Determination and Significance of Resilient Bimodular Properties of Stabilized Soils

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The differences between the values of resilient moduli of stabilized soils measured by using repeated compression, tension, flexure, and split-tension tests and the actual resilient moduli in tension and compression are explained analytically. Results indicate that values of resilient moduli measured by using repeated tension and compression tests agree well with actual values for specimens that have length-to-width ratios of 2 and caps that are at least ten times as stiff as the stabilized material in compression. Repeated flexure tests give values of resilient moduli that are lower or higher than actual values, depending on the dimensions of the specimen, the fixity at points of load application, and the method of calculation used. However, a good estimate of resilient moduli in tension and compression can be obtained for beam specimens that have length-to-width ratios equal to 4 and no lateral supports at points of load application by using simple beam-theory assumptions for a bimodular material (i.e., different moduli in tension and compression). A procedure for the determination of resilient bimodular properties (i.e., resilient tensile and compressive moduli) by using the repeated split-tension test is developed and gives values for resilient moduli in tension and compression that do not deviate from the actual values by more than 16 and 11 percent respectively. The significance of the resilient bimodular properties on the response of stabilized layers under traffic loads is assessed analytically. The results show that tensile stresses and strains on the underside of the stabilized layer are highly influenced by the resilient compressive and tensile moduli of the material.

Stabilized materials such as the cement-treated and lime-treated soils that are used as base and subbase courses in pavement structures are subjected to repeated stresses and strains caused by applied traffic loads. Fatigue cracking in the stabilized layer can therefore occur, and it will hasten the deterioration of the pavement structure. The determination of the resilient moduli of stabilized materials under dynamic loads is essential for the prediction of the response of pavements that contain these materials (1, 2) and is therefore desirable for the development of improved design procedures for pavements.

Resilient moduli have been measured by using repeated compression, tension, flexure, and split-tension tests. In the flexure and split-tension tests, the resilient modulus is estimated by using linear elastic theory and assuming equal moduli of resilience in compression and tension (3, 4, 5). The determination of resilient moduli by using repeated compression tests may give higher or lower values than those obtained by using repeated flexure tests (1, 6, 7).

Data presented by many investigators regarding the stress-deformation behavior of cement-treated and lime-treated soils indicate that these materials may have different moduli in tension and compression (8, 9, 10, 11). In general, cement-treated soils have higher moduli in compression than in tension, under both static and repeated states of stress (8, 9, 10, 11). Strain measurements at the top and bottom of soil-lime beam specimens in flexure suggest that, for these also, the compressive modulus is greater than the tensile modulus.

In this paper, an attempt is made to

1. Explain the differences between the actual and the measured values of resilient moduli of stabilized soils,
2. Suggest a method for the determination of the resilient bimodular properties (i.e., the resilient tensile and compressive moduli) of stabilized soils by using the repeated split-tension test, and
3. Assess the significance of resilient bimodular properties on the response of pavement structures.

METHOD OF ANALYSIS

The finite-element method of analysis (12) was used to determine the response of a stabilized material under repeated loading conditions. The material was assumed to have bimodular properties (i.e., the resilient modulus in tension is different than that in compression). Viscous and inertial effects were assumed to be negligible. An iterative technique was used whereby the resilient modulus in tension (E_t) was set equal to the resilient modulus in compression (E_c) on the first iteration. On successive iterations, E_t was substituted in the direction of principal tension. Three or four iterations were usually sufficient to attain convergence. In the repeated tension, compression, flexure, and split-tension tests (Figures 1 and 2), planar stress conditions were assumed.

