Eccentrically Loaded Surface Footing on Sand Layer Resting on Rough Rigid Base

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Laboratory model-test results are given for the ultimate bearing capacity of an eccentrically loaded rough rigid strip surface footing on sand that has a rough rigid base located at a shallow depth. For centrally loaded footing, the modified bearing-capacity factors calculated from the experimental ultimate loads are compared with the existing theory as presented by Mandel and Salencon. When the ratio of the depth of the sand layer to the width of the footing is smaller than 0.6, the experimental bearing capacity is somewhat lower than that predicted by theory by using the direct shear angle of friction. However, if the experimental variation of the bearing-capacity factor for the centrally loaded footing is assumed to be correct, Meyerhof's effective-area method may be used to estimate the bearing capacity of eccentrically loaded footing.

The ultimate bearing capacity of a shallow continuous footing in a homogeneous soil subjected to centric loading is generally expressed by the following equation:

$$q_u = cN_c + qN_q + \frac{1}{2}\gamma BN_\gamma \qquad (1)$$

where

q_u = ultimate load per unit area of footing,
c = cohesion of soil,
γ = soil unit weight,
B = width of footing, and
N_c, N_q, and N_γ = bearing-capacity factors.

For a footing placed at the surface of a cohesionless soil, $c = 0$ and $q = 0$; hence, Equation 1 will transform to the following form:

$$q_u = \frac{1}{2}\gamma BN_\gamma \qquad (2)$$

A number of solutions for the variation of the bearing-capacity factor N_γ with the angle of friction of the soil have been proposed in the past by such investigators as Terzaghi (1), Meyerhof (2), Caquot and Kerisel (3), and Lundgren and Mortensen (4).

If a rough rigid continuous surface footing that rests on a sand layer is subjected to a uniformly distributed load of magnitude q_u, bearing-capacity failure will take place, and the slip lines will extend to a depth H_{cr} as shown in Figure 1a. However, if we consider a condition in which a rough rigid base is located at a shallow depth H so that $H < H_{cr}$ (Figure 1b), the development of the slip lines at failure will be somewhat affected. Mandel and Salencon (5) have developed a theoretical solution for the ultimate bearing capacity for such cases, and it can be given by the following equation:

$$q_u = \frac{1}{2}\gamma BN'_\gamma \qquad (3)$$

where N_γ' is the modified bearing-capacity factor.

According to Mandel and Salencon (5), for $H \geq H_{cr}$ the value of N_γ' becomes equal to N_γ as determined by Lundgren and Mortensen (4). On the other hand, for $H < H_{cr}$ the modified bearing-capacity factor increases with the decrease of H/B. The theoretical variation of N_γ' with H/B is shown in Figure 2 (5,6) for several values of the soil friction angle ϕ.

Experimental studies in the laboratory have been conducted by Meyerhof (6) and by Pfeifle and Das (7) to compare the experimental values of the modified bearing-capacity factor with the theoretical values presented in Figure 2. This study is not a repetition of any previous work by Das (7).

The purpose of this paper is to present some small-scale laboratory model-test results for the ultimate bearing capacity of eccentrically loaded rough rigid surface footings on sand that have a rough rigid base located at a shallow depth as shown in Figure 3 and to evaluate whether the effective-area concept as suggested by Meyerhof (8) can be used in this case. It must be pointed out that Meyerhof's effective-area concept was originally suggested for conditions in which the rough rigid base is located at great depths (i.e., $H/B > H_{cr}/B$). According to the effective-area concept, if a load is applied on a strip footing that has an eccentricity e measured from the center line, it can be assumed to be equivalent to a strip footing of width $B' = B - 2e$ that has the load applied along the center line as shown in Figure 4.

EXPERIMENTAL PROCEDURE

Model tests were conducted in a box that measured 0.915 m x 0.3048 m x 0.457 m (3 ft x 1 ft x 1.5 ft). The walls of the box were reinforced by steel channels against possible yielding during tests. The bottom of the box was a wooden plank 50.8 mm (2 in) thick. In order to make the bottom of the box rough, a sand-glue mixture was spread over a masonite panel. This was allowed to dry for several days and was then attached to the bottom of the box by means of wood screws. The sand used for the sand-glue mixture was the same sand used for the model tests.

Figure 1. Bearing-capacity failure for rough rigid surface footing on sand.

