State of the Art of Rigid Pavement Design

THE FIRST extensive concrete road system in the United States was constructed in ayne County, Michigan, in 1909. At that time there was no rational design theory in xistence and consequently, design decisions were based entirely on "engineering judg- ient." Since that time civil engineers have continued to grapple with the many prob- ems involved in the design of such pavement. Goldbeck (1) and Older (2) independently eveloped formulas for approximating the stresses in concrete pavements in the early 920's. The best known of these formulas is generally called the "corner formula" and t was the basis for rigid pavement thickness design for many years. In 1926 Wester- aard completed his treatise on the analysis of stresses in pavement slabs (3). The estergaard equations have become the definitive design equations for pavement slabs n the United States.

Although many approaches have been made to the solution of this problem, limita- ions of conventional mathematics, particularly hand solutions, have restricted develop- nents. Owing to the complexity of the problem all of the solutions involve severely imiting assumptions which bypass certain influencing factors that, in reality, are very mportant. As a consequence, none is completely satisfactory.

Several large-scale road tests have been conducted in attempts to bridge this gap etween theory and reality. These include the Bates Road Test in 1922, the Maryland Road Test in 1950, and the AASHO Road Test in 1958-61. All three of these full-scale xperiments have added to the knowledge of pavement design. The AASHO Road Test vas large enough to provide significant information, but even it considered only six asic variables: (a) slab thickness, (b) slab length, (c) axle load, (d) number of axle epetitions, (e) subbase thickness, and (f) jointed reinforced vs jointed plain pavements.

In addition to these road tests, several large tests have been carried out by the U.S. Corps of Engineers for the U.S. Air Force, such as the Sharonville Test Tracks. The esults of these tests have been particularly pertinent to airfield pavements but also hed additional light on highway pavement problems.

Many engineers recognized the magnitude of the problem. The Bureau of Public Roads as well as many state highway departments recognized the importance of the in- luence of environment and climatic factors on the performance of pavements. As a esult there was a proliferation of small research projects, many of them involving 'in-service" pavements. Each of these projects was naturally limited in size and herefore limited in the variables it considered. Furthermore, a tremendous variation n materials and testing methods has made it difficult to correlate the resulting nformation.

Pavement design involves four general classes of variables: (a) load variables, (b) structural variables, (c) regional variables, and (d) performance variables. The 'Guidelines for Satellite Studies of Pavement Performance" (4) discusses these vari- ables in some detail. Major theoretical efforts have been directed toward evaluating structural variables, and the fact that they interact prohibits the consideration of all variables in any single test.

To further complicate the problem, many new techniques have been developed in recent years. Continuously reinforced concrete pavements have been investigated and are currently being used. Continued improvements are being sought in the use of pre- stressed concrete pavements. Other recent developments involve the use of new mate- rials such as expansive cements and synthetic aggregates. In addition, traffic volumes and axle or airplane gear weights have increased. The advent of these new materials and this increased traffic makes the continued improvement and up-dating of pavement design techniques desirable.

All engineering developments combined theory and experience in varying degrees. These developments then lead to design procedures for "current use" but always leave many questions unanswered. A great deal of research is presently in progress toward solving these problems. A critical examination of this research in light of the overall problem and program would be a valuable addition to current techniques.

After considering the overall problem, the work that has been done and that is currently being done, it is valuable to consider the additional research needed. This undoubtedly would include some basic research, some applied research and, in certain instances, some "far-out concepts" which may be very valuable.

In this report four basic types of portland cement concrete pavement will be considered. These are (a) plain concrete pavement, (b) jointed reinforced concrete pavement (c) continuously reinforced concrete pavement, and (d) prestressed concrete pavement. These categories will be carried through each succeeding chapter where appropriate.

In the preparation of this report every effort was made to avoid expressing personal opinions of the authors. The statements made are intended to reflect the information contained in the various references or a reasonable interpretation of them. There was no intention to imply that one design or practice is considered superior to another unless the reference material was cited to support the statement. The reader is urged to personally verify any such statements contained here by consulting the reference item.

HISTORY OF CONCRETE PAVEMENT

Prior to 1890, concrete as known today was not used for building roads (5). In the early 1890's three important "firsts" occurred. In 1891 the first portland cement concrete pavement was constructed at Bellefontaine, Ohio. It consisted of an 8-ft-wide strip of concrete to provide a solid pavement in front of a line of hitching rails (6).

In 1893 Bellefontaine, Ohio, constructed the first full-width concrete pavement, and J. F. Duryea successfully operated the first American-made gasoline-engined automobile (7). Within the following half century paved roadways and motor vehicles became vitally important in the United States.

From 1893 until 1908 less than five miles of concrete pavement were constructed in the entire country, and these were located on city streets or private estates. In 1909 the first mile of rural concrete pavement in the United States was constructed in Wayne County, Michigan, and opened to public traffic on July 4th (8). Approximately four miles of rural road were paved with concrete that year. The mileage of rural concrete roads increased annually; the first big increase came in 1912 when 250 miles were constructed. By 1924 over 31,000 miles of concrete pavement were in use, and construction was proceeding at the rate of 6,000 miles per year (9). During this same period, 1909 to 1924, the production of motor cars increased from about 350,000 to over 3,300,000 per year, and the number registered went from about 306,000 to over 15,400,000 (10).

Neither the vehicle nor concrete pavement is the product of a single inventor; rather they have been developed through the efforts of thousands of technical and nontechnical people. Today's concrete pavements are the result of the accumulated experience of pavement engineers gained by (a) study and appraisal of existing pavements, (b) observation of trial roads and long-range experimental road tests carrying normal traffic, (c) accelerated controlled traffic tests on existing or specially constructed pavement sections, (d) laboratory experimentation, and (e) theoretical and rational analyses.

Between 1893 and 1920 there was very little basic technical information available to highway engineers concerned with concrete pavements. The experimental roads of the period were constructed primarily to determine the most economical designs applicable to local climate, traffic and subgrade conditions (11).

The large-scale road building program which followed World War I intensified the need for engineering data. For example, no test information was available on the supporting strength of subgrade soils, stresses and deflections induced by axle loads of moving vehicles, or effects of temperature and moisture variations on the performance of the pavement (12). Consequently, a nationwide program of research was launched in 1920 by the Bureau of Public Roads and the Highway Research Board. Many universities and state highway departments participated in this endeavor.

Between 1920 and 1923 the State of Illinois constructed and operated the Bates Experimental Road at Bates, Illinois. This was the first controlled traffic road test, and from it came the first design criteria for determining the thickness of a concrete pavement for a known wheel load (2). Other outstanding road tests were conducted in the 1920's and 1930's. The Pittsburg, California, Road Test, started in 1921 and completed in 1922, was designed to determine the efficiency of both reinforced and nonreinforced pavements of varying designs. Although the results were not conclusive, they added to the supply of technical information. They also indicated that longitudinal joints were effective in preventing longitudinal cracks (12).

From 1930 through 1936 the Bureau of Public Roads conducted the Arlington Test. This was an extensive investigation of the structural design of concrete pavement made at Arlington, Virginia. This research work supplied the basis for modern concrete pavement design criteria (13).

In the middle 1930's the use of deicing salts for winter maintenance produced severe scaling of concrete pavement surfaces in the northern states. Numerous road tests were constructed containing various ingredients that might produce scale-resistant pavements. Air-entrained concrete came out of this work. The earliest use of air-entrainment was in New York State in 1938. Between 1938 and 1942 a total of 17 test pavements were constructed in several northern states.

In the late 1930's many highway engineers became concerned about the use of expansion joints where contraction joints were also used. The use of dowels in closely spaced contraction joints and other problems in the joining of pavements were also studied in view of service records. To provide answers to these questions the Bureau of Public Roads authorized the construction of long-range experimental road tests in California, Kentucky, Michigan, Minnesota, Missouri and Oregon (14).

Just prior to World War II, pumping became common on primary systems in many sections of the country. In most cases the pumping developed on roads that carried large volumes of heavily loaded vehicles, and in areas where subgrade materials were mostly claylike or plastic in character. Many pavements on major truck routes were damaged by pumping, while pavements on lightly traveled roads with comparable designs, ages, and subgrade soils did not pump (15). Pumping is usually not as serious a problem in connection with airport pavements.

During World War II there was a significant increase in the volume and weight of truck traffic on highways and a large increase in the wheel load and tire pressure of military aircraft on airport pavements. In addition, adequate maintenance during this period was almost impossible, leaving many of these pavements in serious need of repair and upgrading. Added to this was the need for many miles of additional pavements to satisfy the tremendous growth of both rural and metropolitan areas. Consequently, research programs in all phases of pavement technology were intensified to meet the ever-increasing demands of postwar traffic. Two of these programs are outstanding—Maryland Road Test One-MD and the AASHO Road Test.

The Maryland Road Test One-MD was conducted on a section of existing concrete pavement near La Plata, Maryland, in the summer of 1950. The principal objective was to determine the effects of loads up to 44,800 lb per tandem axle on this pavement. This was the first controlled traffic test made on concrete pavement since the Bates and Pittsburg road tests in 1921-1923 (16).

The AASHO Road Test, sponsored by the American Association of State Highway Officials and other highway agencies, was conducted on a specially constructed section of highway near Ottawa, Illinois, between 1958 and 1961. One of the objectives of the AASHO Road Test as stated by the National Advisory Committee was "to determine the significant relationships between the number of repetitive applications of specified axle loads of different magnitude and arrangement and the performance of different thicknesses of uniformly designed and constructed asphaltic concrete, plain portland cement concrete, and reinforced portland cement concrete surfaces on different thicknesses of bases and subbases when on a basement soil of known characteristics." As far as portland cement concrete pavements are concerned, six basic variables were considered: (a) slab thickness, (b) slab length, (c) axle load, (d) number of axle repetitions, (e) subbase thickness, and (f) presence or absence of steel reinforcement.

Pavement Foundation

The need for uniformity of subgrade support under concrete pavements has long been recognized. Specifications published in 1910 contain a section relating to the preparation of the subgrade (17). However, it was not until 1924-1925 that rapid field methods of identifying the quality of subgrade soils became available (18). Prior to 1925, acceptability of subgrades depended entirely on the judgment of highway engineers. The publication of the Rose reports in 1925 marked a turning point in highway and concrete pavement design. They became the foundation upon which much subsequent subgrade soil research was based. These reports also pointed out that studies were needed to evaluate the influence of subgrade soils on pavement performance. However, it was not until the early 1930's that serious attention was given to such studies (19).

