MATHEMATICAL EVALUATION OF TRAFFIC CONTROL ALTERNATIVES FOR RESTRICTED FACILITIES

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As part of a continuing research study directed to alleviate traffic congestion in the Baltimore Harbor Tunnel, the Howard policy-iteration technique was applied off-line to different traffic control alternatives. Three data-acquisition stations were used inside the tunnel for control purposes. One was at the tunnel's bottleneck, and 2 were upstream of this location. The value of the traffic concentration, which was used as the control variable, at these 3 stations defined 1 out of 18 possible states of tunnel traffic flow. State transitions in the system were assumed to occur every 30 sec. Extensive data collected in the Baltimore Harbor Tunnel were used to determine the state transition probabilities of the system under each alternative. The rewards associated with state transition were obtained by applying a model that considers actual flow at the bottleneck and average speed associated with the flow during each transition interval. Five different control alternatives were considered. One of the alternatives was no control; the remaining 4 were 2- to 4-min cycle lengths of a traffic signal located upstream of the tunnel entrance.

•NORMAL operation of urban freeways is frequently affected by excessive traffic demand. Most drivers have experienced overcrowded highways and delays during morning and evening peak periods. The limited capacity of a highway network is often exceeded by the number of vehicles trying to use the roadway during these periods. As a result, congestion develops and is accompanied by stop-and-go driving conditions. These conditions, in turn, permit fewer vehicles to be served in a given time period. Congestion is significantly more severe for restricted facilities such as tunnels.

Congestion is a daily routine for the Baltimore Harbor Tunnel Thruway (Fig. 1). Traffic is frequently backed up for 2 miles (3.2 km). The problem is especially crucial on weekends when backups extend for more than 4 miles (6.4 km) and affect Interstate highways. This congestion is aggravated further by the fact that the thruway is a toll facility where exit is completely restricted until the toll plaza has been reached.

Research efforts have established that traffic control is an appropriate means to not only improve traffic-flow characteristics during congested periods but also prevent traffic from reaching states of potential congestion (1).

This paper discusses the feasibility of a stochastic or probabilistic approach to reducing traffic congestion on restricted facilities. This probabilistic approach uses the Howard policy-iteration method (2, 3, 4), which is based on the Markov process with rewards. The method was applied to traffic-flow data collected in the Baltimore Harbor Tunnel (5, 6).

LITERATURE

Because we analyzed traffic control alternatives by a Markovian approach in this re-

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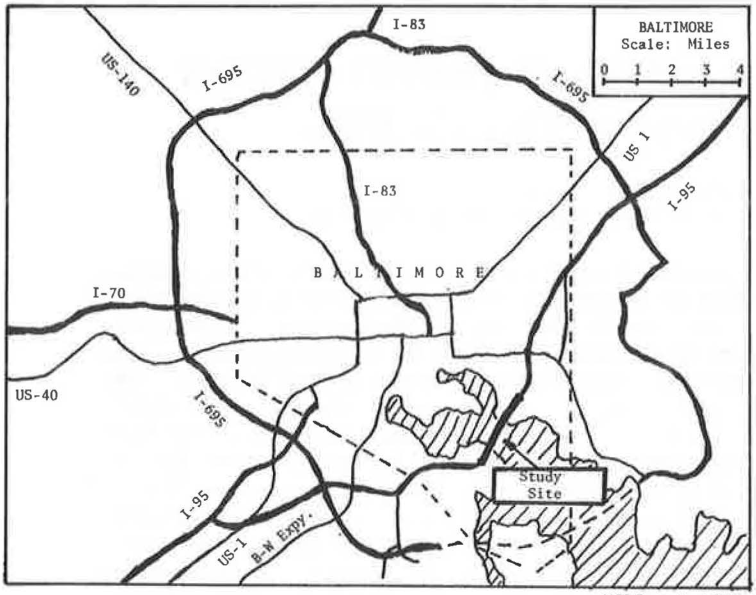
Figure 1. Baltimore Harbor Tunnel.



NOTE: 1 mi = 1.6 km.

search, we dealt with Markovian analysis and control of traffic congestion in a literature review. Because the need for control largely is due to traffic congestion, we undertook a short examination of the causes and consequences of congestion (5, 6).

