approximation, the point-estimate method requires no complex derivations. Yet the point-estimate method is as accurate as the Taylor series approximation.

The normal distribution is not the only type of distribution that can be assumed. Since most geotechnical properties can never take on negative numbers, the lognormal distribution may be a more appropriate model. Another suggestion is to use a symmetrical beta distribution, which is bounded by zero and twice the mean.

In this paper, it was assumed that input variables were symmetrically distributed and, in the case of two or more variables, uncorrelated. However, Rosenblueth's method is not limited by these conditions.

ACKNOWLEDGMENT

We are grateful for the assistance received from members of the Soil Mechanics Bureau of the New York State Department of Transportation, whose director is Lyndon H. Moore.

REFERENCES

1. J.R. Benjamin and C.A. Cornell. Probability, Statistics, and Decision for Civil Engineers. McGraw-Hill, New York, 1970.
2. E. Rosenblueth. Point Estimates for Probability Moments. Proc., National Academy of Sciences, Vol. 72, No. 10, Oct. 1975, pp. 3812-3814.
3. T.W. Lambe and R.V. Whitman. Soil Mechanics. Wiley, New York, 1969.
4. M.E. Harr. Mechanics of Particulate Media. McGraw-Hill, New York, 1977.

Publication of this paper sponsored by Committee on Mechanics of Earth Masses and Layered Systems.

Short-Term Reliability of Slopes Under Static and Seismic Conditions

AKIRA ASAOKA AND DIMITRI ATHANASIOU-GRIVAS

A simplified probabilistic approach to the determination of the short-term ("$\phi_u = 0$") reliability of clayey slopes under static and seismic conditions is presented. The uncertainties associated with (a) the undrained strength of soil and its spatial variation and (b) the analytic procedure used to assess the safety of the slope are considered, and probabilistic tools are introduced for their description and amelioration. The probability of the failure of a slope under static loading is first determined. The effect of an earthquake on the slope is introduced through an equivalent horizontal peak acceleration (deterministic), and the new probability of failure is obtained by using Bayes' theorem. Finally, the developed procedure is illustrated in an example, the results of which are presented and discussed.

The factor of safety F_s of a slope of cohesive soil under undrained ("$\phi_u = 0$") conditions, determined from equilibrium of moments around the center of a circular failure surface (see Figure 1), is given as

$$F_s = R \int_L c_u \, dL / aW \tag{1}$$

where

R = radius of the circular failure surface,
L = length of the failure surface,
c_u = undrained shear strength of soil,
a = distance between W and the center of the circle, and
W = weight of the sliding soil mass.

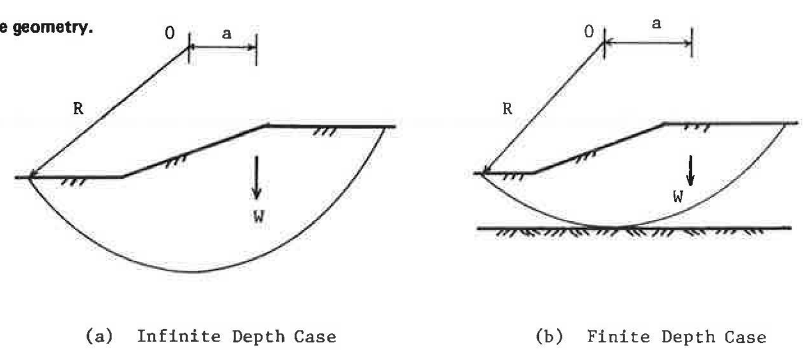
From Equation 1, it is seen that the total undrained shear strength of the slope is obtained by integrating c_u along the length L of the failure surface. If the soil medium is homogeneous and isotropic, then c_u is constant throughout the medium and the total resistance is equal to $c_u L$. In this case, the critical failure surface (i.e., the slip surface for which F_s becomes minimum) can be determined analytically. Thus, by expressing the equilibrium of moments around center O (Figure 1) as $aW = RL\tau$, or

$$aW = RL(N\gamma H) \tag{2}$$

where

τ = mean shear stress along the slip surface,
γ = unit weight of soil,
N = stability number (1), and
H = height of the slope,

Figure 1. Slope geometry.



and substituting Equation 2 into Equation 1, the latter becomes

$$F_s = (1/L \int_L c_u dL)/(N\gamma H) \tag{3a}$$

or

$$F_s = c_u/N\gamma H \tag{3b}$$

From Equation 3b, it is seen that the value of F_s is proportional to c_u while the location of the slip surface within the soil mass is independent of c_u. Charts that provide values for the stability number N as a function of slope angle β (in the case of an infinitely deep medium, as shown in Figure 1a) or angle β and depth factor H/D (in the case of a medium with finite depth, as shown in Figure 1b) can be found in reports by Taylor (1) and Terzaghi and Peck (2).

In the case of low embankments constructed on soft clay deposits, the shear resistance of the fill, within which tension zones commonly develop, may be neglected. Neglect of the embankment shear resistance may also be justified in the case of a stiff cohesive fill or a granular fill placed on soft clay. For these cases, special charts must be used to determine the corresponding values of the stability number N (3).