Repeated Tension and Compression Tests

In these tests (Figures 1 and 2), caps were assumed to be glued at the ends of the specimen (i.e., no relative movement at the interface between the caps and the specimen). In the repeated tension test the load was applied at the center of the caps, whereas in the repeated compression test the load was applied either at the center (Figure 2b) or by means of a loading head (Figure 2a). The interface between the cap and the loading head could be frictionless (Figure 2ai) or fully frictional (Figure 2aii). The nodal points at the interface were assumed to follow the movement of the loading head. For a given cap stiffness (E_{cap}) and a material that has given values of E_t and E_c, the measured resilient moduli can be expressed as

$$(E_c)_M = P/A\epsilon_c \qquad (1)$$

for the repeated compression test and

$$(E_t)_M = P/A\epsilon_t \qquad (2)$$

for the repeated tension test,

where

P = total tensile or compressive load,
A = cross-sectional area of specimen,
ϵ_c = compressive strain based on total deflection across ends of caps or across middle half of specimen,
ϵ_t = tensile strain based on total deflection across ends of caps or across middle half of the specimen,

$(E_c)_M$ = measured resilient modulus in compression, and
$(E_t)_M$ = measured resilient modulus in tension.

Repeated Flexure Test

In this test, third-point loading was used (Figure 1). Top strains, bottom strains, and deflections at the central section of the beam were determined analytically for a given applied load (P) and a material that has a given resilient compressive modulus (E_c) and a given resilient tensile modulus (E_t). The resilient moduli in compression $(E_c)_f$ and in tension $(E_t)_f$ were calculated by using simple beam-theory assumptions as follows:

$$(E_c)_f = (3M/bD^2)\,[(\epsilon_c + \epsilon_t)/\epsilon_c^2] \qquad (3)$$

Figure 1. Finite-element representation of repeated tension and flexural tests.

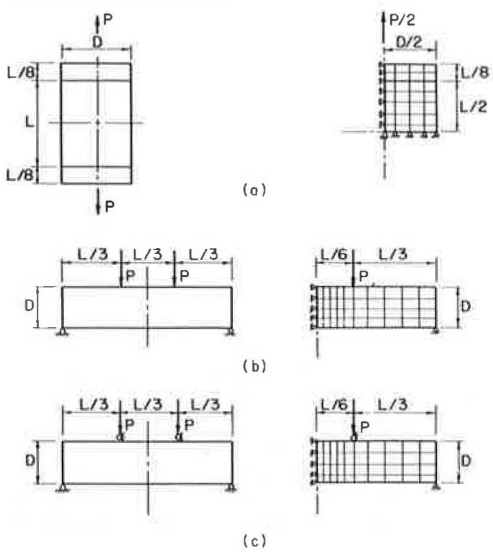
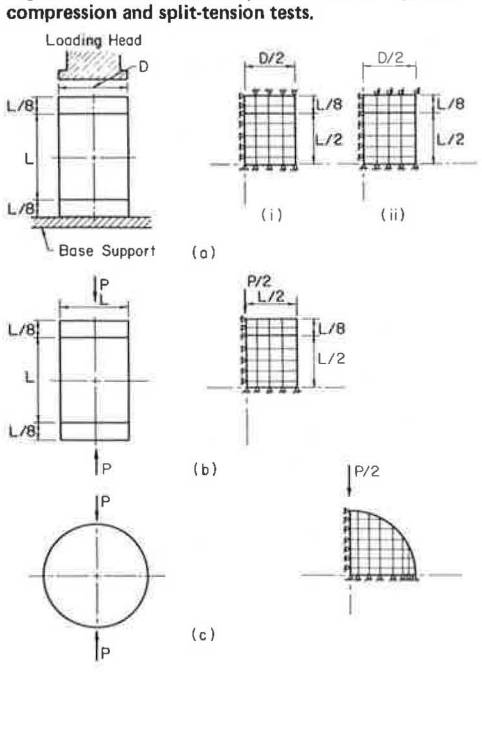


Figure 2. Finite-element representation of repeated compression and split-tension tests.



$$(E_t)_f = (3M/bD^2)\,[(\epsilon_c + \epsilon_t)/\epsilon_t^2] \qquad (4)$$

where

M = moment acting at central section of beam specimen,
ϵ_c = compressive strain at top of central section of beam specimen,
ϵ_t = tensile strain at bottom of central section of beam specimen,
b = width of beam specimen, and
D = depth of beam specimen.

