estimated benefits would appear to exceed the cost.

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Queuing and Search Delays Due to Gasoline Station Closings: Simple Equilibrium Framework

HANI MAHMASSANI AND YOSEF SHEFFI

This paper presents a simple framework for modeling the delays involved in searching for an open gasoline station and queuing at the station in urban areas. It includes an elastic response of the demand for gasoline and solves simultaneously for the number of users per unit of time and the search and queuing delays. The search-time model is based on simple geometric probability considerations, open gasoline stations are modeled as M/G/1 queues, and the demand curve is assumed to be a simple two-parameter curve. The model is concerned only with aggregate averages and not with detailed distribution of the delays. The solution is demonstrated, in a numerical example, as a function of the percentage of open gasoline stations and a demand sensitivity parameter. The example is focused on relative changes (in the output parameters) only, because the model is not calibrated to a particular urban area.

This paper describes a simple equilibrium framework for modeling the delays involved in searching for gasoline and queuing in gasoline stations, including an elastic response of the demand for gasoline. The analysis is macroscopic in scope and aggregate in nature. We deal with the aggregate number of users and average delay only, over a ubiquitous urban area characterized by a random distribution of gasoline stations and a random distribution of trip origins. Furthermore, we deal with relative delays only, since the model is not calibrated to a specific urban area.

The searching time and queuing delay involved in the process of obtaining gasoline can probably be found with a detailed network simulation. With the use of a disaggregate-choice model, the analyst can also determine the number of users in the system at equilibrium. Such methodology can provide microscopic analysis of a variety of policy options. However, it would involve a large data collection effort and a considerable computer budget.

The model presented in this paper is much simpler. Given the area size, the number of gasoline stations, and the percentage of stations that are open, it computes the average delay incurred by a motorist in finding an open station. Also, given the number of users per open station and the stations' average service rate, the model computes the average queuing delay. Lastly, given the search time and queuing delay, a simple demand curve is used to determine the number of users (customers) in the system. Naturally, the queuing delay grows as the number of users grows, and the number of users decreases as the queuing (and search time) grow. Thus, the queuing delay and the number of users in the system have to be solved for simultaneously, and their solution is referred to as the equilibrium queuing delay and the equilibrium number of users.

By use of this model one can investigate the effect of the percentage of open stations on the delays and number of users as well as the effect of other parameters, such as the service variability and the shape of the demand function for gasoline.

The strength of our approach lies in its simplicity. All the calculations can be performed with the aid of a programmable pocket calculator, and most of the major factors of the problem are included in the analysis.

THE TIME SPENT IN SEARCH

Consider an urban area of size A that has n_o gasoline stations located at random throughout the area. Assume that users (gasoline seekers) act independently of each other. Each user starts from a random point in the area and travels to the nearest gasoline station. If the station is closed, he or she keeps searching until the nearest open gasoline station is found. In this section we derive an expression for the mean time spent in the search (i.e., from the origin until an open station is found).

We start by developing an expression for the mean distance that a user has to travel in order to visit m stations, assuming an area of size A and a total number of stations n_o ($m < n_o$). From geometric probability considerations (1), the distance between a randomly selected point and the closest of a set of n_o points (D_{n_o}) is

$$D_{n_o} \cong \theta\sqrt{2\pi A/n_o} \qquad (1)$$

where Θ is a network structure coefficient. For right-angle distance, $\theta \simeq 1/4$; for airline distance, $\theta \simeq 1/5$ (2). Equation 1 holds only for $n_o \rightarrow \infty$; however, it is accurate to within 1 percent for all values of n_o (3) for areas that are approximately circular or square.

Thus Equation 1 is the distance traveled to the first gasoline station. If this station is closed, the customer travels to the next gasoline station that is the closest out of a set of n_o-1 remaining possibilities. (A cluster of gasoline stations at a particular location is considered as one gasoline station for the purpose of this discussion. Therefore, n_o is more exactly the number of clusters of gasoline stations.) Thus, the expected distance to the next stations (D_{n_o-1}) is

$$D_{n_o-1} \cong \theta\sqrt{2\pi A/n_o - 1} \tag{2a}$$

In general, after i stations have been visited, the distance to the closest one [out of the remaining (n-i) stations] is

$$D_{n_o-i} \cong \theta\sqrt{2\pi A/n_o - i} \tag{2b}$$

Thus, the total distance traveled in order to visit m stations in an area of size A with network struture coefficient Θ and n_o stations [$D_m = D_m(n_o,A,\Theta)$] is given by

$$D_m(n_o, A, \theta) \cong \theta\sqrt{2\pi A} \sum_{i=0}^{m-1} (1/\sqrt{n_o - i}) \tag{3}$$

Note that the argument used to derive Equations 2 and 3 is not entirely correct because the locations of the second and successive stations visited are not independent of the first station visited. However, as argued by Daganzo and others (3) this causes two errors that tend to cancel each other and the total error introduced by Equation 3 is less than 5 percent.

