

Determining the Lateral Deployment of Traffic on an Approach to an Intersection

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An analytic model that predicts the lateral (i.e., lane-specific) deployment of traffic on an approach to an intersection is described. The basis for this model is a variation of Wardrop's first principle: that every motorist will select a lane on an approach consistent with his or her intended turn maneuver and with any specified lane channelization so as to minimize his or her perceived travel time. The specified conditions include the approach geometry; lane channelization, if any; specification of control policy at the intersection; service rate for each turn-movement component of the traffic stream; and turn percentages of the traffic stream discharging from the approach. The model yields the proportion of the total traffic volume entering the approach that is deployed in each lane, stratified by turn movement. The model is one component of a larger analysis that predicts the capacity of an approach and, by aggregation, intersection capacity. A brief outline of this capacity model is also included.

Intersection capacity, as defined in the 1965 Highway Capacity Manual (HCM), "actually represents individual approach capacity", whereas intersection service volume "usually refers to service volume on a particular approach" (1, p. 111).

Many investigators have developed analytic models to estimate approach capacity, while others have developed computational procedures for the practicing engineer. An excellent bibliography of this past work is available elsewhere (2). There still remained, in my view, a need to develop an analysis that includes an explicit description of all of the important factors that influence approach capacity. These factors are the following: geometrics, lane channelization, service rate for through vehicles, service rate for unimpeded right turners, service rate for unimpeded left turners, gap acceptance for left turners, percentage of left turners, percentage of right turners, oncoming approach geometrics, opposing volume, opposing service rate for through vehicles, cycle length, duration of green phase, signal phasing, and right turn on red.

In previous work, some of these factors have often been disregarded, or simplifying assumptions have been applied to reduce the complexity of the real-world process. Examples of such simplifications include the assignment of

1. "Passenger-car-equivalence" (PCE) factors to account for the different service rates associated with different turn movements,

2. "Left-turn factors" based on opposing traffic volumes to account for the dependence of left-turn service rates on oncoming traffic volume, and

3. Traffic volume on an approach to specific lanes in a somewhat arbitrary manner.

The assertion of simplifying assumptions in order to reduce a complex process to a form that is analytically tractable is, of course, a necessary and acceptable practice. But the impact of these assumptions on the efficacy of the results obtained must always be a potential source of concern, since it is generally very difficult--perhaps impossible--to assess the quantitative impact of such assumptions.

BACKGROUND

The model discussed here is an outgrowth of a project sponsored by the Federal Highway Administration (FHWA) to develop a macroscopic simulation program

(TRAFLO). Since the integrity of such a model is strongly dependent on the accuracy of estimates of approach service rates, a survey of existing models was undertaken to identify one that would be acceptable for this application. No treatment that possessed the required generalism and precision could be located. Consequently, work was begun on developing a satisfactory capacity model for TRAFLO. This work resulted in the identification of the interdependence of the determination of approach service rates and the determination of the lateral deployment of traffic, by lane, on an approach.

Stated formally, the approach capacity (\bar{s}) can be expressed as follows:

$$\bar{s} = \sum_{\text{over all lanes } i} \bar{s}_i \quad (1)$$

$$\bar{s}_i = \sum_{\text{over all movements } x} \bar{s}_i^x$$

$$\bar{s}_i^x = f_i(P_i^x; V_j) \quad (2)$$

and

$$P_i^x = g_i(\bar{s}_i; V_k) \quad (3)$$

where

\bar{s}_i = service rate for all vehicles discharging lane i ,

\bar{s}_i^x = service rate for vehicles discharging lane i by executing maneuver x ($x = L, T, R$),

P_i^x = proportion of vehicles discharging lane i that are executing maneuver x ($x = L, T, R$),
 L, T, R = left-turn, through, and right-turn maneuvers, respectively, and

V_j, V_k = sets of variables that may be treated as parameters, i.e., independent variables.