A modulus of resilience in flexure (E_f) was also calculated from central beam deflections by using simple beam theory, but assuming that the resilient moduli in tension and compression were equal (i.e., $E_c = E_t$). E_f can be written as

$$E_f = (23/648)(PL^3/dI) \qquad (5)$$

where

P = applied vertical load,
L = length of beam specimen,
d = deflection at center of beam, and
I = moment of inertia of beam cross section.

The ratios $(E_c)_f/E_c$, $(E_t)_f/E_t$, and E_f/E_t were calculated for a given E_c/E_t and compared with the measured and actual values of the resilient moduli.

Repeated Split-Tension Test

In this case (Figure 2c), the modulus of resilience (E_s) was determined by using the relationship developed by Schmidt (4)

$$E_s = P(\nu + 0.2732)/(t\Delta) \qquad (6)$$

where

ν = Poisson's ratio,
t = thickness of specimen,
P = applied vertical load, and
Δ = lateral deformation.

The value E_s was determined analytically for a material that has a given resilient modulus in tension (E_t) and a given resilient modulus in compression (E_c).

RESULTS

The results of analyses that compare the calculated and the actual resilient moduli are summarized below.

Repeated Tension and Compression Tests

The effects of sample size, method of load application, and the rigidity of the caps on the measured resilient moduli in tension and compression are shown in Figures 3 and 4.

In the repeated tension test, the results indicate that, for $E_{cap}/E_c > 10$, the calculated resilient modulus is essentially the same as the true resilient modulus in tension (i.e., E_t) for specimens that have a length-to-width ratio (L/D) of 2. Thus, the deformations across the ends (including the caps) or across the middle half of the specimen were used to calculate this modulus

Figure 3. Comparison between measured and actual resilient moduli in repeated tension test.

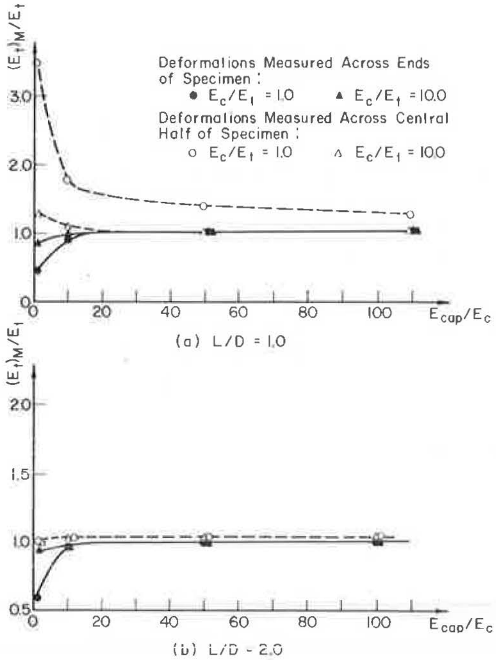
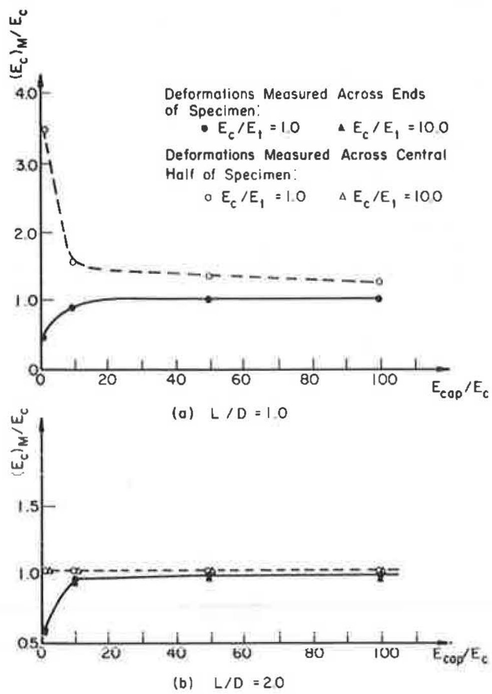


Figure 4. Comparison between measured and actual resilient moduli in repeated compression test.