Subbases were used under concrete pavements as early as 1894, when alleys in Boston were constructed following sidewalk procedures (8-in. cinder subbase under 5-in. concrete). One of the first uses of subbases under street pavements was in Richmond, Indiana, in 1896. It consisted of 10 in. of compacted stone rubble under 5-in. and 6-in. thick concrete pavements (20). Specifications published in 1910 contain sections pertaining to subbases. It was specified that subbases "... of clean, hard, suitable material, not exceeding 4 in. in largest dimension ... be used ... where required; that they have a minimum thickness of 6 in. and "be thoroughly rolled and tamped" (17). The use of subbases to overcome the effect of poor subgrade soils was recommended by Rose in 1925 (18). By the end of World War II the majority of states were specifying subbases to prevent pumping and to aid in maintaining the structural capacity of pavements (21). However, this was not a common practice for city street and airfield design.

Pavement Thickness

A tabulation of data on 29 concrete roads constructed in Ohio in 1911, 1912, and 1913 shows that a wide variety of thicknesses were used. Those with uniform cross sections ranged in thickness from 6 to 7 in. Those with nonuniform cross sections were of thickened-center design with thicknesses of 6-7-6 or 5-7-5 in. It was felt that the greater center thickness would prevent longitudinal cracking (22). Prior to World War I most concrete highway pavements were constructed to a uniform thickness of 4 to 6 in.

The thickened-edge design was developed in 1920 as a means of strengthening pavement edges. This type of cross section proved superior to comparable cross sections of uniform thickness in the relatively narrow pavements tested in the Bates Road Test (1920-1923) and the Pittsburg, California, Road Test (1921-1922). As a result it was quickly adopted and by 1934 41 states were using some form of thickened-edge design (21). The thicknesses of these nonuniform sections were usually 7-6-7, 8-6-8, and 9-7-9 or 9-6-9 in. As the use of wider pavements became more prevalent, some dissatisfaction with the section developed. By 1945 the states were about evenly divided between uniform-thickness and thickened-edge designs. Since 1956, all states with concrete pavement specifications have required uniform cross sections (21). Thickened-edge pavements are used in airfield pavement design to take care of edge stresses.

Pavement Type

Concrete pavement is generally classified as either plain, reinforced, or continuously reinforced. The first plain pavement (in Bellefontaine) has already been mentioned (6). One of the first reinforced pavements was constructed in 1908. This road was 24 ft wide, 11 miles long, contained superelevated lanes, and was placed in two courses to a total thickness of 5 in. (7). Reinforcement was recommended in 1914 to counteract cracking caused by thermally induced expansion and contraction (23). In 1916 it was recommended that all concrete roads be reinforced and specifications were written to cover several important design problems (24). In 1931 the common pavement slab in use in many states was of the thickened-edge design containing 30 to 60 lb of steel, wire mesh or bar mat per 100 sq ft (25).

The concept of continuous reinforcement was first tried experimentally by Indiana in 1938. This project led to the construction of test sections in Illinois in 1947, Texas in 1949, California in 1949, and New Jersey in 1947.

Jointing Practice

Early pavements were constructed without joints, but it soon became evident that it was necessary to introduce transverse joints to control transverse cracking. In 1914 the American Concrete Institute recommended that transverse joints should be not less than $^1/_4$ in. nor more than $^3/_8$ in. wide and should be placed across the pavement perpendicular to the centerline and not more than 35 ft apart. Randomized joint spacing is used in California (13, 19, 18, 12 ft) to avoid resonant response from vehicles.

As early as 1914 Illinois tried skewed joints with a skew angle of 60 degrees with the centerline of the roadway. Adjacent joints were skewed in opposite directions to make the irregularities less noticeable and to reduce cumulative vibrations (26). California constructed an experimental section of pavement with skewed joints in 1932. Skewed joints are now standard in California, Idaho, Colorado, and several other states.

Following World War I the use of steel dowels and proprietary devices to provide load transfer increased (27). Weakened plane contraction joints in which load transfer is achieved by aggregate interlock were introduced in 1919 (28) and have gained considerable acceptance through the years. Aggregate interlock has been the major means of load transfer for the closely spaced contraction joints of plain concrete. Mechanical load transfer devices are used universally in the joints separating the relatively long slabs of reinforced concrete pavements (21).

Early concrete pavements were constructed without longitudinal joints. As wider slabs were constructed and traffic increased, meandering longitudinal cracks developed near the centerline. Some engineers contended, even before 1914, that a pavement over 12 ft wide should have a longitudinal joint down the middle. They contended that dividing the pavement into two sections would serve to reduce warping stresses and to control longitudinal cracking, and would be more economical than building "thickened-center slabs" (26). The effectiveness of such joints in preventing longitudinal cracks in both plain and reinforced pavements was indicated by the Pittsburg, California, Road Test (12). During the 1920's the use of longitudinal contraction joints was generally adopted.

The occurrence of blowups in early pavements that had been in service for five or more years was considered evidence that concrete pavements required expansion joints to protect them from compressive stresses. By 1934 expansion joints were in general use, but with considerable variation in joint spacing among the state highway departments.

In 1934 the BPR required expansion joints at not more than 100-ft spacing and contraction joints at intervals of not more than 30 ft on all federal-aid road construction. Largely because of troubles experienced with construction and maintenance, some state engineers expressed a desire to increase the spacing of expansion joints or to omit them altogether (29). Based on experimental projects in 1940 where expansion joints were spaced from 100 ft to one mile apart, it was concluded that pavement design with contraction joints only could be adopted without fear of blowups. A great variety of expansion and contraction joint arrangements were used by the states in the period immediately after World War II. Since then the use of expansion joints has progressively declined except at structures and abutments. A study of pavement blowups made in Indiana was reported in 1945 (116). On the basis of this report the State of Indiana abandoned the use of expansion joints in concrete pavements except at ends of structures. A number of states followed this procedure within the next few years.

THEORIES FOR CONCRETE PAVEMENT DESIGN

Load Stresses

Engineers often speak of the theories of pavement design. In reality the numerous so-called theories all spring from a single theory, the theory of elasticity. A brief review of the pertinent aspects of the problem will be helpful in understanding subsequent work.

A pavement slab is variously called a slab-on-foundation, a pavement slab, a slab, or a plate-on-foundation. Regardless of the name applied, a pavement slab can be considered to be a plate with various support conditions. Timoshenko (30) distinguishes three kinds of plate bendings: (a) thin plates with small deflections, (b) thin plates with large deflections, and (c) thick plates. Since the deflections of pavement slabs are small in comparison with their thickness, a satisfactory approximate theory of bending of slabs by lateral loads can be developed by assuming that (a) there is no deformation in the middle plane of the slab (this plane remains neutral during bending), (b) planes of the slab initially lying normal to the middle plane of the slab remain normal after bending, (c) the normal stresses in the direction transverse to the slab can be disregarded (this assumption is necessary in the analysis of bending of the plate as will be seen later; approximate corrections can be made to account for pressures directly under the transverse load). With these assumptions all components of stress can be expressed in terms of the deflected shape of the slab. This function must satisfy a linear partial differential equation which, together with the boundary conditions, completely defines deflection w. The solution of this differential equation gives all necessary information for calculating the stresses at any point in the plate.

The approximate theories which define the behavior of thin slabs become unreliable for slabs of considerable thickness, particularly in the vicinity of highly concentrated loads. In these cases thick plate theory must be applied which considers the problem of the slab as a three-dimensional problem of elasticity. The stress analysis of such cases is complex and, according to Timoshenko, the problem is completely solved for only a few particular cases. In some instances the necessary corrections to thin plate theory are introduced at the points of application of concentrated loads. This is desirable in pavement slabs and has been discussed by Westergaard (3).

Timoshenko (30) has derived a differential equation which describes the deflection surface of pavement slabs subjected to loads applied perpendicular to their surface. This equation can be stated as

$$\frac{\partial^2 M_x}{\partial X^2} + \frac{\partial^2 M_{yx}}{\partial X \partial Y} + \frac{\partial^2 M_y}{\partial Y^2} - \frac{\partial^2 M_{xy}}{\partial X \partial Y} = q - kw \tag{1}$$

where M_x is the bending moment acting on an element of the plate in the x direction, M_y is the bending moment acting on an element of the plate in the y direction, M_{xy} is a twisting moment tending to rotate the element about the x-axis (clockwise positive), M_{yx} is a twisting moment tending to rotate the element about the y-axis, q is the applied lateral load, k is the support strength of the subgrade, and w is the deflection at any point.

By introducing the appropriate moment equations (31, 32) the expression obtains

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) = q - kw \tag{2}$$

According to Timoshenko (30) this equation was obtained by LaGrange in 1811. The history of this development is given in Todhunter and Pearson's "History of Elasticity." The solution is complex and significant strides were not made until the early 1920's.

Corner Formula

The equations developed in the early 1920's by Goldbeck and Older for approximating the stresses in concrete pavement slabs are empirical in nature. They apply only under grossly simplified conditions. The best known of these formulas is generally called the "corner formula" and is expressed as

$$\sigma_c = \frac{3P}{h^2} \tag{3}$$

where

σ_c = maximum tensile stress in pounds per square inch in a diagonal direction in the surface of the slab near a rectangular corner;
P = static load in pounds applied at a point at the corner; and
h = depth of the concrete slab in inches.

This formula was derived using the assumptions of point load applied at the extreme corner and no support from the subgrade. The fiber stresses in the surface of the slab are assumed to be uniform on any section at right angles to the corner bisector.

Strain measurements taken on the Bates Road Test in 1922-23 appear to confirm the corner formula. Obviously the assumption of point load and load applied at the extreme corner were not correct for the Bates test sections. It is interesting to note that in spite of this there was reasonably good comparison. This good agreement could be partly due to the high impact transmitted to the slabs with the solid rubber tires used in the Bates test or to the fact that subgrade support may have been very low as assumed by this formula.

Westergaard Solutions

In 1926 Westergaard completed a solution of Eq. 2 and hence for the stresses in concrete pavement slabs. This analysis is concerned with the determination of maximum stresses in slabs of uniform thickness resulting from three separate conditions of loading: (a) load applied near the corner of a large rectangular slab (corner load); (b) load applied near the edge of a slab but at a considerable distance from any corner (edge load); and (c) load applied at the interior of a large slab at a considerable distance from any edge (interior load). In his solution of this problem, Westergaard made the following important assumptions:

1. The concrete slab acts as a homogeneous isotropic elastic solid in equilibrium.
2. The reactions of the subgrade are vertical only and they are proportional to the deflections of the slab.
3. The reaction of the subgrade per unit of area at any given point is equal to a constant k multiplied by the deflection at the point. The constant k is termed "the modulus of subgrade reaction" or "subgrade modulus," and is assumed to be constant at each point, independent of the deflection, and to be the same at all points within the area of consideration.
4. The thickness of the slab is assumed to be uniform.
5. The load at the interior and at the corner of the slab is distributed uniformly over a circular area of contact; for the corner loading the circumference of this circular area is tangent to the edge of the slab.
6. The load at the edge of the slab is distributed uniformly over a semicircular area of contact, the diameter of the semicircle being at the edge of the slab.