The functional relations $f_i()$ and $g_i()$, in aggregate, form a system. Because these functions are interdependent and highly nonlinear and are not all expressible explicitly in an algebraic format, it is necessary to solve the system in an iterative manner.

The system consists of an assemblage of interrelated models:

1. One model calculates the lateral deployment of vehicles on an approach. The lane-specific service rates \bar{s}_i for all lanes are presumed to be known. Other required information includes approach-specific turn fractions, approach geometry, and lane channelization. This model yields the values of P_i^x .

2. One model calculates the service rate for traffic discharging as a mix of left-turning and through vehicles from an approach lane. The proportions of turn-movement-specific vehicles on this lane (P_i^x) are presumed to be known. Other required information includes mean acceptable gap for left turners, unimpeded discharge headways for through and left-turn vehicles, opposing traffic

volume and service rate, and signal-control timing. This model yields the values of \bar{s}_i^x and \bar{s}_i for this lane.

3. One model calculates the service rate for traffic discharging as a mix of right-turning and through vehicles from an approach lane. The proportions of turn-movement-specific vehicles on this lane (P_i^x) are presumed to be known. Other required information includes pedestrian intensity, unimpeded discharge headways for through and right-turn vehicles, and signal-control timing. This model yields the values of \bar{s}_i^x and \bar{s}_i for this lane.

The procedure for applying these models as a system is outlined below:

1. Estimate approach service rates for each lane (\bar{s}_i);
2. Apply the lateral deployment model to calculate the turn-maneuver proportions (P_i^x) for each lane;
3. Apply the left-turn model, given P_i^x for the inside lane, to calculate the values of \bar{s}_i^x and \bar{s}_i for that lane;
4. Apply the right-turn model, given P_i^x for the outside lane, to calculate the values of \bar{s}_i^x and \bar{s}_i for that lane;
5. Compare these computed values of \bar{s}_i with those used for the prior iteration and, if any service rate differs significantly from its previous value, continue the iteration by returning to step 2 above; and
6. At convergence, all values of \bar{s}_i and P_i^x are known.

The above models are designed for signal-controlled approaches and for uncontrolled approaches. Additional models were developed for sign-controlled approaches. Documentation of the entire model is given by Lieberman and others (3).

Fortunately, this iterative procedure is stable and rapidly convergent, and the entire process is computationally efficient. It is interesting to note that a procedure similar in concept (although less detailed), and one that is also rapidly convergent, is presented in the Swedish Capacity Manual (4).

This paper describes the first of the three models outlined above.

MODEL FORMULATION

The lateral--i.e., lane-specific--deployment of traffic on an approach is assumed to satisfy the following variation of Wardrop's first principle: that every motorist will select a lane on an approach consistent with his or her intended turn maneuver and with any specified lane channelization so as to minimize his or her perceived travel time. On this basis, it will be shown that the following factors influence the lateral deployment of traffic on an approach: (a) specified turn proportions (T_L and T_R) for the traffic discharging from the approach; (b) number of lanes (N) on the approach at the stop line; (c) specified lane channelization, if any; and (d) defined service rates for traffic on each lane (\bar{s}_i).

The analysis is developed in stages, and the following approach configurations are treated:

1. An approach with a single lane ($N = 1$);
2. An approach with two lanes ($N = 2$), neither of which is channelized;
3. An approach with more than two lanes, none of which is channelized; and
4. An approach with at least two lanes, one or

more of which are channelized exclusively for turning vehicles (an immediate extension of approach 3 above, not presented in this paper).

Approach with a Single (Unchannelized) Lane

For an approach with a single (unchannelized) lane, since there is only one lane, the solution is trivial:

$$p^L = T_L; p^R = T_R; p^T = 1 - p^L - p^R \quad (4)$$

where p^L , p^R , and p^T are the proportions of vehicles in the subject lane that execute left turns, right turns, and through movements, respectively.

