

Cost Estimating Under Uncertainty: Issues in Bridge Management

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ABSTRACT

Bridge management involves making decisions related to selecting the most cost-effective bridge improvement strategies at both the project and network levels. Typical cost variables include agency costs, user costs, discount rates, inflation, or interest rates, life-cycle costs, etc. Lack of reliable sources for accurate deterministic cost data has been identified as one of the shortcomings of current models of bridge management. Uncertainties can easily lead to making the wrong decisions, especially in selecting the best from pairs of closely ranked competing strategies. Historical records can be used to generate probabilistic estimates. Expert opinions may also be used to suggest subjective estimates or used to complement estimates obtained from historical records. This paper presents an overview of this uncertainty problem in cost estimating of bridge decision variables and also discusses suggestions on how to handle the uncertainty using analytical tools such as the fuzzy sets and probability theories. The decision-making algorithms in some existing bridge management systems and cost models are used to illustrate the suggested uncertainty-handling methods and also show how the uncertainties can affect the overall decision.

INTRODUCTION

Cost-effectiveness is a major criterion in decision-making under bridge management, especially when evaluating feasible improvement strategies for each bridge on a network of highways. Each strategy is typically evaluated on a basis of its life-cycle costs, i.e., analysis of the anticipated stream of agency costs during its expected life, considering the time value of money. This brings into consideration the following factors: the estimated costs of improvement actions (maintenance, rehabilitation, or replacement); timing of these anticipated actions; discount rates; and the expected improvement in bridge condition after implementing the actions (e.g., extension in service life). There are uncertainties involved in the life-cycle cost analysis. Two of these uncertainties involve estimates of cost and estimated timing of improvement actions, e.g., how many years from now to rehabilitate a bridge. If the cost estimates are derived from historical data, there will be a statistical randomness inherent in these estimates. To complement or even substitute for these historical data based estimates, expert opinions are typically used, introducing some subjectivity into the final estimates. Regarding the timing of improvement actions, the bridge deterioration model has to be considered to predict the expected condition of a bridge at the specified time in the planning horizon that will require the particular improvement action. This is also an uncertain process.

Apart from the basic life-cycle cost analysis, cost is also considered as a decision variable in the multi-criteria approach to evaluating bridge improvement activities. The

traditional approach of priority ranking as a basis of funding bridge projects is multi-criteria in nature [FHWA 1987]. Evaluation of bridge improvement projects based on a computed benefit index (utility) for each feasible strategy has also been demonstrated as a good decision model for bridge management [Sobanjo 1991, 1993]. The benefit index, based on an established list of criteria such as traffic volume, improvement in structural capacity, clearance, etc., will typically tend to suggest selection of improvements similar or close to replacement of the bridge because of the expected large increase in the level of service (LOS). Therefore computing a ratio of the benefit index to the project cost is a more realistic and effective way of evaluating each feasible strategy. The Incremental Cost Benefit Ratio technique (INCBEN) [Farid et al. 1988; McFarland et al. 1983] is also a good and acceptable decision making model of evaluation of bridge projects. INCBEN considers a benefit index for each feasible improvement strategy, relative to the cost of the project. Life-cycle cost analysis can also be utilized to compute agency net benefits and used for comparing alternative bridge improvement strategies. As mentioned earlier for the life-cycle cost analysis, the cost estimates in all these cases will have uncertainties in them. In terms of timing the improvement action, if decisions are to be made on a multi-period basis, i.e., in the future, the predicted deterioration comes into consideration again, when computing the benefit index.

For the various scenarios described above: life-cycle cost analysis and an estimate of the benefit index, the sources of uncertainty in cost estimates and timing of the actions can be described in two forms—statistical data randomness and subjectivity. Estimates derived from available historical data have randomness that can be adequately handled in a decision analysis through the use of probability theory. The bridge deterioration process is a stochastic process and it has been modeled as such in many of the modern bridge management systems. On the other hand, subjectivity in the estimates, introduced by the use of expert opinions, can be accounted for using the concepts of fuzzy sets theory. Use of simulation models may also be justified in some cases. There are also situations when both combined concepts of probability and fuzzy sets are applicable. This paper discusses in the following sections an attempt to demonstrate these issues of uncertainties as described above and also presents the application of probability and fuzzy sets theories for handling the uncertainties in some decision making models of bridge management.

