SIGNIFICANCE OF CYCLIC CONFINING STRESS IN REPEATED-LOAD TRIAXIAL TESTING OF GRANULAR MATERIAL

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A number of research projects have been concerned with determining the nonlinear stress-strain characteristics of granular materials by using repeated-load triaxial tests. Improvements in materials testing equipment have led to experiments in which the confining stress and the vertical stress were applied cyclically. There is an apparent difference in the resilient Poisson's ratio determined under constant and under variable confining stresses. This difference is best explained by considering the behavior of granular materials in terms of volumetric stress-strain and shear stress-strain relationships. By doing this, one can show that Poisson's ratio is a function of the ratio of volumetric strain to shear strain. Resilient strains were investigated separately from permanent strain because the permanent strain developed depended on the loading history of the sample and because the resilient behavior was not affected by any previous subfailure stress applications. To keep materials testing techniques simple, a constant confining stress equal to the mean value of a cyclic confining stress can be considered to be equal to the cyclic confining stress for determining the resilient modulus and permanent deformation.

NONLINEAR stress-strain characteristics of asphalt pavements in which a granular layer is the major structural component are important in design calculations (1). A number of research projects have been concerned with determining these characteristics by using repeated-load triaxial tests (2, 3, 4). In most of these, the confining stress remained constant, but the vertical stress or the deviator stress was applied cyclically. Improvements in materials testing equipment have led to some experiments in which the confining stress has also been cycled, but relatively few results from such tests have been reported (3). If a constant confining stress is adequate, then this is a desirable simplification of materials testing techniques.

Allen and Thompson (3) have shown, from a preliminary series of tests on various granular materials, that there is an apparent difference in the resilient properties determined under constant and under variable confining stress tests. This paper discusses this difference and explains it by using a more fundamental approach to stress-strain relationships than has previously been considered in pavement design. It is an extension of the work described by Brown (5) and incorporates some detailed modifications to the suggested stress functions for interpretation of the results. Further details of the basic philosophy are given in another paper (6).

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STRESS CONDITIONS

The stress-strain characteristics of granular materials, particularly when nonlinearity is most marked, can best be expressed in terms of the volumetric stress-strain and shear stress-strain relationships.

For the axisymmetric conditions applying to the triaxial compression test, the relevant stress and strain expressions are

$$p = \tfrac{1}{3}(\sigma_1' + 2\sigma_3') \tag{1}$$

for mean normal stress,

$$v = \epsilon_1 + 2\epsilon_3 \tag{2}$$

for volumetric strain,

$$\tau = \left(\sqrt{2}/3\right)(\sigma_1' - \sigma_3') \tag{3}$$

for octahedral shear stress, and

$$\gamma = \left[\left(2\sqrt{2}\right)/3\right](\epsilon_1 - \epsilon_3) \tag{4}$$

for octahedral shear strain, where σ_1' and σ_3' are the major and minor principal effective stresses respectively, and ϵ_1 and ϵ_3 are the corresponding strains that can be either resilient or permanent.

Effective stress, $\sigma' = \sigma - u$, where σ = total stress and u = pore pressure, is important, although it is frequently difficult to determine for pavement design. This arises because pore pressure can only be reliably determined in saturated materials, and granular materials are often tested partially saturated. Drained tests, which allow time for pore pressure dissipation, can overcome this problem (5). Alternatively, testing of dry materials eliminates the problem altogether.

In the triaxial compression test, the major total principal stress is normally the axial stress, and the minor one is the confining stress. Deviator stress defined as

$$q = \sigma_1' - \sigma_3' \tag{5}$$

is customarily used instead of the expression for shear stress.

Further definitions are required in applying these principles to repeated-load tests. These are shown in Figure 1, which plots mean normal stress p against deviator stress q. The particular stress path followed in a test can be specified by the following factors:

q_r = height of the cyclic deviator stress pulse,
p_r = height of the cyclic mean normal stress pulse, and
p_m = mean level of mean normal stress.

This situation applies in repeated-load tests in which the deviator stress is cycled between zero and some maximum value. If the minimum value is greater than zero, then a further term, q_o, is required to define the stress paths. The foregoing definitions differ from those used by Brown (5) for the suggested expression for p.