When the relationship between variables is probabilistic or random, stochastic models are used. These models can be independent if the outcomes of the experiments do not influence each other and they can be Markovian if the outcome of 1 experiment is directly dependent on the preceding experiment.

Markov processes have been used to correlate successive headways in traffic streams with traveling platoons (7) and in traffic-merge problems (8). In 1967 Jewell (9) recognized the potential of Markovian approaches to traffic-flow theory. In 1972 Haefner and Warner (10) applied the Howard policy-iteration method with rewards to a hypothetical traffic control case. In 1973 Carter and Palaniswamy (5) formulated a conceptual approach to the analysis of traffic control alternatives in the Baltimore Harbor Tunnel. This formulation was further explored the same year by Palaniswamy (11) who suggested a reward structure that could be applied when data became available.

During the studies described by Carter and Palaniswamy (5) and Palaniswamy (11), the Baltimore Harbor Tunnel was divided into 4 major sections:

1. Queue area (upstream of and including the toll plaza),
2. Merge area (immediately downstream of the toll plaza),
3. Ramp area (joins the merge area to the tunnel), and
4. Tunnel (restricted facility).

The tunnel (Fig. 2) was further divided into downgrade, level, and upgrade sections.

During heavy demand, the critical bottleneck was at the foot of the upgrade; this finding agrees with the results obtained in previous New York tunnel studies (12, 13, 14, 15, 16, 17, 18). The rest of the study therefore was directed toward improving traffic flow at the bottleneck.

Data for earlier studies at the Baltimore Harbor Tunnel were collected at 7 stations (Fig. 2). Station 5 was located at the critical bottleneck. A description of the data collection equipment, the detail setup, and problems encountered is given elsewhere (5).

Figure 2. Plan and profile views of Baltimore Harbor Tunnel with station locations.

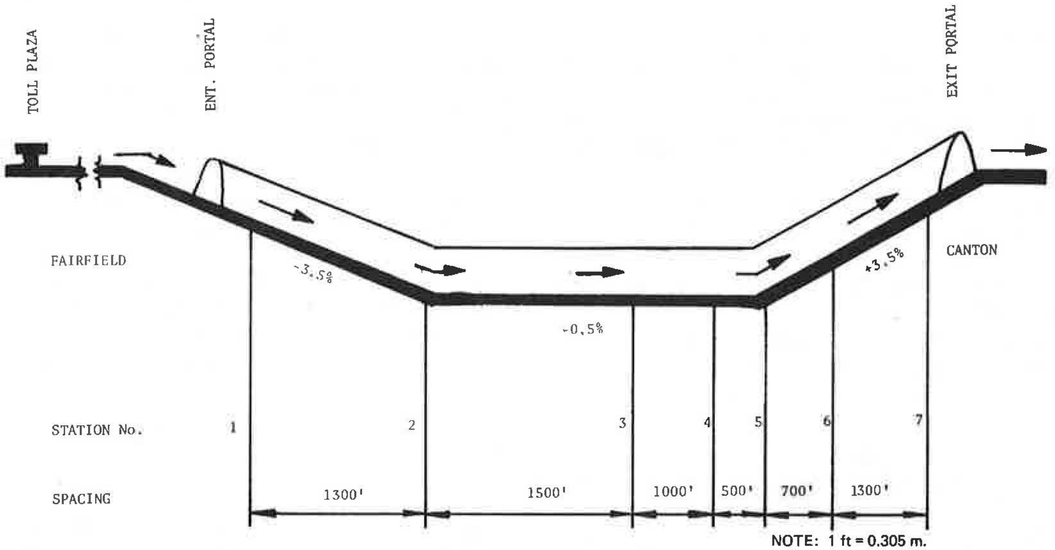


Figure 3. Location of signals and signs used for alternatives.

