PROBABILISTIC APPROACH TO PREDICTION OF CONSOLIDATION SETTLEMENT

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The method of derived distributions is used to develop a probabilistic model for predicting the total settlement in a compressible clay layer in terms of uncertain soil compressibility and loads. The settlement ratio (total settlement divided by thickness of compressible layer) is a function of two independent random variables (compressibility factor and load factor). The compressibility factor is a function of two dependent random variables (compression index and initial void ratio), and the load factor is a function of two independent random variables (total stress at the midheight layer and the preconsolidation stress). The compressibility factor can be described by a normal distribution, and the load factor by a lognormal distribution. The derived distribution of the settlement ratio is also well approximated by a log-normal distribution that approaches a normal distribution as the number of soil samples taken for the settlement prediction increases. Graphs are developed to estimate the settlement ratio parameters in terms of the average dry density of the soil. The effect of the number of samples and the vertical and horizontal correlation on the density function of the settlement ratio is also evaluated.

•MANY design decisions in foundation engineering are made with a great deal of uncertainty (4, 19). Although the determination of reliable and representative settlement parameters for a soil deposit is of fundamental importance in the design of foundations and earthworks, the nature of the soil suggests that these parameters should be described by probability distributions. Furthermore, a probabilistic approach to settlement problems in geotechnical engineering is useful because it provides a systematic insight into the ranges of uncertainty that may be expected for a particular type of settlement problem. Wu and Kraft (22), Folayan, Höeg, and Benjamin (8), Kay and Krizek (12, 13), and Padilla and Vanmarcke (17) have investigated such problems in considerable detail; however, results are generally qualitative because several important uncertainties are not taken into account. Accordingly, this work is directed toward combining the subjective judgment of the engineer with information deduced from collected data to develop the probability distribution for total settlement. The applied loads and the soil compressibility are random variables, and the probability density function for each random variable that affects the settlement prediction is derived. Graphs are presented to help the engineer quantify the risks and economies involved in a decision related to settlement prediction.

PROBLEM DESCRIPTION

The virgin settlement S of a statistically homogeneous compressible soil stratum under-

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lying a foundation is usually computed by use of the equation

$$S = H\left(\frac{C}{1+e}\right)\log\left(\frac{p_o + \Delta p}{p}\right) \tag{1}$$

where H is the thickness of the compressible layer, C is the compression index of the soil, e is the initial void ratio of the soil, p_o is the overburden stress at the midheight of the compressible layer before loading, Δp is the increase in the vertical stress at the midheight of the compressible layer due to the applied load, and p is the preconsolidation stress at the midheight of the compressible layer; in the case of the normally consolidated clay, p equals p_o. For convenience, equation 1 may be rewritten in the form

$$R = \frac{S}{H} = \frac{KL}{2.303} \tag{2}$$

where

$$K = \frac{C}{1+e} \tag{3}$$

$$L = \ln\left(\frac{p_o + \Delta p}{p}\right) = \ln\left(\frac{Q}{P}\right) = \ln(M) \tag{4}$$

The compressibility factor K can take on values between 0 and 1, and the value of the loading term L depends on the magnitude and configuration of the dead and live loads. In contrast to the normal approach to settlement prediction, which involves a deterministic analysis whereby the physical characteristics of the compressible soil are assumed to be constants, probability distributions are used in this study for both the compressibility factor and load ratio to provide appropriate ranges of settlement prediction. Uncertainty associated with equation 1 will not be considered so that the problem can be kept tractable.

SOIL DATA REDUCTION

The probabilistic approach adopted was applied to data from over 700 consolidation tests on undisturbed soils of alluvial, marine, aeolian, and residual origin (2). About three-quarters of these data were obtained from Greece and its environs (tests were performed continuously for about 10 years by Kotzias-Stamatopoulos in Athens), and the rest of the samples were obtained from different parts of the United States (tests were performed by Soil Testing Services, Northbrook, Illinois, and Harza Engineering Company, Chicago). The same test procedure and size of specimen were used in all cases.

The statistical parameters for the soil compressibility parameters and the associated frequency histograms are given in Table 1 and Figure 1 respectively. The whole population was divided into five different groups (A through E, as given in Table 2) to obtain soil groups with similar compressibility properties. These groups were based on the dry density γ_d of each soil sample because dry density is known to be highly correlated with compressibility.

