OPTIMAL CARGO VEHICLE FLOW PATTERNS FOR INLAND WATERWAY SYSTEMS

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In this paper is solved the following multicommodity, mixed fleet transportation problem: Given origin-destination matrices for two commodities, the first of which can be moved in both open hopper and covered hopper barges and the second of which must be moved in covered hoppers, find minimum cost origin-destination flows for loaded and empty hopper barges such that all commodities are moved and flow conservation conditions at each port are satisifed. A linear programming model of this problem is developed, and an efficient solution technique is presented. The model is then used to derive optimal barge flows for an inland waterway system. The effect of this flow optimization on system operations is then investigated, with the aid of an inland waterway simulation model.

•A PHENOMENON common to freight transport systems is that the prevailing commodity flow patterns often dictate the movement of empty cargo units. This, in turn, has important implications regarding the demands placed on the transportation system.

Consider, for example, an inland waterway system. The demand for freight transportation that the waterway must serve is most readily expressed as a matrix (X_{ijk}), the elements of which specify the tons (megagrams) of commodity k that will be shipped from port i to port j during some designated time period. To analyze the operation of this waterway requires that the port-to-port movements of barges, both loaded and empty, that must occur in order to provide for the indicated commodity tonnage flows be determined.

The realities of equipment movement impose an important constraint on the solution of this problem, which may be termed the balance principle: The numbers of barges of each type that depart from and arrive at each port must be equal. That is, a steady-state system cannot have equipment sources or sinks. Some common equipment usage phenomena readily visible on the waterways, such as the ingenuity of the carriers in their attempts to garner backhauls to avoid moving empty barges, pose further difficulties. As a case in point, consider covered hopper barges and open hopper barges. Grain must be protected from the elements and thus must be shipped in covered barges. Many other bulk commodities, such as coal or sand and gravel, are transported in open hoppers. However, these latter commodities can also be moved in covered hoppers if it is convenient to do so. A prime example of this double-duty use of covered hoppers occurs on the Mississippi River, where barges that move grain downstream are used to haul coal northward. The major difficulty involved in incorporating these considerations into the predicted vehicle flow pattern is in determining when such double-duty barge use is possible and convenient (i.e., economically attractive).

STATEMENT OF THE PROBLEM

The specific problem investigated in this paper may be defined as follows: Given origin-destination (O-D) matrices for two commodities, the first of which can be moved in both open hopper and covered hopper barges and the second of which must be moved in covered hoppers, find minimum cost O-D flows of loaded and empty hopper barges such that all commodities are moved and flow conservation conditions are satisfied at each port.

Similar transportation flow problems, usually in the context of fleet scheduling, have been investigated, and a fairly comprehensive literature review is available elsewhere (1). The linear programming (LP) model makes use of some of the ideas presented by Schwartz (2), Laderman et al. (3), Rao and Zionts (4), and Gould (5).

LINEAR PROGRAMMING MODEL

The following variables are used in the formulation of the LP problem:

N = number of ports in the system,
F_{ijk} = number of type k barge loads available for shipment from port i to port j, rounded to the nearest integer,
X_{ijk} = number of loaded type k barges that will move from i to j,
Y_{ijk} = number of empty type k barges that will move from i to j,
c_{ijk} = cost per barge of moving loaded type k barges from i to j,
d_{ijk} = cost per barge of moving empty type k barges from i to j, and
k = 1 for open hopper barges and 2 for covered hopper barges

where subscripts i and j have the range $1, \ldots, N$, $i \neq j$.

Then the linear programming problem may be stated as follows: Find nonnegative values of X_{ijk}, Y_{ijk} such that

$$\text{Min } Z = \sum_{i \neq j}^{N} \sum^{N} \sum_{k}^{2} c_{ijk}X_{ijk} + d_{ijk}Y_{ijk} \tag{1}$$

subject to

$$X_{ij2} \geq F_{ij2} \tag{2}$$

$$X_{ij1} + X_{ij2} = F_{ij1} + F_{ij2} \tag{3}$$

$$\sum_{j \neq i}^{N} (X_{ijk} + Y_{ijk}) - (X_{jik} + Y_{jik}) = 0 \tag{4}$$

X_{ijk} and Y_{ijk} are, of course, the decision variables, i.e., the loaded and empty O-D barge flows to be found. The condition that all covered hopper loads must move in covered hopper barges is expressed by equation 2, and equation 3 states that all O-D commodity flows must be satisfied. Note that, if $X_{ij1} < F_{ij1}$, the latter constraint requires that X_{ij2} exceed its lower bound. That is, some open hopper loads would then move in covered hopper barges. Equation 4 ensures that the number of barges of each type originating at each port is matched by an equal number of terminations. The objective, equation 1, is to minimize total transport costs.

