Abridgment

Thermal Stresses and Deformations in Nonprismatic Indeterminate Composite Bridges

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A review of the literature indicates that thermal stresses warrant increased consideration in design (4, 6). This paper presents a method for determining thermally induced stresses and strains in composite girder bridges and structures with nonprismatic sections subjected to any temperature distribution. It also gives the expected stress range of a typical four-span composite highway bridge with a concrete deck and steel stringer at two arbitrary temperatures.

SLAB AND BEAM STRESSES AND STRAINS

Remove intermediate supports and end restraints, if any, and assume that the composite bridge structure is a simply supported structure in which the slab is separated from the beam. Let the slab be completely restrained in the longitudinal and transverse directions and subjected to temperature effects.

Although it acts as a simple beam before boundary conditions and compatibility are introduced, the slab strains freely in the longitudinal direction. To achieve free movement and satisfy equilibrium, a longitudinal force P'_{ts_k} and a moment M'_{ts_k} are superimposed on the thermally induced force and moment that resulted from the restrained condition.

For the interface strains and curvatures to be compatible, an interface force system of shears and couples, similar to those of Zuk (7) and Aleck (1), must be applied at the ends of prismatic, or constant curvature, segments of the separated slab and beam as shown in Figure 1. The final strains, stresses, curvatures, and deflections may be found after the magnitudes of the interface forces are determined and all deformations have been found to be consistent with the boundary conditions.

The final slab strain at any section is the sum of the strain under unrestrained movement and that resulting from equilibrium and compatibility, which, for a slab in plane strain, is

$$\sigma_{xs_k} = -[\alpha_s E_s T_{s(y)_k}/1 - \mu_s] + (P'_{ts_k}/A_s) + (M'_{ts_k} y_s/I_s) + [(1/A_s) + (cy_s/I_s)]F_k - (y_s/I_s)Q_k \quad (1)$$

where

- s = slab,
- k = segment, if nonprismatic,
- σ_{xs_k} = final slab stress in a determinate member,
- α_s = thermal coefficient of expansion,
- E_s = modulus of elasticity,
- $T_{s(y)_k}$ = change in temperature as a function of vertical distance,
- μ_s = Poisson's ratio,
- P'_{ts_k} and M'_{ts_k} = force and moment superimposed on restrained separated slab to achieve free movement,
- A_s = cross-sectional area of slab,
- I_s = moment of inertia of slab,
- c = half of depth of slab (+ up or down),
- y_s = distance measured from centroidal axis, and
- F_k and Q_k = interface shear force and moment.

In a similar manner, the final beam strain of a determinate member at any section is the sum of the strain under unrestrained movement and that resulting from equilibrium and compatibility, which, for a beam in plane stress, is

$$\sigma_{xb_k} = -\alpha_b E_b T_{b(y)_k} + (P'_{tb_k}/A_{b_k}) + (M'_{tb_k} y_{b_k}/I_{b_k}) + [-(1/A_{b_k}) + (\bar{d}_k y_{b_k}/I_{b_k})]F_k + (y_{b_k}/I_{b_k})Q_k \quad (2)$$

where b = beam and $\bar{d}_k$ = distance from centroidal axis to top of beam (+).

COMPATIBILITY OF COMPOSITE SECTION

For the original composite structure, compatibility requires that the slab strain and curvature be equal to the beam strain and curvature at the interface.

Equating longitudinal interface strains and simplifying yield

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$$\left\{[n(1-\mu_s^2)/A_s] + 1/A_{b_k} + [n(1-\mu_s^2)c^2/I_s] + (\bar{d}_k^2/I_{b_k})\right\} F_k + \left\{\bar{d}_k/I_{b_k} - [n(1-\mu_s^2)c/I_s]\right\} Q_k = [(P'_{tb_k}/A_{b_k}) - (M'_{tb_k}d_k/I_{b_k})] - n(1-\mu_s^2)[(P'_{ts_k}/A_s) + (M'_{ts_k}c/I_s)] \qquad (3)$$

where n = modular ratio E_b/E_s.

Equating curvatures of the slab and the beam at the interface and simplifying yield

$$\left\{\bar{d}_k/I_{b_k} - [n(1-\mu_s^2)c/I_s]\right\} F_k + \left\{1/I_{b_k} + [n(1-\mu_s^2)I_s]\right\} Q_k = [n(1-\mu_s^2)/I_s]M'_{ts_k} - (1/I_{b_k})M'_{tb_k} \qquad (4)$$

Figure 1. Interface shears and moments on separated slab and beam.

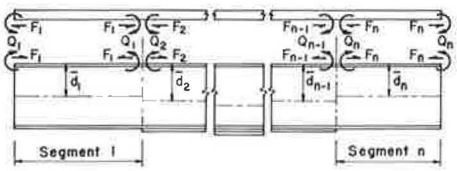


Figure 2. Longitudinal unit stresses resulting from assumed constant temperature gradient in slab and beam.