We now derive the probability mass function of the number of stations visited (n) if only n_p out of the total of n_o stations are open. Assuming that stations operate independently and that the weak law of large numbers applies, the probability of finding a given station open is $P = n_p/n_o$. This is also the probability that the first station visited is open. The conditional probability of finding the second station open given that the first one is closed, Pr(2nd open|1st closed), is given by

$$\Pr(\text{2nd open}|\text{1st closed}) = n_p/n_o - 1 \tag{4a}$$

Note that Pr(2nd open|1st closed) $\neq$ Pr(1st closed) since the number of open stations is finite and as the search proceeds the probability of finding an open station increases. Thus,

$$\Pr[\text{ith open}|\text{1st through (i-1)th closed}] = n_p/n_o - i+1 \quad \text{for } i \leq n_o - n_p + 1 \tag{4b}$$

For $i = n_o - n_p + 1$, the above probability is 1. This corresponds to a case where $(n_o - n_p)$ stations have been visited (and found closed). The remaining n_p stations are open, in accordance with our initial assumption, and thus the next one visited would be open (number $n_o - n_p + 1$ in the search process) and the search terminates. In other words, $1 < n < (n_o - n_p + 1)$ where n is the number of stations visited.

The probability of visiting n stations (P_n) equals the probability of finding the first (n - 1) stations closed and the nth one open. From Equation 4b and our independence assumption, the probability mass function of n is given by

$$\begin{aligned} P_1 &= \Pr[\text{1st open}] = n_p/n_o \\ P_2 &= \Pr[\text{2nd open}|\text{1st closed}] \times \Pr[\text{1st closed}] \\ &= (n_p/n_o - 1)\,[1 - (n_p/n_o)] \\ P_3 &= \Pr[\text{3rd open}|\text{2nd and 1st closed}] \times \Pr[\text{2nd closed}|\text{1st closed}] \\ &\quad \times \Pr[\text{1st closed}] = (n_p/n_o - 2) \times [1 - (n_p/n_o - 1)] \\ &\quad \times [1 - (n_p/n_o)] \end{aligned} \tag{5a}$$

$$P_n = (n_p/n_o - n+1) \times \prod_{i=1}^{n-1} [1 - (n_p \div n_o - i+1)] \quad \text{for } 1 < n \leq n_o - n_p + 1 \tag{5b}$$

$$P_n = 0 \quad \text{for } n > n_o - n_p + 1 \tag{5c}$$

From the probability mass function of n (given by Equations 5a-5c), the mean number of stations visited (n) is given by

$$\bar{n} = \sum_{n=1}^{n_o-n_p+1} n \times P_n \tag{6}$$

We can now derive the mean distance traveled until the first open station is found, by substituting the distance traveled in order to visit n stations (from Equation 3) for n in Equation 6. The mean distance traveled in search for gasoline [$d(n_o,n_p,A,\Theta)$] is

$$d(n_o, n_p, A, \theta) = \theta\sqrt{2\pi A} \sum_{n=1}^{n_o-n_p+1} P_n \times \sum_{i=0}^{n-1} 1/\sqrt{n_o - i} \tag{7}$$

When the probability mass function of n is substituted from Equation 5, the distance becomes

$$d(n_o, n_p, A, \theta) = \theta\sqrt{2\pi A}\left((n_p/n_o^2) + \sum_{n=2}^{n_o-n_p+1} \left\{(n_p/n_o - n+1)\prod_{j=1}^{n-1}\left[1 - (n_p/n_o - j+1)\right] \times \sum_{i=0}^{n-1} (1/\sqrt{n_o - 1})\right\}\right) \tag{8}$$

The average driving time to find gasoline (t_d) is derived by dividing the average distance by the average network speed (V) [i.e., $t_d = t_d(n_o,n_p,A,\Theta,V) = d(n_o,n_p,A,\Theta)/V$].

In order to demonstrate the effect of the number of open stations on the search time, the percentage increase in search time versus the percentage of open stations is depicted in Figure 1. In this figure, as well as in all of the following numerical examples, we use an area of A = 259 km² (100 miles²), a right-angle (dense-grid) network structure (i.e., $\Theta = 1/4$), a total number of gasoline stations of $n_o = 150$, and an average network speed of V = 48 km/h (30 miles/h).