At this point, the reader well versed in mathematical programming techniques might ask why the decision variables are not constrained to have integer values. Indeed, this would be a desirable outcome, for the existence of noninteger X- and Y-values might make the LP solution somewhat difficult to interpret. Further, the flow constraints, F_{ijk}, have been defined to be integers.

The major reason for not requiring that the variables have integral values is that, for most practical problems, the X- and Y-values will be so large that rounding of the

LP solution will be an acceptable procedure. In addition, it is advantageous to avoid the usually troublesome complexities of integer programming at this stage of model development.

SOLUTION TECHNIQUE

The LP problem presented in equations 1 to 4 can readily be solved by the simplex method. The special structure of the model, however, leads directly to an easily obtainable feasible solution and thus greatly reduces the number of simplex iterations required to achieve optimality.

The most obvious and intuitive starting point is to set $X_{ijk} = F_{ijk}$. That is, all type k barge loads should initially be assigned to barge type k. This immediately guarantees that equations 2 and 3 will be satisfied. Initial Y_{ijk} values can then be found by solving two linear programming transportation problems (LPTPs).

Define the demand for empty type k barges at port i as

$$B_{ik} = \sum_{j \neq i}^{N} (X_{ijk} - X_{jik})$$

for i = 1, . . ., N and k = 1, 2. The following demand and supply vectors can then be derived:

$$\begin{aligned} D_{ik} &= B_{ik},\ B_{ik} > 0 \\ &= 0,\ B_{ik} \le 0 \\ S_{ik} &= -B_{ik},\ B_{ik} < 0 \\ &= 0,\ B_{ik} \ge 0 \end{aligned}$$

Hence, vectors D_k and S_k and matrices Y_k and d_k collectively define an LPTP, which can be stated as follows: Find Y_{ijk} subject to

$$\text{Min} \sum_{i \neq j}^{N} \sum^{N} d_{ijk} Y_{ijk}$$

$$\sum_{j \neq i}^{N} Y_{ijk} = S_{ik}$$

$$\sum_{i \neq j}^{N} Y_{ijk} = D_{jk}$$

$$Y_{ijk} \ge 0$$

for i, j = 1, . . ., N, i $\neq$ j and k = 1, 2.

After the two LPTPs stated above are solved by using any standard transportation

algorithm, the initial basic feasible solution to the overall LP problem is complete. A relatively small number of simplex iterations, again based on any conveniently accessible LP package, will then produce the optimal solution.

APPLICATION: THE ILLINOIS-MISSISSIPPI WATERWAY SYSTEM

In this section, the LP model is applied to the problem of deriving optimal hopper barge flows for an inland waterway system. The results obtained with the model are examined in two stages. First, the optimality characteristics of the LP solution itself are explored. Second, an inland waterway simulation model is used to study the impact of barge flow optimization on the operation of the system. Before these topics are discussed, a brief description of the system characteristics is supplied.

Description of the System

The waterway system chosen for this application is a 10-lock subsystem composed of the Illinois Waterway and an adjacent portion of the Upper Mississippi River. This system has been the subject of several previous studies (6, 7, 8, 9, 10, 11, 12). Consequently, the data needed for the study were readily available.

The Illinois Waterway extends for approximately 326 miles (524 km) from Chicago to its confluence with the Upper Mississippi River near Alton, Illinois. Seven locks and dams (L&D) are located along the river at Lockport, Brandon Road, Dresden Island, Marseilles, Starved Rock, Peoria, and LaGrange, each of which is a single-chamber facility 600 ft (183 m) long and 110 ft (34 m) wide.