For the three cases given and the appropriate assumptions as listed, the following expressions for stress were developed by Westergaard:

$$\sigma_i = 0.31625 \frac{P}{h^2} \left[4 \log_{10}\left(\frac{\ell}{b}\right) + 1.0693\right] \tag{4}$$

$$\sigma_e = 0.57185 \frac{P}{h^2} \left[4 \log_{10}\left(\frac{\ell}{b}\right) + 0.3593\right] \tag{5}$$

$$\sigma_c = \frac{3P}{h^2} \left[1 - \left(\frac{a\sqrt{2}}{\ell}\right)^{0.6}\right] \tag{6}$$

where

P = point load, in pounds;
σ_1 = maximum tensile stress in pounds per square inch at the bottom of the slab directly under the load, when the load is applied at a point in the interior of the slab at a considerable distance from the edges;
σ_e = maximum tensile stress in pounds per square inch at the bottom of the slab directly under the load at the edge, and in a direction parallel to the edge;
σ_c = maximum tensile stress in pounds per square inch at the top of the slab, in a direction parallel to the bisector of the corner angle, due to a load applied at the corner;
h = thickness of the concrete slab in inches;
μ = Poisson's ratio for concrete (taken as 0.15 in these equations);
E = modulus of elasticity of the concrete in pounds per square inch;
k = subgrade modulus in pounds per cubic inch;
a = radius of area of load contact in inches (the area is circular in the case of corner and interior loads and semicircular for edge loads);
b = radius of equivalent distribution of pressure at the bottom of the slab = $\sqrt{1.2\, a^2 + h^2} - 0.675\, h$; and
ℓ = the radius of relative stiffness = $\sqrt[4]{\dfrac{Eh^3}{12\,(1-\mu^2)\,k}}$.

Modifications to the original 1926 equations were made by Westergaard in 1933, 1939 and 1947. The 1933 modifications were concerned primarily with interior loads and will not be discussed here.

Slab on Elastic Solid

Other major theoretical work applicable to concrete pavement design involves the solution of large slabs supported on an elastic solid. Work in this area has been done by Hogg (33) and Holl (34) among others. More complicated loading cases have been considered by Volterra (35) and Bergstrom (36).

The case of a large slab supported by an elastic solid layer of finite thickness is perhaps more realistic than the semi-infinite solid case. Pickett (37), Holl (38), and Burmister (39) have considered this case among others. The mathematics of these solutions are quite complex and some assumptions are necessary to make solution possible.

Pickett et al (40) contributed greatly to the solution of these slab-on-elastic-solid problems with their work at Kansas State College, published in 1951. They considered the equations governing the slab on an elastic foundation. Where possible they solved these equations in closed form; where solutions were not obtainable they developed numerical techniques for evaluating the equations. They further developed influence charts to make the results readily usable by practicing engineers. They also extended the methods to rectangular slabs supported on elastic solid layers of finite thickness.

Layered System Analyses

Some authorities consider that pavement behavior is more closely approximated as a three-layered system than as a slab-on-foundation problem. Classical work in the analysis of a two-layered system has been done by Burmister (41) and by others. The analyses of three-layered systems are much more complex than single- or two-layered systems. Because of the great number of variables involved, solutions have only been made for surface deflections between the center of the circular, uniform, vertical load and for certain stresses and strains beneath the same load. These structures cover a wide range of values of the physical constants which are applicable to ordinary pavement structures. Burmister (42) laid much of the groundwork for the solution of two elastic layers on a semi-infinite elastic subgrade. He did not, however, provide numerical evaluation of the deflection or stress. Evaluation of these stresses has been accomplished by Hank and Scrivner (43) and Peattie and Jones (44) among others. To date these equations have been used primarily in the evaluation and analysis of flexible pavements. In reality they are probably more applicable and can be more helpful in the design of concrete pavements than in the so-called flexible pavements. At the present time, however, they await further evaluation and application.

Analysis of Finite Pavement Slabs With Discontinuities and Nonuniform Pavement Support

The theories described previously involve single-load application, and the solution of the equations imposes limiting assumptions on realistic pavement problems. All the solutions involve uniform slab thickness, uniform homogeneous isotropic (or special-case orthotropic) slabs, uniform foundation support, and certain uniform or special-case loading conditions. Lateral loads are considered, but in-plane forces and applied couples or moments cannot be handled.

Hudson and Matlock (32) have developed a method for analyzing slabs which is not limited by many of these previous limitations. The method involves the formulation of the problem by finite element techniques and the solution of the resulting equations by numerical methods in a large digital computer. The general method is suitable for analyzing orthotropic and isotropic slabs and is not limited by discontinuities. The method allows considerable freedom in plan configuration loading, flexural stiffness and boundary conditions. Three principal features are incorporated into the method: (a) the plate is defined by a finite element model, and the components of this model are grouped for analysis into orthogonal systems of beam-column elements and forces; (b) each individual line-element of the two-dimensional system is solved rapidly and directly by recursive techniques; and (c) an alternating-direction iterative method is utilized for coordinating the solution of the individual line-elements into the slab solution. The method necessitates the use of high-speed digital computational equipment, but programs for the solution have been developed and debugged and are available for solution of all classes of problems.

Other Work

In the 1930's, F. T. Sheets introduced an equation containing a constant, c, which was equated to the value of k as employed by Westergaard. The Sheets equation can be written as

$$\sigma_c = \frac{2.4 P (c)}{h^2} \tag{7}$$

This equation is reported to give stresses which are in good agreement with those obtained at the Bates Road Test. However, this equation is no longer in general use and does not contain all the variables of interest to the designer.

Many subsequent stress equations are based on some modification of the original Westergaard equation. The major work which resulted in these modifications included the Kelly equation developed as a result of the BPR Arlington Test, the Spangler equation developed as the result of the Iowa State College tests and the Pickett equations developed as the result of additional mathematical analysis.

Special Theories Applicable to Particular Pavement Types

The first portion of this section described the various load-stress equations developed for use in concrete pavement design. These equations have general applicability for determining pavement thicknesses required to resist certain traffic loads. In the design of particular pavement types, however, certain of these equations are used more than others. In addition, special theories involving the design of reinforcement and prestressing are sometimes involved. This part discusses the application of load-stress theories and other special theories which are needed for the individual pavement types.

Plain Concrete Pavements—Plain concrete pavements are ordinarily designed utilizing one of the basic theories described earlier. Probably a majority of the pavements have been designed using Westergaard's corner loading formula with or without subgrade support under the corner. In some cases a completely unsupported corner with no load transfer is utilized, but in some cases partial transfer of the load to an adjacent slab through some type of load transfer is considered, thus reducing required pavement thickness. Plain pavements require no other special theories in design.

Jointed Reinforced Concrete Pavement—Thickness design of jointed reinforced concrete pavement is basically the same as that for plain concrete pavement with the exception that some designers use Westergaard's edge loading formula for highway design and his modified edge loading formula for airport design. Those who use corner loads always consider some load transfer since common design dictates mechanical load transfer devices between slabs of reinforced pavement.

Reinforcement design for rigid pavement is based on the concept that since it is often not economically possible to prevent the formation of cracks, it is necessary to control the opening of cracks in such a manner that the original load-carrying capacity of the slab is preserved. If the crack is permitted to open, contact between the faces of the crack is lost, with a corresponding loss in shearing resistance, and continued application of load results in progressive breakage. The function of the steel reinforcement is to hold the interlocking faces of the crack in tight contact and thus render the crack shear-resistant. As a consequence, load transfer is maintained across the crack and both slab ends act together as a load approaches. This design is based on the concept that this action maintains the structural integrity of the slab and prevents excessive deflections.

Since the principal function of steel reinforcement in rigid pavement is to hold the interlocking faces of the concrete at a crack in tight contact, it is only necessary to furnish sufficient steel area to resist the forces tending to pull the crack faces apart. These forces develop when the slab tends to shorten as a result of a drop in temperature, concrete shrinkage, and/or a reduction in moisture content. As the slab contracts, the movements are resisted by the friction between the slab and the underlying subgrade or subbase. The resistance to movement produces a direct tensile stress and may cause the concrete to crack. As soon as the concrete cracks, the tensile stress is transferred to the steel reinforcement.

Since the crack can occur at any location in the slab, the steel must be designed to resist the maximum frictional force that can be developed in any one section of the slab. This force is considered to be a maximum, for a slab of given length, at a distance from the nearest free edge equal to half the slab length. Using the notation given below, the maximum force is equal to $L/2 \times Fw$ pounds per foot of width. The tensile resistance of the reinforcing steel per foot of width is equal to $A_s f_s$. Equating these external and internal forces gives

$$A_s = \frac{FLw}{2f_s} \tag{8}$$

where

A_s = required area of steel per foot of width or length, sq in.;
F = coefficient of subgrade friction;
L = distance between free joints, ft;
w = weight of slab, lb per sq ft; and
f_s = allowable tensile stress in steel, psi.

Continuously Reinforced Concrete Pavement—Continuously reinforced concrete pavement is a relatively new concept of pavement design in that transverse joints long considered essential in the construction of concrete pavement are eliminated. In their place the pavement is allowed to crack in a seemingly uncontrolled random pattern and these cracks are held tightly together by continuous steel reinforcement.

The same theory for thickness design can be used for continuous pavements as for jointed pavements, if it is first considered that the concrete and steel are going to act separately. The purpose of the concrete is to carry the wheel load, and the purpose of the steel is to keep cracks tightly closed so that there is effective load transfer across these cracks. If the cracks are kept tightly closed so that no water will seep through them to harm the subgrade, then pumping and other detrimental effects are reduced.

By removing the joints in the pavement it is theoretically possible to design continuous pavements based on interior loads or edge loads. Ledbetter and McCullough (46) show that using the interior load design results in a 20 percent savings in concrete thickness over the regular or jointed pavement designed for corner loading. The same equations cited for use with jointed pavements can be used for continuous pavements by making proper choice of loading conditions.