Most repeated-load test results reported in the literature have characterized the resilient response of the material in terms of the resilient modulus M_r and the resilient Poisson's ratio ν_r, which are defined respectively as

$$M_r = \frac{q_r}{\epsilon_{1r}} \tag{6}$$

and

$$\nu_r = -\frac{\epsilon_{3r}}{\epsilon_{1r}} \tag{7}$$

When cyclic confining stress is applied, the generalized Hooke's law equations are used and give

$$\epsilon_{1r} M_r = \sigma_1 - 2\nu_r\sigma_3 \tag{8}$$

and

$$\epsilon_{3r} M_r = \sigma_3(1 - \nu_r) - \nu_r\sigma_1 \tag{9}$$

Solving for M_r and ν_r gives

$$\nu_r = \frac{\sigma_1\epsilon_{3r} - \sigma_3\epsilon_{1r}}{2\sigma_3 \cdot \epsilon_{3r} - \epsilon_{1r}(\sigma_1 + \sigma_3)} \tag{10}$$

and

$$M_r = \frac{(\sigma_1 - \sigma_3)(\sigma_1 + 2\sigma_3)}{\epsilon_{1r}(\sigma_1 + \sigma_3) - 2\epsilon_{3r}\sigma_3} \tag{11}$$

(Total stresses have been indicated since these have normally been used. ϵ_{1r} and ϵ_{3r} refer to resilient values of the principal strains.)

This second situation treats M_r as being analogous to Young's modulus and assumes the material is linear elastic, an assumption that is widely known to be false for granular materials. The first situation, constant confining stress, treats M_r as something nearer to a shear modulus than Young's modulus.

If elastic constants are to be defined for nonlinear materials, perhaps as functions of stress level, then the bulk and shear moduli are better to use than the resilient modulus (or Young's modulus) and Poisson's ratio. They are defined as

$$K = \frac{p_r}{v_r} \tag{12}$$

for the bulk modulus and

$$G = (\sqrt{2}/3)\,(q_r/\gamma_r) \tag{13}$$

for the shear modulus, where v_r and γ_r are the terms of equations 2 and 4 in terms of resilient principal strains.

There are three advantages to using G and K for nonlinear materials: No assumptions of linear elastic behavior are used in their calculation, the volumetric and shearing components of stress and strain are separated from each other, and they have a more realistic physical meaning in a three-dimensional stress regime than Young's modulus and Poisson's ratio have.

None of the previous expressions for elastic constants has included the third term defining stress path, p_m. Brown (5) has shown that σ_3' is a parameter that can be used for normalizing the applied cyclic stresses in constant confining stress tests. The term, p_m, could equally well be used and is more appropriate in the context of the stress path in Figure 1 and for tests with cyclic confining stress.

The strain values analyzed in this work were the equilibrium values, i.e., the relatively constant values achieved after 10^4 to 10^5 stress cycles (5).

EXPERIMENTS

These experiments are an extension of those described by Brown (5). The same material was tested: a ≤5-mm-sized particle of well-graded crushed stone, 100 mm wide and 100 mm long.

The only significant difference between this experimental technique and that described by Brown (5) was that, in addition to the vertical strain measured with an LVDT, lateral strains were determined by induction coils. This technique presented some difficulty and is described elsewhere (7).

Resilient strains were investigated separately from permanent strain. Considerably more information about resilient strains as opposed to permanent strain could be obtained from an individual sample because loading history was shown to affect permanent but not resilient strains. This is shown in Figures 2 and 3. The permanent strain (Figure 2) resulting from a successive increase in the stress level is considerably smaller than the strain that occurs when the highest stress level is applied immediately. Resilient strain, expressed as resilient modulus (Figure 3), does not show this effect.

Therefore, considering resilient behavior, tests were carried out on two samples as given in Table 1. These samples gave similar results and, therefore, when the data were considered in terms of volumetric and shear strains, the results from only one sample were plotted.