Also included in the system is a 56-mile (90-km) segment of the Upper Mississippi River, beginning just above L&D 25 and ending below L&D 27 near St. Louis. The former lock consists of a single 600- by 110-ft (183- by 34-m) chamber; L&D 27 has a 1,200- by 110-ft (366- by 34-m) main chamber and a 600- by 110-ft (183- by 34-m) auxiliary chamber. L&D 26, which is just below the mouth of the Illinois River, has one 600- by 110-ft (183- by 34-m) chamber and a second chamber that is 360 ft (110 m) long and 110 ft (34 m) wide. This lock is currently processing traffic at the rate of about 3,000,000 tons (2700 Gg) per month, making it one of the busiest facilities on the inland waterways. Long delays and queues are commonplace at L&D 26, and plans are under way to replace it with a larger facility (12).

Figure 1 shows a diagram of the system. As can be seen, 15 ports were included in the system: 12 internal ports and three end ports at the system boundaries. Commodity flows among these ports for the year 1968 were analyzed in this study. This was the base year used in the previous studies referenced above.

The commodity movements that were considered are summarized as follows (1 ton = 0.9 Mg):

Commodity	Total Tonnage
Grain	14,818,000
Coal	12,146,000
Petroleum	12,085,000
Cement, stone, sand, and gravel	5,863,000
Sulfur	381,000
Iron and steel	2,382,000
Industrial chemicals	2,380,000
Agricultural chemicals	1,989,000
Other selected	1,832,000
Miscellaneous	2,201,000
Total	56,077,000

Grain, which must move in covered hopper barges, is the principal southbound commodity; it originates at points along the Illinois and Upper Mississippi Rivers and is shipped to Lower Mississippi River ports. Coal and petroleum are the most significant northbound flows. Coal generally moves in open hopper barges, although it can be (and sometimes is) moved in covered hoppers. Petroleum is shipped in several types of tank barges.

Grain, coal, and petroleum collectively account for about 70 percent of the commodity movements in the system. Lesser amounts of sulfur, construction materials, iron and steel, industrial chemicals, and agricultural chemicals are also shipped, primarily in open hopper barges and tank barges.

Table 1 gives some characteristics of the barge and towboat fleet in use on the system. It was assumed throughout this study that all hopper barge commodities move in jumbo barges 195 ft (59.4 m) long by 35 ft (10.7 m) wide, at an average loading of 1,300 tons (1180 Mg).

Application of the LP Model

A period of analysis of 44,000 min (approximately 1 month) was selected for this study. The requisite barge flow inputs were obtained by dividing annual tonnage flows for 1968 by 12 and then by 1,300 (the assumed average barge load). The resulting flow matrices contained about 1,700 loaded open hopper barge movements and 1,000 covered hopper barge loads. (Tank barge flows were not included in this part of the study because they were assumed to be noninterchangeable.)

It was assumed in this study that barge movement costs are a linear function of interport distance. If m_{ij} is the mileage between ports i and j, the corresponding cost functions are as follows:

$$c_{ij1} = 20 + 3.6\ m_{ij} \tag{5a}$$

$$c_{ij2} = 25 + 4.0\ m_{ij} \tag{5b}$$

$$d_{ij1} = 4 + 0.9\ m_{ij} \tag{5c}$$

$$d_{ij2} = 5 + 1.0\ m_{ij} \tag{5d}$$

This means that the cost of shipping commodities in covered hopper barges is assumed to be on the order of 3 to 3½ mils per ton-mile (0.2 cent per g-km), which is reasonably accurate.

It must be noted at this point that ports 14 and 15 were located approximately halfway between end points of the system (Figure 1) and New Orleans and Minneapolis respectively to reflect the fact that actual commodity origins and destinations are distributed along the Mississippi River and its tributaries. This approximation must be kept in mind when the study results are reviewed, and the transportation costs for various barge flow patterns must be interpreted in accordance with the limitations imposed by this assumption.

Based on the input data given above, the initial basic feasible solution contained about 1,000 empty barge movements for each barge type. The corresponding total cost was as follows:

Barges	Cost (dollars)
Loaded	11,312,100
Empty	2,294,479
Total	13,606,579

This initial solution provides a convenient standard against which to measure the LP results. This is so because the actual system operates somewhat less efficiently than this (i.e., empty barge flows actually exceed those included in this solution), but this standard of efficiency could feasibly be approximated by the operators, given certain economic inducements.

The LP problem remaining after the initial basic feasible solution contained 558 variables and 99 constraints. The optimal solution was achieved after 42 simplex iterations. The resulting total cost was $12,134,594, which corresponds to a cost savings of $1,471,985.