The percentage increase in the average search time is given by 100 x $[t_d(n_o,n_p) - t_d(n_o,n_o)]/t_d(n_o,n_o)$, where the arguments A, Θ, and V were omitted from the notation of $t_d(. . .)$ since their values are fixed at the above-mentioned levels. Note that these notations are used only for clarity of presentation. The expanded expression for the percentage increase in delay is 100 x $[t_d(n_o,n_p,A,\Theta,V) - t_d(n_o,n_o,A,\Theta,V)] \div t_d(n_o,n_o,A,\Theta,V)$. As Figure 1 illustrates, the model predicts that, when 50 percent of the gasoline stations are open, the search time is approximately twice its value when all stations are open. When the number of open gasoline stations decreases, the search time increases sharply.

A more interesting result is depicted in Figure 2, where the percentage increase in delay is plotted again (the curve labeled "no information"). However this time the delay can be compared to a situation

Figure 1. Percentage increase in the average search time as a function of the percentage of open stations.

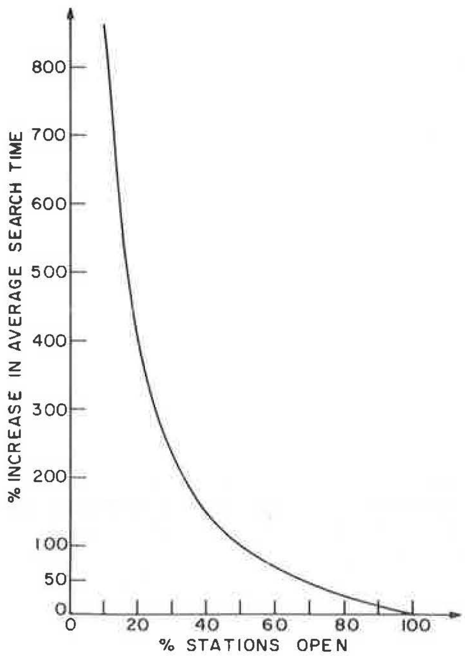
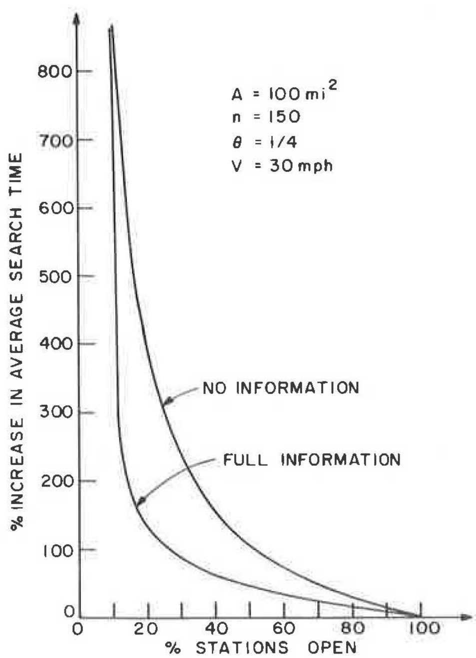


Figure 2. Impact of full information about station closings on the search time.



where the motorists have complete information regarding which stations are pumping gasoline and which are closed. To model the complete-information case one can assume $n_o = n_p$ (since motorists would not visit closed stations). In other words, the percentage increase in delay is given by 100 x $[t_d(n_o,n_p) - t_d(n_o,n_o)] \div t_d(n_o,n_o)$. [Note that from Equation 1, $t_d(n_p,n_p) = 1/V\sqrt{2\pi A/n_p}$.] This curve is labeled "full information" in Figure 2.

The full-information situation may correspond to a continuous broadcast of the location of open gasoline stations. As seen from the figure, the value of the information increases rapidly as the percentage of open stations decreases. Thus, for example, when 50 percent of the stations are open, the information on the locations of open gasoline stations can eliminate more than half of the time (and gasoline) wasted in searching for an open station.

THE TIME SPENT IN QUEUE

So far we have ignored any interaction among users and the familiar effect of queues at the station. However, before we adopt a familiar queuing model in order to model the time spent in queue, we have to clarify the meaning of the term gasoline station as used in this paper.

The definition of a station bears on the accuracy of the assumption that stations are distributed randomly, since stations tend to be clustered at intersections. Thus, for our purposes, a station may represent a cluster of stations (i.e., stations at all corners of a given intersection may be represented as one station cluster or simply as one station).

