

# Analysis of a Horizontally Curved Bridge Through a Geometric Structural Model

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Mathematical analysis is an important adjunct to experimental work and forms a guide to the design and interpretation of tests. The inverse is also significant, and a theory based on mathematical analysis needs to be verified by experimentation or prototype testing. In the study described, a mathematical model was developed and a physical model was designed and used. The physical model was used to verify the mathematical analysis and to provide evidence of the action of horizontally curved highway bridges.

The studies were particularly planned to develop information on the stress and deformation characteristics of a small-scale structure proportioned to the major dimensions of the prototype span—Ramp CBW over Huyck Stream of the Mall Arterial Highway, Interstate 540 in Albany, New York. This single span contains four welded plate girders 1.5 m (60 in) deep, interconnected by seven radial cross frames and a lower horizontal bracing system between each exterior and first interior girder. The radii of the inner and outer girders are 45.7 and 53 m (150 and 174 ft) with lengths of 26.8 and 31.1 m (88 and 102 ft). The deck is a reinforced concrete slab of 19-cm (7.5-in) design thickness placed at 21.6 cm (8.5 in) on corrugated forms.

The small-scale structure was mathematically and experimentally analyzed for three dead load conditions and five live load conditions. The analytical and experimental results of each of these loading conditions were compared for stress, moment, and deformation across four sections of the structure. In addition, end reactions were determined as were forces in members of the diaphragms. Significant comparisons between theory and small structure testing under wet concrete dead loading and five live loadings are given in Tables 1 and 2, and comparisons of transverse distribution of moments at the center section for theory, small structure testing, and prototype testing are given in Table 3.

## MATHEMATICAL ANALYSIS OF HORIZONTALLY CURVED BRIDGE AS THREE-DIMENSIONAL STRUCTURE

A three-dimensional method of analysis previously developed in the Department of Civil Engineering at Syracuse University was extended and adapted for analysis of the structural frame of horizontally curved girder bridges, with and without an integral deck. With this analysis each member of the structure is considered to be acting normally and to be located in its true position in the structure. Analysis is performed by a computer program (1).

The input to the program consists basically of the geometrical and physical properties of the bridge and the applied loadings. Program output includes the rotations and displacements of the joints of the bridge and the internal forces and moments in all members of the bridge. The program is general and can accommodate most configurations of curved or straight bridges, both with and without a concrete deck.

## DESCRIPTION OF THE SMALL-SCALE STRUCTURE

The mathematical analysis was verified by experimentation on a small-scale structure that was geometrically scaled to the field prototype bridge. The design of the small-scale structure was done in accordance with similitude relations (2) by using dimensions in length, mass, and time, and this design was reproduced at a scale of 1 to 8.33 to extremely close tolerances. All steel members of the prototype were type 304 stainless steel members in the small-scale structure. Concrete was used for the deck. The framing of the small-scale structure is shown in Figure 1.

Three dead load tests were performed on the structure without deck. After the dead load testing was completed, stainless steel shear studs were welded to the top flange of each girder and corrugated sheet metal forms were placed on the girders. After the concrete deck was cured, the sidewalks were placed.

## EXPERIMENTAL PROGRAM

The assembled bridge was supported at one end by four load cells, one under each girder. Loads were applied to the structure through gravity. One hundred fifty-four electrical resistance strain gauges were bonded to the structure at critical locations and terminated in signal conditioning units or switching and balancing units. Vertical and rotational deformation measurements were made by using dial gauges.

The small-scale structure was analyzed for deflections, axial forces, bending moments, and torsional moments by the three-dimensional method of analysis. The same eight independent sets of loadings used for mathematical analysis were applied to the scale structure. These consisted of (a) supplemental loading for steel framing, (b) wet concrete, (c) wet concrete followed by supplemental loadings, and (d) live loads in five different positions. All loadings of the bridge model were done in increments of one-fourth of the total load to be added.

Table 1. Comparison of results of analytical and experimental wet concrete loading at midspan of bridge.