A dramatic reduction in the flow of open hopper barges and empty covered hopper barges was achieved by applying the LP model. This is demonstrated in Table 2, which gives total hopper barge flows for the initial basic feasible solution and the optimal solution. It must be noted that this solution is likely to be sensitive to the end port location assumption mentioned above. That is, it is assumed here that covered hopper destinations match open hopper origins beyond the system boundaries closely enough to allow the optimal solution to be implemented.

Cost savings were achieved in the LP solution by allocating open hopper loads to covered hopper barges that would otherwise move empty. As a result of this process, more than 1,000 hopper barge movements, which is about one-quarter of the initial total flow, were eliminated. This should produce a decrease in traffic congestion in the system. The significance of this effect is studied below.

EFFECTS OF FLOW OPTIMIZATION ON SYSTEM OPERATIONS

To determine whether the reduced barge traffic predicted by the LP model would effect a corresponding decrease in towboat delays, we observed the simulated operation of the system under the load patterns produced by the initial and optimal LP solutions respectively. The main reason for simulating the initial flows was to establish a datum against which the performance of the system in processing the LP flows could be measured. The waterway systems simulation model (WATSIM) developed at the Pennsylvania State University (13) was used for this experiment.

Simulation Runs

WATSIM was developed at the Pennsylvania State University during the period 1968-1971 as a general-purpose inland waterway system simulator. WATSIM accepts as input a chronologically ordered list of tows that are to be processed during the simulation. The other major inputs to WATSIM are a system description and a set of frequency distributions for the various components of the locking cycle for each lock chamber. The model outputs statistics on the traffic processed at each lock in the system, including the associated service and delay times. Printouts of selected tables at various intervals during one simulation run may be obtained if desired.

The simulation input data for this experiment were the same as those used for previous simulation studies of the Illinois-Mississippi system (8). Identical tank barge movements were input for both runs. Hence, the only difference between the two runs was in the hopper barge movements.

The simulation period for each run was 44,000 min, preceded by a 4,000-min warm-up period. Intermediate output was obtained every 4,000 min; hence, 11 observations of

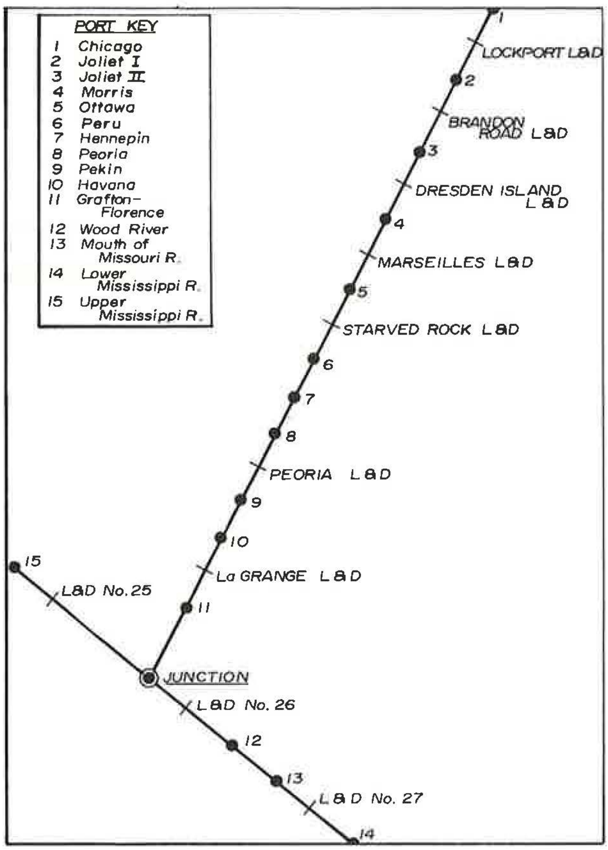

Figure 1. Illinois-Mississippi 10-lock subsystem.

Table 1. Fleet characteristics for the Illinois-Mississippi system.

Barge Type	Commodities Carried	Average Tons per Barge[a]	Average Flotilla Size
Open hopper	Coal	1,300	
	Cement, stone, sand, and gravel		
	Iron and steel		
	Industrial chemicals (50 percent)		8
	Agricultural chemicals		
	Other		
	Miscellaneous		
Covered hopper	Grain	1,320	
Tank I	Petroleum	2,000	3
Tank II	Sulfur		
	Industrial chemicals (50 percent)	2,100	3.5

Note: 1 ton = 907 kg.