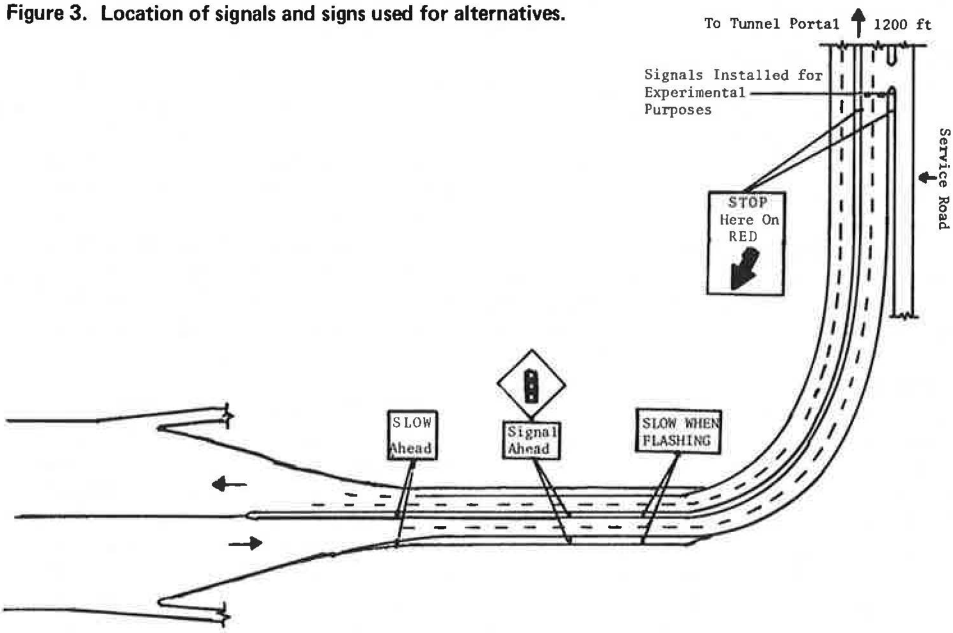


Table 1. Description of alternatives.

Alternative	Cycle Length (sec)	Green (sec)	Amber (sec)	Red (sec)
1	No control		0	0
2	120	109.2	3.6	7.2
3	160	147.2	4.8	8.0
4	180	169.2	3.6	7.2
5	240	225.6	4.8	9.6

STUDY METHODOLOGY

Gonzalez (6) gives a brief description of the Howard policy-iteration method with rewards. A more complete treatment of the subject can be found elsewhere (2, 3, 4, 19).

It is convenient to think of a Markov process as a sequence of states through which a system passes stochastically at successive points in time (8). The states are the various possible conditions in which the system might be at any instant of time. Each state must be uniquely described by the values of a variable or set of variables. When the values of the describing variables change from those of one state to those of another, a state transition is said to have occurred. State transitions can be considered to occur at discrete time intervals.

As the system passes from state i to state j, it earns a reward the value of which depends on states i and j. If several alternatives are examined, each alternative will have its own state transition behavior and its own reward. The Howard policy-iteration method finds the best alternative associated with each state of the system, given the transition probabilities and the rewards associated with each state.

Description of Alternatives and Data Collection

Metering was accomplished with a pretimed signal located in the ramp about 1,200 ft (366 m) upstream of the tunnel entrance. Figure 3 shows a general layout of the site and the warning signs used in conjunction with the metering experiments. The 5 alternatives used are given in Table 1.

Because of the experiences of the Port Authority of New York and New Jersey with the 1-min cycle and because of the high capacity of the Baltimore Harbor Tunnel, it was felt that the 2-min cycle should be the minimum.

Data obtained for each of the alternatives were collected under the same circumstances of heavy traffic demand. Demand was determined by the length of the queue upstream of the tunnel proper. The queue had to extend to the point where the SLOW AHEAD sign was located. If this condition of demand was not met, no data were collected. Because the data collection period coincided with the energy crisis, the condition of not enough demand was the rule rather than the exception.