Design of Steel Reinforcement—The purpose of the continuous steel reinforcement is not to prevent cracking. Its purpose is to hold the cracks which do result tightly closed and provide practical continuity of slab across the crack. The design of this steel is often based on the basic relationship proposed in 1933 by Vetter (47). This relationship states that the percent of steel required to control volume change is equal to the tensile strength of the concrete divided by the tensile strength of the steel. This approach is discussed thoroughly by Ledbetter and McCullough (46). It is appropriate to point out that this relationship indicates that stronger concrete requires more steel in order to be adequately designed, that is, in order to resist contraction of the concrete.

To illustrate the theory involved here, assume a temperature drop which causes the concrete to contract. Restrained contraction induces tensile stresses in the steel at cracks and builds up tensile stresses in the concrete between cracks until the tensile strength of the concrete is reached. At this point the concrete cracks again, thus reducing the stress in the steel at the existing crack. With this concept, less steel than heretofore considered necessary is needed because only enough steel is required to insure that the concrete tensile strength is reached before the steel stress at the crack reaches the yield point or some percent of the yield point.

In the 1962 AASHO Interim Guide for the Design of Rigid Pavement Structures (111) the Committee on Design recommended that the percentage of longitudinal steel in a CRC pavement be determined by correlating a number of variables (such as tensile strength of the concrete, yield strength of the steel, frictional resistance of the subbase) and by using engineering judgment based on experience. In general, the theoretical minimum percentage of longitudinal steel (P_s) may be determined by

$$P_s = (1.3 - 0.2F) \frac{S_t}{f_s - N S_t} \times 100 \qquad (9)$$

where

P_s = ratio of area of longitudinal steel to area of concrete, percent;

F = coefficient of friction between pavement and subbase;

S_t = tensile strength of concrete, psi (about 0.4 S_c, modulus of rupture);

f_s = allowable working stress in steel, psi;

N = E_s/E_c;

E_c = modulus of elasticity of concrete, psi; and

E_s = modulus of elasticity of steel, psi.

The formula is based on the assumptions (a) that sufficient bond area is provided to develop the full working stress of the steel, and (b) that adequate load transfer is provided at transverse construction joints. The frictional factor, F, depends upon the surface smoothness of the subbase immediately beneath the rigid pavement. The value of F may range between 1 and 2, with 1.5 commonly used. In general, the friction factor may be set according to the following standards:

F = 1.0 for smooth-textured subbase surface,
F = 1.5 for medium-textured subbase surface, or
F = 2.0 for rough-textured subbase surface.

Equation 9 will generally govern the percentage of longitudinal steel required in a continuously reinforced pavement. Under severe temperature variations or when unusual material properties are encountered, a formula which considers the thermal coefficient may be required:

$$P_s = (1.3 - 0.2F) \frac{S_t}{2\,(f_s - T\epsilon E_s)} \times 100 \qquad (10)$$

where T = temperature range in degrees F, and ϵ = thermal coefficient of concrete and steel. The percentage of steel should be calculated by both formulas, under the special conditions outlined, and the highest value used in design.

Prestressed Concrete Pavements—Prestressed concrete pavements are more complex to design than conventional pavements. Design problems are increased because prestressed pavements contain many features not used in other pavement types. For example, special devices are needed to prestress the concrete; friction-reducing layers are necessary to reduce subgrade restraint stresses in relatively longer slabs; special transverse joints may be needed to allow large horizontal movements with controlled vertical movements; and end abutments may be required. Other methods for determining load stresses and deflections are not adequate for design of prestressed pavements. In the design of the other pavement types, it is assumed that load stresses and deflections remain within the elastic range of the concrete and therefore design methods based on elastic theory can be used. Prestressed concrete pavement studies (48, 49, 50) concerned with load response indicated that prestressed pavements can adequately support interior and edge loads of greater magnitude than those causing bottom surface cracks. These bottom cracks serve as partial plastic hinges under passage of load. As the load is removed, the cracks are closed by the prestressing forces. In the design of prestressed pavements it is thus necessary to determine stresses and deflections after bottom surface cracking or for conditions not covered by elastic theory. Theoretical methods for determining these values have been reported for the case of interior loading by Levi (51), Cot and Becker (52), and Osawa (53). These analyses are limited to a concrete slab supported by a dense liquid subgrade and prestressed equally in longitudinal and transverse directions. The principal difference between the method used by Levi and the methods used by Cot and Becker and Osawa is the type of cracking assumed at the bottom surface of the slab. Levi assumed an initial circular bottom surface crack to simplify the computation of moments from slopes by use of the reciprocal theorem. In the methods used by Cot and Becker and Osawa, it was assumed that bottom surface cracking occurred in a radial pattern. The radial crack assumption seems more reasonable because the elastic theory shows the tangential moment to be greater than the radial moment near the loading point. The Cot and Becker solution selects a length for the bottom surface radial crack and determines the load required to produce a crack of this length. In contrast, the Osawa solution is a step-by-step procedure whereby an increment of load results in an incremental increase in the length of the bottom surface radial cracks. Each of these theoretical solutions provides a method for predicting the load causing top surface circular cracking. Top cracking occurs at a load of more than twice that causing bottom cracking in most prestressed pavements and is considered to be an indication of pavement failure.

In prestressed pavement design, consideration must also be given to stresses other than those caused by wheel loads. A method (54) based on both theory and results of pavement research has been proposed for computing subgrade restraint and temperature warping stresses in 400- to 800-ft long prestressed slabs. These stresses were combined with wheel load stresses to prepare recommendations for required amounts of longitudinal and transverse prestress in highway pavements.

Because prestressing a concrete pavement increases its load capacity, the thickness of a prestressed pavement may be less than other rigid pavement types for support of equal loads. The loss of all prestress at any section of a prestressed pavement could result in a failure caused by insufficient thickness to support design traffic loads. Therefore, it is essential that a certain minimum amount of prestress be maintained at all times and at all locations of a prestressed pavement. To assure that prestress is maintained, some additional prestress should be applied initially to compensate for certain losses that will occur during and following construction. These losses could result from elastic shortening, creep, shrinkage of the concrete, relaxation in the steel, anchorage losses, tendon friction in post-tensioned systems, and hygrothermal contraction. Theoretical methods for determining these losses have been developed for most prestressed members. Accepted procedures for determining such losses in prestressed pavements have been summarized elsewhere (55).

ACCOMPLISHED RESEARCH AND SERVICE EXPERIENCE

Since the first appearance of concrete pavements there has been a continuous effort to improve design. This research effort includes (a) laboratory experimentation, (b) theoretical analysis, (c) performance studies of existing pavements under actual service conditions, (d) observations and tests on specially constructed pavements for a long period of years under normal traffic, (e) accelerated testing of selected sections of existing pavements and specially constructed pavements under controlled traffic, and (f) accelerated testing with specially built load carts on prototype pavements.

Observations of pavement performance under actual service conditions and normal traffic comprise many variables that are difficult to analyze and correlate with other factors under study. These observations must often continue for several years before the results can be evaluated.

All rigid pavement research emphasizes the interrelationship of traffic loading, environment and subgrade support. Each plays its part in prolonging or reducing the service life of the pavement and each has been the subject of much study and experimentation.

Traffic Loading

Magnitude of wheel loadings, wheel distribution and high airplane tire pressure and the number of load repetitions are the most important factors that influence the useful life of a properly designed and constructed pavement. Although this statement is self-evident, design criteria combining these factors could not be established until research from experimental tests became available.

Early Road Test—The Bates Road Test demonstrated how an increase in the magnitude of the wheel load (dual solid rubber tires) and an increase in the load applications reduced the service life of pavements varying in type and thickness. It produced the first rigid pavement design critéria based on wheel load and allowable stress in the concrete.

In order to obtain up-to-date information on the relative effects of load repetitions corresponding to the large volumes of heavy trucks using the highways after World War II, the controlled traffic test, Road Test One-MD (16), was conducted on a section of an existing highway in Maryland. This test established that a 32-kip tandem axle caused greater distress than an 18-kip single axle; similarly a 44.8-kip tandem-axle load was more severe than a 22.4-kip single-axle load.

Data from Road Test One-MD showed that stresses and deflections measured at vehicle speeds of 40 mph were about 20 percent less than those at creep speed. It was found that wheel placement at 30 in. from the pavement edge gave edge stresses and deflections about 50 percent less than those recorded with the wheel 6 in. from the edge.

AASHO Road Test—The most comprehensive accelerated test under controlled traffic to date is the AASHO Road Test (56) at Ottawa, Illinois (1958-1961). The test pavements were subjected to over one million repetitions of single-axle loads ranging from 2 to 30 kips or tandem-axle loads from 24 to 58 kips. Analysis of the results produced rigid pavement performance equations which make it possible to determine the tandem-axle load in terms of equivalent single-axle load. These performance equations have been extended (57, 111) to cover the conversion of mixed traffic into equivalent single-axle loads. Equivalence factors have been developed which make it possible to estimate the effects of different axle loadings on the pavement structure. Since these equations include the effect of load repetitions, they furnish a basis for estimating the design requirements for a pavement to remain serviceable for a given number of years under the anticipated traffic as determined by traffic surveys. Also, they provide a basis for estimating the service life of existing pavements under fixed or changing traffic conditions.

U.S. Corps of Engineers Tests—During the period 1941 to 1956 the Corps of Engineers conducted for the Air Force surveys of existing pavements and accelerated testing with controlled traffic on existing pavements and on specially constructed prototype pavements (58, 59, 60, 61, 62). Some basic findings with respect to traffic loadings discussed by Mellinger, Sale and Wathen (62) are:

1. The repetitive traffic of slow-moving aircraft is the most severe loading to which airfield pavements are subjected; this occurs on taxiways, runway ends and aprons.
2. Impact on normal landings can be ignored due to lift on the wing surfaces at aircraft landing speeds.
3. Early studies indicated that 5000 coverages were representative of 10 to 20 years of pavement life.
4. Later studies showed that the heavier aircraft tended to confine their travel to the central portion of a taxiway, resulting in channelized traffic indicating that 30,000 coverages could be expected in 10 to 20 years of pavement life.

The term coverage is defined as a sufficient number of vehicle operations to produce one application of the design load over the entire width of the traffic area. In the tests described, a coverage was equivalent to three repetitions of the wheel loading (62).

Theories give concrete stresses on the basis of one application of the load. Therefore a design factor must be added to compensate for fatigue of the concrete due to load repetitions as well as other factors to compensate for the limited efficiency of load transfer devices at joints, temperature stresses, impact and eccentric loading (particularly in the case of loading by airplane landing gears). The design factor being principally related to the life expectancy of the pavement (the Corps of Engineers uses 25 years for roadways with normal maintenance) makes it possible for the designer to relate pavement stresses of various magnitudes to an equivalent number of applications of a constant stress.