Investigators (2, 11, 15, 16, 22) have reported different distributions to fit almost every

Table 1. Statistical parameters of all samples.

Soil Property	Mean	Median	Standard Deviation	Coefficient of Variation	Skewness	Kurtosis	Number of Samples
Compression index	0.20	0.16	0.14	0.73	1.98	7.98	720
Initial void ratio	0.762	0.680	0.316	0.415	1.307	4.69	723
Compressibility factor	0.104	0.092	0.057	0.548	1.387	6.26	720
Preconsolidation stress, kg/cm^2	2.38	2.00	0.146	0.62	1.47	6.05	707

Note: 1 kg/cm^2 = 9.8 Pa.

Figure 1. Frequency histograms for soil compressibility properties of all samples.

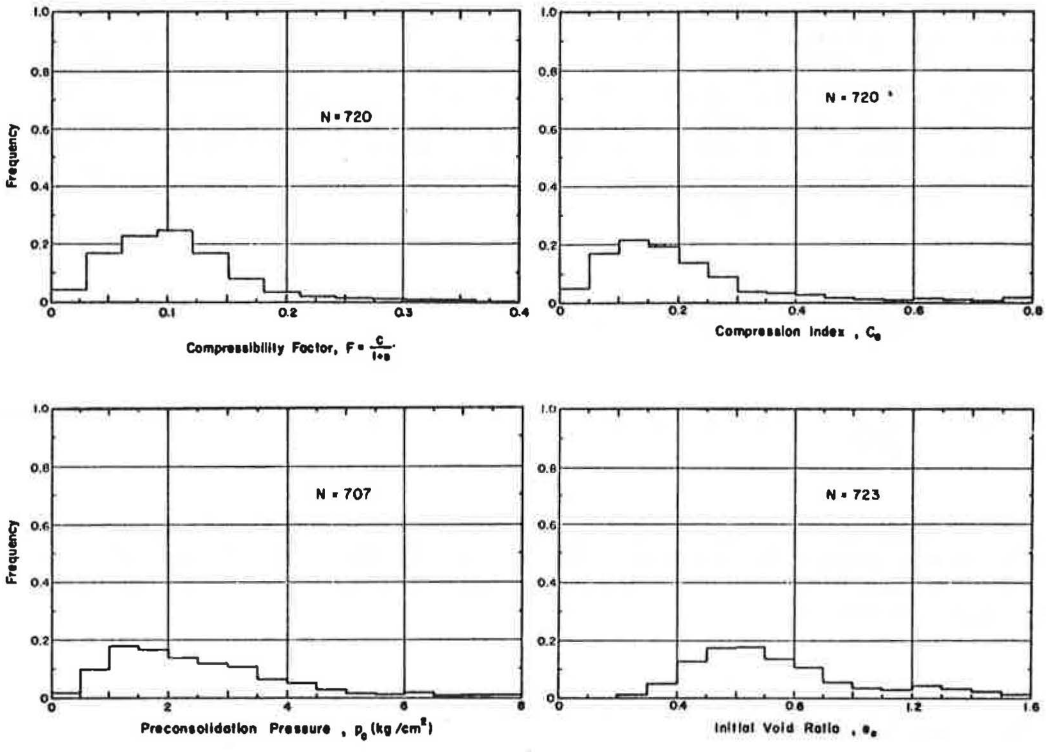
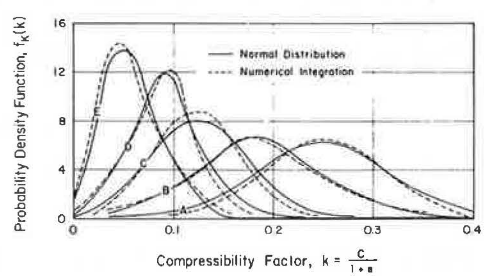


Table 2. Subdivision of soils.