Approach with Two Unchannelized Lanes

For an approach with two unchannelized lanes, the right-turning volume $Q_R = T_R Q$, the left-turning volume $Q_L = T_L Q$, and the through volume $Q_T = (1 - T_L - T_R)Q$, where Q is total entering volume. It follows immediately that

$$Q_1 = Q_R + (1 - p_2)Q_T \quad (5)$$

and

$$Q_2 = Q_L + p_2 Q_T \quad (6)$$

where Q_i = traffic volume discharging from lane i ($i = 1, 2$), where lane 1 is the outside (curb) lane and lane 2 is the inside (median) lane; and p_2 = proportion of through traffic that discharges into (i.e., selects) lane 2 (the inside lane).

The above expressions can be written as follows:

$$Q_1 = Q [T_R + (1 - p_2)(1 - T_L - T_R)] \quad (7)$$

$$Q_2 = Q [T_L + p_2(1 - T_L - T_R)] \quad (8)$$

These equations assume that all right-turning vehicles will select the outside lane and all left-turning vehicles the inside lane. Note that the condition $Q = Q_1 + Q_2$ is satisfied by these equations.

Two conditions must be considered: (a) $1 - T_L - T_R = 0$ and (b) $1 - T_L - T_R > 0$.

The first case asserts that no vehicles execute through movements on leaving the approach (i.e., $Q_T = 0$). In this case, the value of p_2 has no meaning since $Q_1 = Q T_R$ and $Q_2 = Q T_L$; that is, the inside lane contains only left-turning vehicles (if any), and the outside lane contains only right-turning vehicles (if any). Consequently, $p_2^L = p_1^R = 1$.

The second case is more interesting, since the parameter p_2 exists and must be evaluated. Let \bar{s}_i = mean service rate for traffic discharging from lane i in vehicles per second of green time ($i = 1, 2$). These service rates must be known--i.e., must be determined by other models (see the paper by McShane and Lieberman elsewhere in this Record). Furthermore, let \bar{h}_i = mean discharge headway for traffic discharging from lane i [$\bar{h}_i = (1/\bar{s}_i)$] and t_i = time to discharge the vehicles in lane i in one signal cycle C , measured from the start of the green ($i = 1, 2$). By definition, $t_i = Q_i \bar{h}_i C$.

The cited variation of Wardrop's law is stated formally as

$$\left. \begin{aligned} t_1 &= t_2 \\ \text{or} \\ Q_1 \bar{h}_1 &= Q_2 \bar{h}_2 \end{aligned} \right\} \quad (9)$$

Substituting Equations 7 and 8 yields

$$[T_R + (1 - p_2)(1 - T_L - T_R)] \bar{h}_1 = [T_L + p_2(1 - T_L - T_R)] \bar{h}_2 \quad (10)$$

Solving for p_2 ,

$$p_2 = [(1 - T_L)\bar{h}_1 - T_L\bar{h}_2] / [(1 - T_L - T_R)(\bar{h}_1 + \bar{h}_2)] \quad (11)$$

subject to $0 < p_2 < 1$.

Now derive the expressions for P_i^L , P_i^R , and P_i^T by using Equations 7 and 8. For the outside lane ($i = 1$),

$$P_1^L = 0; P_1^R = (Q_R/Q_1) = T_R / [T_R + (1 - p_2)(1 - T_R - T_L)] \quad (12)$$

For the inside lane ($i = 2$),

$$P_2^R = 0; P_2^L = (Q_L/Q_2) = T_L / [T_L + p_2(1 - T_R - T_L)] \quad (13)$$

In either lane,

$$P_i^T = 1 - P_i^L - P_i^R \quad (14)$$

Approach with More Than Two Unchannelized Lanes

For an approach with more than two unchannelized lanes ($N > 2$), as before, assume that all turning vehicles select their respective lanes, either inside (median) or outside (curb). The through vehicles can deploy over all N lanes in a manner to be determined.