STRENGTH VARIABILITY WITHIN NATURAL DEPOSITS

In general, shear strength receives different values at different locations within a given soil deposit. This variation of strength is due to the inherent variability of the soil material, a fundamental cause of uncertainty in geotechnical engineering.

Let x and x' denote the horizontal coordinates of two points, A and A', respectively, that have a vertical distance equal to Δz. This is shown in Figure 2. If μ and σ denote the mean value and standard deviation, respectively, of shear strength c_u along the x-direction, while $r(\Delta z)$ denotes the value of the vertical autocorrelation coefficient r (4) for the two points A and A', one has that

$$E[c_u(x)] = \mu \tag{4a}$$

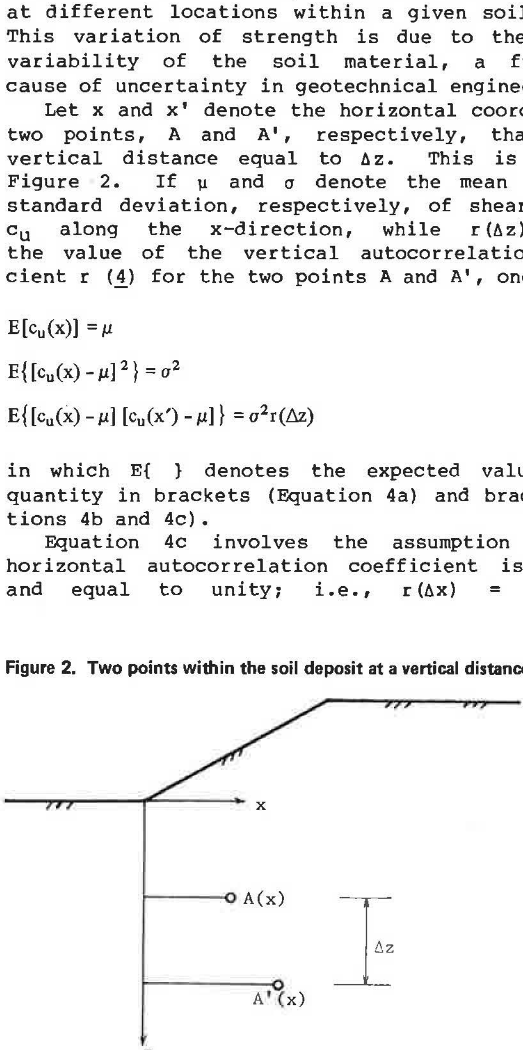
$$E\{[c_u(x) - \mu]^2\} = \sigma^2 \tag{4b}$$

$$E\{[c_u(x) - \mu][c_u(x') - \mu]\} = \sigma^2 r(\Delta z) \tag{4c}$$

in which E{ } denotes the expected value of the quantity in brackets (Equation 4a) and braces (Equations 4b and 4c).

Equation 4c involves the assumption that the horizontal autocorrelation coefficient is constant and equal to unity; i.e., $r(\Delta x) = 1$, where

Figure 2. Two points within the soil deposit at a vertical distance ΔZ.



$\Delta x = x - x'$. This corresponds to a soil deposit formed through a horizontal accumulation process (e.g., an alluvial clay deposit). Moreover, assuming that the process is stationary, one finds that $|r(\Delta z)| \leq 1$ and $r(\Delta z) = r(-\Delta z)$.

The mean value μ, variance σ^2, and autocorrelation coefficient $r(\Delta z)$ of the undrained shear strength c_u of a soil deposit can be determined through a statistical analysis of measured values of $c_u(x)$ and under the assumption that the process described by Equations 4 is ergodic. Thus, the expected values that appear in Equations 4 can be determined as spatial averages [e.g., $\mu = (1/V)\int_V c_u(x)dV$, where V denotes a volume of soil mass, etc.].

From Equations 4, the mean value and variance of the quantity $(1/L)\int_L c_u dL$, which appear in Equation 3a, can be written as

$$E[(1/L)\int_L c_u dL] = \mu \tag{5a}$$

$$Var[(1/L)\int_L c_u dL] = \sigma^2/\delta \tag{5b}$$

in which, Var[ ] denotes the variance of the quantity in brackets and δ is equal to

$$\delta = L^2/\int_L \int_{L'} r(\Delta z) dL dL' \tag{6}$$

Physically, quantity δ represents the number of statistically independent layers that a given soil deposit may be considered to consist of. From Equation 6, it is seen that the determination of the numerical value of δ requires a previous knowledge of the autocorrelation coefficient $r(\Delta z)$ of the shear strength c_u.

By combining Equations 3 and 5, it is found that the mean value and variance of the factor of safety F_s are equal to

$$E[F_s] = \bar{F}_s = \mu/N\gamma H \tag{7a}$$

$$Var[F_s] = \sigma_{F_s}^2 = \sigma^2/\delta(N\gamma H)^2 \tag{7b}$$

Furthermore, if it is assumed that c_u follows a normal distribution (5,6), then the integral of c_u is also normally distributed and, therefore, F_s is a normal variate. This may be written symbolically as

$$F_s = N(\bar{F}_s, \sigma_{F_s}) \tag{8}$$

where the right-hand side denotes the normal distribution with mean $\bar{F}_s$ and standard deviation σ_{F_s}.