One of the main objects of these experiments was to investigate whether the confining stress really needs to be cycled to obtain realistic material behavior. The tests were, therefore, planned in such a way, that comparisons could be made between behavior under a certain cyclic confining stress and under a constant confining stress. The majority of tests were, therefore, carried out with constant confining stresses equal to the mean cyclic values. This is shown in Figure 4 and differs from the procedure adopted by Allen and Thompson (3), whose constant value equaled the peak cyclic value.

The difference between these two approaches is shown in Figure 5 in which the stress paths are compared. Our philosophy results in the same mean value of p and p_m for both types of test; Allen and Thompson have a higher value for p_m in all their constant confining stress tests. This would be expected to result in a stiffer material, a fact that their results largely confirmed. Some additional tests were performed by using a relatively high cell pressure to extend the range of stress ratios considered.

Figure 1. Stress path in repeated-load test.

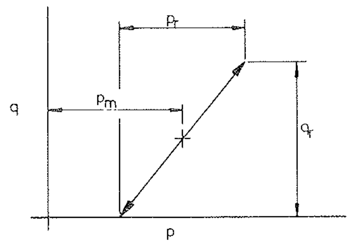


Figure 2. Effect of loading history on permanent strain.

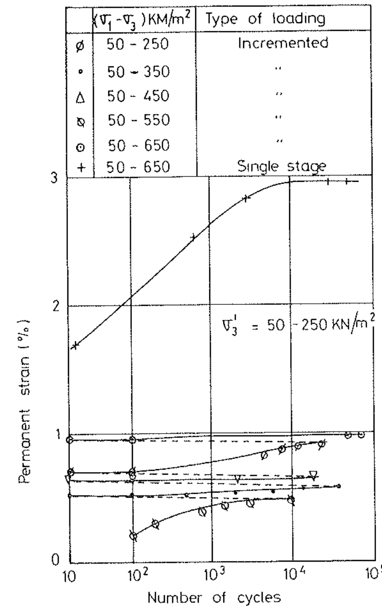


Figure 3. Effect of loading history on resilient modulus.

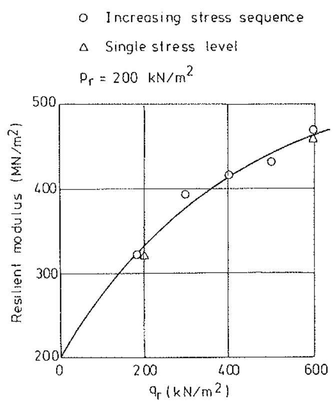
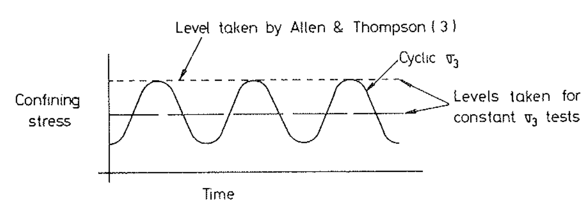


Table 1. Data for resilient strain tests.

Cell Pressure (kPa)	Mean (kPa)	Deviator Stress (kPa)		
		Test 1	Test 2	Test 3
200 to 250	225	0 to 200	0 to 300	0 to 400
150 to 300	225	0 to 200	0 to 300	0 to 400
50 to 400	225	0 to 200	0 to 300	0 to 400
0 to 100	50	0 to 200	0 to 300	0 to 400
50 to 150	100	0 to 200	0 to 300	0 to 400
100 to 200	150	0 to 200	0 to 300	0 to 400
50	50	0 to 200	0 to 300	0 to 400
100	100	0 to 200	0 to 300	0 to 400
150	150	0 to 200	0 to 300	0 to 400
225	225	0 to 200	0 to 300	0 to 400
300	300	0 to 200	0 to 300	0 to 400
400	400	0 to 200	0 to 300	0 to 400

Figure 4. Modes of application of confining stress.



A series of samples was also tested to determine the effects of cyclic cell pressure on the development of permanent strain (Table 2). These were tested without lateral strain measurement. For these tests, a single level of cyclic deviator stress was applied.

