = kx^3$. It may be noted parenthetically that in our tests k is 23,100, 23,600, 22,200, and 21,100 psi for mixes I to IV respectively. The difference between these values, averaged at 22,500 psi, and Powers' value of 29,000 to 34,000 psi is believed to be due to the difference in the shape of the specimens used: 2-in. by 9 ¼-in. cylinders and 2-in. cubes respectively.

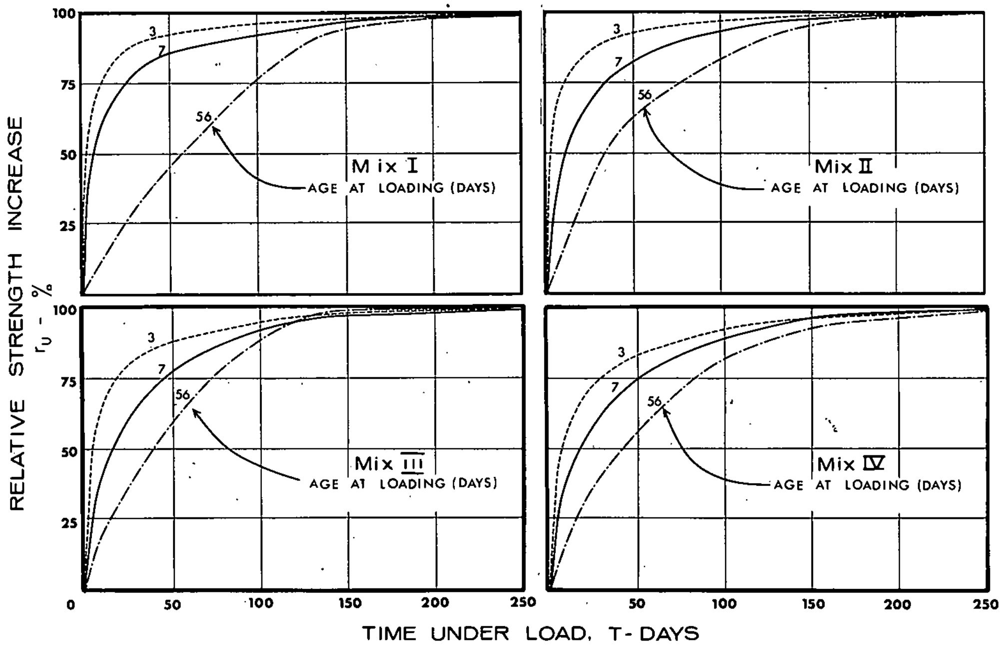

Figure 3. Relation between relative strength increase, r_u, and time under load for different ages at loading.

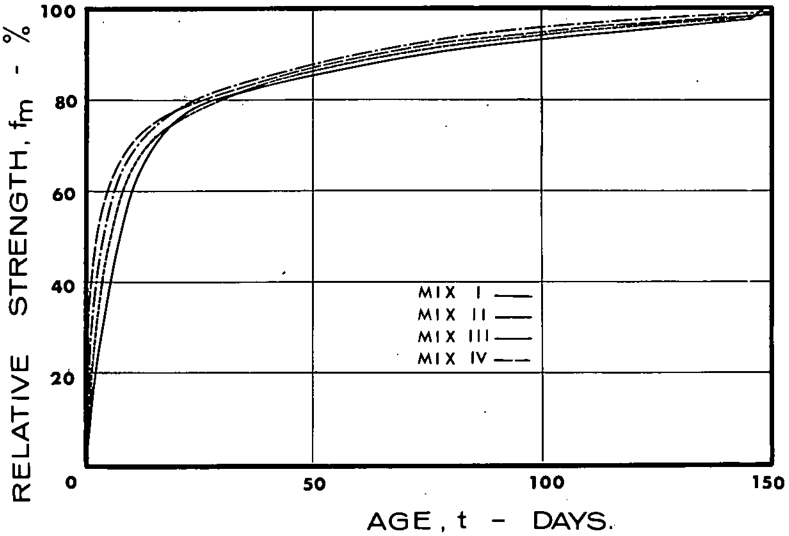

Figure 4. Ratio, f_m, of strength at time t to the strength at 250 days.

In the present tests u'_m was not determined, strength being measured only up to the age of 250 days. Taking the strength at that age as u_m, we can plot

$$f_m = \frac{u}{u_m} \tag{4}$$

for the various mixes (Fig. 4). It can be seen that f_m is higher the lower the water-cement ratio, i.e., a mix with a lower water-cement ratio achieves at an early age a greater proportion of its final strength than a mix with a higher water-cement ratio. It follows that at later ages mixes with a low water-cement ratio gain relatively less strength than those with a higher water-cement ratio; in other words, the latter mixes gain strength at a more steady rate. All this applies of course to curing at 95 percent relative humidity.

It may be recalled that Ali and Kesler (8) used a similar approach by expressing creep as a function of the degree of hydration of cement paste in terms of a compliance factor β, which represents the ratio of the deformation of the gel component of the concrete to the deformation of a hypothetical specimen of pure gel, subjected to the same stress as the concrete. The ratio $\frac{c}{\sigma\beta}$ (where c is creep, and σ is the applied stress) is independent of the mix composition or degree of hydration so that the ratio is a function of T only.

The use of strength ratios rather than measures of degree of hydration in the present paper is thought justified on the grounds of the ease with which strength can be determined.