Group	Dry Density (g/cm^3)	Number of Samples
A	$\gamma_d \le 1.00$	9
B	$1.00 < \gamma_d \le 1.25$	81
C	$1.25 < \gamma_d \le 1.50$	153
D	$1.50 < \gamma_d \le 1.75$	315
E	$1.75 < \gamma_d$	165

Figure 2. Probability density function of compressibility factor for different soil groups.



soil property. The general concensus obtained from their studies is that the inherent variations of most of the soil properties can be well explained by either a normal or log-normal distribution. In this work the normal probability density function will be used to describe the inherent variability of the compression index C, initial void ratio e, and the preconsolidation pressure p. Although the histograms in Figure 1 indicate some positive skewness for these properties, this skewness was significantly reduced when the soils were subdivided into groups according to dry density.

COMPRESSIBILITY OF SOIL

As a consequence of the natural processes involved in the formation of soil deposits, the inherent variability of soil, both in material properties and geometry, is well recognized as an important source of uncertainty. Additional uncertainty is introduced by the design and conduct of laboratory tests (9) and the interpretation of data. For a complete probabilistic approach to settlement prediction, one must have reliable estimates of the probabilities of the compression index and the initial void ratio; then, the methods of derived distributions (18, 21) or simulation can be used to find the probabilistic description of the compressibility factor. Since the compression index and initial void ratio have been assumed to be normally distributed, the following probability density functions can be written:

$$f_C(c) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_C} \exp\left[-\frac{1}{2}\left(\frac{C - m_C}{\sigma_C}\right)^2\right] \tag{5}$$

$$f_e(e) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_e} \exp\left[-\frac{1}{2}\left(\frac{e - m_e}{\sigma_e}\right)^2\right] \tag{6}$$

in which m_C and m_e denote the mean values, and σ_C and σ_e denote the standard deviations for the compression index and initial void ratio respectively. If a new variable, $g = 1 + e$, is defined, g has the same distribution as e, but it is shifted by 1; therefore, $m_g = m_e + 1$ and $\sigma_g = \sigma_e$.

Joint Probability Density Function of Compression Index and Initial Void Ratio

To determine a probability statement for the compressibility factor K, one must find an expression for the joint density function of C and e. The results of a two-way χ^2 test (10) are as follows: For the compression index and initial void ratio of the 82 samples $\chi^2 = 30$, $\nu = 19$, and $P_\nu(\chi^2) = 24$ percent. The results indicate that the joint probability density function of C and e may be assumed to follow the bivariate normal distribution, which can be written in the form

$$f_{C,e}(C, e) = \frac{1}{2\pi\sigma_C\sigma_e\sqrt{1 - \rho^2}} \exp\left\{-\frac{1}{2}\left[\left(\frac{C - m_C}{\sigma_C}\right)^2 - \frac{2\rho(C - m_C)(e - m_e)}{\sigma_C\sigma_e} + \left(\frac{e - m_e}{\sigma_e}\right)^2\right]\right\} \tag{7}$$

where ρ is the correlation coefficient between C and e. It is a characteristic of the normal distribution that $f_{C,g}(C, g)$ is identically equal to equation 7 with e replaced by g and m_e and σ_e replaced by m_g and σ_g.

Probability Density Function of Compressibility Factor

The method of derived distributions enables the probability distribution of K, $f_K(k)$, to be written as

$$f_K(k) = \int_{-\infty}^{\infty} g f_{C,g}(kg, g) dg \tag{8}$$

where $f_{C,g}(C, g)$ is the joint probability distribution of C and g. Since the void ratio cannot be negative, equation 8 simplifies to

$$f_K(k) = \int_{1}^{\infty} g f_{C,g}(kg, g) dg \tag{9}$$

The lower limit for equation 9 can be replaced by zero without affecting the result appreciably, because the probability of g being less than 1 ($e < 0$) is negligible for practical values of m_g and σ_g. By substituting equation 7 into equation 9, by rearranging the power of the exponent, and by using the relations between the moments of e and g, we can rewrite equation 9 in the form

$$f_K(k) = A \exp(-I) \int_{0}^{\infty} g \exp(-Eg^2 - Gg) dg \tag{10}$$

where

$$A = \frac{1}{2\pi\sigma_g\sigma_C \sqrt{1 - \rho^2}} \tag{11}$$

$$E = B\left(\frac{k^2}{\sigma_C^2} - \frac{2\rho k}{\sigma_C\sigma_g} + \frac{1}{\sigma_g^2}\right) \tag{12}$$

