Abridgment

Stability Charts for Effective Stress Analysis of Nonhomogeneous Embankments

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New stability charts have been developed for the effective stress analysis of nonhomogeneous embankments subjected to seepage and seismic conditions. These charts are applicable to soils that have a small effective cohesion, which is the case encountered most frequently in engineering practice. The procedure is based on the normal method and is unique in that, although a large number of factors are considered, only a limited number of charts are needed. The theory by which these charts have been developed, the suggested procedures for their use, and their application to practical cases are presented. The factors of safety determined by using the charts are compared with those obtained by using the available computer programs, based on both the simplified Bishop and the normal methods. When the most-critical failure surface is a shallow circle, the factor of safety determined by using the charts agrees closely with the computer solutions. However, if the most-critical failure surface is a deep circle, the factor of safety determined by using the charts will be somewhat smaller than that obtained by using the computer solution based on the simplified Bishop method but slightly greater than that based on the normal method. Compared with the simplified Bishop method, the use of the stability charts is therefore conservative.

In a previous paper (1), I presented two new charts for the stability analysis of earth embankments. The chart for short-term stability is based on a total stress analysis, uses $\phi = 0$, and can be applied to a nonhomogeneous slope composed of various layers. The chart for long-term stability is based on an effective stress analysis, uses given values of $\bar{c}$ and $\bar{\phi}$, and is applicable only to a homogeneous slope that has a ledge at a considerable distance from the surface. It was indicated that the assumption of a homogeneous slope for effective stress analysis was not a serious limitation because the long-term shear strength parameters (i.e., $\bar{c}$ and $\bar{\phi}$) for most soils might not change significantly and average values could easily be estimated. However, if the strength parameters for different materials in different parts of the slope are significantly different, it will be difficult to obtain average values. Another difficulty in the use of the chart for effective stress analysis is the estimation of pore pressures. Unless the phreatic surface and the location of failure circle are known a priori, pore pressures cannot be estimated with certainty.

The purpose of this paper is to present an additional chart that can be used for the effective stress analysis of both homogeneous and nonhomogeneous slopes subjected to steady-state seepage and seismic conditions. The method requires an iterative determination of the factors of safety for a number of potential failure circles, so that a minimum factor of safety can be obtained. [Due to space limitations, only one chart and a simple example will be presented here; additional charts and detailed procedures for their use are given elsewhere (2).]

The chart presented here can be used only where the effective cohesion of the materials is small. These materials include granular soils and normally consolidated clays. The potential failure surfaces through these materials generally consist of shallow circles, so only a few charts involving shallow circles are needed. It is believed that the assumption of a small cohesion is realistic and can be used in many practical cases.

DESCRIPTION OF METHOD

Figure 1 shows a slope that has a height H and an outslope S:1 (horizontal: vertical). It is assumed that the effective cohesion of the soil in the slope is small, so that the most-critical failure surface is a shallow circle, the two endpoints of which lie at a distance of 0.1 SH from the top edge and the toe. This assumption of 0.1 SH, i.e., one-tenth the horizontal distance between the edge and the toe, is arbitrary. In fact, I have developed other charts that have endpoints passing through or at varying distances from the edge and the toe, so the factor of safety for any given circle can be determined. However, it has been found that the factor of safety for most slopes can be estimated by using this assumption.

When a failure circle is assumed, the average shear stress developed along it can be determined by equating the moment at the center of the circle due to both the weight of the sliding mass and the corresponding seismic force with that due to the average shear stress distributed uniformly over the failure arc. This developed shear stress is proportional to the unit weight of the soil and the height of the slope and can be expressed as

$$\tau = (\gamma H/N_s) + (C_s \gamma H/N_e) \qquad (1)$$

where

τ = developed shear stress,
γ = total unit weight of soil,
N_s = stability number,
C_s = seismic coefficient (the ratio between seismic force and weight), and
N_e = earthquake number.

Both N_s and N_e depend on the geometry of the slope and the location of the circle. The average shear strength along the failure surface varies with γ and H and, according to the Mohr-Coulomb theory, can be expressed as

$$s = \bar{c} + [(1 - r_u)\gamma H \tan\bar{\phi}]/N_f \qquad (2)$$

where

s = shear strength,
$\bar{c}$ = effective cohesion,
r_u = pore pressure ratio (ratio between the pore water pressure and the overburden pressure),
$\bar{\phi}$ = effective angle of internal friction, and
N_f = friction number (which also varies with the geometry of the slope and the location of the circle).

The factor of safety (F) is the ratio between the shear strength and the shear stress. By dividing Equation 2 by Equation 1, F can be expressed as

$$F = [(\bar{c}/\gamma H) + (1 - r_u)\tan\bar{\phi}/N_f]/[(1/N_s) + (C_s/N_e)] \qquad (3)$$

Figure 1. Potential failure circles in a typical slope.

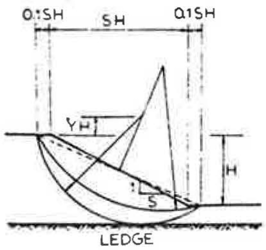


Figure 2. Stability chart.