(Figure 3b). However, for specimens that have L/D = 1, the values of the resilient modulus determined by using vertical deformations across the middle half of the specimen are higher than the actual values of the resilient modulus. On the other hand, if the vertical deformations across the ends of the specimen are used, there is good agreement between the measured and actual moduli for E_{cap}/E_c>10 (Figure 3a).

In the repeated compression test, the results are similar to those obtained in the repeated tension test if the vertical load is applied at the center of the caps (Figure 4). However, if the load is applied through a loading head (Figure 2a), the measured value of the resilient modulus will be essentially equal to the actual value and independent of the frictional conditions at the caps, provided E_{cap}/E_c>10 (Table 1).

Repeated Flexure Test

Two loading conditions were investigated in this test:

1. Third-point loading with no lateral restraints (Figure 1b) and
2. Third-point loading with full lateral restraint (Figure 1c).

The measured values of the resilient modulus in tension $(E_t)_f$ and in flexure (E_f) are compared with the actual resilient modulus in tension (E_t), whereas the measured values of the resilient modulus in compression [i.e., $(E_c)_f$] are compared with the actual resilient compressive modulus, (i.e., E_c). Results are shown in Figures 5, 6, and 7 for different values of L/D and can be summarized as follows:

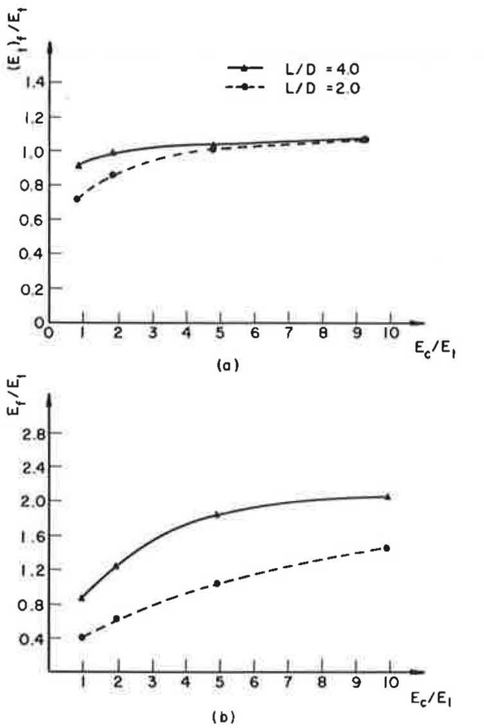
1. $(E_t)_f/E_t$ and E_f/E_t generally increase as E_c/E_t increases, but are smaller for L/D = 2 than for L/D = 4 (Figure 5).
2. The value of E_f can be higher or lower than E_t, depending on the dimensions of the beam specimen (i.e., L/D) and the lateral restraints at points of load application (Figures 5b and 6b).
3. Values of $(E_t)_f$ determined by using beam specimens that have L/D = 4 vary from 0.9 E_t at E_c/E_t = 1 to

Table 1. Effect of friction between cap and loading head on $(E_c)_M/E_c$.

	$(E_c)_M/E_c$ [a]				$(E_c)_M/E_c$ [b]			
	E_c/E_t = 1		E_c/E_t = 10		E_c/E_t = 1		E_c/E_t = 10	
E_{cap}/E_c	L/D = 2	L/D = 1	L/D = 2	L/D = 1	L/D = 2	L/D = 1	L/D = 2	L/D = 1
Full Friction at Cap Interface								
1	0.80	0.81	0.80	0.81	1.01	1.05	1.01	1.05
10	0.98	0.99	0.98	0.99	1.02	1.06	1.02	1.06
50	1.0	1.01	1.0	1.0	1.02	1.06	1.02	1.06
100	1.0	1.0	1.0	1.0	1.03	1.06	1.0	1.01
No Friction at Cap Interface								
1	0.80	0.80	0.80	0.80	1.0	1.0	1.0	1.0
10	0.98	0.98	0.98	0.98	1.01	1.05	1.02	1.05
50	1.0	1.0	1.0	1.0	1.02	1.06	1.02	1.06
100	1.0	1.01	1.0	1.01	1.02	1.07	1.02	1.07

[a] Based on deformations measured across ends of specimen.
[b] Based on deformations measured across middle half of specimen.