In case a ($1 - T_L - T_R = 0$), following the same rationale as for the two-lane approach,

$$P_N^L = P_1^R = 1 \quad (15)$$

and there is zero flow in the middle lane(s); $m = 2, \dots, N - 1$. In case b ($1 - T_L - T_R > 0$), let p_i equal the proportion of through vehicles on the approach that select border lanes $i = 1$ and $i = N$. The remaining through vehicles are assumed to deploy uniformly over the middle lanes m . Then

$$Q_i = Q_R + p_i Q_T \quad (16)$$

$$Q_N = Q_L + p_N Q_T \quad (17)$$

$$Q_m = (1 - p_1 - p_N) Q_T / (N - 2) \quad m = 2, 3, \dots, N - 1 \quad (18)$$

which can be written as

$$Q_i = Q [T_R + p_i (1 - T_R - T_L)] \quad (19)$$

$$Q_N = Q [T_L + p_N (1 - T_R - T_L)] \quad (20)$$

$$Q_m = Q [(1 - p_1 - p_N) (1 - T_R - T_L)] / (N - 2) \quad m = 2, 3, \dots, N - 1 \quad (21)$$

Proceeding as before,

$$t_1 = t_m = t_N \quad (22)$$

where $t_i = Q_i \bar{h}_i C$.

Asserting that $t_1 = t_m$ and substituting Equations 19 and 21, then asserting that $t_N = t_m$ and substituting Equations 20 and 21, yield

$$[T_R + p_1(1 - T_R - T_L)] \bar{h}_1 = [(1 - p_1 - p_N)(1 - T_R - T_L) / (N - 2)] \bar{h}_m \quad (23)$$

$$[T_L + p_N(1 - T_R - T_L)] \bar{h}_N = [(1 - p_1 - p_N)(1 - T_R - T_L) / (N - 2)] \bar{h}_m \quad (24)$$

Solving these two equations simultaneously and recognizing that $\bar{s} = l/h$ yield

$$p_1 = p_N (\bar{s}_1 / \bar{s}_N) + [T_L / (1 - T_R - T_L)] (\bar{s}_1 / \bar{s}_N) - [T_R / (1 - T_R - T_L)] \quad (25)$$

and

$$p_N = [1 / (1 - T_R - T_L)] \{ [1 / (N - 2) (\bar{s}_m / \bar{s}_N) + 1 + (\bar{s}_1 / \bar{s}_N)] - T_L \} \quad (26)$$

Then, with Equation 25,

$$p_1 = [1 / (1 - T_R - T_L)] \{ [p_N(1 - T_R - T_L) + T_L] (\bar{s}_1 / \bar{s}_N) - T_R \} \quad (27)$$

Finally,

$$p_m = (1 - p_1 - p_N) / (N - 2) \quad (28)$$

All values of p_i are subject to the condition $0 < p_i < 1$.

It is instructive to reexamine Equation 11, which can be written as follows:

$$p_2 = [1 / (1 - T_R - T_L)] \{ [1 / 1 + (\bar{s}_1 / \bar{s}_2)] - T_L \} \quad (29)$$

It is seen that this expression is identical to Equation 26 for $N = 2$. It can also be shown that Equation 27 is valid for $N = 2$. Consequently, Equations 26-28 and the condition for p_i ($0 < p_i < 1$) are applicable for all approaches where the number of unchannelized lanes is two or more.

To calculate the lane-specific mix of traffic, proceed as follows:

$$P_N^L = T_L / [T_L + p_N(1 - T_R - T_L)] \quad P_N^R = 0 \quad (30)$$

$$P_1^R = T_R / [T_R + p_1(1 - T_R - T_L)] \quad P_1^L = 0 \quad (31)$$

$$P_m^L = P_m^R = 0 \quad m = 2, 3, \dots, N - 1 \quad (32)$$

For any lane i ,

$$P_i^T = 1 - P_i^L - P_i^R \quad (33)$$

Note that $P_i^x = 0$ whenever $T_x = 0$ ($x = L, R$).