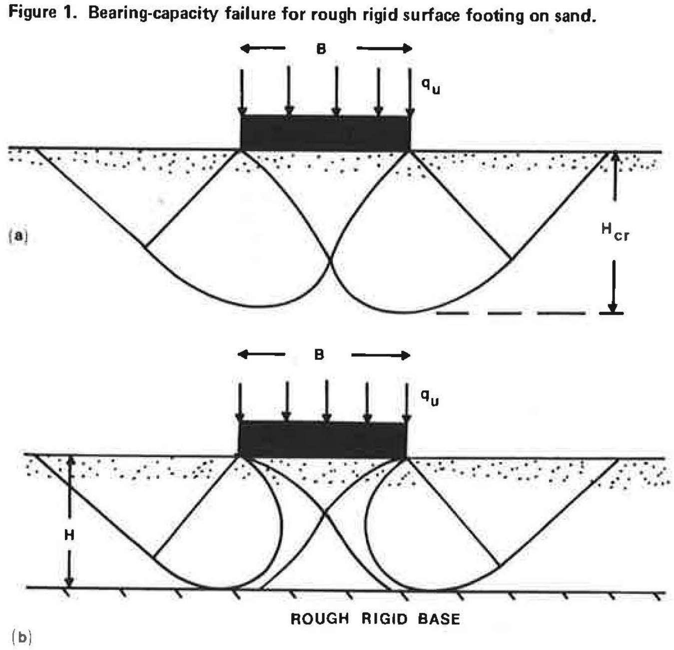


Figure 2. Variation of N_γ' with H/B and ϕ.

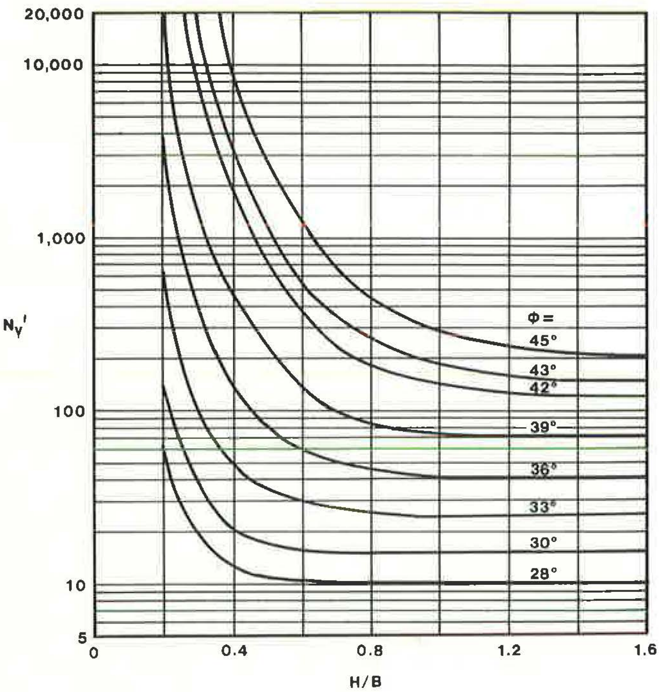
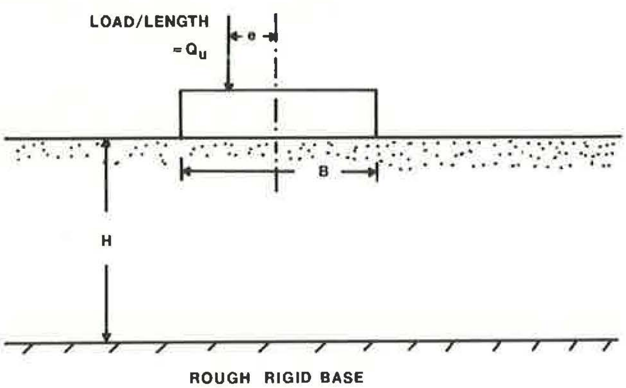


Figure 3. Geometric parameters for bearing-capacity tests with eccentric loading and rough rigid base at shallow depth.

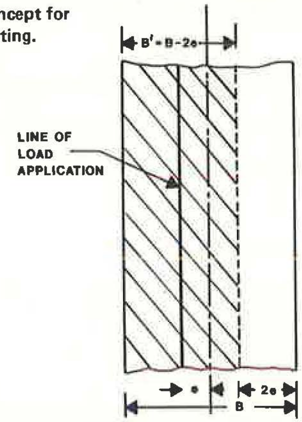


The sand used for the test had 100 percent passing through a no. 10 sieve, 52 percent passing through a no. 40 sieve, and 4 percent passing through a no. 200 sieve. For the model tests, the sand was compacted in small layers in the test box to the required depth to an average unit weight of 16.0 kN/m³ (101.8 lbf/ft³). This yielded an average relative density of compaction of 80 percent. At this compaction, the triaxial angle of friction was determined to be 39° and the direct shear angle of friction was 42°.