When the Howard policy-iteration technique is used, the system must dwell in as many states as possible so that state transitions can occur over an ample range. This was insured in this study by the way the traffic metering was started every day of data collection. Initially, each alternative was carried out and data were collected without discontinuity in or stoppage of traffic before metering began. This resulted in congested starting conditions. A medium level of starting congestion was obtained by stopping the traffic at the signal for 90 sec. This stoppage greatly relieved the state of congestion inside the tunnel, but the time was not long enough to have the tunnel completely cleared of vehicles. The lowest level of congestion was obtained by stopping the traffic for as long as was necessary to allow the traffic already in the tunnel to clear station 5, the bottleneck location. An observer located at this station would radio to a police car adjacent to the signal when this occurred, and traffic then would be released. This procedure was followed for each of the alternatives to provide, when possible, similar conditions for each alternative. When similar conditions were attained, data were collected. A large amount of data had to be collected to obtain the transition probabilities associated with each alternative. If a small sample was used, possible state transitions might not be observed; in the final analysis these transitions would be treated as nonexistent.

Where measurements are to be made is another important factor. It is recommended that 1 of the locations be at the bottleneck because capacity is lowest at this point and shock waves that lead into congestion most likely will originate there.

Description of the system, state, and control operation becomes better as the number of data collection stations increases. In the Baltimore Harbor Tunnel 3 stations were used: 1 in the downgrade (station 1), 1 in the level section (station 3), and 1 at the bottleneck (station 5). The equipment used in the data collection consisted of highintensity light sources placed on the upper portion of the side wall of the tunnel and directed at photoconductive cells under the pavement. The data were recorded and stored on magnetic tape. All of the details concerning collection, storage, and manipulation of data, including several problems encountered in the installation of the data acquisition system, are explained by Carter and Palaniswamy (5).

State Definition

Because concentration, K, is a quantitative measure of congestion (20), it is appropriate to use it as the control variable. States then can be defined in terms of concentration values at certain locations within the tunnel.

If a large number of states are used, a complicated and costly control algorithm could result (10). On the other hand, if few states are used, the description of the tunnel's state of congestion can be obscured to the point where situations requiring control would be overlooked. Such a case would be a nonoptimal situation.

A careful study of the volume-concentration-speed (Q-K-V) relationships for the traffic stream in the Baltimore Harbor Tunnel revealed that concentrations of 55 vehicles/mile (34 vehicles/km) and more were typical of unstable conditions; concentrations of 40 vehicles/mile (25 vehicles/km) were characteristic of stable, uncongested flows. The final state definition was obtained by combining 3 substates, 1 from each of the 3 stations.

For station 1, 3 possible substates were defined (1 vehicle/mile = 0.62 vehicle/km):

1. $0 < K < 40$ vehicles/mile,
2. $40 \leq K \leq 55$ vehicles/mile, and
3. $55 < K$ vehicles/mile.

For station 3, 2 possible substates were defined (1 vehicle/mile = 0.62 vehicle/km):

1. $0 < K < 55$ vehicles/mile, and
2. $55 \leq K$ vehicles/mile.

Station 5 is critical to the operation of the whole system and therefore was assigned 3 possible substates (1 vehicle/mile = 0.62 vehicle/km):

1. $0 < K < 40$ vehicles/mile,
2. $40 \leq K \leq 60$ vehicles/mile, and
3. $60 < K$ vehicles/mile.

The higher limit for station 5 reflects the fact that observed concentrations at station 5 were consistently higher than they were at the other stations.

According to this scheme, the number of states for the tunnel as a whole is 18 ($3 \times 2 \times 3 = 18$). Figure 4 shows the possible combinations of substates and states. Note that state 1 has the lowest concentration values throughout the tunnel, and therefore reflects the least congestion. State 18 reflects the most congestion. This enumeration of states does not necessarily mean that the higher the state number is, the greater is the degree of congestion. For example, it is not necessarily true that state 7 is more congested than state 6. State numbers, then, are more matters of mathematical convenience than they are matters of actual desirability.

Transition Probabilities

When the alternatives were being carried out, all traffic incidents were noted. The time of occurrence and duration of each incident also were recorded. A close comparison of these notes to the time and characteristics of the data stored on magnetic tapes was used to eliminate data that were not directly a result of the specific alternative being tried. This procedure was carried out for each station for each day of data collection. The usable data, in 30-sec averages, were then stored by alternatives on different magnetic tapes. A check was made to verify that the same number of observations was taken at each of the 3 stations and that the observations were taken simultaneously.