Autocorrelation Coefficient of Strength

The autocorrelation coefficient $r(\Delta z)$ of c_u along the vertical direction can be conveniently expressed as an exponential function of Δz. On the basis of data corresponding to marine clay deposits found in Japan, Matsuo and Asaoka (7) determined the following expressions for $r(\Delta z)$:

$$r(\Delta z) = \exp(-|\Delta z|/\ell) \tag{9}$$

where the correlation length ℓ was reported to vary between 0.625 and 1.25 m, for the case of normally consolidated alluvial clay deposits.

When the correlation length and the thickness of a clay deposit are known, from Equation 9 one can determine the corresponding values of the autocorrelation coefficient r.

Number of Independent Layers

For a given value of the autocorrelation coefficient

r, from Equation 6 one can determine the number of independent layers (i.e., quantity δ) of which a clay deposit consists. Thus, for example, if r is given as $r(\Delta z) = \exp(-|\Delta z|)/0.91)$ and the depth of the soil deposit is equal to 8 m, the quantity $8^2/\int_0^8\int_0^8 \exp(-|z - z'|/0.91)dzdz' \approx 5$ determines that the 8-m-thick clay deposit is equivalent to five independent clay layers, each of which has a thickness equal to 8/5 = 1.6 m. This is shown in Figure 3. The number of independent clay layers corresponding to three correlation lengths and various depths of the clay deposit is given below:

Thickness of Clay Deposit (m)	No. of Equivalent Layers ℓ = 0.625 m	ℓ = 0.91 m	ℓ = 1.25 m
4	3.8	2.8	2.3
6	5.4	3.9	3.0
8	6.9	5.0	3.8
10	8.5	6.1	4.6
12	10.1	7.1	5.4
14	11.7	8.2	6.2

Moreover, if for this example the radius of the critical slip circle is R = 14 m, then the segments of the slip surface within the five layers have lengths (Figure 3) equal to ΔL_1 = 3.6 m, ΔL_2 = 4 m, ΔL_3 = 4.8 m, ΔL_4 = 5.6 m, and ΔL_5 = 12.8 m and the total length L of the slip surface is equal to 30.8 m. In this case, Equation 6 reduces to

$$\delta = L^2/\sum_{i=1}^{5}(\Delta L_i)^2 \tag{10}$$

from which, by substituting the values for L and ΔL_i, i = 1, ..., 5, it is found that

$$\delta = 30.8^2/(3.6^2 + 4^2 + 4.8^2 + 5.6^2 + 12.8^2) = 3.84 \tag{11}$$

Finally, an intuitive interpretation of δ can be obtained as follows.

Suppose that a given soil medium consists of n statistically independent and homogeneous layers, each of which has the same width ΔL along the slip surface and the same mean value μ and standard deviation σ for the undrained strength c_{u_i}; i = 1,...,n. If one expresses the quantity $(1/L)\int_L c_u dL$ as

$$(1/L)\int_L c_u dL = (1/n\Delta L)\sum_{i=1}^{n} c_{u_i}\Delta L = (1/n)\sum_{i=1}^{n} c_{u_i} \tag{12}$$

then the expected value of $(1/L)\int_L c_u dL$ is equal to

$$E[(1/L)\int_L c_u dL] = E\left[(1/n)\sum_{i=1}^{n} c_{u_i}\right] = (1/n)E\left(\sum_{i=1}^{n} c_{u_i}\right)$$
$$= (1/n)\sum_{i=1}^{n} E[c_{u_i}] \tag{13}$$

or, because $E[c_{u_i}] = \mu$, i = 1,...,n,

$$E[(1/L)\int_L c_u dL] = (1/n)n\mu = \mu \tag{14a}$$

Similarly, its variance is

$$\mathrm{Var}[(1/L)\int_L c_u dL] = (1/n^2)\sum_{i=1}^{n}\sigma^2 = (1/n^2)n\sigma^2 = \sigma^2/n \tag{14b}$$

If one compares Equation 14b and Equation 5b, it is seen that δ represents the number of statistically independent layers that are equivalent to the soil profile described by Equations 4a-4c.

UNCERTAINTY IN ANALYTIC PROCEDURE

As was mentioned in the introduction to this paper, the short-term stability of a slope (e.g., an embankment rapidly constructed on a saturated clay deposit) is commonly analyzed by using the ϕ_u = 0 method and assuming a circular slip surface. The successful performance of the structure, however, depends on how successfully the design engineer has identified and accounted for all of the uncertainties that are involved at the various stages of the project. These include site selection and investigation, sampling, testing, analysis, and design as well as the details associated with construction. Moreover, in simplifying the actual soil conditions so that a stability analysis can be applied, additional errors and uncertainties are introduced. These are inevitably reflected in the numerical value of the factor of safety of the slope.

Let F_s denote the value of the factor of safety, given by Equations 3a and 3b, and ε the magnitude of the error associated with this value. The "true factor of safety $\tilde{F}_s$" is then equal to

$$\tilde{F}_s = F_s + \epsilon \tag{15}$$

Failure should occur when the true factor of safety receives a value smaller than--or, at most, equal to--one, i.e., $\tilde{F}_s \leqslant 1$ (although the value of the safety factor F_s may be larger than one, i.e., $F_s > 1$).