A more difficult problem is the definition of the service rate of gasoline stations. In this case the proper unit of service may be the single pump (or island) rather than a station or a station cluster. In other words, the distribution of pumps per station (or station cluster) should be accounted for in the computation of the service rate. The problem is compounded by the fact that the distribution of open pumps may be different from the distribution of pumps. In the absence of accurate data, such sophistication of the model seems hardly worthwhile, and thus we use the average service rate (μ) to represent the service rate per station (cluster).

Thus we model a gasoline station as an M/G/1 queuing process, where M is the arrival process, G is the general distribution of service time, and 1 is the number of servers for the queue. In other words, a queue with Poisson arrivals, a general distribution of service time, and one server. The basic result from queuing theory (4) that we use is that the average time spent in the queue, for an M/G/1 queuing system (t_q) is given by the Pollaczek-Khintchine mean value formula:

$$t_q = (\rho/\mu + \lambda\sigma^2)/2(1-\rho) \quad (9)$$

where $\rho = \lambda/\mu$ is the utilization factor and σ_s^2 is the variance of the service time distribution.

In Equation 9, the arrival and service rates (λ and μ, respectively) are measured in customers per time unit (e.g., customers per hour), and the model assumes that customer arrivals follow a Poisson process with mean λ and that the service time per customer is distributed with mean $1/\mu$ and variance σ_s^2.

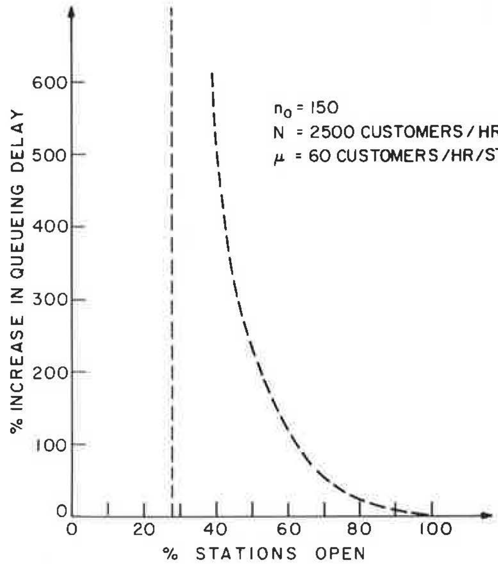
As mentioned in the beginning of this section, the mean service time is taken as the average per station (cluster) across the study area and assumed to be applicable to all stations in the area.

The average arrival rate is given by the rate of customers looking for gasoline divided by the number of open gasoline stations. If we let N denote the total areawide number of customers per time unit, the average (across stations) arrival rate is given by

$$\lambda = N/n_p \quad (10)$$

where n_p is the number of open gasoline stations.

Figure 3. Percentage increase in the average queuing delay as a function of the percentage of open stations.



When λ is substituted by Equation 10 in Equation 9, the queuing delay as a function of the above-mentioned parameters becomes

$$t_q = t_q(N, N_p, \mu, \sigma_s^2) = (N/n_p) \times \left\{(\mu^{-2} + \sigma_s^2)/2[1 - (N/n_p \cdot \mu)]\right\} \tag{11}$$

Figure 3 depicts the percentage increase in the queuing delay with n_p stations open compared with the queuing delay with n_o stations open {i.e., $[t_q(\cdot n_p \cdot\cdot) - (t_q(\cdot n_o \cdot\cdot)]/t_q(\cdot n_o \cdot\cdot)$ for given values of N, μ, and σ_s^2}. In order to conform to our earlier notation, the abscissa in Figure 3 is given in terms of the percentage of open stations (i.e., 100 n_p/n_o) for n_o = 150. The other values for which the graph is drawn are N = 2500 customers/h and μ = (60 customers/h)/station. Note that the variance in service time has no effect on the percentage increase of delay even though the delay itself would increase substantially with increasing variance in service time.

As is evident from Figure 3, the relative delay increases rapidly as the percentage of open stations decreases. Note that for N = 2500 customers/h and μ = (60 customers/h)/station, the delay approaches infinity as the percentage of open stations decreases to 28 percent (since with n_o = 150, this implies an arrival rate that equals the service rate). In general, the queuing delay would approach infinity as $n_p \rightarrow N/\mu$.

Thus, Equation 11 expresses the queuing delay as a function of the number of open stations, the service time distribution parameters, and the total number of customers searching for gasoline per hour.