Finally, the data were processed to determine the state of the system at any given time interval, $t = T$, and at the next interval $t = T + 30$ sec. From this determination the sample size of each individual state transition was obtained. The state transition probabilities then were calculated by dividing the sample size of the individual state transitions by the total number of transitions from that state.

The rewards associated with these transitions were obtained simultaneously.

Reward Structure

It has been mentioned that the system can be described by 18 states. Each of these states has control alternatives associated with it. When 1 of the alternatives is chosen for a given state i, a decision has been made for that state. The set of decisions for all states is called a policy (4). The optimal policy is that which maximizes the gain, g, or average return, per transition. The object of the Howard process is to define such a policy.

A reward is associated with the transition from one state to another every 30 sec. This reward can be considered to be vehicles processed by the facility, savings in travel time, increased speed or safety, or any other meaningful traffic-related variable or combination of variables.

Drew and Keese (21) suggested a measure of performance that simultaneously involves flow, or volume, and speed. This parameter is called the kinetic energy, E_k, of the traffic stream and is given by the product of flow and speed. The reward structure to be proposed for the Howard process involves flow, speed, and concentration. A further discussion of some traffic-flow concepts will help the reader to understand this structure.

Consider the Q-K and V-K curves for a given facility (Figs. 5 and 6). Maximum flow, Q_M, is reached at a certain value of concentration K_M, optimum concentration, (Fig. 5); K_M at the same time determines the theoretical speed at which the flow is maximized, V_M (Fig. 6). When E_k is maximized the corresponding flow will be Q_M', which is less than Q_M. The concentration K_M' will be less than K_M and the velocity V_M' will be higher than V_M (21).

Regression analysis was used to test the fit of data from the Baltimore Harbor Tunnel to the Greenshields (22), Greenberg and Daow (18), and Underwood (23) V-K models. The Greenshields model was selected on the basis of its estimation of Q_M, V_M, and K_M parameters and according to its coefficient of determination, R^2, and standard error of estimate, S_e. The equation obtained is as follows:

$$V = V_f(1 - K/K_j)$$

where

V_f = free-flow speed [56.8 mph (90.9 km/h)];
K_j = jam concentration [112.8 vehicles/mile (70.94 vehicles/km)];
$R^2 = 0.86$; and
S_e = 5.79 mph (9.3 km/h).

Some of the special values associated with this equation are

1. Q_M = 1,604 vehicles per hour (vph);
2. V_M = 28.4 mph (45.4 km/h);

Figure 4. Definitions of states.

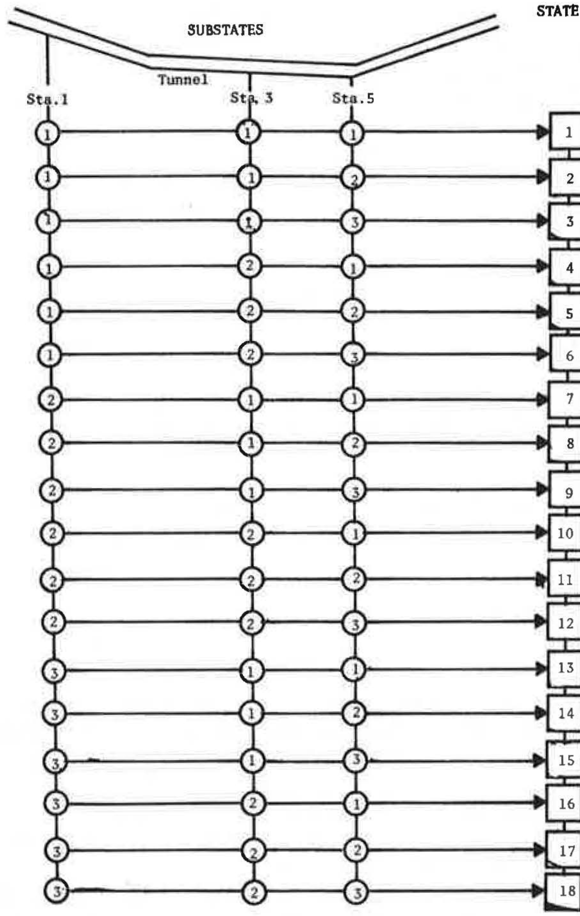


Figure 5. Volume-concentration curve.