Table 1 gives some of the main factors on which the value of the error term ε depends. A positive or negative sign for the associated errors ε_i is also indicated. In general, it is not possible to determine the exact value of each ε_i, since this depends on the inherent heterogeneity of soil, human factors, or some other agent of chance.

An alternative method in estimating the total error is to determine ε on the basis of previous case studies of slopes that have failed. For these, the factor of safety F_s and its true value $\tilde{F}_s$ are both known and ε can be determined from Equation 15 as $\varepsilon = \tilde{F}_s - F_s$. Since, for actual failures, $\tilde{F}_s$ = 1, one finds that

$$\epsilon = 1 - F_s \tag{16}$$

A statistical analysis of the results of a large number of case studies (8,9) has indicated that the error term ε receives values between -0.1 and 0.1 and follows closely a uniform distribution. The histogram of the values taken by ε is shown in Figure 4. A similar calculation of ε that made use of the data reported by Nakase (3) provided a somewhat smaller range for its variation (-0.04 to 0.07).

In this study, the error term that appears in Equation 15 is considered to be a uniformly distributed random variable within the interval -0.1 and 0.1.

PROBABILITY OF FAILURE OF SLOPES

Static Case (Prior Probability of Failure)

The true factor of safety $\tilde{F}_s$, given in Equation 15 as $\tilde{F}_s = F_s + \varepsilon$, is the sum of two random variables: the factor of safety F_s and the error term ε. Failure is defined as the event whereby $\tilde{F}_s$ receives values smaller than--or, at most, equal to--unity; i.e.,

$$\text{Failure} = [\tilde{F}_s \leqslant 1] = [F_s + \epsilon \leqslant 1] \tag{17}$$

Let G_o denote the specific conditions (e.g.,

Figure 3. Number of independent layers of a soil deposit obtained from the autocorrelation coefficient.

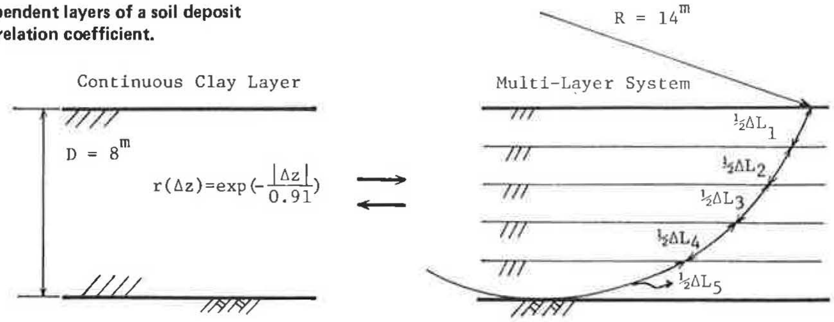
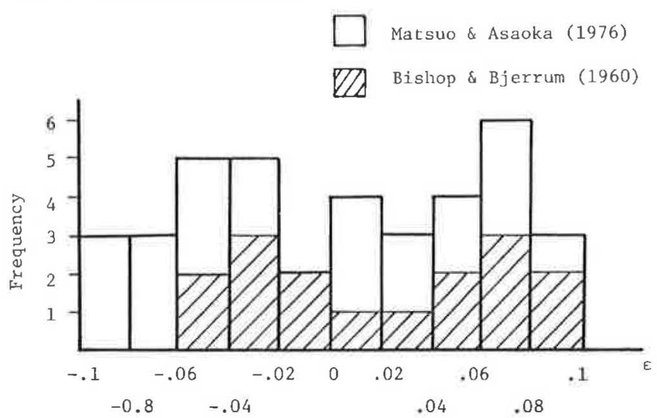


Table 1. Main factors that influence the error term.

Symbol	Factor	Associated Error
1	Stress relaxation of soil sample	$\epsilon_1 > 0$
2	Sampling disturbance	$\epsilon_2 > 0$
3	Plane strain (actual) and axial symmetric strain (testing)	$\epsilon_3 > 0$
3'	Plane-strain strength and vane strength	$\epsilon_{3'} > 0$
4	Progressive failure	$\epsilon_4 < 0$
5	Anisotropy of strength	$\epsilon_5 < 0$
6	Three-dimensional failure	$\epsilon_6 \gtrless 0$
7	Noncircular slip surface	$\epsilon_7 \gtrless 0$
8	Neglect of friction angle of soils	$\epsilon_8 > 0$
9	Tension crack at top of slope	$\epsilon_9 < 0$
10	Strength of embankment	$\epsilon_{10} > 0$
11	Loading rate (consolidation)	$\epsilon_{11} > 0$
12	Loading rate (creep)	$\epsilon_{12} < 0$

Figure 4. Histogram of error term ϵ.



geometry, material, boundary loading, etc.) of a slope before the occurrence of an earthquake and let $p_f(G_o)$ denote the probability of failure of the slope under conditions G_o. From Equation 17, one finds that $p_f(G_o)$ is equal to $p_f(G_o) = P[\text{failure}] = P[\tilde{F}_s \leq 1]$, or

$$p_f(G_o) = P[F_s + \epsilon \leq 1] \tag{18}$$

in which P[ ] denotes the probability of the occurrence of the event in brackets.

