further study of the influence of seam yielding appears beneficial.

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Design Method for Concrete Pipe Under High Fills

J. NEIL KAY AND STEPHEN J. HAIN

A method is proposed for design of reinforced concrete pipe installed in embankment conditions under high fills. At the current stage, the method is restricted to pipes bedded in the natural soil. The method widely used at this time has seen little change since its introduction more than 40 years ago, and recent research has indicated that it is highly conservative for present practice. A study of the complex soil-structure interaction system has been made by using the finite-element method. A simplified linear-elastic approach has enabled comparison of three-edge bearing test conditions with those encountered by a pipe in the field in such a way as to provide curves suitable for a rapid design procedure. Justification for such an approach can be obtained from review of field response data for such conditions. For a large-scale field study the method predicts the cover height to produce cracking within about 20 percent, whereas the conventional procedure provides a safety margin in the ratio of 4.6:1.

On the basis of his own work and that of his colleagues, Spangler (1) proposed a method for concrete pipe design. He recognized the interaction effects of the pipe, the underlying soil, the pipe bedding, and the compacted fill and suggested a parameter, the settlement ratio r_{sd}, defined as follows:

$$r_{sd} = [(S_m + S_g) - (S_f + d_c)]/S_m$$

where

S_m = compression of soil on either side of pipe,
S_g = settlement of natural ground adjacent to conduit,
S_f = settlement of conduit into its foundation, and
d_c = change in vertical height of conduit.

The settlement ratio was used in conjunction with the projection ratio, the lateral pressure coefficient, the soil friction coefficient, the soil unit weight, the height of soil cover, and the pipe diameter to obtain an equivalent total load on the pipe (W_c). On the basis of bedding type, a load factor L_c was estimated. The required class of pipe was then determined from the three-edge bearing strength (S_{eb}) as follows:

$$S_{eb} = W_c/L_f$$

The theory was based on the ultimate resistance to movement on boundaries of a rectangular prism of soil above the pipe. Crude guidelines for selection of r_{sd} were suggested by Spangler based on back-calculation from field tests (2), but the basis for proposals for appropriate values of lateral earth pressure coefficient, friction angle, and load factor, all extremely uncertain, is not so evident. Review of the field tests indicates that the proposed values for r_{sd} are questionable, particularly in the light of the other uncertainties.

Nearly five decades and many computer hours of concrete pipe studies later, the Spangler approach remains the conventional approach to the problem, in spite of the fact that capabilities for analysis are now many times greater than those that were available to Spangler. Some finite-element studies of the buried reinforced concrete pipe problem have been completed (3-5), but they either constitute a parametric study of particular aspects of the problem or offer a complex programming procedure for the system that is not usable for ordinary design purposes.

In this work certain simplifications have been made to enable presentation of results from a comprehensive finite-element study in a simple graphical form. Both the pipe and the soil are assumed to deform in a linear manner under load. Some nonlinearity is to be expected for both the pipe and the

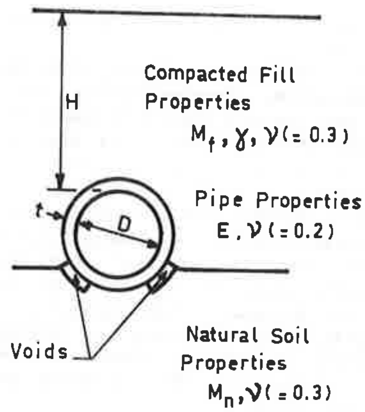


Figure 1. Details of pipe problem.