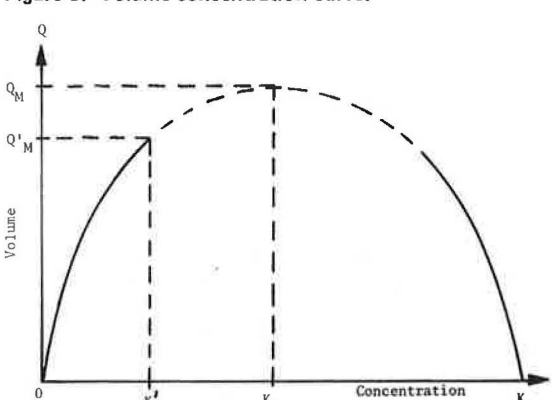
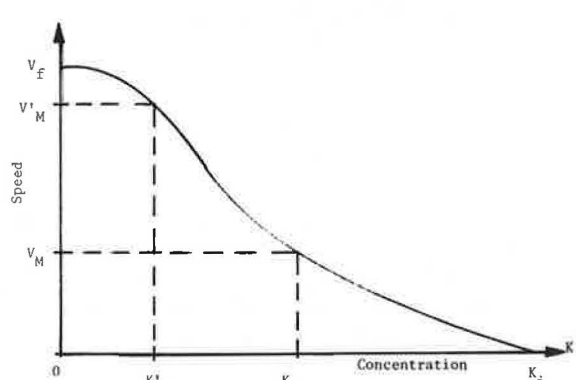


Figure 6. Speed-concentration curve.



3. K_M = 56.5 vehicles/mile (35.03 vehicles/km);
4. Q'_M = 1,425 vph;
5. V'_M = 37.9 mph (60.6 km/h); and
6. K'_M = 37.7 vehicles/mile (23.37 vehicles/km).

The reward, r_{ij}, associated with a transition of the system from state i to state j, is the sum of 2 values:

$$r_{ij} = Q + B \tag{1}$$

where

Q = actual flow processed by the bottleneck during the transition time interval; and
B = bonus, which is dependent on solely the average speed of Q during the same interval.

Maximum bonus, B_M, is assigned to a speed of V'_M, the theoretical speed associated with maximum E_k. Its value is given by the following equation:

$$B_M = 2(Q_M - Q'_M) \tag{2}$$

Any other average V for the traffic stream in a given interval will have a B that always is smaller than B_M. This B is equal to zero under 2 different circumstances: $V = V_M$ and $V = V_f$.

1. If $V = V_M$, the flow is unstable, and a small increase in traffic demand might be accompanied by a large decrease in speed and a large increase in concentration (11). Therefore, congestion is very likely to occur.
2. If $V = V_f$, extremely high headways between vehicles are more likely to occur, which in turn mean smaller actual flows. It should be noted that V possibly could be greater than V_f. In such cases B = 0.

For any other V values, B is expressed by the following linear relationships:

$$B = \frac{B_M(V_f - V)}{V_f - V'_M},\quad V > V'_M \tag{3}$$

$$= \frac{B_M(V - V_M)}{V'_M - V_M},\quad V \leq V'_M \tag{4}$$

Note that Eq. 4 implies negative Bs for speeds below V_M. This is logical because slow speeds at the bottleneck are highly undesirable and therefore should be penalized.

For the Baltimore Harbor Tunnel, the special values associated with the equation from Greenshields' model should be used in Eqs. 2, 3, and 4. The maximum bonus was obtained by using Eq. 2.

$$B_M = 2\ (1{,}604 - 1{,}425) = 358 \text{ vph}$$

$$= (\text{about 3 vehicles/30 sec})$$

The relations among Q, K, V, and B for the Baltimore Harbor Tunnel are shown in Figure 7.

Rewards are obtained by adding the observed flow at station 5 and the bonus, which is obtained from Eqs. 3 and 4.

Results and Interpretation

The state transition probability and reward matrices for each alternative are the necessary inputs to the Howard policy-iteration algorithm. Gonzalez (6) developed a computer program to analyze these data. By using the policy-iteration procedure, he was able to obtain the optimal decision matrix given in Table 2. The expected immediate rewards and relative values also were obtained and are given in Table 2.