Figure 5. Comparison between measured and actual resilient tensile moduli in repeated flexure test.



1.05 E_t at E_c/E_t = 10, if the applied load is not laterally restrained (Figure 5a). However, if full lateral restraints are assumed, $(E_t)_f$ will vary between 0.70 E_t and 1.10 E_t depending on the value of E_c/E_t. For values of $E_c/E_t > 2$, $(E_t)_f/E_t$ = 1.10 (Figure 6a).

4. $(E_c)_f$ is essentially the same as E_c for L/D = 4 if the applied load is not laterally restrained. However, $(E_c)_f$ can be much higher than E_c if the applied load is laterally restrained (Figure 7).

Figure 7. Comparison between measured and actual resilient compressive moduli in repeated flexure test.

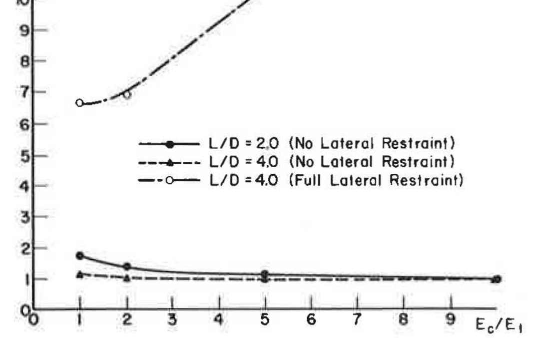
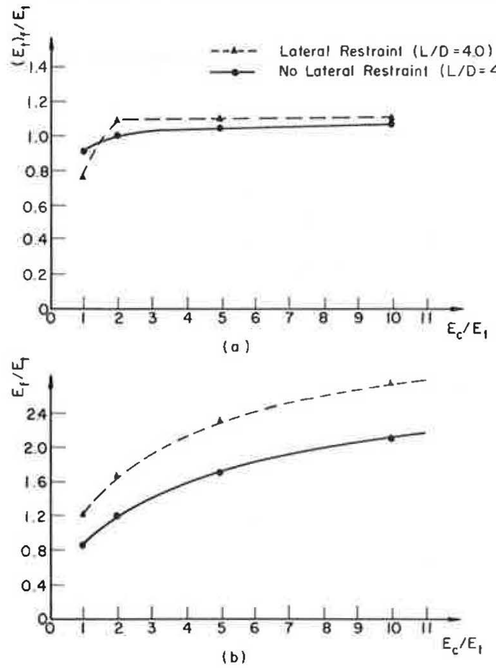


Figure 6. Effect of lateral restraints of applied vertical load on measured tensile moduli in repeated flexure test.



Repeated Split-Tension Test

For this test, the variation of E_s/E_t and E_c/E_t is shown in Figure 8. Analysis of the results leads to the following conclusions:

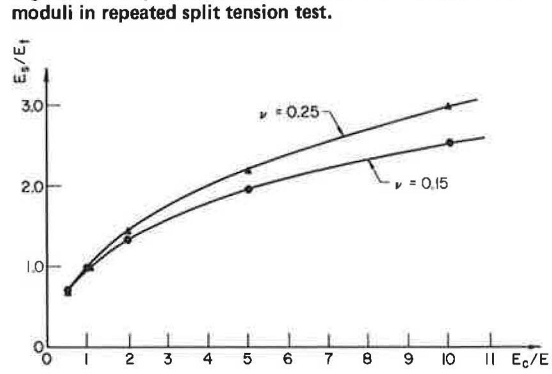
1. E_s/E_t increases as E_c/E_t increases. For a given E_c/E_t, the value of E_s/E_t is greater for higher values of Poisson's ratio.