$$G = 2B\left[\left(\frac{\rho m_g}{\sigma_g\sigma_C} - \frac{m_C}{\sigma_C^2}\right) k + \left(\frac{\rho m_C}{\sigma_g\sigma_C} - \frac{m_g}{\sigma_g^2}\right)\right] \tag{13}$$

$$I = B\left(\frac{m^2}{\sigma_g^2} + \frac{m_C^2}{\sigma_C^2} - \frac{2\rho m_g m_C}{\sigma_g\sigma_C}\right)$$

$$B = \frac{1}{2(1 - \rho^2)} \tag{14}$$

The analytical solution for equation 10 is

$$f_K(k) = \frac{A \exp(-I)}{2E} \exp\left[G^2/(8E)\right] D_{-2}\left(\frac{G}{\sqrt{2E}}\right) \tag{15}$$

where D_{-2} is the parabolic cylinder function, defined as

$$D_i(z) = \frac{\exp(-z^2/4)}{\Gamma(-i)} \int_0^{\infty} \exp(-zx - x^2/2)\, x^{-i-1}\, dx \quad \text{for } i < 0 \tag{16}$$

in which i and z denote the parameters of the function and Γ is the gamma function. The probability density function of the compressibility factor is given by equation 15 and is shown in Figure 2 for the different soil groups and a correlation coefficient of 0.8 between the compression index and the initial void ratio. The results of the Kolmogorov-Smirnov one-sample test are given in Table 3 and indicate that the compressibility factor satisfies the normal probability density function for the different soil groups.

The corresponding fitted normal distributions of the compressibility factor for all soil groups are also shown in Figure 2. In view of the good agreement between the actual distribution of K calculated from equation 15 and the normal distribution, the latter will be used for the soil compressibility in evaluating the probability density function of the total settlement ratio R. Finally, the probability function of K can be written as

$$f_K(k) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_K} \exp\left[-\frac{1}{2}\left(\frac{k - m_K}{\sigma_K}\right)^2\right] \tag{17}$$

in which m_K and σ_K denote the mean and standard deviation respectively.

LOAD FACTOR

Applied Loads and Overburden Stress

The applied loads acting on a structure can frequently be treated as random quantities (5, 17, 20), and in the last few years the concept of load variability has been introduced into structural engineering problems through building codes and safety investigations. Since the total applied load is the sum of many relatively small and independent loads, the distribution of the load-induced increase in the vertical stress at the midheight of the compressible layer can be assumed to follow a normal probability density function. Since there is little evidence on which to base a representation of the uncertainty associated with the estimation of stresses in the ground, the value of the overburden stress p_o is assumed to be deterministic. Consequently, the sum of the overburden stresses and the stresses caused by the added loads will follow a normal distribution, and the probability density function of the sum Q can be written as

$$f_Q(q) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_Q} \exp\left[-\frac{1}{2}\left(\frac{q - m_Q}{\sigma_Q}\right)^2\right] \tag{18}$$

where m_Q and σ_Q are the mean and standard deviation respectively.

Preconsolidation Pressure

The preconsolidation stress p may be due to a variety of factors (14): overburden that causes subsequent erosion, desiccation due to exposure of the surface, sustained seepage forces, and tectonic forces due to movement in the earth's crust. The combined influence of these uncertainties in the determination of the preconsolidation stress is assumed to be a normal distribution. Therefore, the uncertainty associated with the preconsolidation stress can be expressed as

$$f_P(p) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_P} \exp\left[-\frac{1}{2}\left(\frac{p - m_P}{\sigma_P}\right)^2\right] \tag{19}$$

where m_P and σ_P are the mean and standard deviation respectively.