If $p_{F_s}(f_s)$ and $p_\epsilon(\epsilon)$ represent the probability density functions of F_s and ϵ, respectively, and $p_{F_s}(f_s)$ is a normal distribution and $p_\epsilon(\epsilon)$ a uniform distribution, the probability of failure $p_f(G_o)$ of the slope can be found from Equation 18 as follows:

$$p_f(G_o) = \iint_{F_s+\epsilon\leq 1} p_{F_s}(f_s)p_\epsilon(\epsilon)df_s d\epsilon$$

$$= \int_{-\infty}^{\infty}\int_{\epsilon_{min}}^{1-f_s} p_{F_s}(f_s)p_\epsilon(\epsilon)d\epsilon df_s$$

$$= \int_{-\infty}^{\infty} p_{F_s}(f)P_\epsilon(1-f)df_s \tag{19a}$$

or, equivalently,

$$p_f(G_o) = \int_{\epsilon_{min}}^{\epsilon_{max}} p_\epsilon(\epsilon)P_{F_s}(1-\epsilon)d\epsilon \tag{19b}$$

in which $P_\epsilon(\ )$ and $P_{F_s}(\ )$ denote the cumulative distribution functions of ϵ and F_s, respectively, and ϵ_{min} and ϵ_{max} the lower and upper limits of ϵ.

The probability of failure $p_f(G_o)$, given by Equation 19a or 19b, corresponds to the conditions G_o that prevailed before the occurrence of an earthquake. In this sense, $p_f(G_o)$ is said to provide the prior probability of failure.

Seismic Case (Posterior Probability of Failure)

When an earthquake occurs, an additional loading is applied on the soil mass that constitutes the slope. The new probability of failure of the latter, called the posterior probability of failure, can be determined (by using Bayes' theorem) as follows.

Let G_1 denote the new conditions on the slope (i.e., the initial conditions G_o plus the seismic load). The posterior probability of failure, denoted as $p_f'(G_1)$, is the probability of failure of the slope under conditions G_1, given that the slope was safe under conditions G_o (i.e., before the occurrence of the earthquake). This is formally written as $p_f'(G_1) = P[\text{failure under } G_1 \mid \text{slope safe under } G_o]$, or

$$p_f'(G_1) = P_F[G_1 \mid \tilde{F}_s > 1 \text{ under } G_o] \tag{20}$$

in which P_F denotes the probability of failure and $\tilde{F}_s > 1$ is the condition of safety (before the earthquake).

The procedure followed in determining the numerical value of $p_f'(G_1)$, for a given earthquake, is described below in the illustrative example.

Illustrative Example

The safety of the slope shown in Figure 5 is investigated before and after the occurrence of an earthquake. The slope has a height H = 6 m and angle $\beta = 30°$, the thickness of the clay deposit is 6 m, and the unit weight of the material is 1.7 t/m³. It is given that the undrained strength

c_u of the clay is normally distributed, with a mean value $\bar{c}_u$ = 2.2 t/m² and a standard deviation σ_{c_u} = 0.66 t/m². The error ε associated with the conventional factor of safety F_s is uniformly distributed between -0.1 and 0.1.

One wishes to determine (a) quantity δ (the number of independent layers), (b) the probability of failure of the slope, and (c) the new value of probability of failure of the slope for the case where an earthquake (deterministic) occurs and causes a horizontal acceleration to the soil mass equal to 0.2 g, where g is acceleration of gravity.

Solution

Number of Independent Layers

The soil profile consists of eight clay strata, each of which has a thickness equal to 1.5 m (Figure 5). The location within the soil profile of the failure surface is also shown in Figure 5. The lengths ΔL_i, i = 1,..., 8, of the eight segments of the failure surface are as follows: ΔL_1 = 2.2 m, ΔL_2 = 2.4 m, ΔL_3 = 2.5 m, ΔL_4 = 2.5 m, ΔL_5 = 5.7 m, ΔL_6 = 6.4 m, ΔL_7 = 7.2 m, and ΔL_8 = 15.7 m. The total length L of the failure surface is L = 44.6 m.

From Equation 10 we find that $\delta = L^2/\sum_{i=1}^{8} (\Delta L_i)^2$, where L^2 = 1989 m² and $\sum_{i=1}^{8} (L_1)^2$ = 395.9 m². Therefore,

$$\delta = 1989/395.9 = 5.03 \approx 5 \tag{21}$$

Probability of Slope Failure

For the given slope geometry, the value of the stability number N (1) is equal to N = 0.172. Since the undrained shear strength c_u has mean value and standard deviation equal to $\mu = \bar{c}_u$ = 2.2 t/m² and σ_{c_u} = 0.66 t/m², respectively, from Equations 7a and 7b, one finds that the mean value $\tilde{F}_s$ and standard deviation σ_{F_s} of the factor of safety F_s are

$$\bar{F}_s = \mu/N\gamma H = [2.2/(0.172)(1.7)(6)] = 2.2/1.75 = 1.26 \tag{22}$$

$$\sigma_{F_s} = \sigma_{c_u}/\sqrt{\delta}(N\gamma H) = 0.66/\sqrt{5}(1.75) = 0.17 \tag{23}$$

Therefore, the probability density function (pdf) of F_s can be written symbolically as

$$p_{F_s}(f_s) = N(1.26, 0.17) \tag{24}$$

i.e., p_{F_s} is normal with mean 1.26 and standard deviation 0.17.