2. The calculated resilient modulus (i.e., E_s) can be higher or lower than E_t, depending on the value of E_c/E_t. For example, if ν = 0.15, E_s will vary from 1.0 E_t at E_c/E_t = 1.0 to 2.5 E_t at E_c/E_t = 10.

DETERMINATION OF RESILIENT BIMODULAR PROPERTIES BY USING REPEATED SPLIT-TENSION TEST

In the repeated split-tension test, the vertical deformation (d_v) under a given applied load and the corresponding lateral deformation (d_h) are determined analytically for a material that has bimodular properties (i.e., $E_c \neq E_t$). E_s is calculated by using Equation 6, and the values of E_s/E_t are determined for a given d_v/d_h. Figure 9

Figure 8. Comparison between measured and actual tensile moduli in repeated split tension test.



shows the variation of E_c/E_t and of E_s/E_t as a function of d_v/d_h, for a bimodular material for which Poisson's ratio = 0.20.

For a given repeated load, d_v and d_h (the recoverable deformations) can be determined experimentally. The corresponding ratios E_c/E_t and E_s/E_t can be estimated by using Figure 9 and E_s computed by using Equation 6. From E_c/E_t, E_s/E_t, and E_s, the values of E_c and E_t can be determined. In this method, $\nu = 0.20$ is assumed [many investigators have reported, ν-values for cement- and lime-treated soils between 0.15 and 0.25 (7, 13)]. The table below compares the values of E_t and E_c calculated at $\nu = 0.20$ ($\bar{E}_t$ and $\bar{E}_c$) with values of E_t and E_c calculated at $\nu = 0.15$ and $\nu = 0.25$ for different values of d_v/d_h.

	$\nu = 0.15$		$\nu = 0.25$	
d_v/d_h	$E_t/\bar{E}_t$	$E_c/\bar{E}_c$	$E_t/\bar{E}_t$	$E_c/\bar{E}_c$
3	0.95	0.99	1.03	0.98
4	0.94	0.98	1.06	1.01
5	0.90	0.96	1.06	1.01
6	0.90	0.96	1.10	0.95
7	0.87	0.96	1.14	0.98
8	0.86	0.96	1.14	0.94
9	0.85	0.98	1.16	0.89
10	0.84	1.01	1.16	0.92

These results indicate that the errors involved by using $\nu = 0.20$ would not exceed 16 percent in case of E_t and 11 percent in case of E_c.

Stress intensities for which E_c and E_t were determined are summarized in Table 2. In the case of E_c, the stress intensity is equal to the average vertical compressive stress acting along the vertical diameter and, in the case of E_t, the stress intensity is equal to the average tensile stress along the horizontal diameter. Table 2 also gives suggested values for stress intensities in the repeated tension, compression, and flexure tests.

The use of this method is illustrated by calculating E_c and E_t values for a clayey-gravel lime-treated soil tested statically in split tension, from data given by Tulloch and others (14). Horizontal deformations, vertical deformations, and static load at failure are used in the computations; the results (Figure 10) indicate that E_c/E_t varies between 1 and 10.

Figure 9. Suggested curves for use in determining resilient bimodular properties in repeated split-tension test.

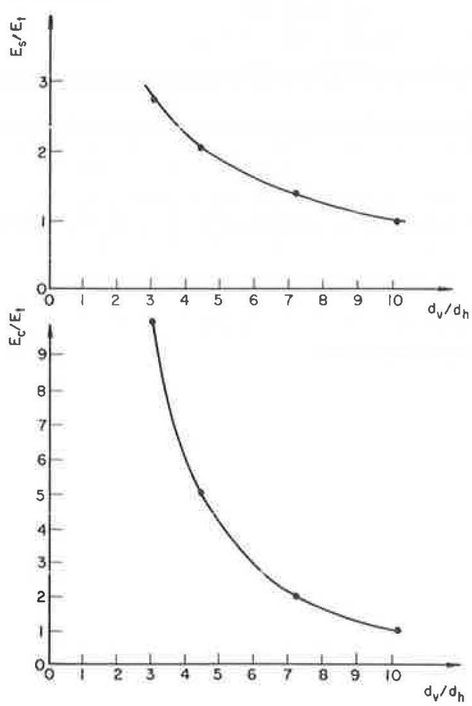


Figure 10. Variation of tensile and compressive elastic moduli for clayey-gravel lime-treated soil.