BRIDGE LIFE-CYCLE COST ANALYSIS

Life-cycle cost analysis can be simply defined as an economic evaluation of an infrastructure over a desired service life, taking into consideration all the costs incurred and benefits gained by the owner over this period, before computing an “equivalent cost” estimate. These costs include the following general classes: initial costs, maintenance costs (annual), future costs (singular), and salvage value. The “equivalent cost” estimate is computed by converting the stream of all the time-related costs to a single equivalent value such as the present worth, annual worth, and future worth. Feasible improvements to a bridge can then be evaluated or compared, using any of these “equivalent costs” as found appropriate. Bridge management models typically employ the present worth as the “equivalent cost,” derived from the following equations which can be found in most economic analysis textbooks:

$$P = F(1+i)^{-n} \quad (1)$$

$$P = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right] \\ = [A/i] [1 - (1+i)^{-n}] \quad (2)$$

$$P = CI \quad (3)$$

where

P = Present worth equivalent,

F = Single future cost (salvage),

A = Annual uniform series of maintenance costs,

i = discount rate,

n = timing of a particular bridge improvement action within the economic planning horizon,

CI = Initial cost.

The bridge maintenance cost, in reality, will have a gradient series distribution, but the uniform series has been adopted here for a relatively simpler and direct analysis. Based on Equations 1, 2, and 3 above, the traditional life cycle cost algorithm for computing the Present Worth Costs of a stream of present, annual, and future costs is given as:

$$P = \sum [CI] + \sum [F(1+i)^{-n}] + \sum [A/i][1 - (1+i)^{-n}] \quad (4)$$

Probabilistic Approach

If all the decision variables in Equation 4 above can be estimated from reliable and available historical data, a probabilistic model can be formulated to estimate an expected value of the present worth or long term cost. Based on a statistical analysis of the historical data, a probability density function (pdf) can be fitted for each cost variable. The parameters determined for the pdf can then be utilized to estimate the cost variable's expected value, variance, etc. With emphasis on each type of cost as the only uncertain variable, i.e., assuming that the other variables are estimated with perfect precision, Equation 4 becomes

$$P_E = \sum [CI_E] + \sum [F_E(1+i)^{-n}] + \sum [A_E/i][1 - (1+i)^{-n}] \quad (5)$$

where the subscript E denotes expected value of the particular cost variable. This Equation 5 only considers uncertainty in the cost variables. The Pontis software, a new bridge management system developed by the FHWA and currently used by many state transportation agencies, considers uncertainty in the timing of the bridge improvement activity [Golabi et al. 1993]. In this case, a bridge deterioration model is applied to reflect the timing of the required improvement action, indicating the probabilistic estimate of the

bridge element being in that condition state at that specific time. With a background of Markov Chain deterioration model and the Dynamic programming optimization theory, Pontis calculates long term costs, based on the stream of life-cycle costs from the following optimality equation [Golabi et al. 1993]:

$$V(i) = \min_a \left[C(i, a) + d \sum_j P_{ij} V(j) \right] \quad (6)$$

where

$V(i)$ = Total expected long term discounted cost,

i = condition state of bridge element,

a = the set of feasible improvement actions for bridge element in condition state i ,

$C(i, a)$ = expected cost for an improvement action a in condition state i ,

d = the discount or present worth factor, [defined as $d = 1/(1 + int)$ where int = interest rate]

j = successor condition state of bridge element one year after action a is taken,

P_{ij} = the probability that the bridge element will transition from state i to state j in one year after action a is taken, and

$V(j)$ = Total expected long term discounted cost next year if state j occurs, calculated recursively from Equation 1.

Though the term “expected cost” is used in defining the cost variable of this Equation 6 [Golabi et al. 1993], only the uncertainty (randomness) in terms of the action timing has been accounted for. The cost estimate is still assumed precise. A suggested improvement will be to actually derive a probabilistic estimate of each cost variable, i.e., pdf, expected value, variance, etc. These probabilistic estimates can then be incorporated into Equation 6.