RESILIENT MODULUS AND POISSON'S RATIO

Hicks (2) and Brown (5) have shown that the resilient modulus depends on the magnitude of both confining and deviator stresses. Testing samples by using a constant deviator stress made it possible to examine the effects of both mean level and amplitude of the cyclic confining stress and to compare the results with those obtained by using a constant confining stress. Figure 6 shows that it was the mean value of the confining stress that determined the resilient modulus for a given deviator stress and that results based on cyclic confining stress were compatible with those based on a constant value. The amplitude did not have any quantifiable effect on the resilient modulus. The results obtained by using the two modes of applying confining stress were apparently much less compatible for Poisson's ratio. Under constant confining stress, Figure 7 shows that Poisson's ratio varied from less than 0.1 to more than 0.5 when dilation of the samples was occurring. However, under a cyclic confining stress, the variation was much less; most of the points lie in the region between 0.1 and 0.2. Furthermore, the stress dependence (Figure 7) is different for the two test situations.

VOLUMETRIC AND SHEAR STRAINS

The more fundamental approach to stress-strain relationships discussed in this paper was used to explain the apparent difference in behavior under cyclic and constant confining stress conditions indicated by the Poisson's ratio results in Figure 7.

A unique stress-strain relationship existed if v_r was plotted against p_r normalized with respect to p_m as shown in Figure 8. Except where dilation and the resultant negative volumetric strains are occurring, the relationship is continuous, and points for both constant and cyclic confining stresses lie on the same curve. The level of q_r does not seem to have any systematic effect on this relationship.

All the tests carried out under constant confining stress lie on the lower portion of the curve. This fact will be shown to have significance when the Poisson's ratio anomalies of Figure 7 are explained. The slope of the line in Figure 8 is a measure of the volumetric compliance, which is the reciprocal of the bulk modulus. As p_r increases, the bulk modulus decreases; as p_m increases, the bulk modulus increases. Considering the relative effects of the mean level and the amplitude of applied deviator stress, this effect is similar to that discussed by Brown (5).

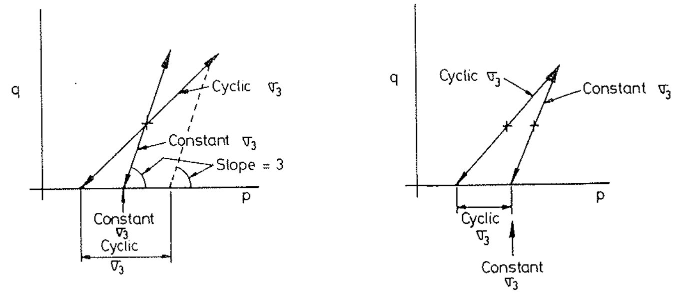
Figure 9 shows the resilient shear strain plotted against deviator stress, which is again normalized with respect to p_m. A single relationship results for both types of tests, and this was apparently unaffected by changes in p_r. The two points where excessively high shear strains occur coincide with the points on Figure 8 and indicate sample dilation.

The gradient of the curve shows that, as q_r increases, there is a slight decrease in the shear modulus and that, as p_m increases, there is an increase. In contrast to the volumetric stress-strain results of Figure 8, the range of shear strains is similar for both types of tests.

Figures 8 and 9 show that the constant confining stress tests resulted in only a small range of volumetric strains; however, the shear strains covered a larger range. Figure 10 shows that this caused the different ranges of values for Poisson's ratio shown in Figure 7 since high values resulted when the shear strain was high relative to the volumetric strain and that this only occurred in the constant confining stress tests.

Figure 7 shows that Poisson's ratio was determined over the same range of (q_r/σ_3') for both types of tests. This follows from the way in which the experiments were planned inasmuch as q_r and σ_3' are the stresses actually applied in the tests. However,

Figure 5. Stress paths indicating effects of mode of application of confining stress.



(a) Constant σ_3 equal to mean cyclic value

(b) Constant σ_3 equal to peak cyclic value

Table 2. Data for permanent strain tests.

Cell Pressure (kPa)	Mean (kPa)	Cell Pressure (kPa)	Mean (kPa)
200 to 250	225	100 to 200	150
150 to 300	225	225	225
50 to 400	225	250	250
0 to 100	50	300	300
50 to 150	100	400	400

Note: Cyclic deviator stress for all tests = 0 to 200 kPa.