The Corps of Engineers developed a method of correlating traffic volume and load intensity with equivalent 18,000-lb single-axle dual wheel load operations (119, 150). Factors can be developed based on statistical analysis of the relationship of operations to width of pavement, width of tire contact area, number of wheels on the axle, spacing of wheels and degree of wander to produce one coverage. Airfield pavement data are available to 30,000 coverages and can be extended on the basis of the fatigue characteristics of concrete for greater vehicular traffic. This establishes the coverage design factor as it affects the design thickness obtained from stress consideration.

Using the Westergaard analysis for loading tangent to a free edge and incorporating a dynamic factor of 1.55, the Corps of Engineers would require a thickness h for 5000 coverages:

$$h = \left[\frac{6P}{\sigma}\left(1.55\,\frac{M}{P}\right)\right]^{1/2}$$

where P is the wheel load, σ is the design stress in the pavement, M/P is the maximum moment per pound of wheel load induced by all wheels on the axle, which is a function of A/ℓ_2 (A is the tire contact area and ℓ the radius of relative stiffness) for a given wheel spacing. During the anticipated 25-year life, the number of coverages will be a factor of the assumed 5000, and likewise the thickness will be a related percent of h ranging from 82 percent for 1 to 158 percent for 300,000,000 equivalent coverages (from the minimum of the low range to the maximum of the high range).

Environment

Pavement performance, like human behavior, is influenced by environment. Environmental factors such as frost, rainfall, and temperature, although variable in intensity from year to year, are constantly at work. Thus it is necessary to adjust designs accordingly. Statistical data and general information on frost, rainfall and temperature are available in publications of the Department of Agriculture (63) and the U.S. Weather Bureau.

Frost—Although the effects of frost action on pavements cannot be separated from temperature and rainfall, it is the most severe of the environmental factors and has been the subject of a large volume of research in the laboratory and in the field (64, 65, 66, 67, 68). Formulas have been developed for predicting the depth of frost penetration, and the Corps of Engineers (69) has produced empirical curves giving the relationship between frost penetration and freezing index.

Rainfall—The influence of rainfall on the stability and strength of the supporting medium (subgrade and subbase) has long been recognized, and rainfall records are normally included in important pavement research. Although it has not been possible to make a quantitative determination of the effects of rainfall, it is well established as a factor in (a) the moisture content of the subgrade and subbase (strength and volume change), (b) the elevation of the water table, (c) the intensity of frost action, (d) erosion, (e) pumping, and (f) infiltration.

The moisture content of the concrete slab will vary with rainfall and will affect the expansion and contraction.

Temperature—Air temperature has a direct influence on both the rigid slab and the supporting medium. Design of the horizontal slab dimensions is controlled by the permissible change in length and width resulting from changes in air temperature. These changes also control the design of steel reinforcement.

Variations in temperature between the top and bottom of the rigid slab affect warping and curling and, consequently, deflections.

Intensity of frost action is directly related to air temperature and will influence the subbase design.

Structural Variables

Subgrade—Ever since 1920 there has been a continuing effort to advance our knowledge of subgrade soils and the application of this knowledge to the design of pavements. Standard methods of surveying, sampling and testing of soils have been adopted by AASHO (70). Three soil classifications based on these standards, or modifications of them, have been established. In general, the highway departments and the Bureau of Public Roads use the AASHO classification which is based primarily on grain size, liquid limit and plasticity index. The Federal Aviation Agency, using the same tests and methods, has set up a classification (71) that is applied to pavement design on civil airports. The Unified System, also based on grain size and plasticity tests, is used by the Corps of Engineers (72), the Department of the Air Force and the Bureau of Yards and Docks (73) for military pavements.

These classifications, which combine a knowledge of the physical properties of the soil as determined by laboratory tests and their performance as subgrades under pavements, serve to identify a subgrade soil as having physical properties similar to those of known behavior. Therefore, they can be expected to furnish the same degree of stability under the same conditions of moisture and climate.

The soil profile, determined by borings, shows the arrangement of the different so layers, the elevation of the water table and the other properties that supply informatio on the need for subsurface drainage and its design.

With a knowledge of environmental factors such as topography and climate the engi neer can utilize the information from the subgrade survey and the classification tests estimate the susceptibility of the soil to volume change, frost action and pumping. Als these data disclose the existence of conditions of nonuniform support, detect resilient soils that are subject to detrimental deflection and rebound, and indicate the feasibilit of improving the stability of the soil by means of additives.

There are a number of other tests (74) that can be performed in place and in the laboratory to estimate the strength and supporting power of the soil. The most widely used test for rigid pavements is the plate loading test to determine the modulus of sub grade reaction referred to as the k value. As a result of research (75) the 30-inch diameter plate has been most widely adopted for the test with a given loading procedu keyed to the results.

The subject of expansive subgrade soils has been the basis for many investigations A bibliography on this subject is provided in reference numbers 123 through 149 inclusiv

Subbase—Some years ago, rigid pavements designed strictly on the basis of the weight distribution and repetition of loads were considered adequate to carry any traff regardless of the type of subgrade soil on which they were placed. With the rapid increase in the volume of heavy vehicles it became evident that the service life of the pavement was greatly affected by the stability and strength of the subgrade soil. As a result, practically all heavy-duty rigid highway pavements and a few airport pavement include a subbase as part of the structure. The subbase may consist of one or more layers of granular or stabilized material, properly compacted, between the subgrade and the rigid slab. This subbase may serve one or more of the following purposes: (a to provide uniform support; (b) to increase the supporting power above that provided b the subgrade soil; (c) to minimize the detrimental effects produced by volume changes in the subgrade; (d) to minimize or eliminate the detrimental effects of frost action; an (e) to prevent pumping.

Granular materials, dense-graded and open-graded, are most commonly used to ac complish these purposes. In many cases they are available in such quality that the onl requirements are uniform distribution and good compaction. Marginal materials may be improved by stabilization to meet specific requirements.

Field and laboratory investigations (56, 76, 77, 78, 79) have shown the effectivene of granular materials in overcoming subgrade problems, provided they have the prope physical properties, are adequate in thickness, and properly compacted. Highway departments and airport authorities have adopted standard specifications (70, 73, 80, 81 82) covering composition, plasticity index and compaction.

Thickness requirements are determined largely by experience. Four- to nine-inch thicknesses are common for the purpose of providing uniformity of support and strengt Greater thicknesses, 12 in. or more, may be required on highly active clays, very un stable soils in poorly drained locations, and highly resilient soils. Some wheel loads encountered on airports may also require thicknesses greater than 12 in.

Subbase design for frost action and pumping are special situations and are covered in the discussions that follow.

Frost Action—Frost action in relation to rigid pavements refers to the heaving whic takes place when the ground freezes and loss of stability when thawing occurs. The mechanics of frost heave, the loss of strength under thawing conditions and methods of prevention have been rather well covered in the literature (64, 65, 66, 67, 68). There is general agreement that the occurrence and magnitude of detrimental frost action depends on the character of the soil, the depth and rate of freezing, and the availability of water. Frost heave studies have disclosed that very fine sands, silts, and some clays are the worst offenders, while well-drained sandy and gravelly soils are not sub ject to heaving. Serious differential heaving was observed where soils such as silts and very fine sands were found as pockets surrounded by well-drained sandy soils.

Heaves were encountered also in sandy and gravelly materials where they were maintained in a saturated condition by blocked drainage or a water table close to the surface.

Detrimental frost action can be reduced in most granular materials if underdrains are installed at proper locations so as to intercept and remove water that would otherwise tend to collect in them.

Other frost-susceptible soils are sometimes excavated to a specified depth and replaced with non-frost-susceptible material or are blanketed with a layer of this material. Among highway departments the thickness of selected material ranges from one-half to the full depth of frost penetration—mostly from one-half to two-thirds.

More interest is being shown among highway departments in the use of freezing index data to determine depth of freezing and required thickness of selected material. This method, used by the Departments of the Army and Air Force and the Navy Bureau of Yards and Docks, is described in the pavement design manuals of these agencies (69, 73) as well as in a paper by Linell, Hennion and Lobacz (67).

Pumping—The problem of rigid pavement pumping took on such proportions in 1942 that the publication of "Wartime Road Problem No. 4, Maintenance Methods for Preventing and Correcting the Pumping Action of Concrete Pavement Slabs" was published in October of that year. The HRB Committee responsible for this report sponsored further studies and issued a final report (83) in 1948. This report defined pumping as the "ejection of water and subgrade soil through joints, cracks and along the edges of pavements caused by downward slab movement actuated by the passage of heavy axle loads over the pavement after the accumulation of free water on or in the subgrade." In 1957 Yoder (84) reported the results of a study of pumping of highway and airport pavements. These reports and others (76, 77, 78, 79) agree that three basic conditions must be present to create pumping. They are (a) frequent heavy loads, (b) fine-grained soils that will go into suspension with water, and (c) free water under the pavement.

Early investigators agreed that in many instances pumping was eliminated when granular subbases were included as a part of the rigid pavement structure. However, Yoder (84) points out that large volumes of heavy trucks will cause pumping if the granular subbase has an excessive amount of particles passing the No. 200 sieve. He indicates that 10 percent passing the No. 200 sieve may be the limit under severe conditions. Pumping development on Road Test One-MD (16) supports these findings.

On the AASHO Road Test (56) all failures in rigid pavements were preceded by pumping of material from beneath the concrete slabs. Generally, this material consisted of subbase gravel including the coarser fractions. Pumping of embankment soil was generally confined to those sections constructed without subbase. Severe pumping of the subbase material was experienced only in sections with the two thinner slab thicknesses. However, some pumping appeared in all but one of the sections. Very little material pumped through joints or cracks. The major pumping was along the pavement edge. The importance of a subbase layer was reiterated at the AASHO Road Test even though there was no difference in performance for subbase thicknesses from 3 to 9 in. There was, however, a significant improvement in the use of a granular subbase as opposed to placing the pavement directly on clay.

It appears from all these studies that (a) the subbase should be well graded from coarse to fine and that the percentage passing the No. 200 sieve can vary depending on the maximum size of the graded material; and (b) in open-graded materials the minus No. 200 material should be kept to a minimum with a maximum of 10 percent while up to 15 percent may be permissible in dense-graded materials and up to 10 percent in sandy types.

Stabilization—When available granular materials for subbase do not conform to specification requirements, they can be improved to an acceptable standard by means of stabilization. Mechanical procedures or chemical additives may be used to accomplish this improvement.