Fuzzy Sets Application to Cost Analysis

As demonstrated in Sobanjo [1999] for the economic evaluation of buildings, an algorithm can be formulated for life-cycle cost analysis of bridges by using fuzzy numbers to represent the variables shown in Equation 4. Fuzzy sets theory has been proven as a valuable tool for handling uncertainties due to subjective estimates in decision making models. If we consider a set \mathbf{A} with elements denoted by x . Instead of the $\{0,1\}$ (yes or no) valuation as seen in conventional set theory, if the membership grade $\mu_A(x)$ can have values in the real interval $[0,1]$ according to how much x belongs to this set \mathbf{A} , then the set \mathbf{A} is a fuzzy set [Zadeh 1965]. If the set \mathbf{A} is a set of criteria, $\mu_A(x)$ is the degree to which x satisfies the conditions of \mathbf{A} , or in other words, $\mu_A(x)$ is the “strength” of the statement : “ x belongs to the set \mathbf{A} .”

In fuzzy sets terminology, values that are known precisely are referred to as crisp ordinary numbers, while imprecise values are represented by fuzzy subsets. Basic fuzzy sets theory textbooks and papers explain the general form of a fuzzy subset \mathbf{A} where α is

the degree of belief, or in terms of fuzzy sets, the degree of membership. A_α , the interval of confidence associated with the α is formally referred to as the α -cut. All possible values of A whose degrees of belief are greater than or equal to a specified value α constitute the α -cut, A_α , such that

$$A_\alpha = \{x | \mu_A(x) \geq \alpha\} \quad (7)$$

A fuzzy subset can be defined as a class of objects where there is no sharp boundary between one object belonging or not belonging. For example, let E = set of possible unit costs (\$/SF) estimated for bridge deck rehabilitation, that is,

$$E = \{20, 22, 25, 29, 33, 40, 45, 50\}$$

Let A_1 = An accurate unit cost of bridge deck rehabilitation, and
 A_2 = Maximum unit cost of bridge deck rehabilitation.

Then, A_1 is a fuzzy subset of the set E , while A_2 is a crisp number. So, based on his experience and expertise, a bridge engineer may judge that

$$A_1 = \{20/0.0, 22/0.0, 25/0.6, 29/1.0, 33/0.5, 40/0.2, 45/0.0, 50/0.0\}$$

$$A_2 = \{50/1.0\}$$

For the engineer, the “fuzziness” in the interpretation of the variable “accurate unit cost estimate” is in the interval [22,45]; outside this range, the engineer is certain that the cost is not accurate. Within the fuzzy range, a measure of how certain the engineer is about the possible costs is reflected by the value of the membership function $\mu_A(x)$. Therefore, in the example above, the engineer is most certain about the accuracy of cost estimate \$29/SF.

A concept of the “level of presumption” inferred from the approximate reasoning in humans can be combined with the another notion termed “interval of confidence,” in order to define the concept of a fuzzy number. Again, consider for example that an expert bridge engineer says it will cost between \$25,000 and \$31,000 to splice a structural steel H-piling substructure, and that she is also most certain that the cost is \$28,000. These estimates are two intervals of confidence which can be represented as:

$$A_1 = [\$25,000, \$31,000] \text{ and } A_2 = [\$28,000, \$28,000].$$

Each of these two intervals can be assigned some measure of confidence, based on the bridge engineer’s strength of belief (or conviction). Using a scale of 0–1, this measure, the level of presumption (α), can be assigned a value of 1 for the interval A_2 and 0 for A_1 because the engineer felt stronger on the precision of her estimates in A_2 than those of A_1 . While this example has only two extreme cases to represent only two levels of presumption (0,1), it is possible to have more levels of presumption within the extremes, with corresponding intervals of confidence. Other possible intervals of confidence are: $A_3 = [\$26,000, \$30,000]$ at a level of presumption $\alpha = 0.4$; and $A_4 = [\$27,000, \$29,000]$ at $\alpha = 0.8$. The various intervals and their associated levels of presumption can be

combined to form a graphical representation or distribution of a fuzzy number labeled "Cost Estimate of Splicing H-Piling." The shape of the distributions is defined by a set of membership functions.

We are dealing here in this paper with the imprecision in quantitative values which are subjectively estimated. A triangular fuzzy number (TFN) provides an adequate representation of the uncertainty in such variables. The TFN has a triangular distribution as the membership function. Using three parameters l , m , and h , a TFN can be adequately expressed as a triplet $[l, m, h]$ where $[l, h]$ is the largest α -cut and m is the modal point. The mode m of a fuzzy number is the most possible value under the distribution while l and h represent the lowest and highest possible values respectively. The support is estimated as $(h - l)$.