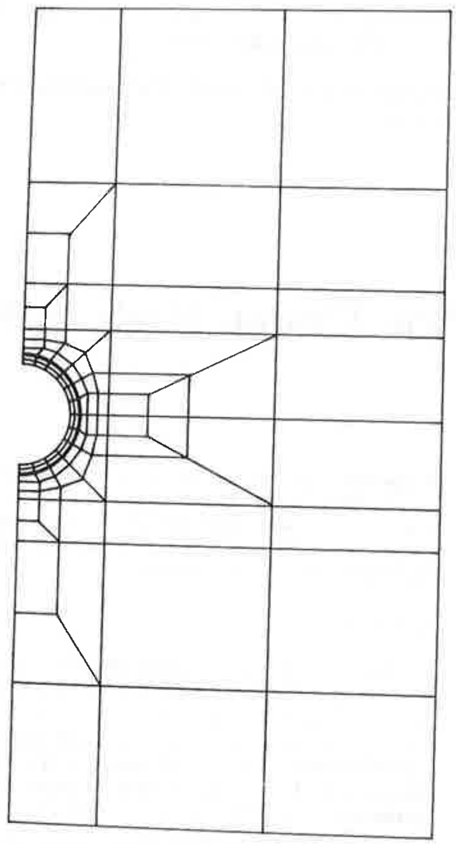


Figure 2. Arrangement of finite-element mesh.

soil, but there is considerable justification for use of the linear model in this context. First, there is considerable uncertainty associated with selection of even average linear parameters. Second, evidence from field measurements of pipe deflections and stresses appears to indicate that a straight line is an equally acceptable fit to results from complex nonlinear models (3).

Only the first of a range of problems is considered in this work. Finite-element results indicated that for cover heights greater than about two pipe diameters, the pipe response associated with a given uniform load increment was constant. The graphs are based on response levels for cover heights beyond two pipe diameters and give conservative results for cases where the cover height is less. A separate study will be needed for cases of cover heights less than two diameters. In many of these cases concentrated vehicle loads rather than fill loads will dominate the design.

In addition, only one type of bedding is considered at this stage--a shaped bedding in the natural soil. The profile is illustrated in Figure 1. The area of shaped-bedding contact has an included angle of 60 degrees. For a further 30 degrees on either side, a void space is introduced to account for the practical difficulties of soil compaction in this area.

FINITE-ELEMENT MODEL

The model considered by the finite-element system was an elastic ring that represented the reinforced concrete pipe contained in a two-layer system of elastic soil as shown in Figure 1. The pipe is bedded over an angle of 60 degrees in the lower stratum and has void spaces on either side to simulate real voids or poor compaction at that location.

The vertical plane of symmetry at the center of the soil-pipe system was used in constructing the finite-element mesh indicated in Figure 2. The side and bottom boundaries both extended to 4.125 diameters' distance from the pipe centerline. The upper boundary was varied from a cover height of 0.75 to 9.5 diameters above the pipe. Eight-node isoparametric elements were used for the soil and the pipe wall, and a quadratic shape function was incorporated to closely approximate the circular shape of the pipe. The pipe wall thickness was 1/16 of the pipe diameter.

It should be noted that it was not possible to vary the ratio of pipe diameter to wall thickness (d/t) for the various finite-element analyses. Such variation would have made the required computing effort prohibitive. However, similitude considerations indicate that independent control of d/t is not necessary in a general sense. Control of the flexibility parameter $[(M_f/E)(d/t)^3$, where M_f is the compacted-fill constrained modulus and E is the pipe Young's modulus] is sufficient. Appropriate variation of M_f and E provided the necessary data for the required range of flexibility parameter. This approach left some error associated with the geometric effects of wall thickness where d/t varied from 16; however, this error will be small. A similar but considerably greater error would have occurred if a thin ring had been used to represent the pipe.

A uniformly distributed load was applied at the upper boundary. This method was selected in preference to a material self-weight approach, because it permitted a simpler evaluation of the problem. The influence of compaction of the fill on either side of the pipe is neglected. This is conservative for a 0.01-in (25-mm) crack-width failure criterion and will have only a small influence at high cover heights. It was found that beyond a cover height of two pipe diameters the response was independent of cover height; that is, for a given change in uniformly distributed load at the upper boundary, there is similar change in pipe diameter or pipe stress level independent of the boundary location. Whether the load was a thin layer of finite thickness with some unit weight or an equal uniformly applied stress at the layer center made little difference.