CREEP AT A CONSTANT APPLIED STRESS (UNADJUSTED TESTS)

It should be noted that the relation between creep and stress-strength ratio postulated in 1959 (1) was derived on the basis of tests on concretes loaded at the age of 28 days. With one exception, loading at 14 days (3), no creep data have been obtained for concretes loaded at an earlier age. The present investigation covers a range of 3 to 56 days at the time of loading, and reveals the influence of the age at loading on creep.

This influence appears to be significantly different from that generally stated by the investigators of the 1930's, who applied the same stress to concretes of different ages. Such a procedure is considered unrealistic as the applied stress should be directly related to the strength of the concrete. This is why in the present tests the stress-

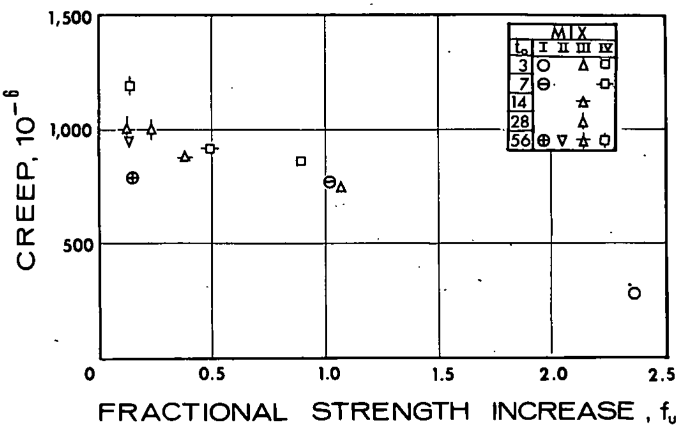

Figure 5. Relation between fractional strength increase, f_u, and creep after 150 days under load for different mixes and ages at loading, t_o.

TABLE 4

CREEP AND AGE AT LOADING (UNADJUSTED TESTS)

Age at Loading t_o (days)	Creep (10^{-6}) After 150 Days Under Load			Creep as Multiple of Creep of Mix I	
	Mix I	Mix III	Mix IV	Mix III	Mix IV
3	375	750	870	2.00	2.32
7	740	(840)	925	1.13	1.25
(56)	790	1030	1160	1.30	1.47

Note: Age of loading in parentheses is for adjusted tests. Creep value in parentheses was interpolated from creep for ages of loading of 3 and 14 days.

strength ratio at the time of loading, t_o, was 0.4 for all values of t_o. The resulting creep for any given mix is greater the larger the value of t_o. This behavior is believed to be due to the variation in the increase in the strength of concrete while the sustained load acts. As shown earlier, f_u is smaller the larger the value of t_o, so that creep is larger the smaller f_u. This can be seen from Table 3, which gives the values of f_u and c for $T = t - t_o = 150$ days for the different mixes and ages at loading t_o from 3 to 56 days. The authors' analysis of Mamillan's (11) data confirms the pattern of influence of the age at loading on creep.

For mixes having different water-cement ratios but loaded at the same age, creep appears to be higher the lower the water-cement ratio. But the lower the water-cement ratio the lower the value of f_u, so that once again the fractional strength increase and creep seem to accord with one another. Figure 5 shows a plot of creep against f_u (for the data of Table 3) when the variation in f_u arises from a change in the water-cement ratio or in t_o; in either case there seems to be the same pattern of the relation.

However, even for $t_o = 56$ days, when f_u is small (Fig. 2), the water-cement ratio seems to affect creep. It is thus possible that an additional factor, probably related to the composition of the cement paste, acts in addition to the influence of f_u.

While the relation between f_u and creep is not of a simple type we should explain that, in fact, a unique relation cannot be expected. This is because creep is affected not only by the magnitude of f_u but also by the rate of its increase; an early increase will have a greater effect on creep than an increase of the same total magnitude but occurring more slowly. It is for this reason that the concept of relative strength increase, r_u, was introduced.

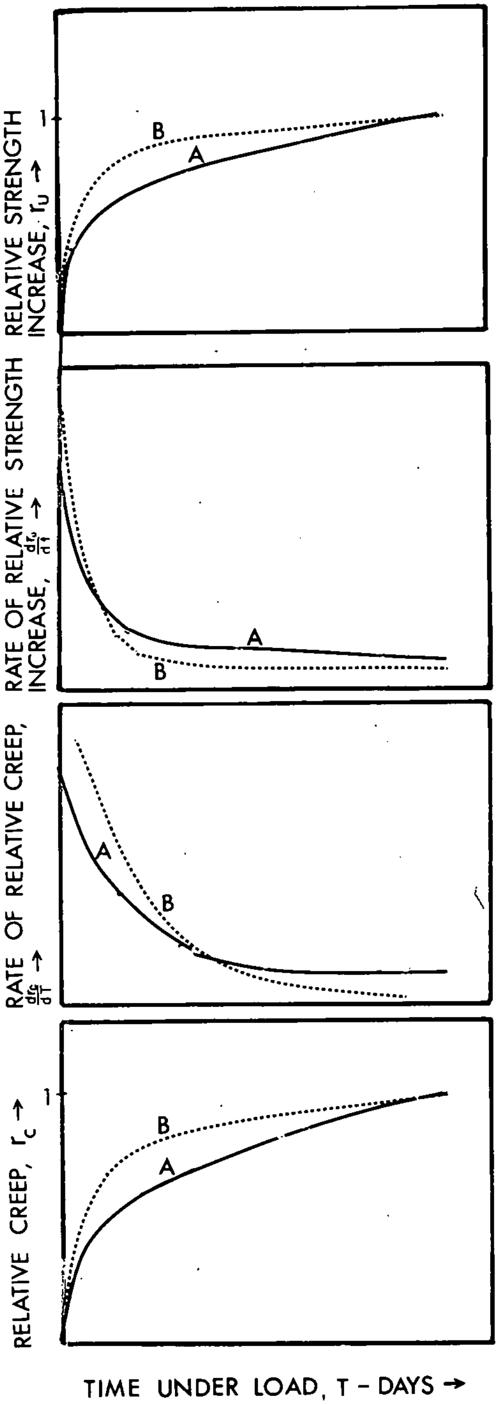

Figure 6. Schematic relations: relative strength increase, r_u, and relative creep, r_c.