The membership function of TFN \mathbf{A} , where $\mathbf{A} = [l, m, h]$, can be defined as:

$$\begin{aligned} \mu_{\mathbf{A}}(x) &= 0, & x < l \\ &= \frac{x-l}{m-l}, & l > x < m \\ &= 1, & x = m \\ &= \frac{h-x}{h-m}, & m > x < h \\ &= 0, & x > h \end{aligned} \quad (8)$$

Based on the concept of interval arithmetic [Kaufmann and Gupta 1988], the following fuzzy mathematical operations may be pertinent to cost analyses in bridge management. Assume a crisp ordinary number, k , a TFN $\mathbf{A} = [l_1, m_1, h_1]$, and a TFN $\mathbf{B} = [l_2, m_2, h_2]$

1. *Multiplication of a TFN by a Crisp Ordinary Number:*

$$k \times \mathbf{A} = k \times [l_1, m_1, h_1] = [kl_1, km_1, kh_1] \quad (9)$$

2. *Division of a Crisp Ordinary Number by a TFN:*

$$\begin{aligned} k \div \mathbf{A} &= k \times \mathbf{A}^{-1} = k \times [l_1, m_1, h_1]^{-1} \\ &= k \times [1/h_1, 1/m_1, 1/l_1] \\ &= [k/h_1, k/m_1, k/l_1] \end{aligned} \quad (10)$$

3. *Division of a TFN by a Crisp Ordinary Number:*

$$\begin{aligned} \mathbf{A} \div k &= \mathbf{A} \times k^{-1} = [l_1, m_1, h_1] \times [1/k] \\ &= [l_1/k, m_1/k, h_1/k] \end{aligned} \quad (11)$$

4. *Division of TFNs:*

$$\mathbf{A} \div \mathbf{B} = [l_1/h_2, m_1/m_2, h_1/l_2] \quad (12)$$

5. *Addition of TFNs:*

$$\mathbf{A} + \mathbf{B} = [l_1 + l_2, m_1 + m_2, h_1 + h_2] \quad (13)$$

6. *Multiplication of TFNs:*

$$\mathbf{A} \times \mathbf{B} = [l_1 * l_2, m_1 * m_2, h_1 * h_2] \quad (14)$$

In the life-cycle costing methodology, it will be necessary to make decisions based on the ranking or comparison of costs of bridge improvement strategies, estimates which are of the form of TFNs. The comparison of TFNs can be mathematically done using two approaches: first, a linear ordering or ranking of the fuzzy numbers based on an equivalent crisp ordinary number; and a “qualified comparison” approach in which the “strength” or “truth values” of the resulting decisions are indicated by the α -cuts of the fuzzy numbers.

Kaufmann and Gupta [1988] discussed the linear ordering of fuzzy numbers using an index called the “removal” or “ordinary representative” of each fuzzy number—the crisp ordinary number equivalent. The “ordinary representative” of a TFN $\mathbf{A} = [l, m, h]$ or A^{ORD} can be computed as

$$A^{ORD} = \frac{(l + 2m + h)}{4} \quad (15)$$

The second approach is the “qualified comparison” approach. To compare two TFNs \mathbf{A} and \mathbf{B} , a “qualified” statement can be made on the relative value of the property represented by these fuzzy numbers. If \mathbf{A} and \mathbf{B} respectively denote the TFN distributions of the numerical values of a property being used to measure and compare two objects (Figure 1), then by graphically comparing \mathbf{A} and \mathbf{B} , a “truth value” can be attached to a statement as to whether one object is better than the other [Watson et al. 1979; Whalen 1987]. Consider the α -cut at which the inside reference lines of the fuzzy distribution intersect. This intersection is at a membership μ_j . By studying the possible values whose degrees of membership (α) are greater than μ_j , i.e., α -cut at μ_j , it could be seen that the lowest possible value for \mathbf{B} is higher than the highest possible value for \mathbf{A} . Using this