Variable	Girder	Three-Dimensional Theory			Experiment
		Condition A	Condition B	Condition C	
Bending moment, J	RC 40	3575	3727	3906	3423
	RC 39	2538	2451	2509	2649
	RC 38	748	700	666	752
	RC 37	643	632	516	644
	Total	7504	7509	7597	7468
Deflection, mm	RC 40	2.845	3.099	3.404	3.861
	RC 39	2.464	2.566	2.718	2.946
	RC 38	2.032	2.083	2.108	2.261
	RC 37	1.600	1.626	1.524	1.473

Note: 1 J = 0.737 ft-lbf; 1 mm = 0.039 in.

Table 2. Bending moments and bottom flange average stresses at center section.

Live Load Position	Girder	Bottom Flange Average Stress (kPa)		Moment (J)	
		Experiment	Theory	Experiment	Theory
1	RC 40	11 549	12 949	1785	2000
	RC 39	9 170	9 439	1062	1096
	RC 38	4 895	5 254	192	203
	RC 37	2 689	-241	113	-101
	Total			3152	3198
2	RC 40	10 467	10 991	1616	1695
	RC 39	10 411	9 067	1198	1051
	RC 38	7 709	7 364	294	283
	RC 37	4 709	3 558	203	147
	Total			3311	3176
3	RC 40	7 309	8 639	1130	1333
	RC 39	8 295	8 557	961	994
	RC 38	10 411	10 170	407	396
	RC 37	7 295	7 909	305	340
	Total			2803	3063
4	RC 40	5 309	6 640	825	1028
	RC 39	7 260	7 612	836	881
	RC 38	12 894	12 211	497	475
	RC 37	10 756	12 990	452	554
	Total			2610	2938
5	RC 40	3 861	5 178	600	802
	RC 39	6 412	6 929	746	802
	RC 38	14 500	13 349	565	520
	RC 37	12 859	16 865	542	712
	Total			2453	2836

Note: 1 kPa = 0.145 lbf/in<sup>2</sup>; J = 0.737 ft-lbf.

## SMALL-SCALE STRUCTURE TEST RESULTS

After each structural test on the small-scale structure, theoretical data were compared with experimental data. Strain gauge readings were converted to stress by using a computer program written for this study. Maximum stresses were computed from a zero base that was the average of the initial and final zero readings.

Bending moments for the dead load conditions were computed by using the stress distribution determined experimentally, and the area of the applicable portion of each member was used to compute the tensile and compressive force resultants. The moments were computed by using these resultants and the lever arm between them. Wet concrete loading experimental bending moments were compared with the analytical moments (Table 1). For this loading condition, the following three conditions of torsional restraint of the girders were studied by using the three-dimensional analyses.