Mechanical stabilization consists in the introduction of a second material in such proportions that the required gradation and physical requirements of the specifications are met.

It may be more economical in a particular area to use an admixture of portland cement, lime or bituminous material to improve substandard or marginal materials. Both lime and portland cement have been widely used to reduce plasticity and as a binder for a great variety of soils and soil aggregate mixtures. Bituminous materials have been used as a binder to produce effective stabilization of some marginal subbase materials.

When granular materials must be hauled long distances, it has been found economical to reduce the subbase thickness and stabilize the subgrade soil. This problem generally arises in areas where the predominant soils are the plastic types, and subgrade stabilization under these conditions is becoming more popular. Portland cement, lime, calcium chloride and combinations of portland cement and lime, mixed with the soil and compacted to a depth of 6 inches, are used for this purpose.

Stabilization is generally covered by highway department and federal specifications. Publications of the Portland Cement Association (85), the Asphalt Institute (86), and the National Lime Association (87) contain recommendations covering the details of testing, design, and construction. Publications of the Highway Research Board (88, 89, 90, 91, 92) contain additional information.

Compaction—Modern construction practice requires density and moisture control when compacting subgrades and subbase material. Ever since Proctor (93) brought out the relationship existing between the soil, the degree of compaction and the resulting density, a great deal of effort has been expended to develop criteria applicable to compaction control for different types of material (94, 95, 96). This has been especially important in connection with the compaction of granular materials for subbases, since vibratory action of traffic can result in differential settlement of the rigid slab if the subbase is not properly compacted. All constructing agencies such as the various highway departments represented by AASHO (70), the Corps of Engineers (80), the Bureau of Yards and Docks (73), the Federal Aviation Agency (81), and the Bureau of Public Roads (82) specify definite compaction requirements (density and moisture content).

Slab Dimensions and Characteristics

Thickness—As noted earlier, the corner formula for estimating the required thickness of a rigid pavement was proposed by Goldbeck (1) in 1919 and later applied by Older (2) in analyzing the results of the Bates Road Test in 1924. Prior to the Bates Road Test most pavement slabs were uniform in cross section and from 4 to 6 in. thick. As a result of the Bates Road Test the thickened-edge cross section was widely adopted and the prevailing thicknesses were 8-6-8 and 9-7-9 in.

In the early 1930's the Bureau of Public Roads conducted extensive investigations on the structural design of concrete pavements (13). Among many other important findings it was concluded that, so long as the basic conditions assumed for the analysis are approximated, the Westergaard theory describes quite accurately the action of the pavement. From tests carried on in the 1940's by the Corps of Engineers (58) a similar conclusion was drawn with respect to airfield pavements.

Accelerated tests under controlled traffic, such as Road Test One-MD (16) and the AASHO Road Test (56) for highway pavements and the Lockbourne (59, 60, 61) and Sharonville (62) tests for airfield pavements, produced information on the influence of dynamic load repetitions on slab thickness requirements. As a result of these investigations, and the fact that higher speeds on highways required wider pavements with the consequence that the wheels of vehicles could operate at a greater distance from the pavement edge, a cross section of uniform thickness has been generally adopted.

With the development of continuously reinforced concrete pavements and the elimination of transverse joints (97) it appears that the thickness of a concrete pavement, continuously reinforced with adequate steel area, can be reduced.

Warping—Kelley (98) has pointed out that as early as 1926 Westergaard presented a theoretical analysis of warping stresses due to temperature but their importance was not generally recognized and it remained for the Arlington tests (13) to demonstrate their magnitude. Based on the theoretical analysis, Bradbury (99) developed general equations for the computation of temperature warping stresses. Using these equations, Kelley shows that the maximum warping stresses can go as high as 275 and 375 psi, respectively, for a 6- and 9-in. slab.

Data from Road Test One-MD (16) show that stresses and deflections caused by loads acting at the corners and edges of a pavement slab were influenced to a marked degree by temperature warping of the slab. For the corner loading the stresses and deflections for a severe downward-warped condition were observed to be only approximately one-third of those for the critical upward-warped condition. For the free-edge loading the effect of temperature warping, although appreciable, was not as pronounced as for the case of the corner loading.

An important observation quoted from the report of Road Test One-MD states, "It is noted that the values of the combined load and warping stresses greatly exceeded one-half of the modulus of rupture value of the concrete. At first thought, and in light of the fatigue properties of the concrete determined from laboratory tests, it would seem that such high stress values would soon cause failures in the structure. However, this was not the case of the sections in the experimental pavement, even after 10 years of service."

All studies (Arlington, One-MD and AASHO) show higher stresses when the slab is warped upward than when warped downward. The warped down condition gave a reduction in stresses on the AASHO Road Test, but the reduction was less on a percentage basis than that on Road Test One-MD.

A report on warping of concrete pavements was published in 1945 (45).

Jointing—In order to control certain stresses adequately, it is necessary to limit the width and length of slabs by means of joints spaced at suitable intervals. Most of the available information relating to joint spacing and joint design has been obtained from field inspections of pavements in service. As a result it is known that horizontal slab dimensions, which determine the spacing of joints, depend primarily on whether the pavement is reinforced or nonreinforced. Other factors may include environment, materials and thickness.

It has been found that the formation of random longitudinal cracks can be eliminated from highway pavements 5 to 10 in. thick if they are divided by longitudinal joints into lanes not over about 12 ft wide.

On airports, where greater thicknesses are common, the design manuals of the Federal Aviation Agency (71), the U.S. Navy (73), and the Corps of Engineers (100) permit the construction of pavement widths of 25 ft without a center joint when the pavement is over 10 in. thick. With respect to transverse contraction joints, nonreinforced slabs may be from 15 to 25 ft long depending on thickness while adequately reinforced slabs may be up to 75 ft long regardless of thickness. Nonreinforced slabs, generally, are limited to lengths of 15 to 20 ft for slabs up to 10 in. thick and from 20 to 25 ft for slabs over 10 in. thick. In practically all cases these are contraction joints. Randomized joint spacing is used by some agencies.

Experimental construction to study the behavior of expansion and contraction joints at different spacings was initiated in 1940 and 1941, and the results after 10 years of service were reported in 1956 (101). In summarizing the reports from the various states, Sutherland (101) stated:

"It was found that in pavements with expansion joints spaced at what was considered to be a desirable interval and intermediate contraction joints at sufficiently close intervals to control transverse cracking there was a tendency for the expansion joints to close progressively and the contraction joints to open progressively with time. These movements progress rapidly during the early life of the pavement and within a few years are of sufficient magnitude to destroy aggregate interlock in contraction joints of the weakened plane type. Where the expansion joints were eliminated or widely spaced there has been little or no tendency for the contraction joints to open progressively.

"On the basis of the 5-year progress reports the practice of many of the states with respect to expansion joints has changed. Today practically every state has eliminated expansion joints in nonreinforced concrete pavements except at structures and other special locations. This has resulted in pavements which offer greater resistance to pumping and faulting because of the better maintenance of aggregate interlock in the contraction joints."

In a study of the structural efficiency of transverse weakened-plane joints, Sutherland and Cashell (102) concluded that aggregate interlock in weakened-plane joints could be expected to break down under the severe hammering of heavy, high-frequency traffic and that where stress control is desirable it is necessary to use some more effective device for load transfer.

Load transfer devices, generally in the form of round, smooth steel dowels, are required at all transverse joints between long reinforced slabs. In 1959 Teller and Cashell (103) reported the results of laboratory tests to develop information on the structural action of load transfer systems under repetitive loading. Of immediate application was their finding that for round steel dowels at a 12-in. spacing in joint openings of ¾ in. or less, the dowel diameter in eighths of an inch should equal the slab depth in inches.

Reinforcement—Reinforcement has been involved in all accelerated tests under controlled traffic from the Bates Road Test (2) to the AASHO Road Test (56). This included Road Test One-MD and tests on airfield pavements at Lockbourne (59, 60) and Sharonville (62). In each of these investigations except Road Test One-MD, both reinforced and nonreinforced pavements were subjected to the prescribed traffic. In all cases except for the AASHO Road Test, the reinforced pavements generally performed better than the nonreinforced.

Under the AASHO Road Test conditions there was no significant difference between the performance of the reinforced and nonreinforced pavements. However, load transfer dowels were used at the joints between the 15-ft nonreinforced slabs as well as between the joints for the 40-ft reinforced slabs. These have been absent from the nonreinforced pavements in other previous investigations.

In the Bates Road Test and the tests at Lockbourne and Sharonville a common finding was that the reinforcing prolonged the life of the pavement after the first crack occurred. An analysis of the Lockbourne and Sharonville tests demonstrated that the steel reinforcement increased the effective thickness of the rigid pavement in proportion to the percentage of steel used in the design.

Based on a very comprehensive survey covering several thousand miles of highways in service, Hogentogler (104) reported in 1925 on the economic value of reinforcement in rigid pavements. Further information on performance of reinforced pavements is reported by Spencer, Allen and Smith (77), by Vogelgesang and Teske (105) and by Velz and Carsberg (106). A significant observation from these surveys and experiments is that the use of distributed reinforcement in the slab prevents faulting and lessens progressive deterioration at cracks between joints. It is also indicated that reliable results are obtained if the required steel area is determined on the basis of the "subgrade-drag" formula using an average coefficient of subgrade friction of 1.5 and an allowable stress in the steel equal to ⅔ to ¾ of the yield strength of the particular grade of steel.

Rigid Overlays

Bradbury (99) reported that a composite pavement consisting of a concrete base and a grout-filled brick surface in the Bates Road Test (2) was similar in its effectiveness to resist flexure due to loads to an equivalent thickness of concrete pavement in accordance with the formula $h^2 = h_b^2 + h_s^2$ where h is the equivalent thickness and h_b and h_s are the thicknesses of the concrete base and the grout-filled brick surface respectively.

Although a number of rigid overlays had been constructed over the years, it was not until the Corps of Engineers (100) initiated a program in the 1950's that data were obtained to formulate design criteria for overlay pavements. This program included theoretical studies and actual field tests. As a result of these investigations the above equation was modified to cover two conditions as follows:

Partial bond: $$h_o^{1.4} = h_d^{1.4} - Ch^{1.4}$$

No bond: $$h_o^2 = h_d^2 - Ch^2$$

where

h = thickness of base slab,
h_o = overlay thickness,
h_d = equivalent single-slab thickness, and
C = coefficient relating to the condition of the base slab.