The elements of the decision matrix correspond to the number of the alternative in the ith state that, in the long run, will maximize g. For example, when the state of the system is 1, the optimal decision is alternative 2, or the 120-sec cycle. Any other alternative will have a reward, but, in the long run, the 120-sec cycle will yield the maximum reward. The same statements hold true for the other states and the associated optimal decision. Note that the optimal policy is made up of 4 different alternatives. Alternative 5, the 240-sec cycle, does not appear.

Gain is $g = 14.10$ units/30 sec, or $g = 14.10 \times 120 = 1{,}692$ units/hour. Note that values are in units/hour rather than vph. This is because the value 1,692 does not mean that a flow of 1,692 vph can be expected. g, as is the reward r^m_{ij}, is made up of 2 parts: actual flow and a bonus according to the speeds at the bottleneck.

The 1,692 units/hour reflect a range of conditions that include a volume of 1,692 vph served by the bottleneck at a speed of 28.4 mph (45.7 km/h) in which the bonus equals zero (Fig. 7) or a reduced volume ($1{,}692 - B_M = 1{,}692 - 360 = 1{,}332$ vph) served at a speed of 37.9 mph (61.0 km/h). Combinations of values between these limits also can occur. Theoretical Q for any given speed will be

$$Q = 1{,}692 - B \tag{5}$$

B is calculated by either Eq. 3 or Eq. 4. For example, at an average speed of 30 mph (48 km/h), the B associated with the traffic stream is given by Eq. 4 as follows:

$$B = 360 \times (30 - 28.4)/(37.9 - 28.4) = 61 \text{ vph}$$

Taking this value to Eq. 5 yields

$$Q = 1{,}692 - 61 = 1{,}631 \text{ vph}$$

Therefore, at 30 mph (48 km/h) a theoretical Q of 1,631 vph can be expected.

It is important to note that, theoretically, volumes even larger than 1,692 vph could be served, but they would be served at speeds lower than those associated with maximum flow [$V_M = 28.4$ mph (45.7 km/h)]. This is due to the nature of the bonus structure, which assigns negative bonuses to these speeds. For example at a speed of 25 mph (40 km/h), Eq. 3 indicates

$$B = 360 \times (25 - 28.4)/(37.9 - 28.4) = -128 \text{ vph}$$

Figure 7. Relationship among volume, concentration, speed, and bonus values.

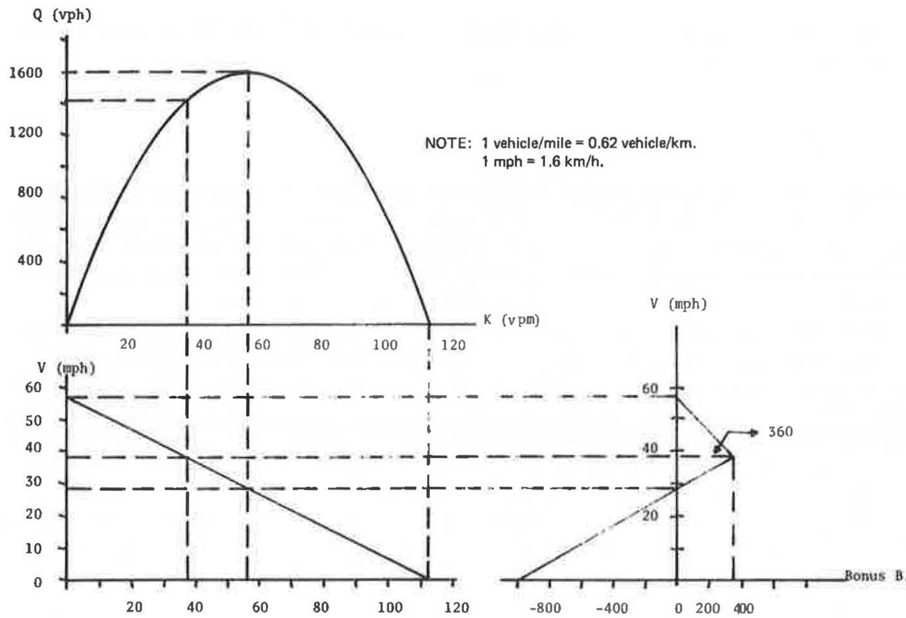


Table 2. Optimal decisions and associated values.