Joint Probability Density Function of Applied Loads and Preconsolidation Stress

Since Q and P are statistically independent random variables that follow a normal distribution, the joint distribution will be a bivariate normal distribution (3) that can be written as

$$f_{QP}(q, p) = \frac{1}{2\pi\sigma_Q\sigma_P} \exp\left\{-\frac{1}{2}\left[\left(\frac{q - m_Q}{\sigma_Q}\right)^2 + \left(\frac{P - m_P}{\sigma_P}\right)^2\right]\right\} \tag{20}$$

Probability Density Function of Load Ratio

From equation 4 the load ratio M is defined as $M = Q/P$, and the method of derived distributions can be used to write the probability distribution of M, $f_M(m)$, as

$$f_M(m) = \int_{-\infty}^{\infty} p f_{Q,P}(mp, p)\,dp \tag{21}$$

where $f_{Q,P}(q, p)$ is the joint probability density function of Q and P. Since the value of P cannot be negative, equation 21 simplifies to

$$f_M(m) = \int_{0}^{\infty} p f_{Q,P}(mp, p)\,dp \tag{22}$$

Substituting equation 20 into equation 22 and rearranging the power of the exponent leads to

$$f_M(m) = \bar{A} \int_0^{\infty} p \exp(-\bar{E}p^2 - \bar{G}p)dp \tag{23}$$

where

$$\bar{A} = \frac{1}{2\pi\sigma_Q\sigma_P} \exp\left[-\frac{1}{2}\left(\frac{m_Q^2}{\sigma_Q^2} + \frac{m_P^2}{\sigma_Q^2}\right)\right] \tag{24}$$

$$\bar{E} = \frac{1}{2}\left(\frac{1}{\sigma_P^2} + \frac{m^2}{\sigma_Q^2}\right) \tag{25}$$

$$\bar{G} = -\left(\frac{m_Q}{\sigma_Q^2} + \frac{m_P}{\sigma_P^2}\right) \tag{26}$$

The analytical solution of equation 23 will yield the probability density function of the load ratio M as

$$f_M(m) = \frac{\bar{A}}{2\bar{E}} \exp[\bar{G}^2/(8\bar{E})] D_{-2}\left(\frac{\bar{G}}{\sqrt{2\bar{E}}}\right) \tag{27}$$

where D_{-2} is the parabolic cylinder function defined by equation 16.

Probability Density Function of Load Factor

Given the distribution function of the load ratio M it is possible to derive the probability density for the logarithm of this ratio. From equation 4

$$L = \ell n(M) \text{ or } M = \exp(L)$$

and the probability density function of L will take the form

$$f_L(\ell) = \frac{dM}{dL} f_M[\exp(\ell)] \tag{28}$$

$$f_L(\ell) = \exp(\ell) f_M[\exp(\ell)] \tag{29}$$

where $f_M \exp(\ell)$ is the probability density function of the load ratio M evaluated at a value equal to $\exp(\ell)$. Figure 3 shows the probability distribution of the load factor L calculated from equations 28 and 29 for mean values of 2, 4, and 6 and for $\frac{\sigma_Q}{\sigma_P}$ equal to 0.75. It appears from Figure 3 that L approximately follows a log-normal distribution, where the value of L is nonnegative and the density function is skewed to the right.

Accordingly, the probability distribution of the load factor will be approximated by the following log-normal distribution in the analysis of settlement prediction:

$$f_L(\ell) = \frac{1}{\ell}\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma_{\ell nL}} \exp\left[-\frac{1}{2\sigma^2_{\ell nL}}(\ell n\ell - m_{\ell nL})^2\right] \tag{30}$$

in which $m_{\ell nL}$ and $\sigma_{\ell nL}$ denote the mean and standard deviation of $\ell n L$ respectively. For a mean load ratio of 1, the exponential distribution shown in Figure 3 described the load factor satisfactorily.

UNCERTAINTY OF SETTLEMENT PREDICTION

Based on the foregoing probabilistic analyses, a normal distribution can be reasonably used to describe the uncertainty associated with the compressibility factor K, and a log-normal distribution can be used to describe the load term L. These two factors will be assumed to be independent in evaluating the effect of their uncertainty on the total settlement ratio R, defined by equation 2. The method of derived distributions will be used to find a probability distribution for R, $f_R(r)$, in the form

$$f_R(r) = 2.303 \int_{-\infty}^{\infty} \frac{1}{L} f_{K,L}\left(\frac{2.303r}{\ell}, \ell\right) d\ell \tag{31}$$

where $f_{K,L}(k, \ell)$ is the joint density function of K and L. Since K and L are assumed to be independent, the joint distribution can be written as

$$f_{K,L}(k, \ell) = f_K(k) f_L(\ell) \tag{32}$$

where $f_K(k)$ and $f_L(\ell)$ are the density functions of K and L respectively. Furthermore, since the value of the compressibility cannot be negative, equation 31 simplifies to

$$f_R(r) = 2.303 \int_0^{\infty} \frac{1}{L} f_K\left(\frac{2.303r}{\ell}\right) f_L(\ell) d\ell \tag{33}$$