The pdf of the error term ε is uniform and equal to

$$p_\epsilon(\epsilon) = 1/[0.1 - (-0.1)] = 5 \quad -0.1 \leq \epsilon \leq 0.1 \tag{25}$$

The cumulative distribution of ε evaluated at 1 - f_s, P_ϵ (1 - f_s), appearing in Equation 19a, is equal to

$$P_\epsilon(1 - f_s) = \begin{cases} 1 & f_s < 0.9 \\ 5(1.1 - f_s) & 0.9 \leq f_s \leq 1.1 \\ 0 & 1.1 < f_s \end{cases} \tag{26}$$

The probability of failure $p_f(G_o)$ of the slope can be determined from Equation 19a as follows:

$$p_f(G_o) = \int_{-\infty}^{0.9} N(1.26, 0.17)df_s + \int_{0.9}^{1.1} 5(1.1 - f_s) \times N(1.26, 0.17)df_s \tag{27}$$

or

$$p_F(G_o) = \Phi(0.9 - 1.26/0.17) + 5 \times 0.17 \int_{0.9}^{1.1} (1.26 - f_s/0.17) \times N(1.26, 0.17)df_s - 5 \times \int_{0.9}^{1.1} (1.26 - 1.1)\ N(1.26, 0.17)df_s \tag{28}$$

in which ϕ( ) denotes the tabulated Gauss function. After performing the indicated operations, it is found that $p_F(G_o)$ = 0.074 = 7.4 percent.

Probability of Slope Failure for Seismic Conditions

The horizontal force acting on the slope because of the earthquake is shown in Figure 6. The new overturning moment M_o' around the center of the failure surface is

$$M_o' = (8 \times W) + (0.2 \times W \times 6) = M_o + 0.2 \times W \times 6 \tag{29}$$

where M_o is the value of the moment without the earthquake force. The percentage of increase of the overturning moment $[(M_o' - M_o)/M_o]$ is 15 percent, and therefore the new safety factor F_s' is 1.15 times smaller than the value of F_s under static conditions; i.e., $F_s' = F_s/1.15$. The new true factor of safety is then equal to $\tilde{F}_s = F_s/1.15 + \epsilon$.

The posterior probability of failure $p_f'(G_1)$ can be determined from Equation 20 as follows:

$$p_f'(G_1) = \iint_{(f_s/1.15)+\epsilon<1} p'(f_s, \epsilon)df_s d\epsilon \tag{30}$$

in which

$$p'(f_s, \epsilon) = \begin{cases} p_{F_s}(f_s)p_\epsilon(\epsilon)/[1 - p_f(G_o)] & f_s + \epsilon > 1 \\ 0 & f_s + \epsilon < 1 \end{cases} \tag{31}$$

The posterior joint probability density function, $p'(f_s, \epsilon)$, is shown in Figure 7.

Figure 8 shows, on a (ϵ, f_s) coordinate system, the intersection between lines [-0.1, 1 - $(f_s/1.15)$] and, [1 - f_s, (0.1)] and the region of ε between [-0.1, (0.1)]. It is seen that the range of variation for f_s is as follows:

Null	:	$f_s < 0.9$
[1 - f, 0.1]	:	$0.9 \leq f_s \leq 1.035$
[1 - f_s, 1 - $(f_s/1.15)$]	:	$0.035 \leq f_s \leq 1.1$
[1 - 0.1, 1 - $(f_s/1.15)$]	:	$1.1 < f_s \leq 1.1$
Null	:	$1.265 < f_s$

Thus, by introducing the expression for p_ϵ in Equation 32 and performing the indicated integration, one obtains

$$\begin{aligned} p_f'(G_1) = & \int_{0.9}^{1.035} \{5[0.1 - (1 - f_s)]/[1 - p_f(G_o)]\}\, p_{F_s}(f_s)df_s \\ & + \int_{1.035}^{1.1} \{5[f_s - (f_s/1.15)]/[1 - p_f(G_o)]\}\, p_{F_s}(f_s)df_s \\ & + \int_{1.1}^{1.265} \{5[1.1 - (f_s/1.15)]/[1 - p_f(G_o)]\}\, p_{F_s}(f_s)df_s \end{aligned} \tag{32}$$

Finally, by substituting into the above expression $p_f(G_o)$ = 0.074 and $p_{F_s}(f_s)$ = N(1.26, 0.17), it is found that $p_f'(G_1)$ = 21.9 percent.

Figure 5. Slope geometry and failure surface of illustrative example.