Continuously Reinforced Pavement

Experimental continuously reinforced concrete pavements were constructed in Indiana (1938), New Jersey and Illinois (1947), California (1949) and Texas (1951). Since these early experiments a considerable mileage of additional experimental pavements as well as standard construction has been built (97). The experimental data and service behavior surveys indicate that the optimum amount of longitudinal steel is that which results in a crack spacing of from 3 to 10 ft. A closer spacing is not necessary and a spacing greater than 10 ft is not desirable. Also, it has been found that crack intervals within this range may be obtained with a steel area of from 0.5 to 0.7 percent of the cross-sectional area of the slab.

Performance studies reveal that (a) the numerous fine transverse cracks are usually not visible to the driver or passenger and that crack widths remain insignificant under all conditions and sealing is not necessary; (b) although the edges of the cracks were under traffic, this is only a surface condition which does not affect the load-carrying capacity or the riding qualities; (c) there is no spalling, faulting or pumping at the tightly closed cracks; and (d) the cracks are held tightly closed, which prevents the passage of water and the infiltration of solid material.

An important finding from the various studies is that longitudinal movements are limited to the end sections (200 to 500 ft) and that the long central portion is, for all practical purposes, fully restrained. This is true regardless of the uninterrupted length of the pavement. Various states and organizations (107, 108) have developed design practices which cover design details such as the calculation of required steel areas, size and spacing of members, lap requirements, construction joints and terminal joints.

Prestressed Pavements

The state of the art of prestressed concrete pavements has been presented quite comprehensively in the report (55) of the Subcommittee on Prestressed Concrete Pavements of the HRB Committee on Rigid Pavement Design. Practically all prestressed concrete pavements up to the present time are to be found in Western Europe and most of these were constructed in the late 1950's. Except for reports that these pavements are performing satisfactorily, no detailed performance data concerning them were available when the Subcommittee report was prepared.

Design theories and construction practices have been developed to the point where prestressed pavements can compete economically against conventional concrete pavements in certain countries in Europe—especially on airports where the reduction in pavement thickness can be more than 50 percent. This reduction could probably apply to highway pavements as well, but because of necessary construction procedures a minimum thickness of about 5 in. seems to be required.

The construction of the few relatively short sections of prestressed concrete pavement thus far in the United States has been rather costly, probably because this is a new concept and there is a lack of construction experience. In addition to the customary construction equipment and processes, a prestressed pavement requires (a) special jointing since transverse joints may be 300 to 800 ft apart, (b) sleeper slabs under the joints, (c) friction-reducing layers between the pavement and the subgrade or subbase, (d) steel tendons and conduits distributed throughout the area of the pavement, (e) filling the conduits with grout after the tensioning is completed, and (f) jacks for stressing the tendons.

An empirical equation has been developed (109) for predicting the top surface cracking load of a prestressed pavement. This equation was based on results of laboratory load tests on 45 small-scale prestressed slabs constructed of gypsum cement. Both pre-tensioned and post-tensioned slabs were loaded statically with loads simulating single-wheel and multiple-wheel gear configurations. During testing, the slabs were supported by a rubber subgrade that had a reaction modulus of 35 pci. The results indicate that, for a single static application at an interior location, the top surface cracking load of a prestressed pavement is dependent on the radio of the radius of the loading area to the radius of relative stiffness.

Because a prestressed pavement will be subjected to many repetitions of moving loads during its lifetime, pavement fatigue must be considered, especially at the working bottom surface cracks. The life of a prestressed pavement subjected to repeated moving loads at interior locations was studied (110) during tests on seven precast, pre-tensioned laboratory model slabs. These tests were designed to determine the relationship between prestressed pavement performance under repetitive moving loads and performance under single static loads as previously established by the empirical equation. Initial visual distress from traffic loads consisted of top surface circular cracking patterns in the traffic area. There was little or no evidence of pavement distress prior to the development of surface cracking. After the circular crack patterns occurred, a few additional repetitions of load generally were sufficient to completely disintegrate the concrete for the full slab depth in the distressed area. Results of these model tests were evaluated to determine design factors for repetitive loading. A graph of design factors vs interior load coverages producing pavement failure was presented for use in predicting prestressed pavement traffic life for different combinations of design variables.

HRB Special Report 78 reviews the progress that has been made during the years since the initial construction of a prestressed airfield pavement (55). It is evident that further improvements are needed, from the standpoint of both design and construction, before all the advantages of this type of pavement can be realized. However, experience in Europe demonstrates that existing difficulties can be overcome and that current design and construction practices can be used with the assurance of satisfactory results.

There is a considerable amount of accomplished research and service experience on concrete pavements that is not included in the reference section of this report. Two possible sources for this additional material would be the U.S. Bureau of Public Roads Office of Research and Development and the Highway Research Information Service (HRIS) of the Highway Research Board.

SOME CURRENT DESIGN PROCEDURES

Because of the wide range of environments and materials encountered in pavement construction it is not surprising that many methods are available for the thickness design of concrete pavements. Most of the methods combine a theoretical determination of the load stresses with an experimental or theoretical approach to the fatigue of concrete. However, in some cases the theoretical analysis has been supplemented or modified by experience gained from field observations of the performance of in-service pavements. To illustrate some of the considerations involved in the thickness design of concrete pavements, a brief description is given of some current design procedures. References to a particular design method should not be considered an endorsement of one method over others, nor should it be construed that a method not included was considered inadequate.

Plain Concrete Pavements

American Association of State Highway Officials—The empirical equation derived from the AASHO Road Test (111) provides a basis for design that incorporates axle load, number of load applications and slab thickness. The equation was derived for fixed values of modulus of elasticity, modulus of rupture, modulus of subgrade reaction, joint design, environmental conditions and test period. The equation expresses the ratio of the loss in serviceability at any time to the total potential loss. The basic equation was extended for general use by incorporating the semi-empirical stress equation developed by Spangler. The Spangler equation is used to linearize the measured strains, and the two combined equations yield a general design equation. The Spangler equation applies to a load at a corner, and for development of the Road Test equation the distance from the corner to the center of the load was taken as 10 in. The dense liquid subgrade theory is assumed, and the strength of the subgrade is expressed by the modulus of subgrade reaction, k. The design nomographs assume constant values of E = 4,200,000 psi and M = 0.20. However, these values can be changed in the design equation. The allowable working stress is set at 75 percent of the modulus of rupture. To simplify the equation a standard single-axle load of 18 kips is used. For other axle loads, conversion factors are provided to convert any given load into equivalent 18-kip single-axle loads. The fatigue of the pavement is considered indirectly in the concept of serviceability by including an estimate of the number of load applications for the design road life. No special provision is made for impact or a safety factor. At present, the design procedure does not include a regional factor. The nomographs are based on a design life of 20 years. For a road life other than 20 years, the daily equivalent load applications are multiplied by a factor equal to the design life in years divided by 20 years.

American Concrete Institute—Recommendations are presented for the design of concrete pavements and bases (122) based on practice proved successful in the United States. Comprehensive directions are given for designing rigid airport and highway pavements or bases for conditions of climate, traffic, available construction materials and equipment, and construction methods of the United States. Included are recommendations for soil foundations, selection of slab dimensions, joints and details for nonreinforced pavements.

Federal Government Agencies—The following agencies have design procedures for plain concrete pavements (details are contained in the references noted): Department of the Navy (73); Federal Aviation Agency (71); and Corps of Engineers (100, 120, 121—used by Departments of the Army and Air Force).

Texas State Highway Department—The Texas design method for rigid pavements (112) is basically an extension of the design developments presented in the AASHO Interim Pavement Design Guide. The variables are magnitude of load, repetition of load, tire pressure, axle type, strength of support media, concrete strength, concrete modulus of elasticity, Poisson's ratio, the continuity condition, friction of the support media and the regional factors.

The pavement structure is designed to encompass external loads introduced to the pavement from wheel loads. The following equation for pavement thickness is similar to that presented in the AASHO Guide except that it was expanded to include concrete modulus of elasticity, total traffic and pavement continuity (jointed or continuous):

$$\log \Sigma L = -8.682 - 3.513 \log \left[\frac{J}{S_x D^2} \left(1 - \frac{2.16a}{Z^{1/4} D^{3/4}} \right) \right] + 0.9155 \frac{G}{B} \tag{11}$$

where

ΣL = number of accumulated equivalent 18-kip single-axle loads;
J = a coefficient dependent upon load transfer characteristics or slab continuity;
S_x = modulus of rupture of concrete at 28 days (psi);
D = nominal thickness of concrete pavement (inches);
Z = E/k;
E = modulus of elasticity for concrete (psi);
k = modulus of subgrade reaction (psi/inch);
a = radius of equivalent loaded area = 7.15 for Road Test 18-kip axles;
$G = \frac{P_o - P_t}{3} = \frac{4.5 - P_t}{3}$;
P_o = 4.5 = serviceability at time zero;

P_t = serviceability at end of time, t; and

$$B = 1 + \frac{1.624 \times 10^7}{(D + 1)^{8.46}}$$

Only one term in the equation has not been evaluated adequately, the continuity or J term. The selection of a value J for design purposes must be postulated on the basis of limited data. The J value for the jointed pavements on the AASHO Road Test is automatically fixed at the value of 3.2 which was used in all correlation work.

Portland Cement Association—The design procedure (113) is semi-empirical, utilizing the mathematical analysis developed by Westergaard (98). The solution of the mathematical analysis has been presented in convenient influence charts (114) for determination of moments for various positions and configurations of loads. The axle is assumed parallel to a joint with no load transfer and with the tire contact areas as close as possible to the joint. Design nomographs have been developed from the influence charts for single- and tandem-axle loads. The nomographs relate stresses, modulus of subgrade reaction k, axle load and slab thickness.

In developing the nomographs, tire imprint areas were varied with the amount of load. Poisson's ratio was given a fixed value of 0.15, and the modulus of elasticity was assumed equal to 4 million psi. These values can be varied in the basic equation. Life expectancy of the pavement is determined by considering the applied stress as a function of the flexural strength of the concrete and the anticipated number of load applications. Loads causing a stress less than one-half the modulus of rupture are not considered in the design. The fatigue resistance consumed by each load level is the ratio, expressed as a percentage, of the expected load repetitions to the allowable load repetitions determined from a fatigue diagram. The sum of the fatigue resistances should not exceed about 125 percent. A fatigue resistance of greater than 100 percent is permissible because a conservative value of flexural strength is used that does not consider the gain in concrete strength with age. Load safety factors are recommended for all highways except residential streets and other light-traffic roads.