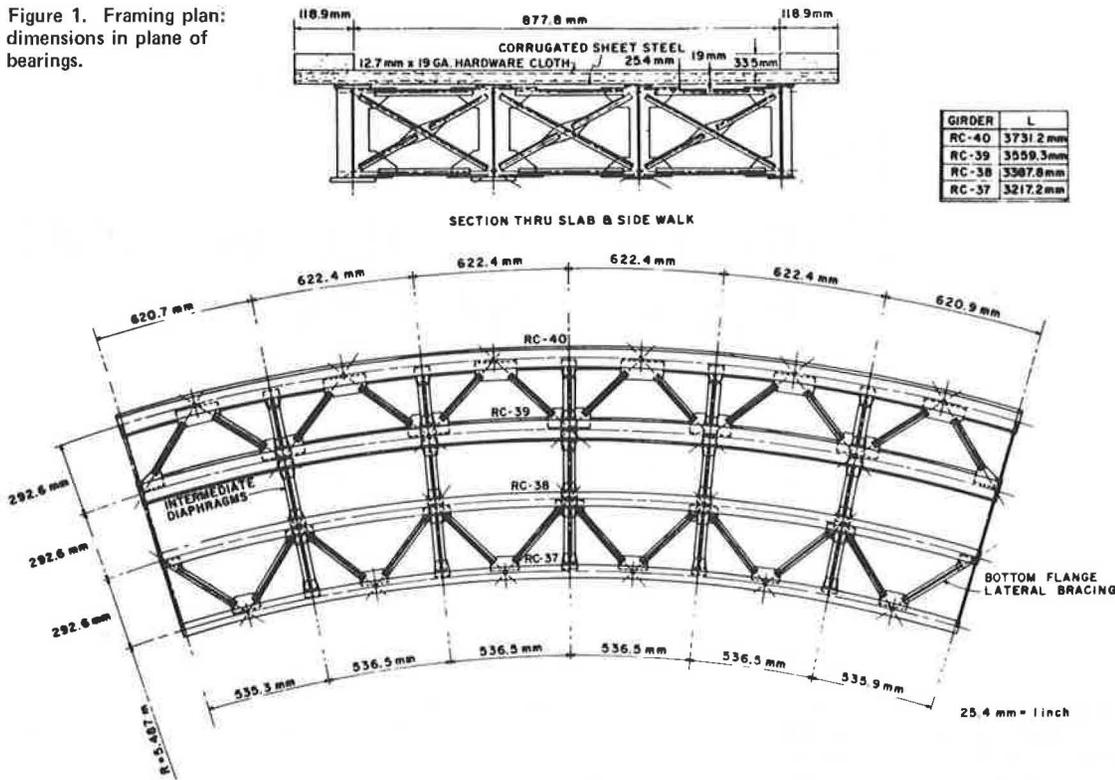
Condition	Description
A	Torsional restraint at each diaphragm and halfway between each diaphragm at each girder
B	Torsional restraint at each diaphragm and at the lateral bracing
C	Torsional restraint at each diaphragm

Five live loadings were applied to the structure, and wheel spacing and intensities were scaled to the Federal Highway Administration test vehicle that was used to test the Huyck Stream prototype bridge in Albany. These loads were placed sequentially across the transverse centerline of the bridge. Theoretical values of lower flange average stresses were determined from the computed moments by using the lower flange section modulus. Experimental moment values were computed by using this same lower flange section modulus and the average experimental lower flange stress. These values are given in Table 2.

Table 3. Transverse distribution of bending moments at center section.

Load Type	Girder	Percentage Distribution of Moment		
		Experiment	Theory	Prototype Field Test
Wet concrete	RC 40	45.8	51.4	47.0
	RC 39	35.4	33.0	37.5
	RC 38	10.1	8.8	9.0
	RC 37	8.6	6.8	6.4
Live load, position 1	RC 40	56.6	62.5	51.9
	RC 39	33.7	34.3	35.4
	RC 38	6.1	6.4	6.6
	RC 37	3.6	-3.1	6.1
Live load, position 2	RC 40	48.8	53.4	45.6
	RC 39	36.2	33.1	35.3
	RC 38	8.9	8.9	9.1
	RC 37	6.1	4.6	10.0
Live load, position 3	RC 40	40.3	43.5	38.5
	RC 39	34.3	32.5	33.3
	RC 38	14.5	12.9	13.4
	RC 37	10.9	11.1	14.7
Live load, position 4	RC 40	31.6	35.0	30.7
	RC 39	32.0	30.0	28.8
	RC 38	19.1	16.2	16.8
	RC 37	17.3	18.8	23.7
Live load, position 5	RC 40	24.4	28.3	25.5
	RC 39	30.4	28.3	27.4
	RC 38	23.0	18.3	17.6
	RC 37	22.1	25.1	29.5

Figure 1. Framing plan: dimensions in plane of bearings.



## CONCLUSIONS

Tests of a small-scale structure, geometrically similar to a field prototype, produced useful and valuable data. This tool, properly and carefully used, provides excellent information for evaluating mathematical analyses. The studies on this structure indicate that there is close correlation between theoretical and experimental moment values, and this is particularly evident in data given in Table 3 in which the distribution of the total moment across the center section to each girder is shown by percentage for wet concrete loading and for five live load positions. These distributions are compared with the results of the field tests on the full-size bridge performed by the New York State Department of Transportation and the Federal Highway Administration (3). As noted in this table, the distribution of moments determined from theory, small-scale structure testing, and field testing of the full-sized bridge is in close agreement.

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