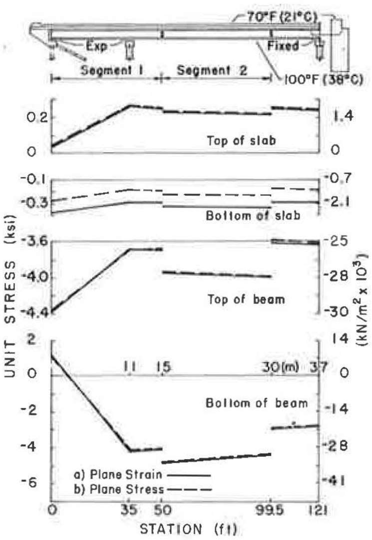
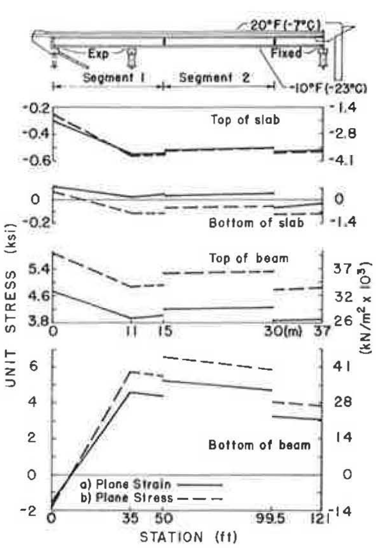


Figure 3. Longitudinal unit stresses resulting from assumed linear temperature gradient in slab and constant gradient in beam.



The unknown quantities F_k and Q_k may be obtained by solving equations 3 and 4 simultaneously. The equations are best solved numerically for specific problems.

With F_k and Q_k known, the longitudinal interface stress in a determinate slab and beam may be obtained from equations 1 and 2 respectively. Final slab and beam stresses in an indeterminate member may be found by superimposing equations 1 and 2 with the elastic stresses obtained from the moment and thrust diagram.

EXAMPLE BRIDGE

Variations of stress resulting from thermal loading may be illustrated by applying assumed temperature gradients to a typical four-span, nonprismatic, six-stringer, continuous, four-lane composite highway bridge with expansion rollers except for the center pier. The reference temperature is 21° C (70° F).

Assumed temperature gradients and a comparison of the thermal stresses calculated for the interior girder for (a) plane strain in the slab in the transverse direction and plane stress in the beam and (b) plane stress in both slab and beam by the theoretical method are shown in Figures 2 and 3.

SUMMARY

The procedure presented provides a practical means for considering nonprismatic bridge structures subjected to thermal loading.

The thermal coefficient of expansion for limestone-aggregate concrete has been substantiated by the authors to be approximately $7.2 \times 10^{-6}/°$ C ($4.0 \times 10^{-6}/°$ F) and was assumed for the example bridge calculations.

Maximum longitudinal stresses in the slab for the two assumed thermal loadings were 1810 kPa (262 lbf/in²) for the constant temperature gradients and -3920 kPa (-568 lbf/in²) for the linear slab and constant beam gradients.

Maximum longitudinal beam stresses were -33 880 and -33 670 kPa (-4910 and -4880 lbf/in²) by methods a and b for the constant gradients. For the linear slab and constant beam temperature gradients, the maximum longitudinal stresses were 36 360 and 45 130 kPa (5270 and 6540 lbf/in²) for methods a and b. The actual stresses would probably be between the values of methods a and b, depending on the actual transverse slab restraint. Comparable significant thermal stresses have also been found by other investigators (2, 3, 5).

Although directed toward thermal behavior, the derivations presented are general in nature and may be used to account for other internal effects, e.g., shrinkage strain, provided an equivalent temperature distribution is used.

REFERENCES

1. B. J. Aleck. Thermal Stresses in a Rectangular Plate Clamped Along an Edge. Trans., ASME, Vol. 71, 1949, pp. 118-122.
2. C. Berwanger. Thermal Stresses in Composite Bridges. Proc., ASCE, Specialty Conference on Steel Structures, Engineering Extension Series, No. 15, Univ. of Missouri—Columbia, June 1970, pp. 27-36.
3. C. Berwanger and Y. Symko. Thermal Stresses in Steel-Concrete Composite Bridges. Canadian Journal of Civil Engineering, Vol. 2, No. 1, 1975, pp. 66-84.
4. J. H. Emanuel and others. An Investigation of Design Criteria for Stresses Induced by Semi-Integral End Bents: Phase I—Feasibility Study. Missouri Cooperative Highway Research Program, Univ. of

Missouri—Rolla, Final Rept. 72-9, 1973.

5. Y. N. Liu and W. Zuk. Thermoelastic Effects in Prestressed Flexural Members. Journal of Prestressed Concrete Institute, Vol. 8, No. 3, June 1963, pp. 64-85.
6. J. C. Reynolds and J. H. Emanuel. Thermal Stresses and Movements in Bridges. Journal of Structural Division, ASCE, Vol. 100, No. ST1, Proc. Paper 10275, Jan. 1974.
7. W. Zuk. Thermal Behavior of Composite Bridges—Insulated and Uninsulated. HRB, Highway Research Record 76, 1965, pp. 231-253.