When $p_N = 0$, i.e., when only left turners occupy lane N , it implies that $t_N > t_m$ and $t_N > t_1$. To calculate p_1 when $p_N = 0$, only the condition $t_1 = t_m$ can be applied. Equation 23 then becomes

$$[T_R + p_1(1 - T_R - T_L)] \bar{h}_1 = [(1 - p_1)(1 - T_R - T_L) / (N - 2)] \bar{h}_m \quad (34)$$

Then,

$$p_1 = 1 / [(N - 2) (\bar{s}_m / \bar{s}_1) + 1] \{ 1 - [T_R(N - 2) (\bar{s}_m / \bar{s}_1) / (1 - T_R - T_L)] \} \quad (35)$$

When $p_1 = 0$, i.e., when only right turners occupy lane 1, it implies that $t_1 > t_m$ and $t_1 > t_N$. To calculate p_N when $p_1 = 0$, only the condition $t_N = t_m$ can be applied. Equation 24 then becomes

$$[T_L + p_N(1 - T_R - T_L)] \bar{h}_N = [(1 - p_N)(1 - T_R - T_L) / (N - 2)] \bar{h}_m \quad (36)$$

Then

$$p_N = 1 / [(N - 2) (\bar{s}_m / \bar{s}_N) + 1] \{ 1 - [T_L(N - 2) (\bar{s}_m / \bar{s}_N) / (1 - T_L - T_R)] \} \quad (37)$$

When $T_L = 0$, it can be shown that

$$p_1 = 1 / [(N - 1) (\bar{s}_m / \bar{s}_1) + 1] \{ 1 - [(N - 1) T_R (\bar{s}_m / \bar{s}_1) / (1 - T_R)] \} \quad (38)$$

Then Equation 31 yields P_1^R and $P_m^L = P_m^R = 0$, where $m = 2, 3, \dots, N$. When $T_R = 0$, it can be shown that

$$p_N = 1 / [(N - 1) (\bar{s}_m / \bar{s}_N) + 1] \{ 1 - [T_L(N - 1) (\bar{s}_m / \bar{s}_N) / (1 - T_L)] \} \quad (39)$$

Then Equation 30 yields P_N^L and $P_m^L = P_m^R = 0$, where $m = 1, 2, \dots, N - 1$.

This analysis also applies, with minimal

modification, to approaches with one or more lanes channelized exclusively for turning vehicles and/or for buses. Hence, the model is completely general for approaches of any geometric configuration and traffic demand of any intensity. Note that traffic volume is not a parameter--i.e., it does not appear in the equations yielding p_1 and P_1^x .

The foregoing analysis has led to the development of formulas that permit the calculation of the lateral (lane-specific) deployment of traffic on an approach. The proportion of through and turning vehicles in each lane is shown to be dependent on known, or estimated, values of (a) number of lanes (N), (b) approach-specific turn-movement percentages (T_L and T_R for left- and right-turning vehicles, respectively), and (c) lane-specific service rates [\bar{s}_N , \bar{s}_m , and \bar{s}_1 for the inside (median), middle (if any), and outside (curb) lanes, respectively].

The values of n , T_L , and T_R are exogenous variables that must be known or estimated by the analyst. The mean value of service rates for through vehicles (s_T) must also be provided by the analyst. In addition, the service rates s_L and s_R , for unimpeded left- and right-turning vehicles, respectively, must be provided by the analyst. All of these service rates should be estimated by direct observation, preferably for the approach under consideration or for a similar approach configuration.

Since the middle lanes service only through vehicles, $\bar{s}_m = s_T$. The inside lane generally services a mix of left-turn and through vehicles. Its service rate (\bar{s}_N) is dependent on known, or estimated, values of (a) unimpeded left-turn service rate (s_L), (b) oncoming traffic volume (Q_{op}), (c) opposing approach geometry (number of through lanes, N_{op}), and (d) proportion of left-turning vehicles (P_N^L) in the lane.