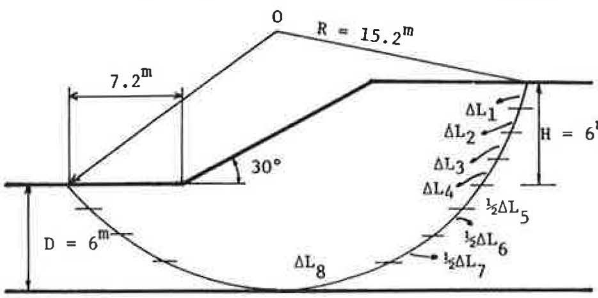


Figure 6. Forces on the slope mass during the earthquake.

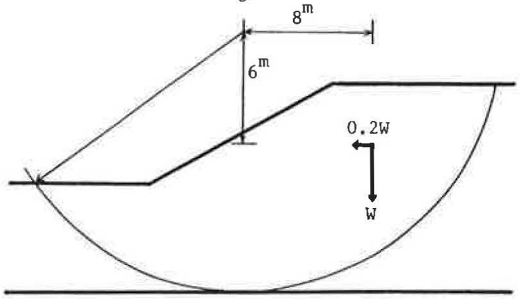


Figure 7. Posterior joint distribution for f_s and ϵ.

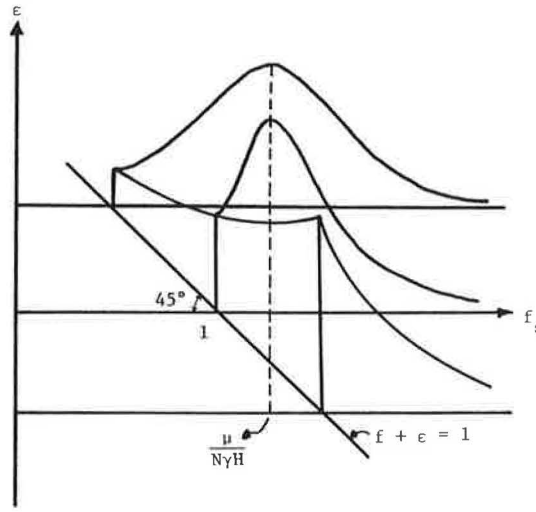
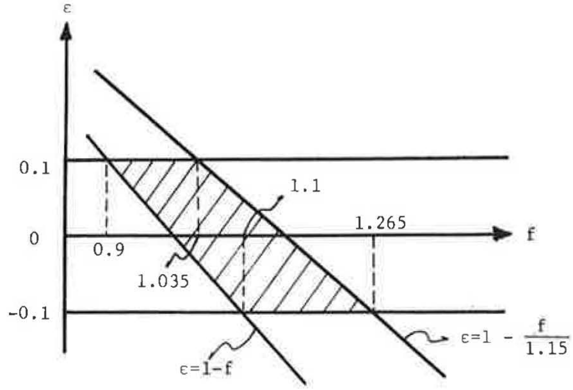


Figure 8. Region for integration of Equation 17.



DISCUSSION OF RESULTS

This probabilistic analysis of the stability of slopes under undrained ($\phi_u = 0$) conditions was formulated on the basis of the following two considerations:

1. That the undrained strength of soil is a random variable with a spatial (vertical) autocorrelation and
2. That the uncertainty associated with the method of analysis used can be accounted for through an error term, the limits and distribution of which can be determined empirically.

The uncertainty in the values of the shear strength c_u of soils is due to the inherent variability of the soil medium. Under the assumption that c_u is a normal variate, it was found that the factor of safety F_s is also normally distributed. This is a reasonable assumption for the case in which the mean value of c_u is at least three times larger than its standard deviation (as in the illustrative example). Otherwise, the assumption of normality would result in a considerable probability of a negative value of c_u, a situation incompatible with the definition of strength. When this is the case, a different (e.g., beta) distribution for c_u must be used (10,11).

The uncertainty in the method of analysis is due to many reasons, the most important of which are given in Table 1. To account for this uncertainty, an error term was introduced and its distribution was determined empirically by using many case records of actual slope failures. It was found that ϵ varies uniformly between -0.1 and 0.1.

By using the developed procedure, one can easily obtain the probability of failure of a slope under undrained conditions without extensive calculations. This is considered to be a great advantage of the method, particularly for its application to common practical problems.

By using Bayes' theorem, it was possible to update the probability of failure of the slope for the case in which additional seismic loading was introduced. It should be noted that the expression for the posterior probability of failure p_f', given by Equation 20, is quite general and can be used to determine the probability of failure during any change in the slope conditions.

In an illustrative example, it was found that the probability of failure of a given slope was 7.4 percent while the corresponding value of the factor of safety was 1.25. When an earthquake occurred, causing a horizontal ground acceleration equal to 0.2 g, the probability of failure (posterior) received a value equal to 21.9 percent. This corresponds to an increase in p_f of approximately a factor of three. For this case, the value of the factor of safety F_s was equal to 1.26/1.15 = 1.10; i.e., F_s decreased by only 13 percent!

A non-Bayesian analysis was also applied for the seismic conditions of the illustrative example. This involved the determination of the probability of failure of the slope by considering that the static and seismic forces acted concurrently. It was found that p_f was equal to 27.7 percent, a value greater than the posterior probability of failure (21.9 percent).