THE DEMAND FUNCTION

The demand function that we describe in this section relates the number of customers in the system (per time unit) to the queuing and search time. It assumes that, as the delays increase, more users would be discouraged and abandon the system. In accordance with the level of analysis of the preceding sections, we have assumed a demand function of the type

$$N = N_o\left\{[t_d(n_o) + t_q(n_o)]/[t_d(n_p) + t_q(n_p)]\right\}^{\alpha} \tag{12}$$

where $t_d(n_o)$, $t_q(n_o)$ are the search and queuing times, respectively, that correspond to n_o gasoline stations open; $t_d(n_p)$, $t_q(n_p)$ are the search and ueuing delay that correspond to n_p (out of n_o) open stations; α is a positive parameter that reflects user sensitivity to search and queuing times; and N_o is the number of users when all the stations are open.

Note that N_o might reflect a demand effect that is not modeled in our work (i.e., the known or expected closings). Since there are times, such as late nights or Sundays, when almost no gasoline station is open, the volume of customers that would have normally (i.e., without gasoline shortage) been serviced during these hours is added to the number of users during business hours. In fact, the basic number of customers (N_o) is probably a function of the expected (or advertised) number of open stations and thus one might interpret Equation 12 as including this effect [through $t_d(n_p)$ and $t_q(n_p)$]. However, without data the use of a more complicated functional form for the demand does not seen warranted, and thus Equation 12 may be interpreted both ways.

The parameter α captures the sensitivity of customers to queuing and search time. This parameter can be estimated by use of standard econometric techniques. However, note that, in a rapidly changing environment, this parameter may be unstable since it reflects the users' belief that gasoline might be more (or less) available at some other time. Due to the high degree of uncertainty associated with α, in this paper we perform a parametric study of the effects of this parameter.

Note also that we assumed α to be independent of $t_q(\cdot)$ and $t_d(\cdot)$, although one may argue that, as the search time increases, customers might accept longer queues. Incorporation of such an effect would lead, again, to a more complicated demand function, which we tried to avoid.

In specifying the demand function we hypothesized that the searching and queuing time have equal effect on the number of users. Naturally one can specify a model with different weights on $t_d(\cdot)$ and $t_q(\cdot)$ and test for this assumption when the model is estimated econometrically.

Figure 4 illustrates the demand function that depicts N/N_o versus $[t_d(n_p) + t_q(n_p)]$, for α = 0.2, 0.5, and 0.8, assuming that $[t_d(n_o) + t_q(n_o)]$ = 1 (for normalization purposes).

Given the area parameters (A, n_o, Θ, and V), the service parameters (μ, σ_s^2), and the demand parameters (N_o, α), we can now solve for the number of users (N) and queuing and searching time (t_d, t_q) as a function of the number of open stations (n_p). This solution specifies an equilibrium situation.

EQUILIBRIUM CONDITIONS

The equilibrium solution (N^*, t_d^*, t_q^*) has to satisfy the following system of equations (see Equations 8, 11, and 12). (We use asterisks to denote the value of these variables at the equilibrium solution.)

$$t_d^* = (\theta/V)\sqrt{2\pi A}\left((n_p/n_o^2) + \sum_{n=2}^{n_o-n_p+1}\left\{[n_p/(n_o-n+1)]\prod_{j=1}^{n-1}\left[1 - [n_p \div (n_o - j+1)]\right] \times \sum_{i=0}^{n-1} 1/\sqrt{n_o - i}\right\}\right) \tag{13a}$$

$$t_q^* = (N^*/n_p)\left\{(\mu^{-2} + \sigma_s^2)/2[1 - (N^*/n_p \cdot \mu)]\right\} \tag{13b}$$

$$N^* = N_o\left\{[t_d(n_o) + t_q(n_o)]/(t_d^* + t_q^*)\right\}^\alpha \quad (13c)$$

Given the area parameters (A, n_o, θ, and V) and n_p, Equation 13a can be computed independently of the remaining two equations (since the search time is independent of the queuing delay and the number of users). Given t_d^*, one can substitute Equation 13b in 13c to get the fixed-point problem:

$$N^* = C_1\left(t_d^* + C_2\left\{N^*/[1 - (N^*/n_p \cdot \mu)]\right\}\right)^{-\alpha} \quad (14)$$

where the constants C_1 and C_2 are given by the demand function parameters and the service time distribution parameters, respectively; i.e.,

$$C_1 = N_o[t_d(n_o) + t_q(n_o)]^\alpha$$

and

$$C_2 = (\mu^{-2} + \sigma_s)/2n_p$$

Many numerical methods can be used to solve for N^* in Equation 14. (We used a bisecting method.) Once N^* is know, t_q^* can be determined from Equation 13b.

In the remainder of this section we show the characteristics of the equilibrium solution as a function of the percentage of open stations, in the context of a simple numerical example.