From Figures 2 and 3 we see that not only is the fractional strength increase greater in mixes with a higher water-cement ratio, but the relative strength increase takes place earlier. Thus when the stress is constant (unadjusted tests) the earlier and greater fractional strength increase in a high water-cement ratio mix has a more significant effect on the reduction of creep than would be the case in a mix with a lower water-cement ratio. On the other hand, in the adjusted tests (see following section), the earlier increase in strength means an earlier increase in the applied stress, and hence an increase in creep.

For a given mix, the influence on creep of age at loading decreases with a decrease in the water-cement ratio (Table 4). This accords with the pattern of strength development in the various mixes as the fractional strength increase, f_u, is lower the lower the water-cement ratio; hence the differences in increase are smaller than in a high water-cement ratio mix. And it is these differences that are reflected in the range of creep values for different ages of loading of the same mix.

We may note that the difference in the behavior of mixes III and IV is small. Since they represent a large part of the range of mixes used in structural concrete, it is not surprising that the influence of the water-cement ratio mentioned above has not previously been clearly observed.

Values of creep for dry storage are given in Table 2; except for the specimens loaded at the age of 28 days (when creep is the same for wet and dry storage), creep as recorded includes drying creep (see Appendix B) and is therefore higher than for wet storage. In all cases of wet and dry storage shrinkage has been deducted from the total nonelastic deformation.

We should now recall the concept of relative creep, r_c, as the ratio of creep after time T under load to the final creep. In practical cases the final creep is unknown, and creep after a finite time (e.g., 150 days) is taken as reference. Thus

$$r_c = \frac{c}{c_{150}} \qquad (5)$$

Figure 6 shows qualitatively the expected relation between r_c and r_u. In the present tests the relation for different ages of loading of the same mix and for different mixes loaded at the same age agrees well with that of Figure 6. Table 5 summarizes