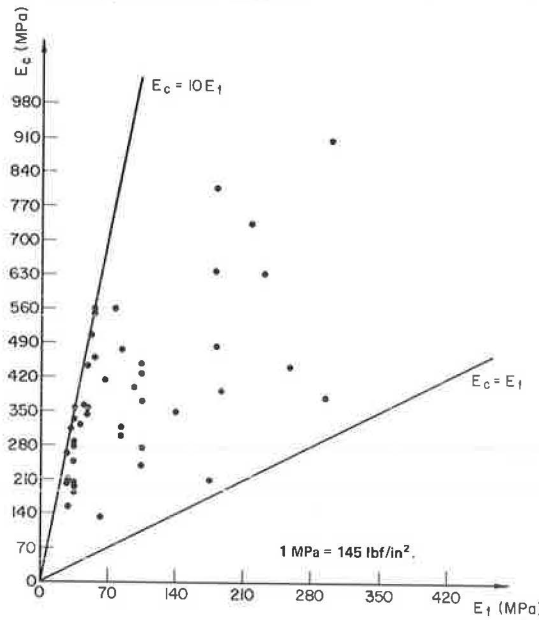
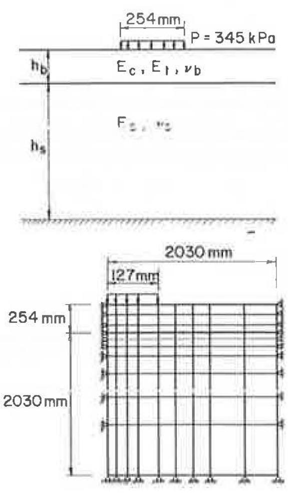


Figure 11. Finite-element representation of pavement section.



Note: 1 mm = 0.039 in and 1 MPa = 145 lbf/in².

Table 2. Suggested stress-intensity values.

		Repeated Flexure Test[a]		Repeated Split-Tension Test	
E_c/E_t	σ_A[b]	$(\sigma_t)_r$[c]	$(\sigma_c)_r$[d]	$(\sigma_t)_s$[e]	$(\sigma_c)_s$[f]
1	P/A	0.90 σ_f[g]	0.88 σ_f	0.65 σ_i[h]	5.55 σ_i
5	P/A	0.66 σ_f	1.37 σ_f	0.46 σ_i	5.90 σ_i
10	P/A	0.60 σ_f	1.72 σ_f	0.38 σ_i	6.20 σ_i

Note: Tensile stress intensity corresponds to measured resilient tensile modulus. Compressive stress intensity corresponds to measured resilient compressive modulus.

[a] L/D = 4 and vertical load is free from lateral support.
[b] Stress intensity in repeated tension or compression test.
[c] Maximum tensile stress at central section of beam.
[d] Maximum compressive stress at central section of beam.
[e] Tensile stress intensity.
[f] Compressive stress intensity.
[g] Flexural stress at central section of beam.
[h] $P_i/\pi R$, where P_i = applied vertical load and R = radius of specimen.

SIGNIFICANCE OF RESILIENT BIMODULAR PROPERTIES FOR RESPONSE OF PAVEMENTS

To investigate the significance of the resilient bimodular properties for the response of pavement structures under repeated wheel loads, a two-layer pavement (i.e., a stabilized layer over a subgrade) was analyzed by using the finite-element method and assuming planar strain conditions. Figure 11 shows a finite-element representation of the section; properties are summarized below (1 mm = 0.039 in and 1 MPa = 145 lbf/in^2).