It was shown that the value P_N^L is a function of, among other parameters, the mean service rate \bar{s}_N of traffic on the inside lane. Yet this service rate is not known a priori, since it, in turn, is a function of P_N^L . This condition requires the application of an iterative procedure that simultaneously produces the values of P_N^L and \bar{s}_N . Similar comments apply to the variables P_1^R and \bar{s}_1 (see Equations 1-3).

This iterative procedure is outlined below for $N \geq 2$:

1. Assert that $P_N^L = 1.0$ and that $P_1^R = 1.0$ and calculate $\bar{s}_1^{(0)}$ on this basis (the superscript attached to \bar{s}_1 denotes the iteration n .) Set $n = 1$.
2. Calculate p_N by using Equation 26. If $p_N < 0$, continue with step 3. Otherwise, continue with step 4.
3. Calculate p_1 by using Equation 35 and continue with step 6.
4. Calculate P_1 by using Equation 27. If $P_1 < 0$, continue with step 5. Otherwise continue with step 6.
5. Calculate p_N by using Equation 37 and continue with step 6.
6. Calculate p_m by using Equation 28 if $N > 2$, and P_N^L , P_1^R , and P_1^T with Equations 30-33.
7. Calculate $\bar{s}_N^{(n)}$ and $\bar{s}_1^{(n)}$ and compare with $\bar{s}_N^{(n-1)}$ and $\bar{s}_1^{(n-1)}$, respectively. If $\bar{s}_N^{(n)} \approx \bar{s}_N^{(n-1)}$

and $\bar{s}_1^{(n)} \approx \bar{s}_1^{(n-1)}$ within an acceptable tolerance, then the procedure is complete. Otherwise, set $n = n + 1$ and continue iterating at step 2 by using the most recent values of $\bar{s}_1^{(n-1)}$

Since the values of $\bar{s}_N^{(n)}$ and $\bar{s}_1^{(n)}$ each increase monotonically as P_N^L and P_1^R , respectively, decrease, the iteration will converge.

In view of the conditions implied by the cited variant of the first principle (i.e., $t_1 = t_2 = \dots = t_N$), it suffices to require that $t_1 = \text{minimum}$. Then, by using Equations 19-22, we may express the above requirement as follows:

$$\tau_N = [T_L + p_N(1 - T_L - T_R)]/\bar{s}_N = \text{minimum} \quad (40)$$

where $\bar{s}_N = (\bar{h}_N)^{-1}$. Similarly,

$$\tau_1 = [T_R + p_1(1 - T_L - T_R)]/\bar{s}_1 \quad (41)$$

and

$$\tau_m = (1 - p_1 - p_2)(1 - T_L - T_R)/(N - 2)\bar{s}_m \quad (42)$$

Note that the first principle is only satisfied for those (unchannelized) lanes that service some portion of the through component of traffic on an approach. A violation of this principle implies that traffic volume for one or more turning movements exceeds approach capacity for that movement. In this case, the value of t for that lane or those lanes will exceed the values of t for the lanes servicing the other turn-movement components of traffic, which implies over-saturated conditions.