[a]8.5-ft (2.6-m) average loaded draft.

Table 2. Hopper barge movements on the Illinois-Mississippi system.

Barge Type	Total Flow Initial	Total Flow Optimal	Barge Type	Total Flow Initial	Total Flow Optimal
Loaded			Empty		
Open hopper	1,719	961	Open hopper	1,175	520
Covered hopper	1,013	1,771	Covered hopper	976	471
Total	2,732	2,732	Total	2,151	998
			Total movements	4,883	3,730

Table 3. Selected simulation results for the Illinois-Mississippi system.

	Run 1: Initial Flows				Run 2: Optimal Flows			
	Total Barges				Total Barges			
Location	Loaded	Empty	Total Tows	ADPT (min)	Loaded	Empty	Total Tows	ADPT (min)
Lockport	995	675	393	58	980	608	382	88
Brandon Road	951	691	290	62	998	663	292	60
Dresden Island	1,024	754	310	24	1,025	700	308	25
Marseilles	964	698	294	34	908	511	276	23
Starved Rock	961	691	297	25	934	483	273	22
Peoria	1,189	999	363	29	1,120	419	296	21
LaGrange	1,107	933	364	23	1,081	506	317	20
L&D 25	742	722	241	11	886	89	156	7
L&D 26	2,018	1,715	609	91	1,988	515	461	48
L&D 27	2,107	1,722	603	2	2,087	516	460	1
Total	12,058	9,600	3,764	38	12,007	5,010	3,221	33

Table 4. ADPT observations and variables for selected locks.

	Run 1: Initial Flows				Run 2: Optimal Flows			
Observation	Lockport	Peoria	L&D 26	System	Lockport	Peoria	L&D 26	System
1	24.3	37.9	41.1	29.0	13.0	4.1	22.3	17.0
2	30.4	10.4	67.0	30.9	14.1	42.5	33.1	27.0
3	24.5	27.9	98.3	32.7	83.7	14.2	21.1	32.6
4	54.7	39.1	84.1	34.6	100.7	24.8	22.4	28.4
5	28.8	47.0	90.1	37.0	20.6	19.6	53.9	40.9
6	76.8	38.8	106.6	47.8	42.2	18.7	52.2	29.1
7	60.0	27.3	40.4	28.8	325.0	20.8	98.9	73.2
8	53.0	5.8	120.0	51.7	75.5	12.3	34.8	25.7
9	63.0	28.8	41.7	27.2	7.2	7.0	33.0	13.1
10	48.0	18.3	17.5	45.8	90.4	30.3	78.3	42.4
11	164.5	27.4	96.2	58.0	80.8	25.6	56.2	34.3
Σ	628.0	308.7	803.0	423.5	853.2	219.9	506.2	363.7
$\overline{X}$	57.1	28.1	73.0	38.5	77.6	20.0	46.0	33.1
S_x	39.7	12.6	33.3	10.6	89.2	10.9	25.0	16.0
$S_{\bar{x}}$	12.0	3.81	10.0	3.18	26.9	3.28	7.54	4.81

Table 5. Results of ADPT hypothesis tests.

Location	$\overline{X}_1 - \overline{X}_2$	T	Significance[a]
Lockport	-20.5	-0.697	0.50[b]
Peoria	8.1	1.63	0.062
L&D 26	27.0	2.16	0.023
System	5.4	0.935	0.192

[a]20 degrees of freedom. [b]Two-tailed test.

system performance were available for each run.

Simulation Results

Selected traffic and delay statistics for each run after 44,000 simulated min of system operation are given in Table 3. There is very little difference between the two runs for the upper reaches of the Illinois River (except for an apparently anomalous delay situation at Lockport). From Marseilles lock, however, and through the rest of the system, fewer empty barges were procedded during the second run than during the first run, which gradually brought down the number of tows processed. The largest decreases occurred on the Mississippi River segment. At L&D 26, for example, in run 2 only one-third as many empty barges were processed and 140 fewer tows than in run 1.

Average delay values do not seem to respond so fast as the traffic data. The only large delay reduction is at L&D 26, where average delay for the second run is only about one-half of that for the first. Smaller reductions, of 11 and 9 min, were noted at Marseilles and Peoria. For an inexplicable reason, average delay at Lockport is 30 min higher for run 2, even though the traffic served there was very similar for both runs. For the system as a whole, average delay per lockage decreased by 4 min from run 1 to run 2.