The accuracy of the finite-element model was established by comparison of results with those obtained from an idealized theoretical model. Burns and Richard (6) have developed a closed-form solution for an elastic tube surrounded by a homogeneous isotropic linear-elastic infinite medium subject to an overpressure. Pipe and soil elements were given similar modulus values in the finite-element and the closed-form models and both were subjected to the same overpressure. Results indicated very close agreement between the two methods, and data generated from the finite-element model should therefore be relatively free from numerical errors.

BASIS FOR CONSIDERATION OF REAL PIPE RESPONSE

An alternative to the traditional 0.01-in crack as the criterion for failure is to use the more conventional methods for general reinforced concrete design. However, the three-edge bearing test associated with this criterion is a widely accepted method of pipe quality control that provides the manufacturer with some flexibility in product design. It would therefore appear to be a desirable procedure and should be maintained. Field experience seems to indicate that crown or invert tension cracking precedes other possible failure modes such as compression or diagonal shear. Continued use of the 0.01-in crack criterion is proposed.

In order to use the linear-elastic ring approach and to equate failure conditions in a three-edge bearing test with failure conditions in the field, a hypothetical outer-fiber stress is used. The conventional approach is to equate failure in the field with failure in the three-edge bearing test in both cases in terms of the 0.01-in crack. The pipe in the three-edge bearing test at the failure condition has associated with it a bending moment at the pipe crown. The hypothetical outer-fiber stress is the outer-fiber stress exhibited by a homogeneous geometrically similar elastic pipe that exhibits the same deformation characteristics when subjected to the same bending moment in the three-edge bearing test. This hypothetical outer-fiber stress is then used to define failure in the elastic ring contained in the finite-element model. Of course, the hypothetical outer-fiber stress value will not exist, because the pipe will have cracked. However, it is postulated that equal hypothetical outer-fiber stresses will represent equal crack widths. This will not be precisely correct, since the differences between field and three-edge bearing test bending-moment distributions will lead to slightly different crack patterns, but the error from this source is likely to be small.

The outer-fiber stress (σ_m) for an elastic beam subject to combined axial force T and bending moment M is given by the following:

$$\sigma_m = (My/I) \pm (T/A) \quad (1)$$

where y is the beam half-depth, I is the beam moment of inertia, and A is the beam cross-sectional area. When the wall of a pipe is subject to combined axial force and bending moment, some modification is necessary owing to the curved-beam effect. This effect is relatively small except for very thick-walled pipes and might well have been neglected, but since very little additional computational effort was necessary, it was included. Curved-beam factors k and j are introduced (7, p. 624) to account for this effect at the inside of the pipe crown as follows:

$$\sigma_m = (kMy/I) - (jT/A) \quad (2)$$

where

$$k = [(1/3)/(d/t - 2/m)] \cdot [(1 - d/t + 2/m)/(d/t - 1)]$$

$$m = \ln[(d/t + 1)/(d/t - 1)]$$

$$j = t/d + 1$$

The symbols d and t refer to mean pipe diameter and pipe wall thickness, respectively.

In the three-edge bearing test a sufficiently accurate solution for the bending moment at the pipe crown (M_c) is given by the following:

$$M_c = (qd^2/2\pi) \quad (3)$$

where q is the three-edge bearing test load per unit length of pipe per unit of pipe mean diameter. The stress at the inside of the pipe crown (σ_i) is given by the following:

$$\sigma_i = kMy/I = (3/\pi)kq(d/t)^2 \quad (4)$$

For equal field and three-edge bearing test outer-fiber stresses, Equations 2 and 4 may be combined and rearranged to give the following:

$$q/\gamma H = 2\pi\{(M/\gamma Hd^2) - [(1/6)(j/k)(t/d)(T/\gamma Hd)]\} \quad (5)$$

The soil unit weight (γ) and the cover height (H) are introduced on both sides of the equation to produce convenient dimensionless parameters. The ratio $q/\gamma H$ becomes a key design parameter and $M/\gamma Hd^2$ and $T/\gamma Hd$ are the dimensionless forms of the data obtained from the finite-element analyses for solving Equation 5.