Finally, to determine the effect on p_f of the assumption that the error term ϵ is uniformly distributed, the illustrative example was also solved for the case in which ε follows a normal distribution. The results are given below:

Loading Condition	Probability of Failure (%) Uniform ε	Normal ε
Static	7.4	7.4
Seismic		
Bayesian	21.9	22.8
Non-Bayesian	27.7	28.5

It can be seen that the two distributions of ε give almost identical values for the probability of failure.

SUMMARY AND CONCLUSIONS

This paper has presented a probabilistic approach to determination of the short-term stability of slopes under static and seismic conditions. Two important uncertainties were considered: (a) the uncertainty in soil strength and its spatial variation and (b) the uncertainty in the method of analysis used. The developed approach was illustrated in an example, and the results obtained were presented and discussed.

On the basis of the analysis and results of this study, the following conclusions are drawn:

1. The probability of failure of slopes can be determined by exploring the uncertainties involved in both material strength and method of analysis.
2. The effect of seismic conditions on the reliability of slopes can be accounted for by using Bayes' theorem.

ACKNOWLEDGMENT

The work discussed in this paper was part of a research study conducted at Rensselaer Polytechnic Institute and partly supported by the National Science Foundation.

REFERENCES

1. D.W. Taylor. Fundamentals of Soil Mechanics. Wiley, New York, 1948.
2. K. Terzaghi and R.B. Peck. Soil Mechanics in Engineering Practice. Wiley, New York, 1967.
3. A. Nakase. Stability of Low Embankments on Cohesive Soil Stratum. Journal of Japanese Society of Soil Mechanics and Foundation Engineering, Vol. 10, No. 4, 1970, pp. 40-64.
4. A. Papoulis. Probability, Random Variables, and Stochastic Processes. McGraw-Hill, New York, 1965.
5. J.A. Hooper and F.G. Butler. Some Numerical Results Concerning the Shear Strength of London Clay. Geotechnique, Vol. 16, 1966, pp. 282-304.
6. P. Lumb. The Variability of Natural Soils. Canadian Geotechnical Journal, Vol. 3, May 1966, pp. 74-97.
7. M. Matsuo and A. Asaoka. Probability Models of Undrained Strength of Marine Clay Layer. Journal of Japanese Society of Soil Mechanics and Foundation Engineering, Soils and Foundations, Vol. 17, No. 3, 1977, pp. 1-16.
8. M. Matsuo and A. Asaoka. A Statistical Study on a Conventional Safety Factor Method. Journal of Japanese Society of Soil Mechanics and Foundation Engineering, Soils and Foundations, Vol. 16, No. 1, 1976, pp. 75-90.
9. A.W. Bishop and L. Bjerrum. The Relevance of the Triaxial Test to Solution of Stability Problems. Proc., ASCE Special Conference on Shear Strength of Cohesive Soils, Boulder, CO, 1960, pp. 437-501.
10. D. Athanasiou-Grivas. Probabilistic Evaluation of Safety of Soil Structures. Journal of Geotechnical Engineering Division, ASCE, Vol. 105, No. GT9, Sept. 1979, pp. 1091-1095.
11. D. Athanasiou-Grivas, J. Howland, and P. Tolscer. A Probabilistic Model of a Seismic Slope Stability Analysis. Renssalaer Polytechnic Institute, Troy, NY, Rept. CE-79-1, 1979.

Publication of this paper sponsored by Committee on Mechanics of Earth Masses and Layered Systems.

Model for Assessing Slope Reliability

KEVAN D. SHARP, LOREN R. ANDERSON, DAVID S. BOWLES, AND RONALD V. CANFIELD

Traditionally, an evaluation of the safety of slopes has been based on computing a safety factor against failure. In computing the safety factor, the geometry of the slope, the soil parameters, and the pore pressures are treated as deterministic quantities even though they are known to be random variables. Vanmarcke has developed a three-dimensional probabilistic slope-stability model that treats shear strength as a random variable. The model uses the probability of a slope failure as an assessment of slope reliability. A probabilistic slope-stability model that is an extension of Vanmarcke's model is presented. The model can accommodate zoned embankments of soil in which the strength is described by the Mohr-Coulomb strength envelope. Autocorrelation functions are used to describe the spatial variation of the mean and standard deviation of the strength parameters, c and tan ϕ. Several examples are presented to illustrate the influence of the choice of the statistical soil parameters on the probability of failure. The results show that the critical failure surface based on the minimum safety factor is not necessarily the failure surface that will yield the maximum probability of failure.

The safety of embankments depends on many factors, including the correctness of design assumptions, the adequacy of quality control during construction, the level of inspection and maintenance, the skill of the operators where the embankment impounds water, and the occurrence of various natural phenomena such as floods, earthquakes, and landslides. A complete evaluation of all of the factors that contribute to embankment safety is very complex, and procedures for developing and using this type of information in benefit/cost analyses are still in the formulative stages. The Federal Coordinating Council for Science Engineering and Technology (1) has identified the application of probabilistic methods and risk analysis to dam project development as an important area that needs research. Although progress is be-