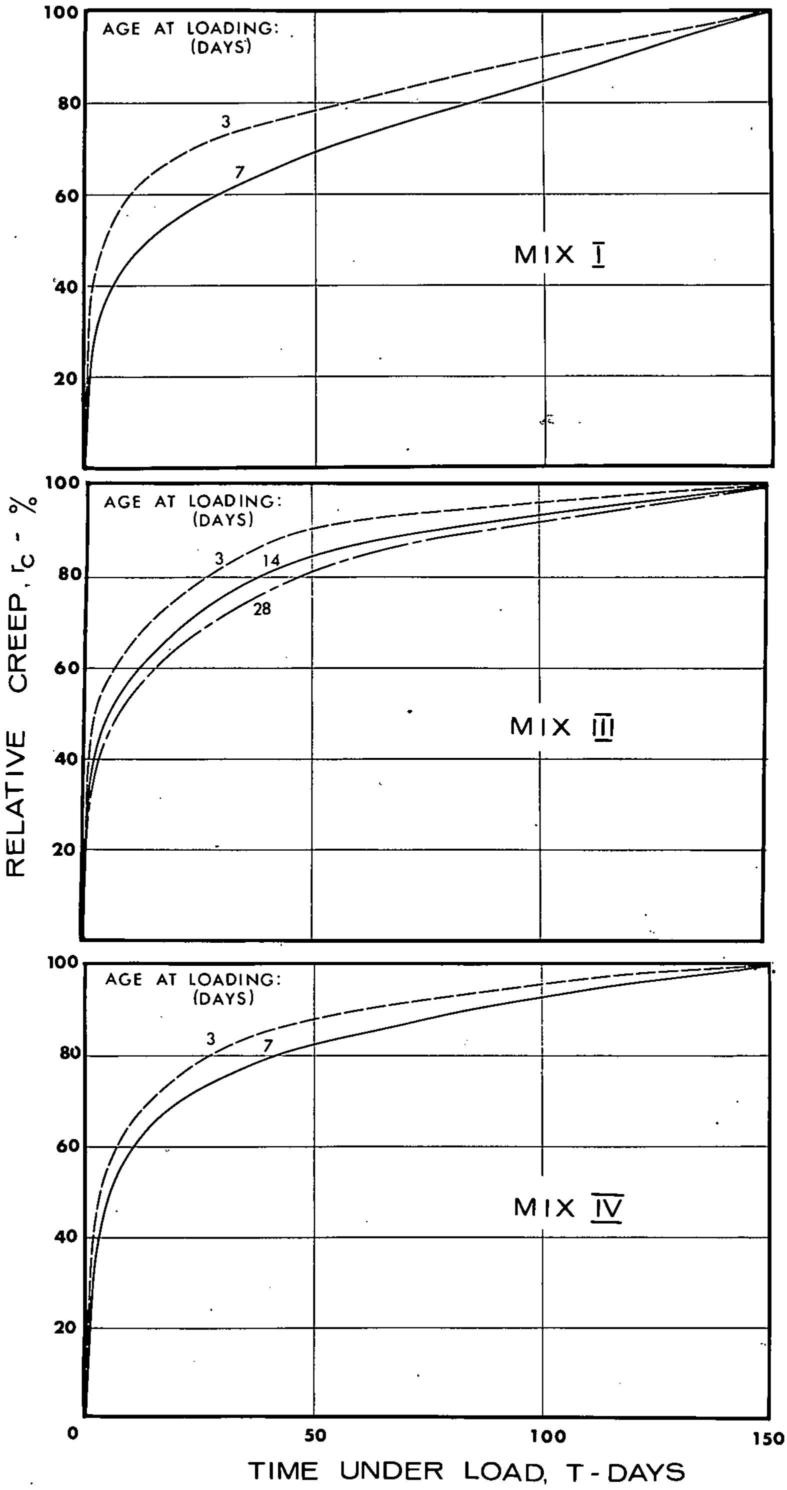

Figure 7. Relation between relative creep, r_c, and time under load for different ages at loading of series B.

TABLE 5

VALUES OF RELATIVE CREEP AS A PERCENTAGE OF CREEP FOR DIFFERENT MIXES AND AGES AT LOADING (UNADJUSTED TESTS)

Age at Loading (days) / Time After Loading (days)	Wet Storage								Mix III[b]	
	Mix I[a]		Mix III[a]				Mix IV[a]		Wet Storage	Dry Storage
	3	7	3	(7)	14	28	3	7	28	28
3	45	32	51	(48)	43	38	51	43	42	34
7	56	43	60	(56)	52	49	61	55	54	48
14	67	50	69	(66)	62	58	70	65	64	61
28	72	60	81	(78)	74	70	81	74	78	75
56	80	72	92	(89)	86	84	89	83	93	91
80									100	100
150	100	100	100	(100)	100	100	100	100		

Note: Values in parentheses were interpolated from data for ages at loading of 3 and 14 days.

[a] After 150 days.
[b] After 80 days.

TABLE 6

SUMMARY OF CREEP DATA FOR SERIES A (ADJUSTED TESTS)

Relative Humidity (%)	Mix No.	Water-Cement Ratio	Age at Loading t_o (days)	Creep (10^{-6}) After Time Under Load T Days					
				3	7	14	28	56	150
95	I	0.86	3	285	420	580	805	1130	(1605)
			7	220	305	390	500	625	860
			21	190	285	370	475	595	810
			56	190	285	370	470	575	790
	II	0.73	3	350	560	775	980	1270	(1680)
			7	260	390	540	680	870	1130
			21	320	430	545	650	785	1015
			56	320	420	495	610	730	970
	III	0.60	3	370	540	740	940	1125	1370
			7	250	420	640	860	1080	1320
			14	230	450	640	820	975	1175
			21	250	450	585	725	880	1110
			56	220	400	520	655	815	1030
	IV	0.47	3	530	690	855	1020	1190	(1490)
			7	530	700	875	1045	1220	(1540)
			21	490	635	795	965	1120	(1405)
			56	490	605	700	815	960	(1160)
50	III	0.60	3	670	920	1170	1130	1470	

Note: The values given are averages for two specimens. Values of creep in parentheses are extrapolated.

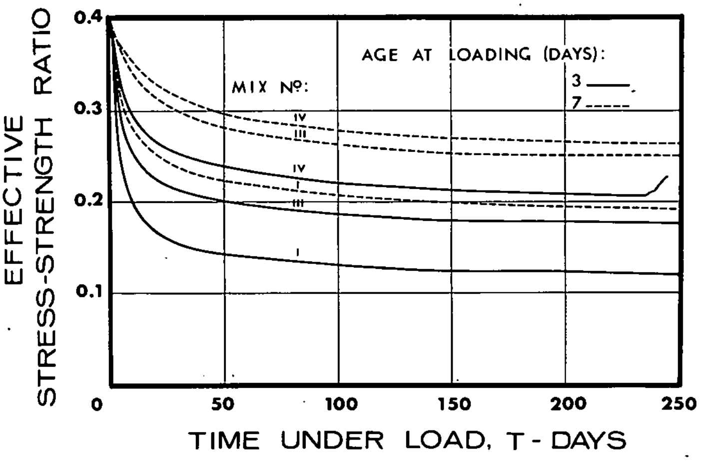

Figure 8. Effective stress-strength ratio for unadjusted tests.

the values of relative creep. Considering different ages at loading for a given mix, r_u is greater the earlier the age at loading; hence, a higher relative creep would be expected. This is confirmed in Figure 7.

Considering different mixes loaded at the same age, r_u is greater the higher the water-cement ratio; hence, a higher relative creep would be expected. Table 5 shows this generally to be the case with mixes III and IV, but mix I shows poor agreement. However, this mix exhibited only a small creep, which made the values of relative creep critical. Thus a small error in the absolute value of creep could lead to a large error in the value of relative creep.

Considering now the dry-stored concrete (Table 5), we observe that it exhibits a higher relative creep than similar concrete stored wet. This accords with Figure 6; it should be noted that dry-stored concrete exhibits retrogression of strength.

It may be relevant to refer now to some tests discussed in an earlier paper (3). It was suggested there that relative creep is independent of the water-cement ratio of the mix. However, in those tests an increase in the water-cement ratio was always accompanied by an increase in the cement paste content, and it now appears that the apparent lack of influence of the water-cement ratio on relative creep was the net result of two compensating tendencies: a small increase in relative creep with an increase in water-cement ratio (established in the present investigation) and a decrease in relative creep with an increase in the cement paste content (3).