By substituting the corresponding expressions for $f_K(k)$ and $f_L(\ell)$ into equation 33, we obtain

$$f_R(r) = 2.303 \int_0^{\infty} \frac{1}{\ell}\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma_K} \exp\left[-\frac{1}{2}\left(\frac{\frac{2.303r}{\ell} - m_K}{\sigma_K}\right)^2\right]$$

$$\frac{1}{\ell} \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_{\ell nL}} \exp\left\{-\frac{1}{2}\left[\frac{1}{\sigma_{\ell nL}}(\ell n\ell - m_{\ell nL})\right]^2\right\} d\ell \tag{34}$$

which, on rearrangement of the exponents, becomes

$$f_R(r) = a \int_0^{\infty} \frac{1}{\ell^2} \exp\left[-\frac{-1}{2\sigma_K^2}\left(\frac{2.303r}{\ell}\right)^2 + \frac{m_K}{\sigma_K^2}\left(\frac{2.303r}{\ell}\right)\right.$$

$$\left.- \frac{1}{2\sigma_{\ell nL}^2}(\ell n\ell)^2 + \frac{m_{\ell nL}}{\sigma_{\ell nL}^2}(\ell n\ell)\right] d\ell \tag{35}$$

where

$$a = \frac{2.303}{2\pi\sigma_K\sigma_{\ell nL}} \exp\left[-\frac{1}{2}\left(\frac{m_K}{\sigma_K^2} + \frac{m_{\ell nL}^2}{\sigma_{\ell nL}^2}\right)\right] \tag{36}$$

A numerical technique was applied to perform the indicated integration, and the probability density function for the settlement ratio R for different soil groups at mean load ratios of 1, 2, 4, and 6 is plotted in Figure 4. As the load ratio M increases and as the dry density of the soil decreases, the mode of the frequency curve is shifted to the right and the mean and standard deviation increase. A heuristic argument, based on the fact that (a) the log-normal distribution can be viewed as a model for the product of independent random variables, and (b) the similarity between the results (Figure 4) and the shape of the log-normal distribution favor the adoption of a log-normal model for the settlement ratio R. Accordingly, the probability density function of R can be expressed as

$$f_R(r) = \frac{1}{r} \frac{1}{\sigma_{\ell nR}} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left[\frac{1}{\sigma_{\ell nR}}(\ell n r - m_{\ell nR})\right]^2\right\} \tag{37}$$

in which $m_{\ell nR}$ and $\sigma_{\ell nR}$ denote the mean and standard deviation of $\ell n R$ respectively.

Analysis Charts

Use of the above probabilistic analysis allows the development of charts to determine the parameters of the log-normal distribution of the settlement ratio for a particular application. These charts (Figure 5) for load ratios 1, 2, 4, and 6 relate the average dry density of a soil to the average and standard deviation of the settlement ratio R.

Effect of Samples

The reduction of inherent uncertainty associated with the determination of R may be approached by calculating both vertical and horizontal correlation structures for the compressibility factor and the preconsolidation stress (17). The vertical correlation is determined by treating each boring (with readings at various elevations) as a sample function of a random process over depth. The autocorrelation as a function of vertical

Table 3. Results of Kolmogorov-Smirnov goodness-of-fit test for compressibility factor.

Group	Number of Samples	Hypothesized Normal Distribution: D_2	Hypothesized Normal Distribution: Probability (percent)
A	9	—[a]	—[a]
B	81	0.06	72
C	147	0.09	26
D	314	0.07	19
E	165	0.08	39

[a] Insufficient data.

Figure 3. Probability density function of load factor for different mean load ratios.