Significance Tests

Because the results discussed above only apply to the operating history of the system at one point in time, nothing can be said as yet about whether the differences noted are significant. Mean tow delay and its associated variance cannot themselves be used for this purpose because the individual tow delay times are autocorrelated. Repeat observations on average delay per tow (ADPT), however, if taken at widely spaced intervals, can be treated as a random sample. Some results obtained by Rao (14), based on a technique developed by Fishman (15), indicate that the 4,000-min intervals used for these runs can be considered to be independent observations. Hence, ADPT values for selected locations for each interval were calculated (Table 4). Sample statistics are also given in the table.

Data for Peoria, L&D 26, and the system as a whole were included to determine the significance of the apparent delay reductions at those locations. Hence, an appropriate hypothesis test is

$$H_0: ADPT_1 = ADPT_2$$

against the one-sided alternative

$$H_1: ADPT_1 > ADPT_2$$

Lockport, on the other hand, was included to examine the anomalous higher delay observed there for run 2. Thus a two-tailed test is more appropriate (there being no a priori expectation concerning the directionality of the inequality condition).

T-statistics for testing the above hypotheses, calculated under the equal variance assumption, and their associated significance levels are given in Table 5. Only the large delay reduction at L&D 26 is highly significant. The 8.1-min savings at Peoria can be accepted as genuine if a 6.2 percent chance of making a type 1 error can be accepted. Systemwide delay reduction fares even worse, and the equality hypothesis cannot be rejected at normal significance levels. Fortunately, the seemingly strange result at Lockport turns out to be spurious, for the equality hypothesis cannot be rejected there, either.

These findings point out one of the difficulties involved in interpreting the results of simulation experiments. A highly insignificant increase in average delay occurred at Lockport because of the "luck of the draw" in the simulation model. This delay increase, however, was large enough to offset a highly significant delay reduction at L&D 26, so that the significance level of the systemwide delay reduction was raised to an unacceptable value. Given these somewhat conflicting results, it is the author's inclination to judge the improvement in system operations to be real, rather than the result of chance occurrences.

SUMMARY

The simulation results indicate that optimization of barge flows can have a substantial effect on system operating performance. For the particular system studied, elimination of a great number of empty barge movements allowed the same tonnage to be serviced with significantly lower delays at the key bottlenecks. Hence, transportation costs were reduced not only through greater equipment use but also through decreased system congestion at critical locations.

These results have several implications regarding effective use of the LP model. From the fleet scheduling viewpoint, it must be realized that optimization of vehicle flows will produce a change in transit times at any service facility that has flow-dependent delays. These changes may be significant enough to alter the unit transportation costs input to the model. Hence, it may be necessary to iterate through the scheduling process several times and to reestimate flow costs for each trial, before a satisfactory equilibrium is attained.

From the planning viewpoint, these results show that system performance indexes are a function of the degree of efficiency of equipment use that is assumed when traffic demand estimates are prepared. The LP model assumes that cooperation among shippers is close enough that optimization of total barge flows can be accomplished. If this degree of cooperation is lacking, flow predictions based on the LP model will underestimate actual demand.

CONCLUSIONS

This paper deals with the general problem of determining the origin-destination flows of cargo vehicles, both loaded and empty, that are required to serve a specified transportation demand matrix. The model derived in the paper applies directly to a particular class of such problems in which one set of commodities must be shipped in a special class of vehicles and the other cargo can be shipped in either the special vehicles or general-purpose vehicles.

The particular application used throughout the paper, that of predicting covered hopper and open hopper barge movements, is only one example of how the model might be used. The problem described by Gould (5) is another. Similar examples include refrigerated and nonrefrigerated trucks or railroad cars; container ships and break bulk ships (one could assume either that containers move only in container ships or that uncontainerized cargo moves only in break bulk ships); and even passenger aircraft and cargo aircraft.

It must be emphasized here that the model was devised for use in the context of transportation system planning. Hence, there is no provision in the model for considering vehicle availability. That is, it is assumed that enough vehicles will be provided so that the predicted number of vehicle trips can take place during the analysis period. For planning purposes, this is of little concern, since future commodity flows will normally not be known precisely enough to warrant a more detailed investigation of vehicle flows.