A confirmation of the influence of the relative strength increase on relative creep has also been obtained from an analysis of Hummel's (9) data, as shown in Table 10. Good agreement can be seen both for different mixes loaded at the same age and for a given mix loaded at different ages.

CREEP AT A CONSTANT STRESS-STRENGTH RATIO (ADJUSTED TESTS)

In these tests (series B) the stress-strength ratio at any time remained at 0.4. The observed creep values are summarized in Table 6, and it can be seen that for any given mix an earlier age at loading leads to a higher creep. Figure 3 shows that the earlier the age at loading the higher the relative strength increase, r_u. Now, because the applied stress is increased in proportion to the increase in strength, a higher relative strength increase means an earlier increase in the applied stress. Thus, although the sustained stress-strength ratio is constant, an earlier increase in stress leads to a higher creep. The qualification in the last sentence has been inserted as it could have been thought that different concretes subjected to a constant effective stress-strength ratio should exhibit the same creep.

Thus, while a greater increase in the relative strength increase reduces creep for the same initial stress-strength ratio, the adjustment of the stress-strength ratio so that it remains at a constant value leads to an increase in creep in such a case. (Fig. 8 shows the effective stress-strength ratio in the unadjusted tests.)

Comparing different mixes loaded at the same age, it appears that creep is higher the lower the water-cement ratio. This is similar to the situation in the unadjusted tests. Thus the characteristics of a mix in relation to creep are qualitatively the same regardless of whether the effective stress-strength ratio drops off or is constant, i.e., whether the concrete is subjected to a constant stress or to a constant stress-strength ratio.

However, for a given mix the influence of the age at loading is such that the greater "natural" reduction in the effective stress-strength ratio with an earlier age reduces creep, but the adjustment of the stress-strength ratio is in effect an over-adjustment as far as constancy of creep is concerned.

We have observed that when the stress-strength ratio is kept constant for any given mix and duration of load the delay in the application of load leads to a decrease in creep. This effect is greatest in mix I and decreases with a decrease in the water-cement ratio. The same is the case with unadjusted tests (see Table 2), but there the effect is smaller, although equally regular.

It is not surprising that the effect of the age at loading is smaller in mixes with low water-cement ratios, because the fractional strength increase, f_u, while the specimen is under load is smaller in these mixes than in those with higher water-cement ratios (Fig. 2). Likewise, the fractional strength, f_m, is higher.

Consider now the relative creep in the adjusted tests. The higher the value of r_u the lower the relative creep, regardless of whether we consider mixes of different water-

TABLE 7

VALUES OF RELATIVE CREEP AS A PERCENTAGE OF CREEP FOR DIFFERENT MIXES AND AGES AT LOADING (ADJUSTED TESTS)

Age at Loading (days) / Time After Loading (days)	Mix I				Mix II				Mix III					Mix IV			
	3	7	21	56	3	7	21	56	3	7	14	21	56	3	7	21	56
3	18	28	23	24	21	23	32	33	27	19	20	23	21	36	34	35	42
7	26	35	35	36	33	35	42	43	39	32	38	41	39	46	46	45	52
14	36	45	46	47	46	48	54	51	54	49	54	53	51	57	57	57	60
28	50	58	59	60	58	60	64	63	69	65	70	65	64	69	68	69	70
56	70	73	74	73	76	77	77	75	82	82	83	79	79	80	79	80	83
150	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100

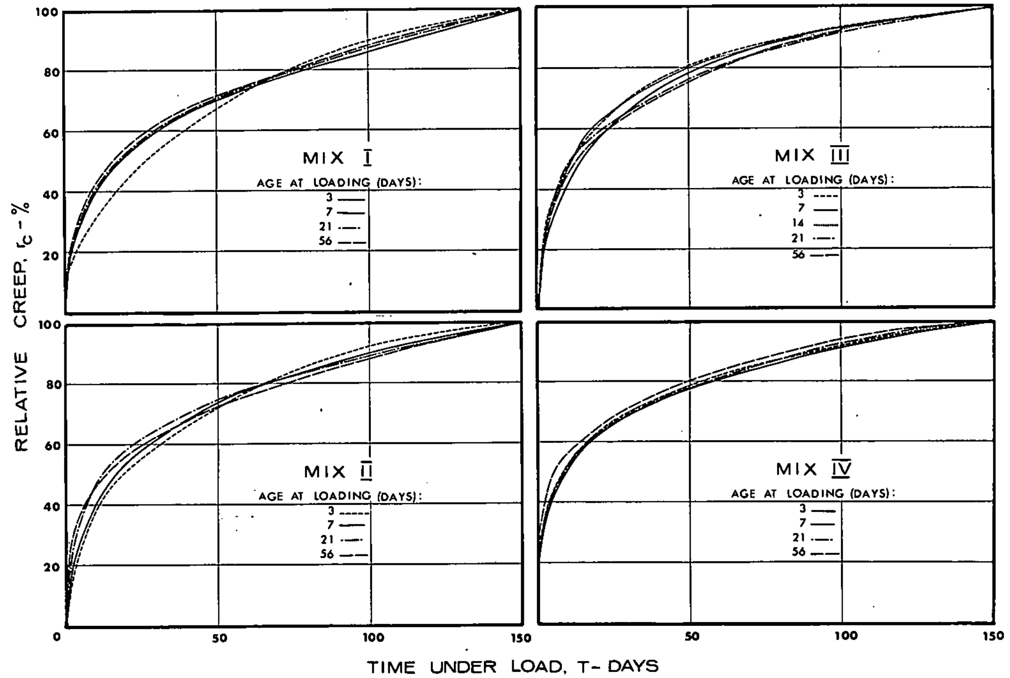

Figure 9. Relation between relative creep, r_c, and time under load for different ages at loading of of series A.