The three-edge bearing test load is not normally defined in terms of the pipe mean diameter (d) but rather the inside diameter (D). The conventional three-edge bearing test load (Q) is related to q by the following:

$$Q = q[(d/t)/(d/t - 1)] \quad (6)$$

Equation 5 now becomes

$$Q/\gamma H = 2\pi \left([(d/t)/(d/t - 1)] \{(M/\gamma Hd^2) - [(1/6)(j/k)(t/d)(T/\gamma Hd)]\}\right) \quad (7)$$

A careful study of previous theoretical work by Burns and Richard (6) together with other considerations of similitude indicated that determination of the moment and axial-force dimensionless parameters depended on variation of the dimensionless parameters $[(M_f/E)(d/t)^3]$ and (M_f/M_n) for a given value of Poisson's ratio ν. M_f is the constrained modulus of the compacted fill, E is the elastic modulus for the concrete pipe, and M_n is the constrained modulus of the underlying soil. Subject to the slight error associated with the location of the pipe-soil interface in the model discussed previously, this enabled solution of the problem over a wide range of d/t values without actual variance of this parameter in the finite-element model. Substitution in Equation 7 of the moment and axial-force parameters from the finite-element analyses gave the numerical data for the general relationship:

$$Q/\gamma H = f[(M_f/E)(M_f/M_n)(d/t)] \quad (8)$$

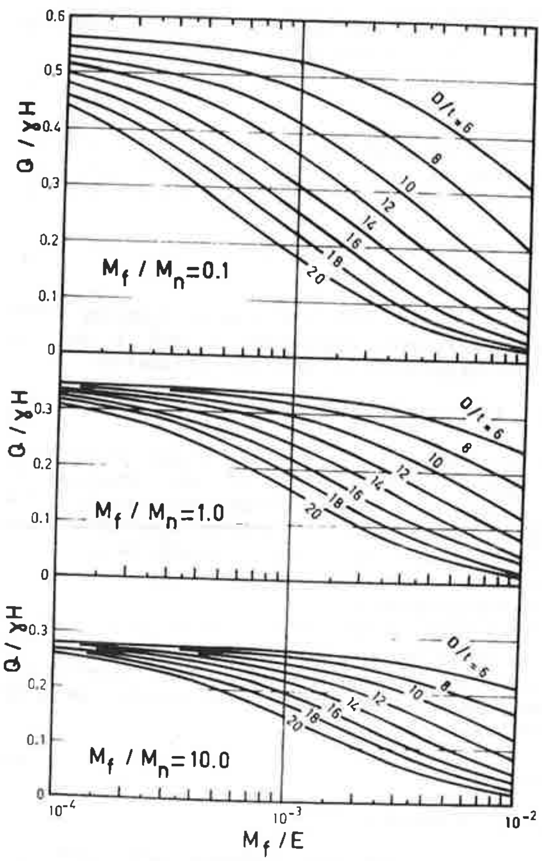
For practical purposes, design curves are more desirable in terms of the pipe inside diameter (D) than the mean diameter (d). The d/t values are converted on the basis of D/t = d/t - 1, and the curves of Figure 3 were plotted in the following form:

$$Q/\gamma H = g[(M_f/E)(M_f/M_k)(D/t)] \quad (9)$$

VALIDITY OF ELASTIC MODEL

Although the approach taken is very much more sophisticated than was possible in the 1930s, the extent of idealization that remains is considerable. Both to achieve a successful analysis and to present