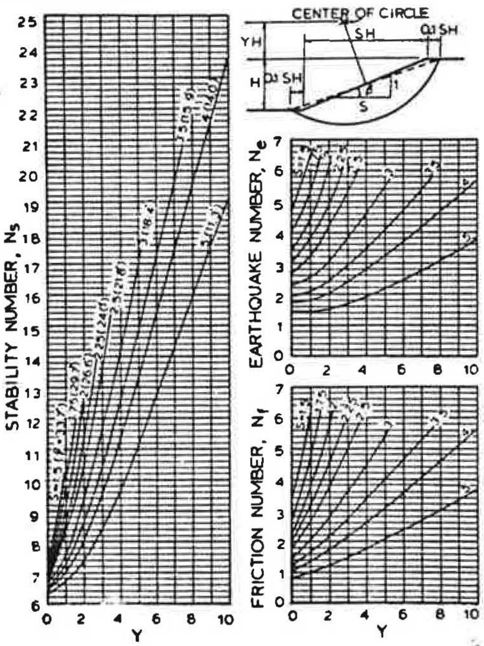
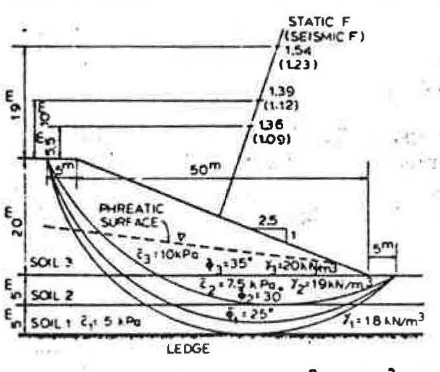


Figure 3. Analysis of nonhomogeneous slope.



Note: 1 m = 3.28 ft; 1 kPa = 20.9 lbf/ft^2; 1 kN/m^3 = 6.37 lbf/ft^3.

Equation 3 shows that F depends on four geometric parameters (H, N_s, N_f, and N_e) and four soil parameters (r_u, γ, $\bar{c}$, and $\bar{\phi}$). N_s, N_f, and N_e can be obtained from the stability chart, and r_u can be determined from the location of the phreatic surface with respect to the failure circle. If the slope is homogeneous, γ, $\bar{c}$, and $\bar{\phi}$ are given directly. If the slope is nonhomogeneous, average values of γ, $\bar{c}$, and $\bar{\phi}$ must be determined. [To facilitate the computation of average soil parameters, a special table and form were developed but are not presented here (2).] The method for computing γ, $\bar{c}$, $\bar{\phi}$, and r_u for a nonhomogeneous slope is illustrated below.

The value of F obtained by using Equation 3 is similar to that obtained by the normal method, which is one of the two methods used in the ICES-LEASE computer program (3) [the other is the simplified Bishop method (4)]. When the pore pressure ratio = 0 or there is no seepage, the normal method and the well-known Fellenius method (5) are identical. When the pore pressure ratio ≠ 0, the normal method differs from the Fellenius method because the former is based on the concept of submerged weight, which acts vertically, while the latter is based on the pore pressure normal to the failure surface. The simplified Bishop method was not used because the assumption that the shear stress varies with F makes it impossible to express F in the simple form shown by Equation 3.

Figure 2 shows N_s, N_f, and N_e in terms of the dimensionless parameters Y and S, where Y = ratio between the distance from the center to the top of the slope and the height of the slope. Because the slope angle (β) is related to S by S = cotβ, the slope angles corresponding to each value of S are also shown.

In using the stability chart, it is necessary to plot a cross section of the slope. A bisector perpendicular to the dashed line is drawn, as shown in Figure 1, and the values of F for several circles that have centers on the bisector are determined and compared. If the ledge or stiff stratum is close to the surface, the circle tangent to the ledge is usually the most critical.

EXAMPLE

Figure 3 shows a 2.5:1 slope, 20 m (66 ft) high, composed of three different soils. Soil 1 has an effective cohesion of 5 kPa (104 lbf/ft^2), an effective friction angle of 25°, and a total unit weight of 18 kN/m^3 (115 lbf/ft^3); soil 2 has an effective cohesion of 7.5 kPa (157 lbf/ft^2), an effective friction angle of 30°, and a total unit weight of 19 kN/m^3 (121 lbf/ft^3); and soil 3 has an effective cohesion of 10 kPa (209 lbf/ft^2), an effective friction angle of 35°, and a total unit weight of 20 kN/m^3 (127 lbf/ft^3). The location of the phreatic surface is as shown. Assuming a seismic coefficient of 0.1, determine both the static and the seismic values of F.

Because the weakest material (soil 1) lies immediately above the ledge, the most-critical circle is probably tangent to the ledge. Thus, a circle that cuts through all three soils is drawn tangent to the ledge and passing through the two endpoints 5 m (16 ft) from the edge and the toe. The center of the circle is 5.5 m (18 ft) above the top of the slope, or Y = 5.5/20 = 0.275. For S = 25 and Y = 0.275, Figure 2 gives N_s = 7.0, N_f = 2.0, and N_e = 2.8.