TABLE 8

COMPARISON OF CREEP VALUES AFTER 150 DAYS UNDER LOAD FOR ADJUSTED AND UNADJUSTED TESTS

Mix No.		I	II	III	IV
Creep (10^{-6})	adjusted	1605	1680	1370	1490
	unadjusted	375	–	750	870
Difference (10^{-6})		1230	–	620	620
Fractional strength increase, f_u		2.22	1.84	1.26	0.92

Note: All data for concrete loaded at the age of 3 days and stored at a relative humidity of 95 percent.

cement ratios loaded at the same age, or the same mix loaded at different ages (Table 7). Figure 9 shows that this pattern of behavior is maintained in all cases except for mix III loaded at different ages. Thus the postulated relation between relative creep and relative strength increase is at least qualitatively correct for a wide range of strength-time curves both for adjusted and nonadjusted conditions.

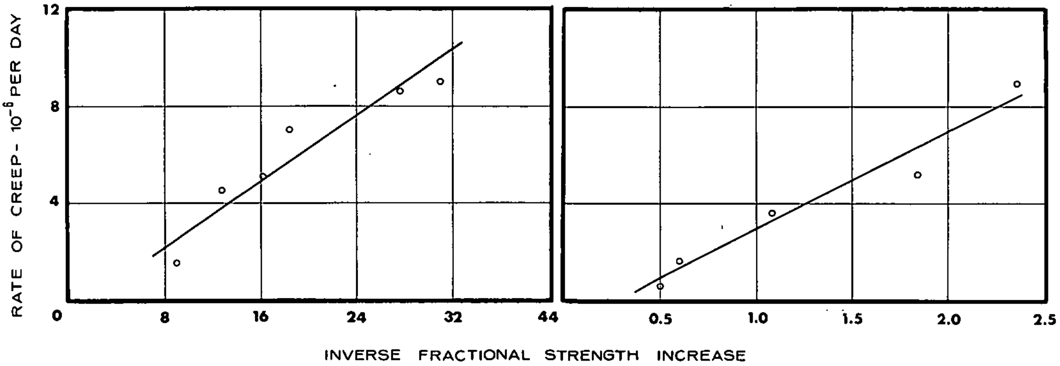

Figure 10. Relation between rate of creep, $\frac{dc}{dT}$ and inverse fractional strength increase, $\frac{1}{f_u}$: (a) mix III, unadjusted, loaded at 56 days; (b) Hummel's (9) data for a mix with a water-cement ratio of 0.55.

It may be interesting to compare the differences in the creep for the unadjusted and adjusted conditions for corresponding mixes and ages at loading (Table 8). The difference is larger the higher the water-cement ratio, this being a reflection of the higher difference in the fractional strength increase and, therefore, in the case of the adjusted tests, of a higher applied stress.

QUANTITATIVE RELATIONS

From the foregoing discussion it can be seen that creep is a function of the fractional strength increase, f_u. This functional dependence was suggested in an earlier paper (1), but only now is a quantitative evaluation possible.

Using the experimental data both of this investigation and of Hummel's (9) tests, the relation between the rate of creep $\frac{dc}{dT}$ and the fractional strength increase, f_u, was found to be approximately hyperbolic.

This would suggest that $\frac{dc}{dT}$ and $\frac{1}{f_u}$ are linearly related (Fig. 10). We can postulate, therefore,

$$\frac{dc}{dT} = -F + k \frac{u_o}{u - u_o} \qquad (6)$$

where F and k are constants. Eq. 6 is valid for $T > 0$ only. We shall now attempt to show that an equation of this form can be derived from considerations of shape of the creep- and strength-time curves.

Now the creep-time relation can be expressed in hyperbolic form, as suggested by Ross (10):

$$c = \frac{T}{a + bT} \qquad (7)$$

where T is time since loading, and a and b are constants.

The assumption of this form of creep-time curve means that

$$\lim_{T \to \infty} \frac{dc}{dT} = 0 \qquad (8)$$

From Eqs. 6 and 8

$$F = k \frac{u_o}{u_m - u_o} \qquad (9)$$

since u_m is the strength at $t = \infty$. Hence, when $T > 0$,

$$\frac{dc}{dT} = k\left(\frac{u_o}{u - u_o} - \frac{u_o}{u_m - u_o}\right) \tag{10}$$

It may be convenient to approximate the strength-time relation also by a hyperbola, and write

$$u = \frac{t}{m + nt} \tag{11}$$

where t = age of concrete, and m and n are constants. Since t_o = age at loading,

$$T = t - t_o \tag{12}$$

From Eqs. 7, 11, and 12

$$\frac{dc}{dT} = -F + \frac{N}{(u - d)} + \frac{M}{(u - d)^2} \tag{13}$$

where N, M, and d are constants. This equation is valid for all values of T. The last term of Eq. 13 is small and can be neglected as the relation between the rate of creep $\frac{dc}{dT}$ and $\left(\frac{1}{u - d}\right)$ was found to be approximately linear.