Figure 6. Resilient modulus versus constant and cyclic confining stress conditions.

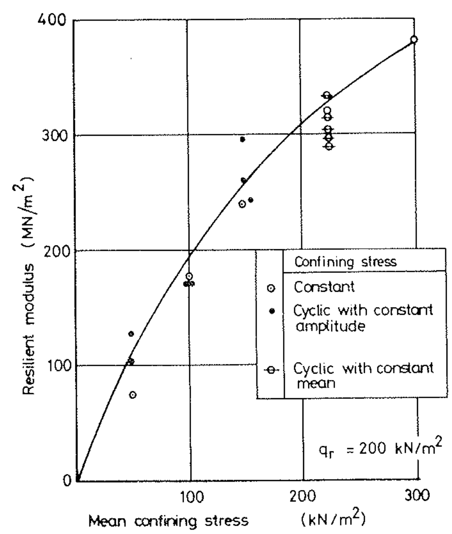


Figure 7. Poisson's ratio versus ratio of deviator stress to mean confining stress.

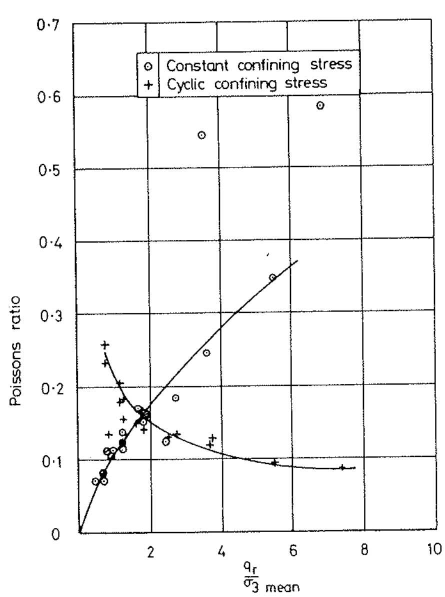


Figure 8. Resilient volumetric strain versus normalized mean normal stress.

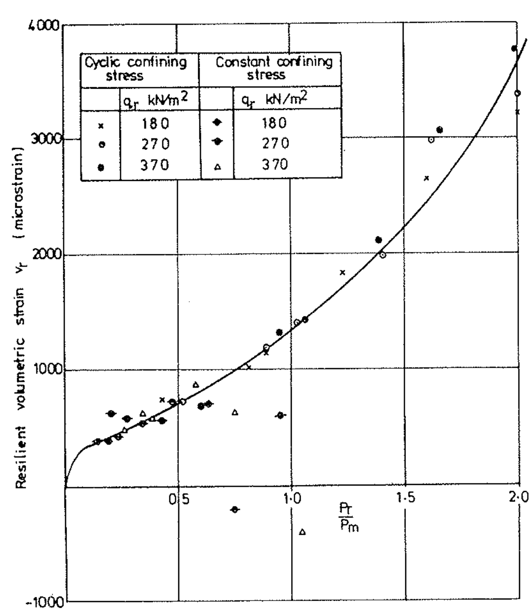


Figure 9. Resilient shear strain versus normalized deviator stress.

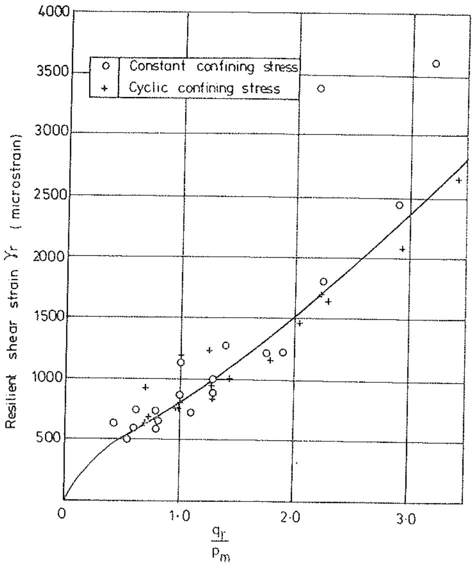


Figure 10. Poisson's ratio versus ratio of volumetric strain to shear strain.