To determine the average soil parameters, the sliding mass is divided into a number of subareas, as shown in Figure 4. The area of each subarea is measured; the sums for soils 1, 2, and 3 are 131, 221, and 534 m^2, respectively.

The average unit weight for the entire sliding mass (γ) = [(131 × 18) + (221 × 19) + (534 × 20)]/(131 + 221 + 534) = 19.5 kN/m^3 (124 lbf/ft^3).

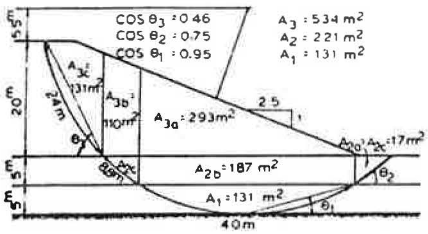
The length of the failure arc through soils 1, 2, and 3 is measured; these values are 40, 17.6, and 24 m (131, 57, and 79 ft), respectively.

$\bar{c}$ = [(40 × 5) + (17.6 × 7.5) + (24 × 10)]/(40 + 17.6 + 24) = 7.0 kPa (146 lbf/ft^2).

Because only the component of weight normal to the failure surface is effective in producing friction, tan $\bar{\phi}$ is determined by multiplying the weight above the failure surface by cos θ, where θ = angle of inclination of the chord, as shown by θ_1, θ_2, and θ_3 in Figure 4.

The weight normal to the failure arc in soil 1 = [(131 × 18) + (187 × 19) + (293 × 20)] × 0.95 = 11 182 kN/m

Figure 4. Area, arc length, and chord inclination of critical circle.



Note: 1 m = 3.28 ft; 1 m² = 10.7 ft².

(771 172 lbf/ft), that in soil 2 = [(2 × 17 × 19) + (110 × 20)] × 0.75 = 2135 kN/m (147 241 lbf/ft), and that in soil 3 = 131 × 20 × 0.46 = 1205 kN/m (83 103 lbf/ft).

Therefore, $\tan\bar{\phi}$ = (11 182 tan 25° + 2135 tan 30° + 1205 tan 35°)/(11 559 + 2135 + 1205) = 0.502.

The average pore pressure can be estimated by using

$$r_u = \text{area of sliding mass under water} \times \text{unit weight of water} \div \text{total area of sliding mass} \times \text{average unit weight of soil} \quad (4)$$

The area of sliding mass under water is measured and found to be 527 m² (5571 ft²).

r_u = (527 × 9.8)/(886 × 19.5) = 0.299.

From Equation 3, the static factor of safety = [[7.0/(19.5 × 20)] + [(1 - 0.299) × 0.502]/2.0]/(1/7.0 + 0) = (0.0179 + 0.1760)/0.1429 = 1.36.

The seismic factor of safety = (0.0179 + 0.1760)/(0.1429 + 0.1/2.8) = 0.1939/0.1786 + 1.09.

Two more circles, as shown in Figure 3, were also evaluated; their factors of safety were greater than the above values, thus confirming that the circle tangent to the ledge is the most critical.

The factors of safety obtained by using the REAME computer program (6) are summarized below:

	Method	
Factor of Safety	Simplified Bishop	Normal
Static	1.508	1.206
Seismic	1.129	1.002

Thus, the normal method yields a factor of safety somewhat smaller than does the simplified Bishop method. It was also found that the discrepancy decreased as the most-critical circle became shallower. The factor of safety determined by using the stability chart always lies between that found by using the normal method and that found by using the simplified Bishop method, as is expected. Compared with the simplified Bishop method, the use of stability charts is conservative.

SUMMARY AND CONCLUSIONS

A new stability chart for the effective stress analysis of slopes is presented. This chart is a valuable supplement to the stability chart presented in a previous paper (1). The advantages of the new chart over the earlier one are that (a) it can be used for both homogeneous and nonhomogeneous slopes that have a ledge or a stiff stratum either close to or far from the surface, (b) it can be used to determine both the static and the seismic factors of safety, and (c) it makes possible a more accurate evaluation of the pore pressure ratio. However, the application of the chart to a nonhomogeneous slope requires the determination of average soil parameters by measuring the arc length and the cross-sectional area of different soils in various regions.

The application of the stability chart is based on the normal method, which is a modified version of the Fellenius method. If the foundation is good or the ledge is near to the ground surface, the most-critical circle will be a shallow circle, and the factor of safety obtained by using the normal method will be only slightly smaller than that obtained by using the simplified Bishop method. If the foundation is poor or the ledge is far from the surface, the most-critical circle will be a deep circle, and the factor of safety obtained by using the normal method will be much smaller than that obtained by using the simplified Bishop method. Because the circle used in conjunction with the stability chart may not be the most-critical circle, the factor of safety determined by using the chart generally lies between the minimum factor of safety obtained by using the normal method and that obtained by using the simplified Bishop method. If the acceptance of a design is based on the simplified Bishop method, the use of the stability chart is conservative.

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