The values of F, N and d were derived from the experimental creep data, and it was found that for all values of $T > 0$, $d \mathrel{\hat{=}} u_o$. Thus, when $T > 0$

$$\frac{dc}{dT} = -F + \frac{N}{u - u_o} \tag{14}$$

Comparing with Eq. 9, when $T \to \infty$, we find

$$N = k\, u_o \tag{15}$$

This is valid for all values of T. Now, when $T = 0$

$$\frac{dc}{dT} = -F + \frac{ku_o}{u - d} \tag{16}$$

Substituting for F from Eq. 9 into Eqs. 14 and 16 we obtain Eq. 10 for $T > 0$, and, for $T = 0$

$$\frac{dc}{dT} \doteq k\left(\frac{u_o}{u - d} - \frac{u_o}{u_m - d}\right) \tag{17}$$

Substituting for u from Eq. 11 into Eq. 10 and integrating, we obtain

$$c = kt\left(\frac{u_o n}{1 - u_o n} - \frac{u_o}{u_m - u_o}\right) + \frac{u_o m k}{(1 - u_o n)^2} \log_e \left[\, t\,(1 - u_o n) - u_o m \,\right] + \text{const} \tag{18}$$

Substituting for u from Eq. 11 into Eq. 17, integrating, and inserting the lower limit values $t = t_o$ and $c = 0$, we obtain

TABLE 9

DATA FOR THE RELATION BETWEEN $\left(\frac{du}{dt}\right)_k$ AND k

Mix No.	Age at Loading t_o	$\left(\frac{du}{dt}\right)_k$ (psi per day)	k (per day)
I	3	89	8.8
	7	60	10.8
	56	1	0.8
III	3	70	12
	14	13.3	4
	28	8.3	2.4
IV	3	43.5	7.5
	7	18.5	2.4
	56	1.6	0.74
Hummel's Type I cement mix	28	0.855	0.75
	90	0.855	0.24
Hummel's Type III cement mix	28	1.14	6.6
	90	0.184	0.24

TABLE 10

RELATIVE STRENGTH INCREASE AND RELATIVE CREEP FOR HUMMEL'S DATA

Hummel's Cement Type	Age at Loading t_o (days)	Time Since Loading, T (days)					
		7	14	28	42	56	200
		Relative Strength Increase, r_u (%)					
I	3	63	74	85	90	92	100
	28	17	23	49	62	73	100
	90	12	20	36	59	70	100
III	3	53	66	79	87	92	100
	28	21	42	64	70	81	100
	90	6	13	38	51	63	100
		Relative Creep, r_c (%)					
I	3	47	59	69	76	81	100
	28	43	49	62	70	76	100
	90	45	54	66	71	75	100
III	3	46	55	65	70	75	100
	28	30	40	55	61	67	100
	90	36	45	54	61	66	100

$$0 = kt_o\left(\frac{u_o n}{1 - dn} - \frac{u_o}{u_m - d}\right) + \frac{u_o mk}{(1 - dn)^2}\log_e\left[t_o(1 - dn) - dm\right] + \text{const} \qquad (19)$$

Since, from Eq. 7, when T = 0, $\frac{dc}{dT} = \frac{1}{a}$ and $u = u_o$, we have from Eq. 17

$$\frac{1}{a} = k\left(\frac{u_o}{u_o - d} - \frac{u_o}{u_m - d}\right) \qquad (20)$$

Providing we know k, this equation will enable us to find the value of d, since u_o and u_m are known ($u_m = \frac{1}{n}$), and a is nearly constant for all creep curves. Hence, by substituting in Eq. 19 the constant of integration can be found.

Eq. 18 then becomes

$$c = u_o mk\left\{\frac{1}{(1 - dn)^2}\log_e\left[t_o(1 - dn) - dm\right] + \frac{1}{(1 - u_o n)^2}\log_e\left[t(1 - u_o n) - u_o m\right]\right\} \qquad (21)$$

To find k we equate the term in parentheses in Eq. 10 to unity. Then, for instance,

$$u = u_o\left(2 - \frac{u_o}{u_m}\right) = u_k$$

For this condition, then, $t = t_k$, $T = T_k$, $\frac{du}{dt} = \left(\frac{du}{dt}\right)_k$, and $k = \left(\frac{dc}{dT}\right)_k$. We have computed the values of these quantities for the present unadjusted tests and for 4 of Hummel's (9) tests. The results are summarized in Table 9. Figure 11 shows that there is a good linear relation between $\left(\frac{du}{dt}\right)_k$ and k for a wide range of mixes. The quantities required can be obtained from actual measurements or, which is more useful, by calculation from Eqs. 7 and 11. It may be noted that the latter method leads to results

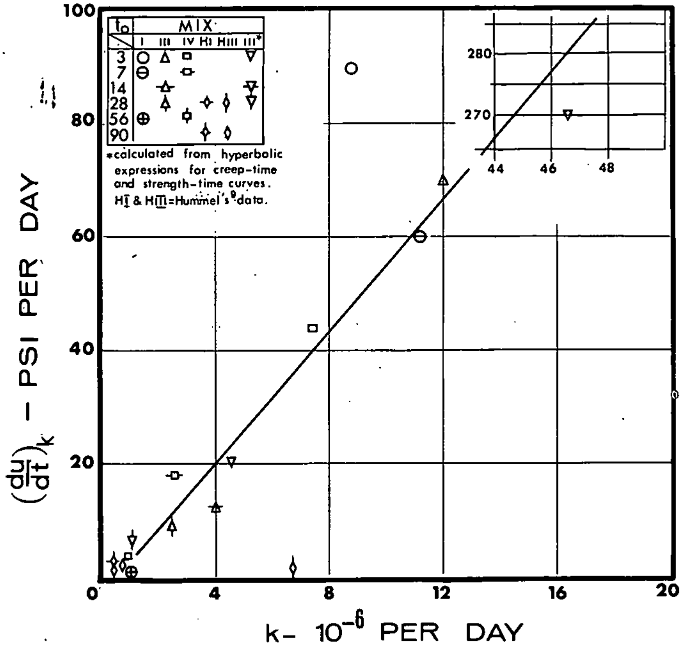

Figure 11. Relation between $\left(\frac{du}{dt}\right)_k$ and k.