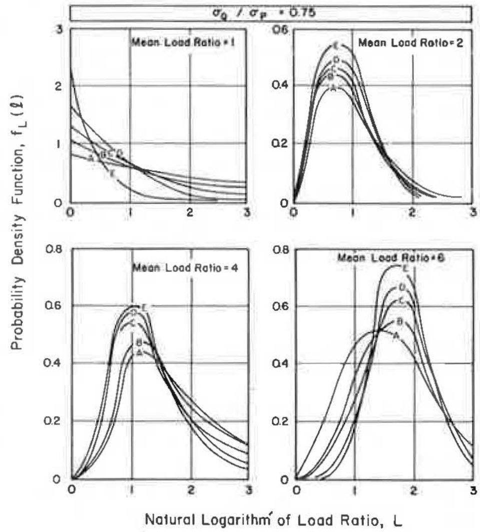


Figure 4. Probability density function of settlement ratio for different soil groups.

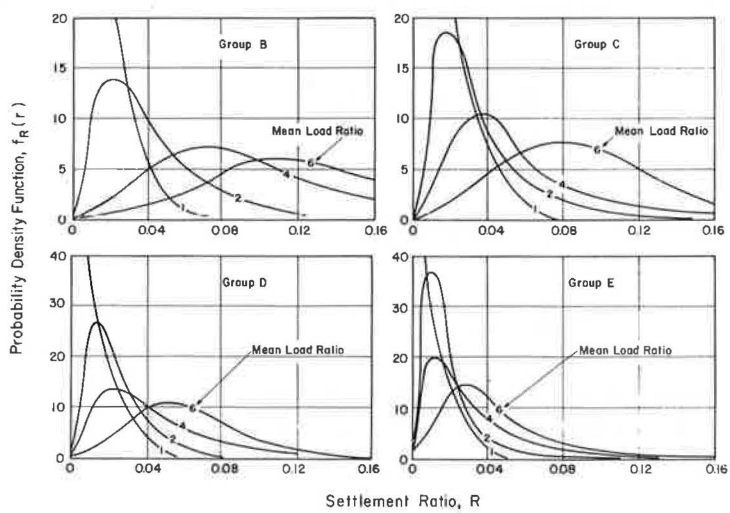
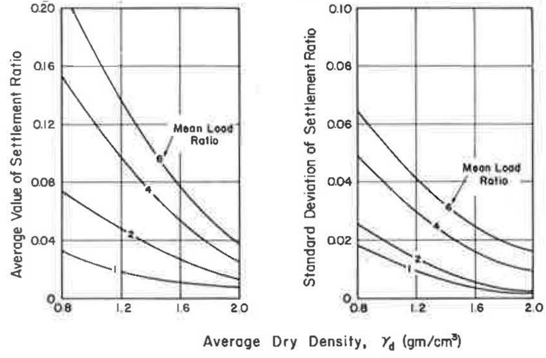


Figure 5. Model parameters of settlement ratio.



separation is then the vertical correlation, and the horizontal correlation can be obtained in a similar manner by treating the readings within a certain layer from all borings as a sample function of a random process over the horizontal position. In the settlement prediction problem, the variability of the average properties beneath a certain foundation, not the point-to-point variability within the soil mass, is needed. The variability of the average is a function of the spatial correlation structure and the number and location of samples (1) and may be related to an equivalent number of independent samples (6). Figure 6 shows the probability density function of the average value of the settlement ratio m_R as well as the density distribution itself, for 1, 4, 9, and 16 independent soil samples from soil group D. It can be seen from Figure 6 that, as the number of samples increases, the uncertainty associated with the value of R decreases. If n is reasonably large, the distribution of the average will be approximately normal with mean m_R and standard deviation $\sigma_R/\sqrt{n}$, where n is the number of independent samples.