Inasmuch as the model is intended for use as a predictive tool, some objection might be raised to applying optimization techniques to obtain a solution. Indeed, in actual practice, vehicle flows are determined by transportation companies or private fleet operators so as to meet individual private objectives, rather than to minimize system-wide costs. If cost minimization can be accepted as the universal privately applied criterion, however, then the solution should not be far removed from what will actually occur.

This point can be argued as follows. Consider first the initial solution. This might correspond to the situation in which each shipper is using his own vehicles (either private or hired) to provide the necessary loaded movements. Now suppose shippers A and B are crosshauling loaded and empty vehicles between points i and j. It will be to their advantage to arrange to use the same vehicles and thus eliminate some of their empty vehicle trips.

What about shipper C, located at point k between i and j? He may be shipping from k to j and returning empties; shipper B is moving empty units from i to j. B and C could obviously reduce costs if B would carry C's loads from k to j. Other things being equal, this again is the sort of solution that tends to be provided by the model. The three movements involved will be replaced by an empty vehicle trip from i to k and a loaded trip from k to j.

The essential point to be made is that systemwide optimization is not necessarily opposed to minimization of individual costs. In fact, a system optimum will normally be composed of a great many solution elements that correspond to private optima as well. Of course, numerous hypothetical counter examples can be constructed, but real-world problems tend to be more like the waterways example presented. Some theoretical support for this line of reasoning is also available in some recent significant findings by Dafermos (16, 17).

As a final note of caution, it is recommended that for planning applications model predictions be compared with actual vehicle flows for the base year of the study. If substantial deviations are found, it will be necessary to use some other technique or to modify the data input to the model so that the ultimate vehicle flow matrix incorporates some of the inefficient vehicle utilization practices that sometimes occur in the real world.

The model can also be used as the first step of a fleet-scheduling model. The second step consists of specifying realizable vehicle itineraries that collectively provide for all of the movements indicated in the solution matrix. Normally more than one set of itineraries will be feasible, and the optimal set will have to be selected so as to satisfy the scheduling objective. If the number of feasible itinerary sets is not too large, a branch-and-bound method can probably be used to find the optimum.

As a second possible procedure, the LP solution matrix can be used as a set of flow constraints for a vehicle-scheduling mathematical program. Any minimum cost vehicle schedule must provide for the loaded and empty movements specified in the solution. The scheduling problem is to allocate specific vehicles to each movement requirement. Hence, given the LP solution, a relatively simple linear program for vehicle scheduling can be devised.

Regardless of whether the optimal vehicle flows specified by the model can be attained in actual practice, they can be used as a basis for measuring the overall efficiency with which a transportation system is being used. For this application, it is necessary to have available a model that analyzes or simulates the performance of the system in serving a particular matrix of vehicle flows. Inasmuch as the model can be used to generate a minimal demand matrix, it follows that system performance measures that are functions of traffic flow will also achieve their extreme values in serving this demand. Actual traffic flows and delay times observed in the field (or values of these quantities predicted by the system model) can then be compared with their optimal counterparts to assess the efficiency of system operations (or the potential effectiveness of plans for increasing utilization efficiency).

As a case in point, consider the Illinois-Mississippi waterway system studied above. For the 1968 commodity flow matrix, lockage delays and number of empty barges processed can be no lower than those observed in the second simulation run. Thus, for example, a tally of empty barges processed at each lock will indicate how effectively towing companies are using their equipment.

Another application of the model is in establishing the minimum capacity that a proposed facility must have if a specified future commodity demand matrix is to be served.

In summary, the model is applicable to many different types of problems, including prediction of cargo vehicle flows, vehicle scheduling, and establishment of minimum system vehicle processing requirements. With simple modifications, the model can incorporate such additional considerations as dedicated equipment, cargo-dependent transportation costs, unequal vehicle capacities, equipment availability, and multiple commodities. Perhaps the most attractive features of the model are its simplicity and its relatively small size. Hence, it could profitably be used to obtain suboptimal or approximate solutions for more complex problems that cannot be solved with complex models because of size limitations.

ACKNOWLEDGMENTS

This research was performed while the author was a research assistant at the Pennsylvania State University. Funding for the study was provided by the U.S. Army Corps of Engineers, North Central Division, and the Pennsylvania Transportation Institute, Pennsylvania State University. Thanks are also due to T. D. Larson, T. B. Davinroy, J. I. Taylor, C. E. Antle, and particularly J. L. Carroll, all of the Pennsylvania State University.

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