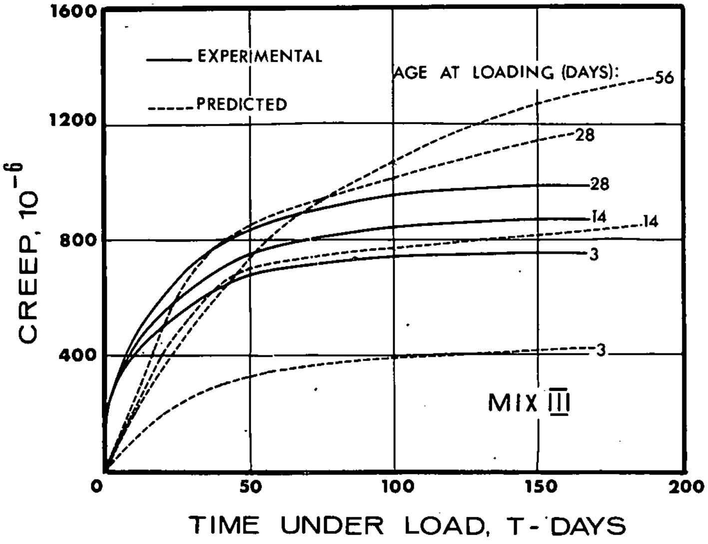

Figure 12. Predicted (Eq. 21) and observed creep.

which fit the relation of Figure 11 well but the actual values of $\frac{du}{dt}$ are in error, owing to the imprecise nature of the hyperbolic equation to strength; it is proposed to revise the equation possibly by replacing it by a polynomial.

Figure 11 can now be used to determine k, given u_o, u_m, and t_o. Eq. 18 can then be used to predict creep for any time under load, T. For example, for mix III loaded at 28 days, $m = 10.80 \times 10^{-4}$ days per psi and $n = 1.27 \times 10^{-4}$ per psi, the strength-time curve being fitted for the interval of t (20, 150) days.

We take $u_m = 7450$ psi, $k = 0.236 \times 10^{-5}$ per day and $a = 11,000$ days. Hence Eq. 21 becomes:

$$c \times 10^6 = 120 + 286 \log_e (0.234t - 6.53)$$

where t is age in days, and stress-strength ratio is 0.4.

The calculated and experimental values of creep are shown in Figure 12. The agreement is reasonably good but it is essential that the strength-time curve be fitted correctly for ages immediately following the time of loading.

CONCLUSIONS

Creep is related to the development of strength of concrete after the application of the load, being higher the lower the fractional strength increase when the applied stress is constant. Since for earlier loading age the increase in strength is greater than for later loading, for a given mix, creep is greater the later the load is applied (for the same initial stress-strength ratio). However, when the stress-strength ratio is maintained constant, the earlier increase in stress in specimens loaded at an earlier age leads to a higher creep.

The rate at which the ultimate creep is reached depends on the relative strength increase of the concrete.

Quantitative relations have been formulated making possible the prediction of creep from the strength-time curve for the concrete. Numerical procedures for such a prediction are being developed.

ACKNOWLEDGMENT

This investigation has been supported by a grant from the National Research Council of Canada, which is gratefully acknowledged. We are also grateful to Mrs. M. Fogarasi and Miss S. M. J. Porter, who assisted in the analysis of the data.

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Appendix A

PROPERTIES OF CEMENT

Bogue composition (%):

C_3S	C_2S	C_3A	C_4AF	MgO	SO_3	Alkalis (as soda equivalent)
54.6	21.0	9.0	5.8	3.52	2.72	0.50

Compressive strength of standard* 2-in. mortar cubes, psi:

1 day	3 days	28 days
2,750	4,730	7,110

GRADING OF AGGREGATE

ASTM sieve size	100	50	30	16	8	4	⅜ in.
Cumulative percentage passing	1.8	8.6	22.8	32.3	39.7	54.5	96.5

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*Canadian Standard A 5-1961

Appendix B

DRYING CREEP OF DRY-STORED SPECIMENS

Mix III, dry-stored, has a 3-day strength of 3100 psi. Using Powers' (5) expression for gel-space ratio, the degree of hydration is estimated to be 65 percent. Since shrinkage was 360×10^{-6}, the drying creep is estimated from Ali and Kesler's (8) expression to be 400×10^{-6}.

The basic creep for loading at 3 days should be the same as for loading at 28 days since there is little difference in the fractional strength increase for the two conditions. Thus

$$\begin{aligned} \text{basic creep} &= 850 \times 10^{-6} \\ \text{drying creep (calculated)} &= \underline{400 \times 10^{-6}} \\ \text{total creep (predicted)} &= 1250 \times 10^{-6} \\ \text{observed total creep} &= 1400 \times 10^{-6} \end{aligned}$$

For loading at the age of 7 days the strength is 3670 psi and the degree of hydration is 70 percent. Shrinkage is 260×10^{-6}, and drying creep is estimated to be 330×10^{-6}. Hence, the predicted total creep is 1180×10^{-6}, which is exactly the observed value.