Jointed Reinforced Concrete Pavements

Wire Reinforcement Institute—Thickness design for reinforced concrete pavement, in accordance with the Wire Reinforcement Institute design procedures, is based on the theoretical analyses of Westergaard modified by empirical relationships obtained from observations of performance under service conditions (99). In this respect thicknesses are the same as for plain pavements, but the basic design includes distributed steel throughout the slab and mechanical load transfer at the joints. This design provides load transfer at all joints and at any cracks that may form between the joints.

The amount of steel, longitudinal and transverse, is determined by the "subgrade drag" formula discussed earlier, which takes into account (a) the length of the slab between free joints, (b) the coefficient of friction between the slab and subgrade or subbase, (c) the weight of the slab, and (d) the allowable stress in the steel. The steel is intended to hold cracks tightly closed and thus maintain the shearing resistance of the slab at a crack. This permits the use of longer slabs (40 to 100 ft) than can be permitted in plain pavements.

American Concrete Institute—Recommendations are given for the design of jointed reinforced concrete pavements and bases (122) based on practice proved successful in the United States. (This is the same reference as shown for plain concrete pavements.)

Texas State Highway Department—Steel reinforcement is used in concrete pavement where the joint spacing exceeds 15 ft. The reinforcement is placed in the slab to hold any cracks that form in the pavement tightly closed. The pavement may thus perform as an integral structural unit. The Texas method covers the design of two of the three basic types of reinforced concrete pavement, i.e., jointed reinforced and continuous reinforced (prestressed concrete pavement is not covered).

The equation for thickness design is the same as used for plain pavements (see Eq. 11). A value of 3.2 is used for the J term. The reinforcement for the jointed concrete pavement is determined by the application of the conventional subgrade drag theory. In essence, the equation is based on the principle of balancing the slab resistance to movement against tensile strength of the steel. The formula is

$$P_s = \frac{L\,F}{2\,f_s} \times 100 \tag{12}$$

where

P_s = required steel percentage, percent;
L = length of slab between joints, feet;
F = friction factor of subbase; and
f_s = allowable working stress in steel, psi.

Portland Cement Association—The PCA design procedure for reinforced concrete pavement does not consider distributed steel to increase the flexural strength of an unbroken slab when used in quantities considered to be within the range of practical economics. The principal functioning of the steel is to hold together the fractured faces of slabs after cracks have formed. Hence, the thickness design procedure of the concrete is the same as for plain concrete pavements. The amount of distributed steel required is based on (a) distance between joints, (b) coefficient of friction between slab and subgrade, (c) the weight of the slab, and (d) the allowable working stress in the steel.

American Association of State Highway Officials—The AASHO procedure does not specify a method for design of thickness of the concrete. The amount of steel is determined by the subgrade drag theory.

Departments of the Army and Air Force—The advantages in using steel reinforcement include a reduction in the required slab thickness. The design procedure of the Corps of Engineers has been developed empirically from a limited number of prototype test pavements subjected to accelerated traffic testing and performance data. Although it is anticipated that cracking will occur in pavements under the design traffic loadings, the reinforcing will hold the cracks tightly closed, which prevents spalling and faulting and provides a serviceable pavement during the anticipated design life. The designer determines the percent of steel required, the thickness of the reinforced rigid pavement and the maximum allowable length of the slabs. The Corps of Engineers uses the following relationship to determine the increase in effective slab thickness h in percent as a function of the amount of steel S each way as a percent of the area:

$$h = 16.0 \times \log_e S + 50$$

This relationship is shown graphically as Figure 8 in the paper by Mellinger et al (62) and has been found applicable also to vehicular traffic.

The maximum allowable slab width or length on Corps of Engineers projects is determined by the equation

$$L = \left[0.00047\,h_r\,(f_s S)^2\right]^{1/3}$$

where h_r is the thickness of the reinforced pavement in inches, f_s the yield strength of the reinforcing steel in psi, and S the percent of reinforcing steel.

It should be noted that, while equal percentages of steel are used in both transverse and longitudinal directions of airfield slabs, vehicular pavements were found to perform satisfactorily for their design life with the percentage of steel used in the transverse direction equal to one-half that used in the longitudinal direction.

The design criteria for reinforced pavements were found by the Corps of Engineers to be subject to the following limitations:

1. No reduction in the required thickness of nonreinforced rigid pavements should be allowed for $S < 0.06$.
2. No further reduction in the required thickness of nonreinforced rigid jointed pavements should be allowed over that computed for $S = 0.50$ regardless of the amount of steel used.
3. The length L should not exceed 75 ft for vehicular and 100 ft for airfield pavements regardless of the values of S, h_r or f_s for jointed pavements.
4. The minimum value of h_r should be 6 in.

Continuously Reinforced Concrete Pavements

American Association of State Highway Officials—The Guide (111) provides an equation for the determination of a theoretical minimum percentage of steel based on (a) the coefficient of friction between slab and subgrade, (b) the allowable working stress in the steel, (c) the tensile strength of the concrete, and (d) the elastic modulus ratio of the steel to that of the concrete.

Under severe temperature variations a formula is suggested which considers (a) the thermal coefficient of concrete and steel, (b) the temperature range, (c) the coefficient of friction between slab and subgrade, (d) the allowable working stress in the steel, (e) the tensile strength of the concrete, and (f) the modulus of elasticity of the steel.

Wire Reinforcement Institute—The selection of pavement thickness is based on observations of the performance of continuously reinforced concrete pavements under service conditions. It takes into account route classification, locality description, controlling vehicle type, and controlling single-axle load. On the basis of these factors, recommendations for pavement thickness vary between 6 and 9 in. In this connection it is suggested that the design methods under development by AASHO be used when they become available. The interim AASHO Guide (111) is an example of a more rational design.

The amount of longitudinal steel, based on a yield strength of 70,000 psi for deformed steel welded wire fabric, is recommended as 0.5 to 0.7 percent of the cross-sectional area of the slab. A proportionate adjustment in percentage is recommended for steels having other yield strengths. The area of transverse reinforcement is determined in the same manner as for jointed concrete pavements.

Recommendations are given for spacing of longitudinal and transverse members, and it is recommended that the steel not be placed below the mid-depth of the slab and that the concrete cover be not less than 2 in.

Concrete Reinforcing Steel Institute—The CRSI design procedure (108) is semi-empirical and utilizes the mathematical analysis developed by Westergaard (3). The equations have been given in influence charts (114) that may be used in the design.

The design procedure assumes that the load is applied at an interior position. The subgrade is assumed to be a dense liquid and its strength is expressed by the modulus of subgrade reaction, k. Design charts are shown based on the assumption that the tire imprint area is a function of the wheel load, and the loaded area is assumed equal to the area of contact of the dual-wheel tires plus the area between them. The equivalent radius is the radius of the loaded area as defined. Poisson's ratio is assumed as 0.15 and the modulus of elasticity is taken as 4 million psi. An allowable working stress of 50 percent of the modulus of rupture is suggested for use. The design wheel load is taken as the design static wheel load increased by a 20 percent load factor allowance. Fatigue is implicitly considered by permitting a working stress of 50 percent. The 20 percent load factor allowance provides for an impact or safety factor. No regional factor is considered.

The amount of steel is based on accepted minimum criteria of (111) 0.7 percent longitudinal steel for a 7-in. slab or (112) 0.6 percent for an 8-in. slab. Deformed reinforcing bars ¾ in. (No. 6) or less in diameter with a minimum yield point of 60,000 psi are recommended.

The reinforcement may be located at a depth from 2½ in. below the surface to mid-depth of the slab. If longitudinal bars are staggered, they should be lapped a minimum of 25 bar diameters; if not staggered, 30 bar diameters.

Texas State Highway Department—Thickness design is based on the same equation used for plain pavements, but a continuity or J value of 2.2 has been established based on comparisons of previous design procedures and performance studies. The design method for continuous reinforced concrete is based on the concept of balancing the internal concrete stresses developed by temperature and shrinkage against the strength of steel. The formula for determining the minimum percent longitudinal steel is

$$P_s = (1.3 - 0.2F) \frac{S'_c}{f_s - NS'_c} \times 100 \tag{13}$$

where

P_s = required steel percentage;

F = friction factor of subbase;

S'_c = tensile strength of concrete, psi;

f_s = allowable working stress in steel, psi (0.75 of yield strength recommended, the equivalent of safety factor of 1.33),

E_c = modulus of elasticity of concrete, psi,

E_s = modulus of elasticity of steel, psi; and

N = E_s/E_c.

PROPOSED AREAS FOR FUTURE RESEARCH

After considering the research now in progress and the overall problem of pavement design, the Rigid Pavement Committee proposes several areas of study for future research. The Committee recommends that any agency conducting research on rigid pavements give serious consideration to encompassing or partially encompassing any of the research problem areas listed.

The various areas of research are outlined as research problem areas with subdivisions describing the problem and defining the objectives of the proposed research.

Research Problem Area No. 1

Problem—Several very significant changes in the design and construction of rigid pavements have taken place in recent years. Evaluation of the effects of these changes has generally been on a state basis or limited to experimental projects. Consequently, a critical need exists for a nationwide appraisal of the performance of postwar rigid pavements, with emphasis on diagnosis of pavement defects.

Objectives

1. Identify and describe the significant changes that have taken place in pavement design in the past 15 years, such as subbase requirements, pavement type, jointing practices, and sealants.
2. Characterize possible defects that may occur in these pavements.
3. Develop guidelines that will provide for (a) a nationwide study of the nature, extent, seriousness and cause of pavement defects, and (b) correlation of these defects

with construction materials, structural components, climate, traffic, soil conditions and other factors.

Research Problem Area No. 2

Problem—Current analysis procedures of pavements subjected to dynamic loads are inadequate. Current methods of basing so-called repetitive load designs on fatigue life of concrete are not satisfactory. Information made available by repetitive load tests such as the AASHO Road Test, while very helpful, has not provided adequate solution to the problem.

Objectives—Establish analysis techniques that will provide a coordinated evaluation of repetitive load variables in pavement design. Utilize these analysis techniques to evaluate existing information, such as that available from full-scale traffic tests (AASHO Road Test) and repetitive load tests on model pavements. Finally, based on these analyses, make recommendations for additional physical testing or other work required to provide the necessary information for correct dynamic design of pavements.

Research Problem Area No. 3

Problem—It is postulated by some that the effect of environment on rigid pavement performance far outweighs the effect of all other variables involved, including the effect of repetitive loads. This is particularly true in the northern part of the United States. Therefore, there exists a great need to evaluate the effects of environment, including climatic and regional variables (moisture and temperature), on pavements on a nationwide basis.

Objectives—Establish the magnitude of various environmental effects on rigid pavement performance. These effects are known to include temperature and moisture as well as the general effects of location which have been termed regional effects in NCHRP Report 2A (4).

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