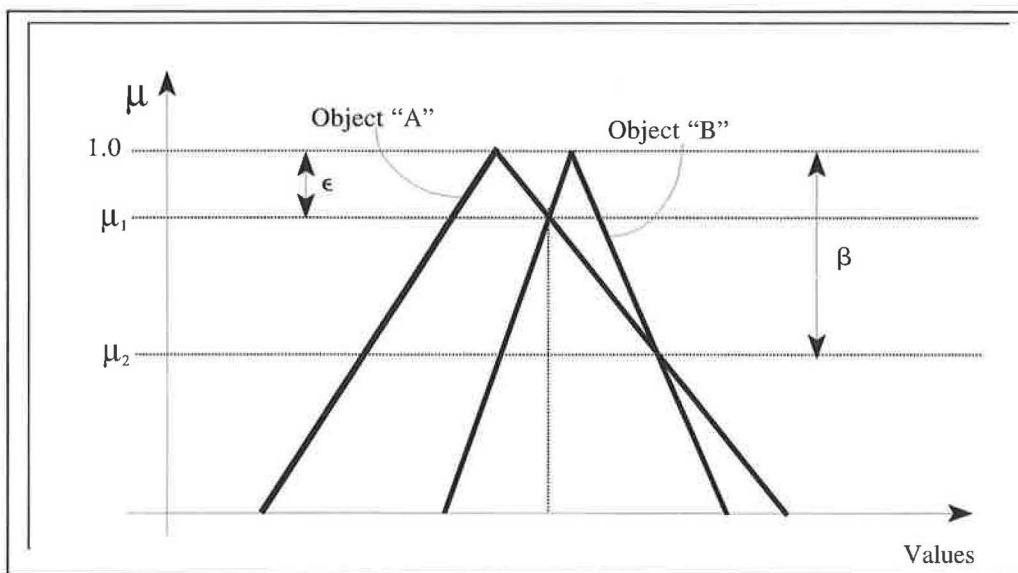


Figure 1: Comparison of triangular fuzzy numbers.

standard of possibility ($\alpha = \mu_1$), it could be said that the property value of B is strictly greater than the property value of A. The “strength” of this statement or its “truth value of strict dominance” is given by the complement of the lowest degree of membership (α) above which the statement is true [Whalen 1987]. Thus, the statement “B has a better property than A” has a “truth value” of $1 - \mu_1$, or as denoted in Figure 1.

Looking again at Figure 1, above the α -cut of μ_2 , an overlap occurs between possible values for these two objects. The highest possible value for B is still always higher than the highest possible value for A, and the lowest possible value for B is still higher than the lowest possible value for A, but the lowest values of B are not higher than the highest values of A. Thus, the statement “B has a property at least as good as A” has a “truth value” of $1 - \mu_2$, or β as denoted in Figure 1. The property being discussed here can represent the cost estimate of a bridge improvement action or the expected present worth of a stream of life-cycle costs.

The “truth value” mentioned above can be computed from the graphical relationship of the TFNs. Consider any two TFNs A and B, where $A = [l_1, m_1, h_1]$, and $B = [l_2, m_2, h_2]$. If $m_2 > m_1$, then the “truth value of strict dominance” as discussed above can be derived from the possibility level μ_1 at which the left reference function of B intersects with the right reference function of A (Figure 1). Above this level α_1 , all possible values of B are greater than all possible values of A. Therefore, the “truth value of strict dominance” is simply the complement of μ_1 , computed as:

$$\varepsilon = 1 - \mu_1 = \frac{1}{h_1 - m_1} - \frac{h_1 m_2 - m_1 l_2}{(h_1 - m_1) + (m_2 - l_2)} - m_1 \quad (16)$$

Referring back again to Equation 4 and representing all variables as TFNs except the discount rate i and the timing, n , which are treated as crisp ordinary numbers, the following algorithm is formulated for computing the fuzzy number (TFN) estimate of the Present Worth of a stream of a bridge's life cycle costs:

$$\begin{aligned} [P_l, P_m, P_h] = & \sum [C_{l_1}, C_{l_m}, C_{l_h}] + \sum [F_l, F_m, F_h] [(1+i)^{-n}] \\ & + \sum [A_b, A_m, A_h] [i^{-1}] [1 - (1+i)^{-n}] \end{aligned} \quad (17)$$

The variables are the same as defined earlier for Equations 1, 2, and 3 except for the addition of subscripts indicating the triplet l , m , and h for a TFN. A numerical illustration is presented in Sobanjo [1999] for life-cycle cost evaluation of buildings, including the computation of fuzzy present worth costs using equation 17 above, and the “qualitative comparison” approach (using Equation 16) for alternative building designs.