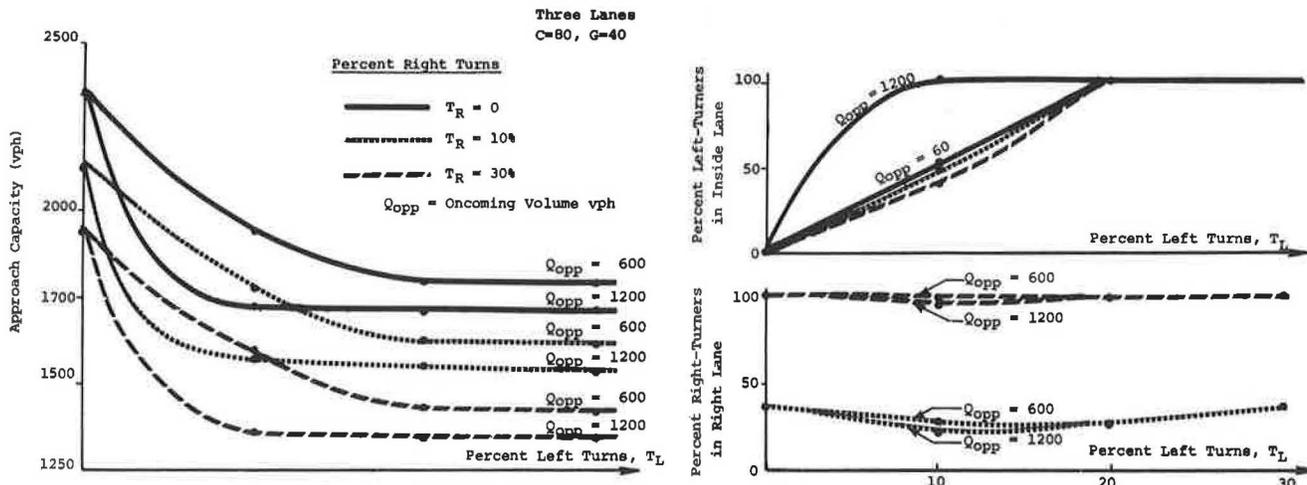
REPRESENTATIVE RESULTS

Consider a three-lane unchannelized approach to an intersection. The two-phase signal control operates on an 80-s cycle with a split of 50 percent. Left turners from the subject approach must contend with oncoming traffic and right turners with pedestrian interference. The model examines the effects on service volumes of varying turn percentages and of opposing volume on the oncoming approach (i.e., approach capacity).

Consider Figure 1 for the case of 10 percent right turners and $Q_{opp} = 600$ vehicles/h. At zero left-turn percentage, both the inside and middle lanes can provide saturation service volume for the through vehicles. The outside lane contains 38.6 percent right turners, and the remainder are through vehicles. The outside lane is less attractive to the through vehicles because the right turners discharge at a lower service volume (it was necessary to implement the entire analytical system of equations, including the service-rate models, to generate these results). Hence, only 15 percent (approximately) of the total through vehicles entering the approach select the outside lane with 85 percent split between the other two lanes.

At 10 percent left turners, the inside lane contains 47.1 percent left turners and 52.9 percent through vehicles when the opposing volume is 600 vehicles/h and 100 percent left turners when the opposing volume is 1200 vehicles/h. Since left turners encounter more impedance than right turners, the through vehicles are less attracted to the inside lane than to the outside lane even when both turn percentages are the same. In this case (opposing volume = 600 vehicles/h), 11 percent of all through vehicles select the inside lane while nearly 25 percent select the outside lane; the

Figure 1. Approach capacity and lane deployment as a function of several factors.



remaining 64 percent select the middle lane. Note that the percentage of through vehicles in the outside lane increased when the incidence of left turners rose to 10 percent.

At 17 percent left turners and an opposing volume of 600 vehicles/h, the inside lane contains only left turners and this movement experiences saturation conditions. That is, the service volume for left turners is at a maximum. Increasing the percentage of left turners entering the approach will create an unstable queue on the inside lane but will not change capacity.

It has been shown that lane deployment and approach capacity (i.e., service rates) are strongly influenced by many factors. The heightened level of detail embodied in this model--and in the associated service-rate models--provides results that cannot be extracted from models that depend on more simplifying assumptions.

A computer program exists that can produce the necessary curves and/or tabulations that will permit the development of a manual procedure to estimate approach, and intersection, capacity, based on this system of analytic models. Results produced by this system have been compared, in a limited study, with data reduced from 16-mm film, and the results are favorable. A more extensive validation effort would provide the necessary evidence for using this system as a reliable tool for estimating approach capacity.

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