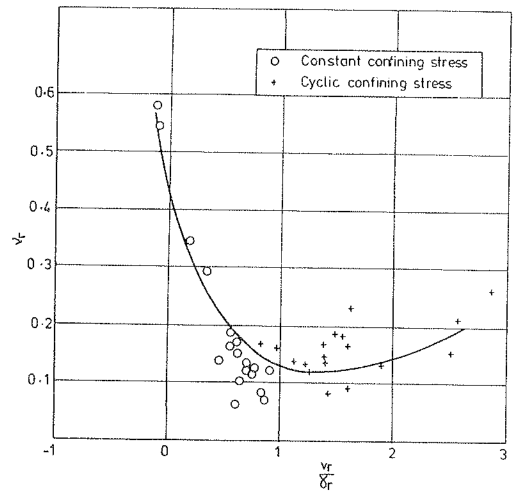


Figure 11. Permanent strain versus ratio of deviator stress to mean confining pressure.

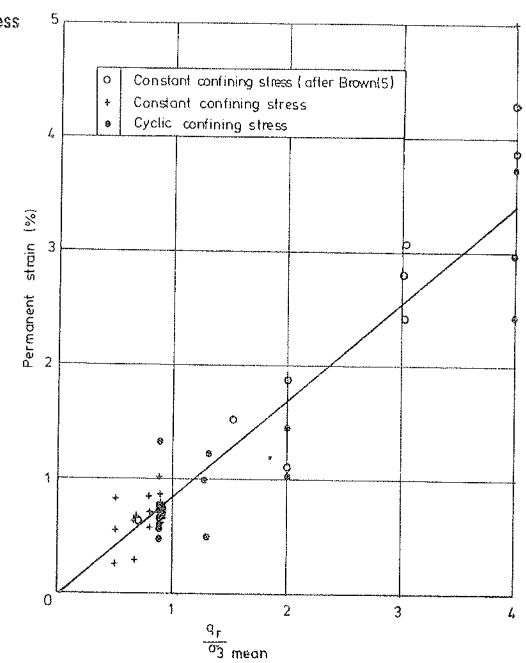


Figure 5a indicates that, for given values of q_r and σ_3', or mean σ_3, for the cyclic tests, different values of p_r can and did arise. Therefore, Poisson's ratio was significantly affected by the value of p_r or the corresponding strain v_r, and M_r and permanent strain are not.

This being the case, constant confining stress tests cannot be used reliably for the evaluation of Poisson's ratio. However, if the results are interpreted in terms of volumetric and shear stress-strain relations (Figures 8 and 9), this simpler test can be satisfactorily used.

PERMANENT STRAIN

Earlier work on this project (5) and that described by Barksdale (8) has shown that under constant cell pressure conditions the permanent strain is a function of the deviator stress normalized with respect to the cell pressure. Figure 11 shows all such results from this project and those obtained under cyclic cell pressure conditions. For these latter tests, the normalizing parameter has been taken as the mean level of cyclic cell pressure. Reasonable correlation was obtained among all the results.

CONCLUSIONS

1. Similar values of both resilient modulus M_r and permanent strain ϵ_p were obtained from cyclic and constant confining stress tests when the constant stress was equal to the mean of the cyclic value.
2. This test equivalence was not valid for the determination of Poisson's ratio; much higher values came from constant confining stress tests.
3. It appears that Poisson's ratio is affected by the magnitude of the cyclic mean normal stress p_r as well as its mean p_m, but both M_r and ϵ_p are only affected by the latter.
4. Results from constant and cyclic confining stress tests were compatible when interpreted in terms of volumetric and shear stress-strain relationships. The only exception to this was when dilation occurred.
5. Bulk and shear moduli, as functions of applied stress, are better than resilient modulus and Poisson's ratio for characterizing nonlinear materials. If this better method is used, the constant confining stress test is considered adequate.
6. Loading history had a negligible effect on resilient modulus, provided subfailure stresses were involved, but had a considerable effect on permanent strain.

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