ESTIMATING BENEFIT INDEX OF BRIDGE IMPROVEMENT

For the purpose of evaluating feasible strategies of bridge improvement, a benefit index is usually computed and then applied in the various decision models such as the multi-criteria utility models, optimization models, incremental benefit cost ratio technique (INCBEN), etc. Sobanjo [1991] presented a multi-criteria utility-based model for

evaluating bridge improvement projects on a multi-period basis, involving the computation of a benefit utility index. Fuzzy sets and the probability theory can be applied to handle uncertainties in the estimates of the decision variables in such models. In this case, the timing uncertainty is handled by a combined fuzzy and probability theory, while the cost estimates are completely based on expert opinion, and treated as fuzzy numbers.

Combined Fuzzy Sets—Probabilistic Approach

Because of the stochastic nature of the bridge deterioration process, any decision made on the bridge, on a long term or multi-period basis, may be classified as Decision Making Under Risk (DMUR), a framework in which the only available knowledge about the outcome condition states is a probability distribution [Bradley 1976]. In order to model the bridge deterioration process, a probabilistic estimate of the expected bridge condition with respect to time can be determined based both on existing bridge inspection records and the bridge engineer's expert opinion.

Under this approach, the crisp probabilities may first be estimated using the statistical analysis of available historical data on the bridge. These crisp probabilities are then modified or replaced directly with fuzzy probabilities in which each of the possible probabilities can be assigned a membership grade (a measure of possibility) to obtain a possibility distribution for each probability estimate. Assuming a TFN for each distribution, the state probability vector can be modified to reflect the bridge engineer's judgment on each state's probability. That is, using the National Bridge Inventory (NBI)'s "0" to "9" condition rating scale,

$$P_n = [(p_{9l}, p_{9m}, p_{9r}), (p_{8l}, p_{8m}, p_{8r}), (p_{7l}, p_{7m}, p_{7r}), (p_{6l}, p_{6m}, p_{6r}), \dots, (p_{2l}, p_{2m}, p_{2r})] \quad (18)$$

where

$$\begin{aligned} P_n &= \text{Fuzzy probability estimate vector} \\ (p_{il}, p_{im}, p_{ir}) &= \text{fuzzy probability estimate of bridge being in a future} \\ &\quad \text{condition state } i \\ p_{il} &= \text{left extreme (least possible) least estimate of } p_i \\ p_{im} &= \text{midvalue (most possible) estimate of } p_i \\ p_{ir} &= \text{right extreme (least possible) largest estimate of } p_i \end{aligned}$$

Also, based on the basic axioms of the probability theory,

$$\begin{aligned} \sum_i p_{im} &= 1 \\ \text{and } 0 &\leq p_i \leq 1 \end{aligned} \quad (19)$$

Generally, at the long range (multi-period) level of multi-criteria decision making, let us assume the bridge engineer is faced with the following situation: the set of feasible bridge improvement alternatives is known; the set of possible outcomes of each alternative under each decision criterion can be estimated by the bridge engineer, but as fuzzy numbers; and the states of nature (possible deteriorated states or condition of the bridge) are not known for sure, but the bridge engineer can identify each possible

condition, assign a probability to the occurrence of each state, and also provide information on the predicted future condition of the bridge in the form of a fuzzy state probability vector. Employing the principle of maximum expected utility, the following steps will make up an algorithm that can be used to evaluate a bridge at project level and select the best improvement strategy.