The model footing was made of a 12.7-mm (0.5-in) steel plate that measured 101.6 mm x 304.8 mm (4 in x 12 in). The bottom of the model footing was made rough by spreading a similar sand-glue mixture as that used for the bottom of the box and allowing it to dry.

Vertical load was applied to the model footing by a hydraulic jack through a steel shaft 38.1 mm (1.5 in) in diameter. The bottom of the shaft was enlarged and rounded. Semicircular grooves into which

Figure 4. Equivalent-area concept for eccentrically loaded strip footing.



the bottom of the shaft would just fit were cut on the top of the footing. This allowed free rotation of the footing during failure. These grooves were cut parallel to the center line of the footing at distances of e = 0, 12.7 mm (0.5 in), and 19.05 mm (0.75 in). During the model tests, the load was measured by a proving ring. The deflection along the center line of the footing was measured by a dial gauge. Tests were conducted at various values of e/B and H/B. Some grain crushing occurred during the tests at lower values of H/B.

MODEL-TEST RESULTS

Diagrams of typical load per unit length versus settlement of footing for the model footing at H/B = 0.375 determined from the laboratory tests are shown in Figure 5. The ultimate load per unit length of the footing at failure Q_u was determined from these diagrams. Figure 6 shows a plot of Q_u versus H/B for all the tests conducted in this program.

For tests in which the loads on the footing were centrally applied (i.e., e/B = 0), the modified bearing-capacity factor can be given by $Q_u = q_u(B)(1) = 1/2\gamma B^2 N'_\gamma$ (Equation 3), or

$$N'_\gamma = Q_u/(0.5\gamma B^2) \tag{4}$$

By using the experimental values of Q_u (for the tests in which e/B = 0) given in Figure 6, the values of N_γ' were determined and are shown in Figure 7. For comparison purposes, the theoretical values of N_γ' (from Figure 2) for ϕ = 39°, 42°, and 43° have been plotted in Figure 7. A comparison of the experimental and theoretical values shows the following:

1. The experimental values of N_γ' are higher than those presented by theory with ϕ = 39°, which is the triaxial angle of friction. However, for tests of continuous footing, the plane strain friction angle should be used. The experimental results of N_γ' are fairly close to those predicted by using friction angles determined from direct shear tests up to a value of H/B $\geq$ about 0.6. In the region of H/B between 0.6 and 0.4, the experimental values are lower than those of the theory (by using direct shear angle of friction).

2. The experimental values of N_γ' for H/B < 0.4 are lower than those predicted by theory by using the triaxial friction angle of 39°. There can be several factors that would cause the type of experimental results obtained here and their derivations from the theory for the zone of H/B < 0.6. They are as follows: (a) the sand-placement technique makes the sand anisotropic; (b) although a

Figure 5. Typical plot of load per unit length of footing versus settlement.

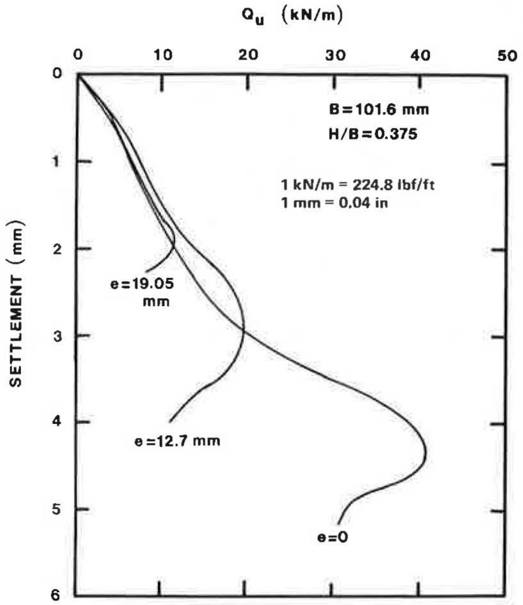
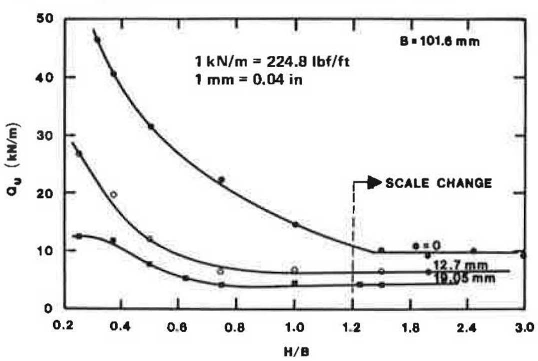


Figure 6. Q_u versus H/B obtained from laboratory tests.



quantitative evaluation was not made, some grain crushing did occur during the tests; and (c) there was curvilinearity of the Mohr-Coulomb failure envelope at very high pressures.