State	Optimal Decision	Expected Immediate Rewards (units/30 sec)	Final Relative Values (units/30 sec)
1	2	14.98	V(1) = 45.45
2	2	14.99	V(2) = 43.86
3	2	12.48	V(3) = 30.30
4	4	14.23	V(4) = 44.38
5	2	12.74	V(5) = 13.32
6	4	10.08	V(6) = 3.91
7	2	14.73	V(7) = 44.25
8	2	15.00	V(8) = 43.71
9	3	11.59	V(9) = 22.37
10	1	13.51	V(10) = 44.22
11	4	12.81	V(11) = 13.17
12	2	10.33	V(12) = 3.10
13	4	16.86	V(13) = 44.81
14	3	13.71	V(14) = 35.33
15	3	9.51	V(15) = 13.53
16	4	11.67	V(16) = 2.18
17	4	10.56	V(17) = 0.68
18	2	9.24	V(18) = 0.00

And the theoretical flow at this speed would be

$$Q = 1{,}692 - (-128) = 1{,}820 \text{ vph}$$

The problem with this flow is that, because it occurs at higher values of concentration at the bottleneck (Fig. 7), vehicles will be packed more closely, and a single slow vehicle in the group might create a general slowdown in the following traffic. That is, a single vehicle might have a shock-wave effect, which in turn might result in a breakdown of flow into stop-and-go conditions. When this situation arises, flow will sharply decrease, and the theoretical expected flow will not be obtained continuously, although it might occur for a certain period of time.

At higher speeds, Figure 7 shows that the concentration is smaller. Headways are larger and the shock-wave effect is more likely to be absorbed. Conditions existing before a vehicle's slowdown are more likely to be restored, and the expected theoretical flow can be obtained. As a result, a smoother flow throughout the facility is more likely to occur.

This is one of the main reasons why the reward structure was chosen in a way that would involve not only actual flow but also traffic speed at the bottleneck. The theoretical maximization of traffic throughput alone might result in a smaller actual flow because of the higher probability of a state of total congestion. This would impede the smooth and continuous movement of traffic through the facility.

It is important to note in Table 2 that gain was obtained with an optimal policy that involved 4 different alternatives that were used according to the state of the system. This gain, therefore, requires a system capable of determining the state of the tunnel at a given moment and transmitting a command to implement the corresponding optimal alternative. This could be achieved by a real-time control system.

CONCLUSIONS

This research has examined the feasibility of applying the Howard policy-iteration method to the evaluation of traffic control alternatives. The system is categorized into 18 different states, which include all of the possible situations encountered by the physical system. Concentration is used as the control variable. The rewards associated with the state transitions are defined in terms of actual flow and average speeds at the bottleneck for 30-sec periods. The Howard method was applied to data collected at the Baltimore Harbor Tunnel for 5 different pretimed metering alternatives (including no control). Because of the lack of on-line hardware, the method could not be tested in real time.

The results obtained for this facility show that

1. The Howard policy-iteration method can be successfully used in evaluating different traffic control alternatives.
2. The optimal policy obtained by this method generally is composed of different alternatives. The results obtained in this research seem to indicate that the policy-iteration method is suitable for application to systems that can continuously monitor the state of the system and subsequently implement the optimal alternative associated with this state.
3. Elimination of nondesirable alternatives is accomplished by the policy-iteration method. These alternatives will not appear in the optimal decision vector. In the case considered in this paper, for example, the 240-sec cycle alternative was eliminated.
4. Normal operation should be used as an alternative. For some states, this situation might be the optimal control alternative.
5. Maximization of expected immediate rewards is not necessarily the best policy in the long run. The long-run criterion should be optimized if the system is to operate for a large number of intervals. This is what the policy-iteration method achieves

when it maximizes the gain.

6. Applying the results obtained by this method to on-line control is the only way to determine with certainty the applicability of the procedure. The theoretical benefits certainly indicate that this should be done.

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