1. Declare the states of nature to be considered in the analysis, that is, possible deteriorated states or condition ratings, s_k , that the bridge or its component can be expected to be at a particular fixed age where $k = 1, 2, 3, \dots, r$.
2. Generate feasible bridge improvement alternative, A_i , and the cost, C_i , of the alternative, where, $i = 1, 2, 3, \dots, m$, and C_i is of the form of a TFN triplet (l, m, h) .
3. Evaluate the score y_{ijk} of bridge improvement alternative A_i under each decision criterion c_j in the state s_k , where $j = 1, 2, 3, \dots, n$, and y_{ijk} is of the form of a TFN triplet (l, m, h) . The decision criteria will include the average daily traffic (ADT), Expected improvement in structural condition appraisal rating, Expected improvement in deck geometry appraisal rating, Expected improvement in clearance appraisal rating, expected improvement in load capacity appraisal rating, Expected improvement in waterway adequacy appraisal rating, Expected improvement in approach roadway alignment appraisal rating, and Expected extension in bridge service life (years) [Sobanjo 1991].
4. Determine the utility u_{ijk} of bridge improvement alternative A_i under each decision criterion c_j in the state k , where $j = 1, 2, 3, \dots, n$, and u_{ijk} is of the form of a TFN triplet (l, m, h) .
5. Compute the weighted utility U_{ijk} for each bridge improvement alternative A_i under each decision criterion c_j in the state s_k , where $j = 1, 2, 3, \dots, n$.

Let w_j = relative weight of decision criterion c_j , such that $w^T = \{w_1, w_2, \dots, w_j, \dots, w_n\}$, and

$$\sum_j w_j = 1$$

$$\text{Therefore, } U_{ijk} = w_j u_{ijk} \quad (20)$$

6. Compute the overall utility U_{ik} for bridge improvement alternative A_i in the state s_k .

$$U_{ik} = \sum_{j=1}^n w_j u_{ijk} \quad (21)$$

7. Compute the expected overall utility U_i for bridge improvement alternative A_i as follows:

Let $p(s_k)$ = the probability of the bridge or component being in state s_k at the age N (yr), and $p(s_k)$ is of the form of a TFN probability triplet (l, m, h) .

Therefore,

$$U_i = \sum_{k=1}^r U_{ik} p(s_k) \quad (22)$$

9. Compute the benefit index as utility per unit cost, \tilde{U}_i for each improvement alternative A_i , where the cost C_i is of the form of a TFN triplet (l, m, h) .

$$\tilde{U}_i = \frac{U_i}{C_i} \quad (23)$$

10. For all the feasible bridge improvement alternatives, repeat steps 3 through 9.
11. Compare the expected overall utility per unit cost, \tilde{U}_i for the various alternatives A_i . Select the best improvement alternative based on the principle of maximum expected utility. That is, choose

$$A_i^* = \left\{ A_i \mid \max_i \tilde{U}_i \right\} \quad (24)$$

12. Rank the preference order of alternatives according to the descending order of \tilde{U}_i .

It is also suggested to perform a “qualitative comparison” of the bridge improvement alternatives using equation 16 presented earlier in this paper.

Fuzzy Sets Modification of INCBEN

The Incremental Benefit-Cost Ratio Technique (INCBEN) is an economic analysis method for selecting, on a network level, a combination of bridge improvement alternatives with the maximum expected benefits [Farid et al. 1988; McFarland et al. 1983]. Basically, the INCBEN investigates the justification of increasing the cost of bridge improvement through the computation of an incremental benefit-cost ratio [Farid et al. 1988]. This is the ratio of the increase in benefits when changing from one bridge improvement to the other, to the associated cost increase [FHWA 1987]. Farid et al. [1988] concluded the satisfactory feasibility of applying of the INCBEN approach in the selection of bridge improvement projects, summarizing the procedure as follows:

1. Sort all mutually exclusive bridges in the order of increasing initial costs.
2. Tentatively accept the first least-cost alternative which is economically justifiable (desirable).
3. Calculate the ratio of the incremental benefits to the incremental costs for the second least-cost alternative. If the ratio equals or exceed 1.0, then discard the alternative accepted previously, and accept the current alternative. This now becomes the base alternative for comparison with the next least-cost alternative;
4. Repeat Step 3 for all alternatives;
5. Select the highest-cost alternative with an incremental benefit-cost ratio of at least 1.0, subject to budgeted cost.

This paper presents an extension of the INCBEN technique by a consideration of TFN representation and a computation of ordinary representatives of the decision variables, including the cost and the associated benefits (utility). It should be noted however that since both benefits and costs are both measured in dollars in the original INCBEN, the benefit cost ratio is dimension less. So setting an acceptable benefit-cost ratio equal to 1.0 as specified in Steps 3 and 5 above is reasonable in the case of the original INCBEN. But if the utils are used to measure the benefits as being proposed in this paper, then an acceptable benefit-cost ratio (utils/cost) has to be set, which may not necessarily be 1.0. Establishing this desired threshold of benefit-cost ratio can be easily done using the level-of-service (LOS) criteria of the transportation agency [Chen and

Johnston 1987]. The desired LOS is then applied to the Multic-Criteria-Utility curves similar to those developed in Sobanjo [1991], to compute a threshold benefit utility index.