Figure 3. Design graphs.



a practical design method, numerous simplifications to material properties are necessary. Many of the difficulties are itemized in the following:

1. Soil uniformity: In the compacted fill, the soil in close proximity to the pipe is likely to exhibit different compressibility characteristics from that further from the pipe. Also, stratum changes and thus compressibility changes may occur in the underlying soil.
2. Anisotropy: Both compacted fills and overconsolidated natural soil exhibit different compressibility properties parallel and perpendicular to the layering or bedding direction.
3. Soil compressibility: The compressibility of soil is nonlinear, stress-level dependent, and time dependent.
4. Pipe stiffness: The pipe exhibits a cracked modulus as well as an uncracked modulus that may differ by as much as a factor of 3 or more. Owing to variation in moment around the pipe, a nonuniform modulus will normally exist.
5. Poisson's ratio: The analyses are based on a Poisson's-ratio value of 0.3 for the soil. Although this is an average value for natural soils, little information is available on appropriate values for compacted soils. It is also likely to vary.
6. Interface slip conditions: Results are based on a no-slip condition at the concrete-soil interface. Fortunately, this error should be small; consideration of stress and strength levels from appropriate theory (8, p. 158) indicates that little slip is likely.

Obviously, the model used has shortcomings, particularly in relation to soil compressibility. Unfortunately, at the present time, very little information is available on which even average soil compressibilities may be based. Present practice in relation to pipe installation does not involve sophisticated soil-property determination, and no change in this situation is likely. Therefore, a study based on an average isotropic linear modulus would seem acceptable for the present. It should be noted that, although similar problems exist for prediction of settlement of building foundations, similar simplifications are made (9).

COMPARISON BETWEEN PREDICTIONS AND FIELD-TEST RESULTS

An extensive program of loading of concrete pipe to failure has been conducted by Davis and others (10-12) of the California Division of Highways. In these studies a series of reinforced concrete pipes with 84-in (2130-mm) diameter and 8-in (200-mm) walls were subjected to loads from compacted fills up to 136 ft (41.5 m) in height. For identical pipes under a variety of bedding conditions, the detailed crack conditions have been recorded and carefully illustrated from the onset of cracking through various stages of further deterioration. These tests offer an excellent opportunity to evaluate the proposed method for a pipe installed under embankment conditions.

In the following, the data associated with a case covered by the design graphs are assembled. A prediction is first made from Figure 3 of the fill height required to cause a 0.01-in crack. Then the outcome of the field tests for this case is reported. Finally, an estimate of fill height is made from a modern concrete pipe users' handbook based on the Spangler approach.

Field-Test Conditions

One of the bedding types studied by Davis and others (10) at Mountainhouse Creek, California, is appropriate for the design graphs prepared at this stage. In Zone 9 the pipe was placed in a shaped bedding and the fill was brought up after placement of the pipe from approximately invert level.

The constrained modulus for the compacted fill (M_f) can be determined approximately from the reported data. Some large-diameter undrained triaxial tests indicated undrained deformation-modulus values in the range 4400-5800 psi (30-40 MPa), but both end-of-construction and long-term values would reasonably be lower than these values. Measured settlement data provide a means for back-calculation of constrained modulus and indicated 3000-4500 psi (20-30 MPa) at the end of construction and 1500-2000 psi (10-15 MPa) in the longer term. The last figures showed reasonable agreement with a prediction of 1500 psi (10 MPa) based on an empirical relationship proposed by Espinosa, Krizek, and Corotis (13). Even though the usual design should be based on the drained-modulus value, the end-of-construction figure is more appropriate for this case, since it corresponds with the time of the observations for comparison of results. A value of 3500 psi (24.2 MPa) is chosen on this basis. The average in-place unit weight of the compacted fill was 130 pcf (20.5 kN/m^3) and the dry weight was 116.5 pcf (18.3 kN/m^3).