ILLUSTRATIVE EXAMPLE

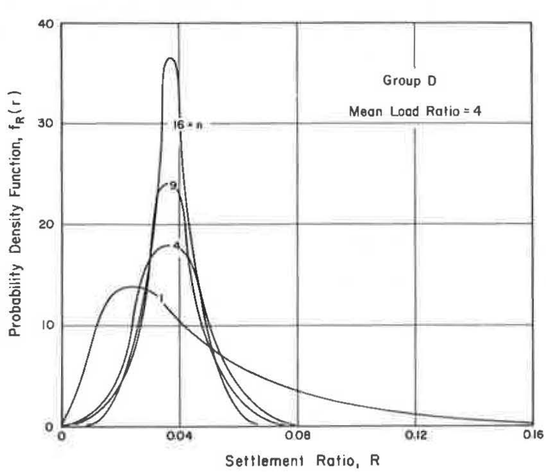
The application of the foregoing procedure can be best illustrated in an example problem. Consider the case where an 18-ft-thick (6-m) compressible layer is loaded one-dimensionally so that the estimated stress at the midheight of the layer due to both the overburden and the applied load is 3,500 lb/ft^2 (170 kPa). Six consolidation tests from samples taken at random in the layer indicate average values of 0.2 for C, 1.54 g/cm^3 for γ_d, 0.75 for e, and 2,500 lb/ft^2 (120 kPa) for p. Determination of the expected settlement due to primary consolidation is desired. According to the deterministic approach, we obtain

$$S = H\left(\frac{C}{1+e}\right)\log\left(\frac{p_o + \Delta p}{p}\right)$$

$$= 18\left(\frac{0.2}{1+0.75}\right)\log(1.4)$$

$$= 3.6 \text{ in. } (9.2 \text{ cm}) \qquad (38)$$

Figure 6. Effect of number of independent samples on probability density function of settlement ratio.



Utilizing the probabilistic approach, we find from Figure 5 that the mean and standard deviation of the settlement ratio are 0.021 and 0.005 for $m_{\gamma_d} = 1.54$ g/cm^3. The average and standard deviation of the total settlement are 4.5 and 1.1 in. (11.5 and 2.7 cm) respectively. The corresponding standard deviation of the natural logarithm of R is 0.241, which is high enough that the log-normal distribution, rather than the simple normal distribution, should be used for the settlement. If the cumulative function of R is evaluated at 0.90, the result will be the settlement for which there is a 90 percent confidence that it will not be exceeded. Since the total settlement is directly proportional to R, the statistics of the total settlement may be used in the log-normal expression to obtain

$$F_u\left[\frac{\ln (S/4.4)}{0.241}\right] = 0.9 \tag{39}$$

or

$$S = 6.0 \text{ in. } (15 \text{ cm})$$

where 4.4 is the median of the total settlement and F_u is the standardized normal cumulative function.

SUMMARY AND CONCLUSIONS

The soil compressibility factor can be quite closely approximated by a normal distribution. Although the most accurate method for determining the mean and standard deviation of the compressibility is to perform a series of consolidation tests on representative undisturbed samples taken at random locations and depths, Elnaggar and Krizek (1) and Azzouz (2) have shown that the compressibility factor is highly correlated to the initial void ratio, and they have provided empirical equations for the determination of the mean. The standard deviation can then be obtained by selecting a value for the coefficient of variation based on a large number of previously obtained test results from soil in the vicinity of the project site. This selection must be tempered by engineering judgment, and the geological aspects of the site must be appropriately considered. The log-normal distribution appeared to be satisfactory for describing the load factor for mean load ratios of 2, 4, and 6. In the case of a mean load ratio of 1, the distribution can be quite closely approximated by an exponential distribution. When the distribution of the load factor was evaluated, σ_Q/σ_P was set equal to 0.75, reflecting somewhat less uncertainty in the sum of the stresses due to the added loads and overburden than in the preconsolidation stress. Finally, the derived distribution for the settlement was in good agreement with the log-normal distribution, the parameters of which were related to the average dry density of the soil at different load ratios.

Based on the probabilistic analyses above, the following conclusions can be made:

1. A prediction model can be developed to yield probabilistic information about the settlement of a clay layer in terms of probabilistic descriptions of soil compressibility and loads.
2. The uncertainty involved in the determination of the soil compressibility can be adequately described by a normal distribution.
3. The log-normal distribution appears to satisfactorily describe the load factor for mean load ratios greater than 1.
4. The total settlement can be expressed in a probabilistic design by a log-normal distribution.
5. As the number of borings increases and the vertical and horizontal correlations are taken into account, the uncertainty involved in the determination of the settlement decreases.

6. If the number of samples is reasonably large, the distribution of the average of the settlement will be approximately normal, and the standard deviation will be inversely proportional to the equivalent number of independent soil samples.

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