An algorithm that can be used to accomplish a fuzzy sets- modified INCBEN analysis (network level) is summarized as follows:

1. Declare the number of bridges N , on the network, and the allocated budget B .
2. At each bridge site i , $i = 1, 2, 3, \dots, N$,
 - a. Generate feasible bridge improvement alternatives, A_j , $j = 1, 2, \dots, n$, and the respective first costs C_j of each alternative, where C_j is of the form of a Triangular Fuzzy Number (TFN) triplet (l, m, h) .
 - b. Evaluate the score of each bridge improvement alternative under each decision criterion and compute the benefit index or overall utility U_j , where U_j is of the form of a TFN triplet (l, m, h) . The steps for computing U_j is similar to those steps in the algorithm described earlier in this paper, using Equations 20–22.
3. At each bridge site i , sort all the alternatives in the order of increasing initial costs C_j . If there are some alternatives with same cost but different benefit values, delete all but for the alternative with highest benefit.
4. At each bridge site i , calculate the ratio of the Benefit (util) to the Cost for each alternative.
5. At each bridge site i , delete alternatives with the Benefit/Cost ratio less than an acceptable benefit- cost ratio in utils/cost, (say γ).
6. At each bridge site i , perform a “qualified comparison” among all the feasible alternatives using the Benefit/Cost ratio. Using equation 16 developed earlier in this paper, estimate the “truth value of strict dominance” ϵ between alternatives. Based on an established “truth value” threshold of say, $\epsilon = \epsilon_0$, identify alternative pairs with “truth values” less than this threshold ϵ_0 .
7. At each bridge site i , refine the estimates of the decision variables for these selected pairs of alternatives with $\epsilon < \epsilon_0$ and repeat steps 2 to 6 above until no pair of alternatives selected has $\epsilon < \epsilon_0$ in a “qualified comparison” procedure.
8. Prepare a list of the remaining feasible alternatives at each bridge site in order of increasing initial costs. Calculate the ratio of the incremental benefit (utils) to the incremental cost for each alternative, i.e., the IncBen/IncCost. Delete alternatives with the IncBen/IncCost less than an acceptable benefit-cost ratio in utils/cost, γ .
9. Compare the initial alternatives, starting with the least cost, to next more expensive alternatives. If the second has an incremental benefit/cost ratio greater than the first alternative, combine the two to form an average incremental benefit/cost ratio.
10. Prepare a list showing remaining feasible alternatives at all the bridge sites, with the respective estimated costs, in a descending order of the incremental benefit/cost ratios.
11. Under the established budget level B , choose the most attractive alternative for each bridge site. Once chosen for a bridge, exclude all other less expensive feasible alternatives for this bridge remaining in the list. Drop the last alternative chosen from the list and continue the selection process adding more projects as the budget level will allow. Many sets of selection may be produced. The set with largest estimate of total benefits is accepted as the optimal solution.

SIMULATION MODEL

An overall decision-making model for both project-level and network-level evaluation of bridge improvement projects can also be simulation-based. Based on a statistical analysis of reliable and available historical data, probability distributions of the decision variables, including costs, can be fitted and their parameters estimated. These information can be utilized in formulating algorithms for a simulation model. The basic computations in the underlying analytical algorithms of bridge deterioration models and life-cycle costing methodologies can be easily adapted to in a simulation model estimate probabilistic outcomes and also make decisions related to cost-effective bridge management.

CONCLUSIONS

Handling uncertainties in decision variables by reflecting their effects on the overall decision output is very important. A set of conceptual algorithms and equations have been presented to illustrate how the uncertainties introduced due to statistical data randomness and subjectivity in estimates of cost variables can be handled in bridge life cycle cost analysis and computation of the benefit index of a feasible bridge improvement action. Utilizing fuzzy numbers, probabilistic estimate or a combination of both, to represent decision variables, computations can be performed to obtain realistic outputs in terms of long term costs , and benefit (utility) values for evaluation and comparison of bridge projects.

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