Property	Value
Stabilized layer	
E_c, MPa	3450
E_t, MPa	3450
or	
E_c, MPa	3450
E_t, MPa	345
ν_b	0.20
h_b, mm	254
Subgrade	
E_s, MPa	69
ν_s	0.48
h_s, mm	2030

The tensile stresses (σ_t) and strains (ϵ_t) at the bottom of the base, the vertical stresses (σ_v) at the top of the subgrade, and the vertical deflections at the pavement surface (d) are compared when E_c and E_t of the base are equal and when E_t is reduced to a value equal to $E_c/10$ (Figure 12). The results indicate that, although reducing E_t increases the vertical deformation of the pavement surface and the vertical stresses on the subgrade surface, its most significant effect is that on the tensile stresses and strains at the bottom of the base. The maximum tensile stress is reduced by a factor of approximately five, whereas the maximum tensile strain becomes twice as large.

Fatigue life of a stabilized layer is a function of the tensile stresses and strains or both on its underside, depending on the fatigue characteristics of the material (5, 15). For example, recent studies (10) indicate that crack initiation and propagation in cement-treated bases due to traffic loads is dependent on the state of stress at the bottom of the base. Therefore, the correct assessment of the resilient tensile and compressive moduli of stabilized layers should lead to a better estimation of the fatigue lives of these layers.

SUMMARY AND CONCLUSIONS

In this paper, an attempt has been made to explain analytically the differences between the actual and the measured values of resilient moduli of stabilized soils and to develop a method for the determination of the resilient bimodular properties (i.e., E_c and E_t) by using the repeated split-tension test. Analysis has also been used to assess the significance of bimodular properties on the response of stabilized layers under traffic loads.

Values of resilient moduli measured by using repeated tension and compression tests agree well with actual values for specimens that have length-to-width ratios of 2 and caps that are at least 10 times stiffer than the compressive stiffness of the stabilized material. Values of resilient moduli measured by using the repeated flexure test and simple beam theory but assuming $E_c \neq E_t$ appear to give a good estimate of the actual resilient moduli for specimens that have length-to-width ratios equal to 4 and no lateral restraints at points of load application. A procedure for the determination of the resilient bimodular properties by using the repeated split-tension test has been developed and gives values for resilient moduli in tension and compression that do not deviate from actual values by more than 16 and 11 percent respectively.

Results of analysis indicate that the resilient bimodular properties have significant effects on the traffic-induced stresses and strains at the underside of stabilized layers in pavement structures. Therefore, the correct assessment of these properties is desirable for the attainment of the goal of an improved pavement design.

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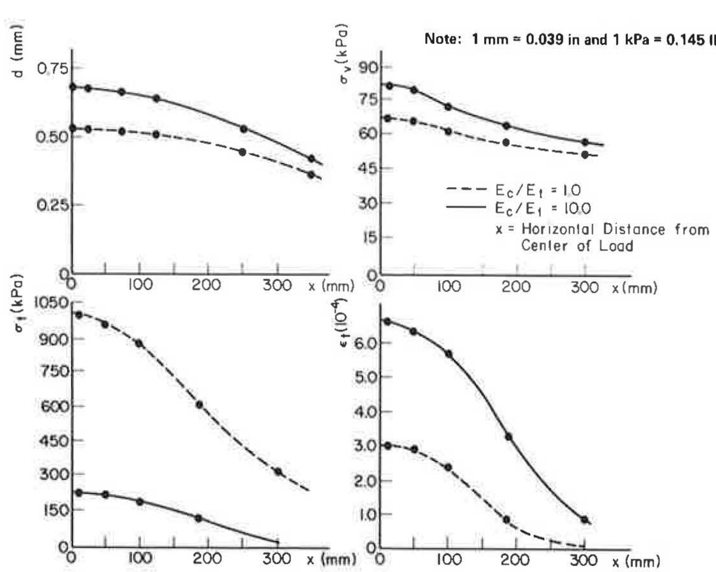
Figure 12. Response of pavement section under traffic load.



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Publication of this paper sponsored by Committee on Strength and Deformation Characteristics of Pavement Sections.