Little information is available on which assessment of the modulus of the in situ soil (M_n) can be made. It is assumed that this determination is the same as that of the compacted fill.

For the pipe, the average 28-day concrete compressive strength was 5200 psi (36 MPa). This corresponds to a Young's modulus of about 3.5 x 10^6 psi (25.2 GPa). The pipe diameter was 84 in and the pipe wall thickness was 8 in. The three-edge bearing test load (Q) for the pipe was specified as 1000 lbf•ft/ft of pipe diameter (48 kN•m/m), but the results of three tests indicated an average strength of 1500 lbf•ft/ft (72 kN•m/m). Again, since the object here is to predict the failure fill height, the latter value is used.

Data summary: M_f = 3500 psi, M_n = 3500 psi, E = 3.5×10^6 psi, D = 84 in, t = 8 in, Q = 1500 lbf•ft/ft, γ = 130 pcf. M_f/M_n = 1; $M_f/E = 10^{-3}$; D/t = 10.5.

Predicted fill height at failure: From Figure 3 ($Q/\gamma H$ = 0.30),

$$H = Q/0.30\gamma = 1500/0.30 \times 130 = 38.5 \text{ ft } (11.7 \text{ m}).$$

Field conditions at failure: From Davis and others (11), Zone 9:

1. Fill height 30 ft (9.1 m), intermittent hairline cracks;
2. Fill height 32 ft (9.8 m), intermittent crack at invert greater than 0.01 in;
3. Fill height 34 ft (10.4 m), intermittent crack at invert averages 0.035-0.55 in (0.9-1.4 mm) localized spall.

Predicted Failure Height: Spangler Approach

Handbooks have been developed by the concrete pipe industry for use in pipe selection. Certain conditions are presumed for the field installation and specific values of the uncertain "Spangler constants" are used to permit more direct selection of the appropriate class of pipe. One such handbook, the Concrete Pipe Guide (14), was published by the Concrete Pipe Association of Australia in 1980. The product of settlement ratio and projection ratio ($r_{sd}p$) is taken to be 0.5 and the product of lateral earth pressure coefficient and coefficient of friction $K\mu$ is taken as 0.1650 for "well compacted clayey sand". For each pipe diameter an allowable fill height is given in terms of trench width, bedding type, and pipe class.

For the case of interest, a type-C bedding and a pipe with an average three-edge bearing strength of 1500 lbf•ft/ft (72 kN•m/m), the allowable fill height is indicated as about 7 ft (2.1 m).

DISCUSSION OF RESULTS

Comparison of the prediction based on the proposed design method with field results shows very reasonable agreement. The predicted height to cause failure of 38.5 ft (11.7 m) versus the actual failure height of 32 ft (9.8 m) indicates that use of the allowable (1000 lbf•ft/ft) in place of the average (1500 lbf•ft/ft) three-edge bearing value may well have provided sufficient safety margin in a design context. However, more information from field tests is required with better information on field-modulus values. It may be for the case studied that the modulus of the natural soil was considerably more than for the fill rather than equal to it as assumed. If it were 35 000 psi rather than 3500 psi, for example, the ratio M_f/M_n is then 0.1 and the predicted fill height to cause failure is 27.3 ft (8.3 m).

The Spangler approach is very clearly grossly conservative for this particular case. The allowable fill height of 7 ft versus a measured failure at 32 ft implies a factor of safety of 4.6.

SUMMARY AND CONCLUSIONS

For the proposed approach to design of concrete pipe, comparisons of predictions based on the method with results measured in one extensive field-test program are very encouraging. However, the approach is based on an idealized model, and the uncertainties associated with the required parameters are not all negligible. Work is required to provide guidelines for selection of typical parameters for design purposes, and more field testing is needed for further evaluation of the method. In addition, extension of the method is required to cover a broader range of cases such as other bedding types and shallow cover conditions.

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