The parameters of our example are as follows:

1. Area size, $A = 259\ km^2$ (100 $miles^2$) with rectangular grid structure of the road network ($\theta = 1/4$);
2. Total number of stations in the study area, $n_o = 150$;
3. Average speed over the network, $V = 48$ km/h (30 mph);
4. Average station service rate, $\mu = 60$ vehicles/h; and
5. The number of users with all the stations open, $N_o = 2500$ customers/h.

The model is solved for three values of α: 0.2, 0.5, and 0.8.

Figure 5 depicts the percentage increase in average queuing time, $100 \times [t_q^*(n_p) - t_q(n_o)]/t_q(n_o)$, as a function of the percentage of open gasoline stations ($100\ n_p/n_o$) and the demand parameter (α). Note that this figure depicts the average waiting time per user (i.e., per waiting customer). The general shape of all the curves shown in Figure 5 is similar to the queuing delay curves shown in Figure 3. However, due to the demand effect, these curves are all asymptotic to the ordinate rather than to N/μ, since N is a decreasing function of n_p as well.

Note that for a given number (or percentage) of open gasoline stations, the delay per user decreases with increasing α. This, of course, is due to the demand effect; as α increases, more users are discouraged by the long search time and queuing time, and the queues decrease.

Figure 6 shows the combined effect of the increased delay and the reduction in the number of users. It depicts the percentage increase in total queuing time; i.e., $100[N^* \cdot t_q^*(n_p) - N_o \cdot t_q(n_o)] \div N_o \cdot t_q(n_o)$, versus the percentage of open stations and the demand parameters. In this case, two opposite effects influence the shape of the total queuing delay. The queuing time per customer, $t_q(\cdot)$, is an increasing function of n_p and the number of customers in the system is a decreasing function of n_p. Thus, their product depends on the input parameters and, in our particular example, on the demand parameter (α).

The effect of the demand parameter is more pronounced in Figure 6 than in Figure 5 since both N and $t_q(\cdot)$ decrease with increasing α. As seen

Figure 4. Relative number of users as a function of the normalized sum of search and queuing time.

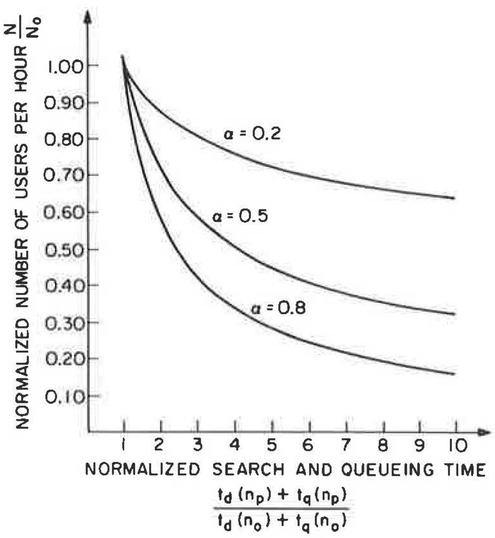


Figure 5. Percentage increase in queuing time per user—equilibrium result.

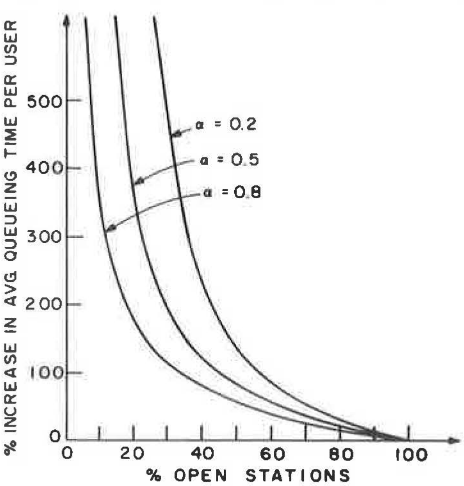


Figure 6. Percentage change in aggregate queuing time—equilibrium result.

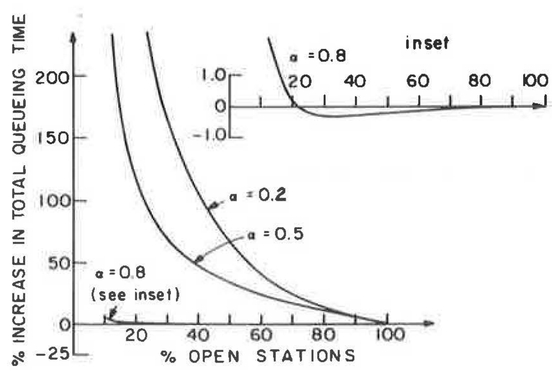


Figure 7. Percentage increase in total search time—equilibrium result.