However, an evaluation of the model-test results shows that, if the experimental variation of N_γ' versus H/B is assumed to be correct, the ultimate bearing capacity of the eccentrically loaded footing can be evaluated approximately by using the effective-area concept presented by Meyerhof (8) for footings in which $H/B > H_{cr}/B$. Hence,

$$B' = B - 2e \quad (5)$$

where B' is the effective width.

For surface footings, the ultimate load per unit length can be expressed as follows:

$$Q_u = \tfrac{1}{2}\gamma B' N_\gamma' (B')(1) = \tfrac{1}{2}\gamma B'^2 N_\gamma'$$

$$N_\gamma' = Q_u/(0.5\gamma B'^2) = Q_u/[0.5\gamma(B - 2e)^2] \quad (6)$$

Note that N_γ' is now a function of H/B'. For centrally loaded footings, H/B' = H/B. By using the values of Q_u and e given in Figure 6 and by using Equation 6, the experimental values of N_γ' for all tests have been determined and are shown in

Figure 7. Comparison of experimental N_γ' with theory (e/B = 0).

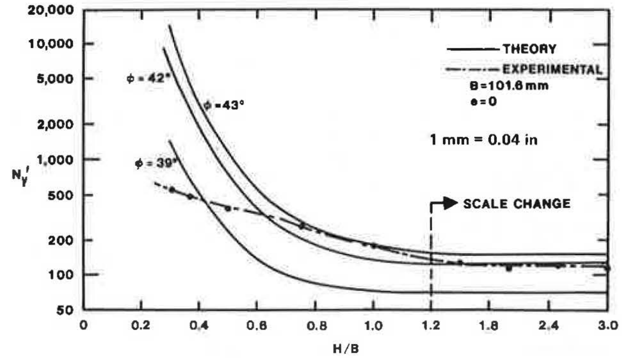


Figure 8. Variation of experimental N_γ' with H/B.

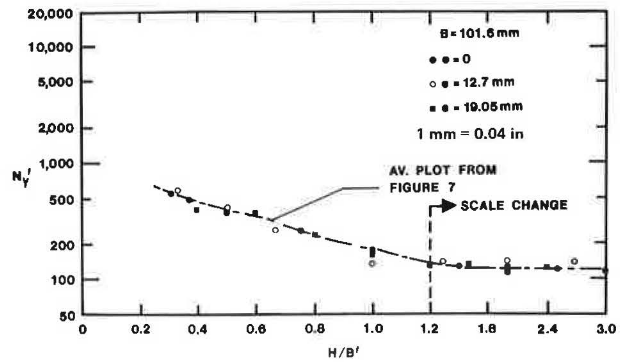


Figure 8. The average experimental plot of N_γ' versus H/B shown in Figure 7 is also replotted in Figure 8 for comparison. It can be seen that, although there is some scattering, the average N_γ' versus H/B' plot is the same for all tests.

CONCLUSIONS

Laboratory model tests have been presented for the ultimate bearing capacity of rough rigid continuous surface footings on sand that have a rough rigid base located at a shallow depth. Based on the model-test results, the following conclusions can be drawn:

1. For centrally loaded footings (i.e., e/B = 0), the experimental modified bearing-capacity factor N_γ' compares reasonably well with the theory (with the assumption that the direct shear angle of friction is valid) only in the range of H/B > 0.6. For H/B < 0.4, the experimental values of N_γ' are lower than those predicted by theory, even by using the triaxial friction angle. Grain crushing, curvilinearity of the Mohr-Coulomb failure envelope at high normal stress, and possible anisotropy in sand due to the placement technique may be responsible for such results.

2. If the experimental variation of N_γ' with H/B' for centrally loaded footings is assumed to be correct, the ultimate bearing capacity of eccentrically loaded surface footings can be reasonably estimated by using the concept of effective area for continuous footing, $Q_u = 1/2\gamma(B - e)^2 N_\gamma'$.

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