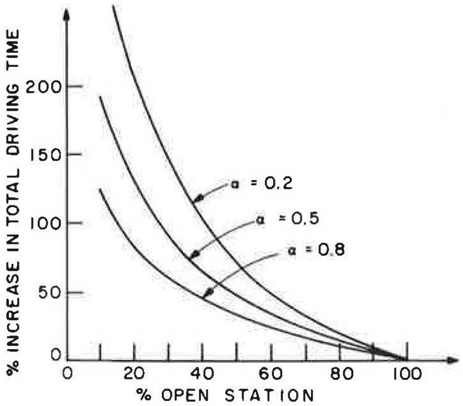
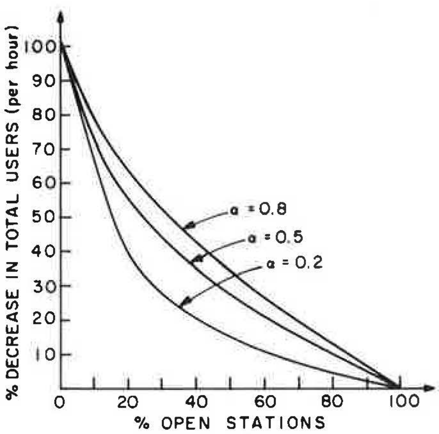


Figure 8. Percentage decrease in the number of users per hour—equilibrium result.



in Figure 6, the curve is almost zero for 10 percent $\leq$ 100 n_p/n_o $\leq$ 100 percent for $\alpha = 0.8$. With such value of α, the relative decrease in the number of customers who search and queue for gasoline approximately offsets the relative increase of the queuing time per customer. However, note that, since the number of open stations becomes very small, the total delay would increase, regardless of α.

The percentage increase in average driving time per customer in searching for gasoline, for this example, is shown in Figure 7. Note that the search time (t_d^*) is not affected by the number of customers in the system and, therefore, Figure 7 applies to our example as the equilibrium solution for t_d.

However, the total search time (for all users in the system) is a function of the percentage of open stations and the demand parameter. As expected, the total search time decreases with increasing percentage of open stations and with increasing α.

The effect of the percentage of open stations on the equilibrium number of users is shown in Figure 8. This figure depicts the percentage decrease in the total number of customers per hour in the system, $100(N_o - N^*)/N_o$, as a function of $100n_p/n_o$ and α. The number of users in the system decreases with decreasing percentage of open stations and with increasing α. Note that these curves are not asymptotic to the ordinate since as $n_p \rightarrow 0$, so does N^* and the percentage decreases as the number of customers approaches 100 percent (i.e., there are no customers in the system).

CONCLUSION

This paper presented a simple model for assessing the delays associated with the search for an open gasoline station and the wait in line at the station and estimates the number of customers in the system.

The model consists of three equations. The first one is the search-time model. Based on geometric probability considerations and the assumption of a random distribution of gasoline stations, this equation relates the area parameters and the number of open stations to the time spent in search for an open station. The area (input) parameters include the area size, the total number of gasoline stations, the network structure parameter, and the average network speed.

The second equation is based on viewing each gasoline station as an M/G/1 queuing system. The equation relates the time spent in the queue to the mean and variance of the service time and to the rate of customer arrival. The arrival rate is, in turn, determined by the ratio of the number (per time unit) of customers in the system to the number of open gasoline stations.

The third equation, the demand function, relates the number of users in the system to the time spent in search and in queue (relative to those times when all the stations are open) and the number of customers in the unconstrained case (where all stations are open).

These three equations have to be solved simultaneously to get the driving time, queuing time, and the number of customers (per time unit) at equilibrium. We demonstrated numerically the solution of these equations for given values of the input parameters (A, N_o, θ, V, μ, and σ_s^2) for the the whole range of n_p (the number of open gasoline stations) and for several values of α, the demand function parameter.

The effect of N_o on the equilibrium solution would be opposite in direction to the effect of α. An increase in α (with everything else remaining constant) means an equilibrium solution that has fewer users in the system and, therefore, lower queuing times. As can be seen from Equation 13c, a decrease in N_o would have a similar effect on the equilibrium solution.

The parameters θ, V, and A affect the equilibrium solution through the search-time equation. As can be seen from Equation 13a, as long as $\theta\sqrt{A}/V$ remains constant, the solution is not affected. If $\theta\sqrt{A}/V$ increases (either θ or A increase, or V decreases) the driving time would increase (holding everything else constant). The effect of increasing search time would diminish the queuing effect (since the area parameters under consideration appear in both the numerator and the denominator of the demand function). However, as can be seen from Equations 13b and 13c , the number of users in the system would somewhat decrease and the queuing time would decrease accordingly.

The effect of increasing the service rate is to reduce the queuing time and therefore to increase the number of customers in the system. A similar effect would be generated by an increase in the variance of the service time. Note, however, that the effect of both of these parameters on the demand is somewhat limited since (as with the area parameters) they affect both the numerator and the denominator of the demand function.

In Figure 2 we showed the effect of full information about station closings in the system on the driving time in search for an open station. This result still applies in an equilibrium framework. However, one can expect long queues and more users than in the no-information situation because a reduction in the search time would generate an increase in customer volume and longer queues.

Once calibrated, the model presented in this paper may be used to obtain first-cut estimates of the effects of relief policies. One such policy directed at reducing the search time is the above-mentioned information dissemination. Another policy, directed at reducing the queuing time, can be a variance-reduction policy, used in the summer of 1979 by many station attendants (i.e., $7 worth of gasoline to every car, or variants of this policy). Such a policy would decrease the queuing time; however, as with all other variables, this reduction is absorbed to some extent by the equilibrium effect (the reduced queuing time encourages more customers to look for gasoline, which causes an increased queuing time). A minimum purchase policy can be modeled by decreasing N_0, which should be interpreted as the number of gasoline trips rather than the number of customers. A maximum purchase policy can be modeled by increasing N_0, which of course would cause increased queues. In appplying our model to the evaluation of an odd-even plan, one might naively assume that N_0, the number of potential gasoline trips per hour, should be halved and therefore (after allowing for the equilibrium effect) queuing time should decrease somewhat. However, under an odd-even plan, individuals' determination to obtain gasoline is drastically increased; or, in terms of our model, α decreases substantially. In other words, under an odd-even plan (which is usually coupled with weekend closing as well), customers are relatively inelastic with respect to queuing delays, and stay in the system, which causes even larger delays.

The model presented in this paper should not, of course, be used for detailed analysis and policy assessment. It is intended more as a framework for thought about the problem and general assessment of the search and queuing delays involved in obtaining gasoline in a situation similar to that of the summer of 1979.

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Demand for Travel and the Gasoline Crisis

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This paper uses traffic count data to estimate and analyze the demand for gasoline and different kinds of work and leisure travel in California from 1970 to 1975. Empirical results of the ordinary least-squares regressions show the price elasticity of gasoline and travel to be quite inelastic—between -0.05 and -0.50. The income elasticities range between 0.5 and 1.5. Furthermore, the results suggest that leisure-oriented travel is less price- and income-sensitive than work-oriented travel. Results also indicate that travel and gasoline are affected by seasonal variations. In addition to the conventional demand analysis, the study investigates the gasoline crisis in California in 1974. During the gasoline crisis, the existence of queuing at service stations suggested that disequilibrium existed in the gasoline market. Due to the difficulty in purchasing gasoline, the true price of gasoline exceeded the actual price paid at the pump. Results show that the true price of gasoline rose from a precrisis price of $0.31/gal to more than $1.00/gal in some instances during the height of the crisis in March 1974. Furthermore, the value that would have been transferred from consumers of gasoline to suppliers was approximately $355 million. This amount, which averages about $27/licensed California driver, could be thought of as a measure of the gross welfare loss of gasoline rationing.

Recent developments in the worldwide energy situation have caused economists to become interested in the demand for both gasoline and automobile travel. In general, studies in this area have estimated the price and income elasticities of demand for gasoline and have come to reasonably consistent conclusions. However, these studies generally are subject to two main shortcomings:

1. By focusing on gasoline demand they are unable to distinguish between different types of automobile travel and

2. They have failed to analyze the period from December 1973 to April 1974 (henceforth referred to as the gasoline crisis), a period when the gasoline market was in disequilibrium.

This paper presents an analysis that overcomes these shortcomings by direct assessment of the demand for automobile travel by the use of monthly traffic counts on the California state highway system. These traffic counts have been disaggregated into urban, rural, weekday, and weekend trips. Furthermore, by employing traffic count locations of different characters near the San Francisco area, the study investigates the demand for recreation (I-80) and commercial and commuter travel (I-580). By using ordinary least-squares regression techniques on the monthly time series data from January 1970 to December 1975, along with seasonal monthly dummy variables, the price and income elasticities of each category of travel with respect to the price of gasoline are determined. In addition, monthly gasoline-crisis dummy variables are used to calculate what shall be called the waiting price for gasoline (due to nonprice rationing) for each month of the gasoline crisis. Once the waiting price of gasoline is calculated, the effects of the disequilibrium situation are investigated empirically by calculating the welfare loss caused by the gasoline crisis. Finally, as a