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Modification of the Asphalt Institute Bituminous Mix Modulus Predictive Equation

JOHN S. MILLER, JACOB UZAN, AND MATTHEW W. WITCZAK

The dynamic modulus test results for five bituminous mix types (crushed stone, gravel, slag, sand-low P_{200}, sand-high P_{200}) were compared with modulus values predicted by using the Asphalt Institute regression model. Results of this comparison showed an excellent correlation for crushed stone, which was the primary mix type in the model development. For other materials, slag and sand, the results were poor. A correction methodology was developed based on statistical methods to minimize the mean square error between the measured moduli and those predicted by using the model. The model parameter used as the means for correction was the percentage of asphalt content, chosen because it had been shown that the asphalt content range used in model development was narrower than the range encountered in the laboratory study and that, mathematically, it had the greatest effect on the resultant modulus of all the model variables. A unique, material-dependent constant was calculated for each of the five mix types in the study that would be subtracted from the actual asphalt content of the mix for calculating the modulus. A more desirable, generalized method for determining the correction constant was developed based on the difference between the Marshall optimum asphalt content and the actual asphalt content of the mix instead of relying on mix nomenclature (e.g., slag asphalt, sand asphalt). The correction scheme produced a correlation coefficient of 0.891 for all 1179 data points used in this study. This is an excellent result for practical engineering applications.

With the trend toward application of elastic theory to problems in flexible pavement evaluation and design, the development of accurate stress-strain relations for asphalt concrete mixtures is important. Furthermore, asphaltic concrete modulus values have been correlated with substitution ratios or layer coefficients in empirical design methods (1,2) so that an accurate modulus characterization serves both methods. However, direct laboratory modulus characterization under conditions similar to those encountered in the field (dynamic repeated loading, temperature, and load rate) normally requires sophisticated and expensive dynamic laboratory equipment (3). To avoid costly laboratory procedures, alternative methods of determining modulus by using physical and mechanical properties of the mixture were developed. Among these methods were use of the Marshall stability-flow quotient and the development of the Shell nomograph and the Asphalt Institute predictive model.

The Marshall stability-flow quotient was suggested by Nijboer (4) and recommended for use in high temperature ranges by Heukelom and Klomp (5). Nijboer's formula was as follows:

$$S_{60°C, 4\,sec} = 1.6\,(\text{stability/flow}) \quad (1)$$

where S is given in kilograms per square centimeter, stability in kilograms, and flow in millimeters. McLeod (6) suggested a variation on the Nijboer formula:

$$\text{Modulus} = 40\,(\text{stability/flow}) \quad (2)$$

where modulus is given in pounds per square inch, stability in pounds, and flow in inches. These formulas use routine laboratory-determined properties of a mix.

The Shell nomograph originally developed by Van der Poel (7) permitted determination of the stiffness of asphalt cement at a particular load rate and temperature as a viscoelastic characteristic as distinguished from elastic modulus. Heukelom and Klomp (5) developed a relation to translate the bitumen stiffness to a mixture stiffness based on volume of aggregate and volume of asphalt cement in the mix. McLeod (6) modified the nomograph by changing the entry temperature criterion. Finally, Claessen and others (8) produced a pair of nomographs to be used together to accomplish the evaluation of bitumen and mixture stiffness used in the current Shell design manual (9).

This paper focuses on the third method for modulus determination, the Asphalt Institute model (10). By examining dynamic modulus values measured in the laboratory for various materials and comparing them with values predicted by the model based on physical properties of the mixture (asphalt content, percentage passing the No. 200 sieve, volume of voids, and asphalt viscosity), the accuracy of the predictor can be established.

RESEARCH PROBLEM STATEMENT

At present, the two predictive models most commonly used are the Shell method and the Asphalt Institute equation. The Shell method is based on stiffness of the asphalt cement determined by entering a nomograph with factors derived from asphalt softening point and penetration and temperature and frequency of loading. This bitumen stiffness is then modified by using a second nomograph to consider volume percentages of aggregate and asphalt cement to give a stiffness or modulus value for the mix.

The Asphalt Institute method was initiated by

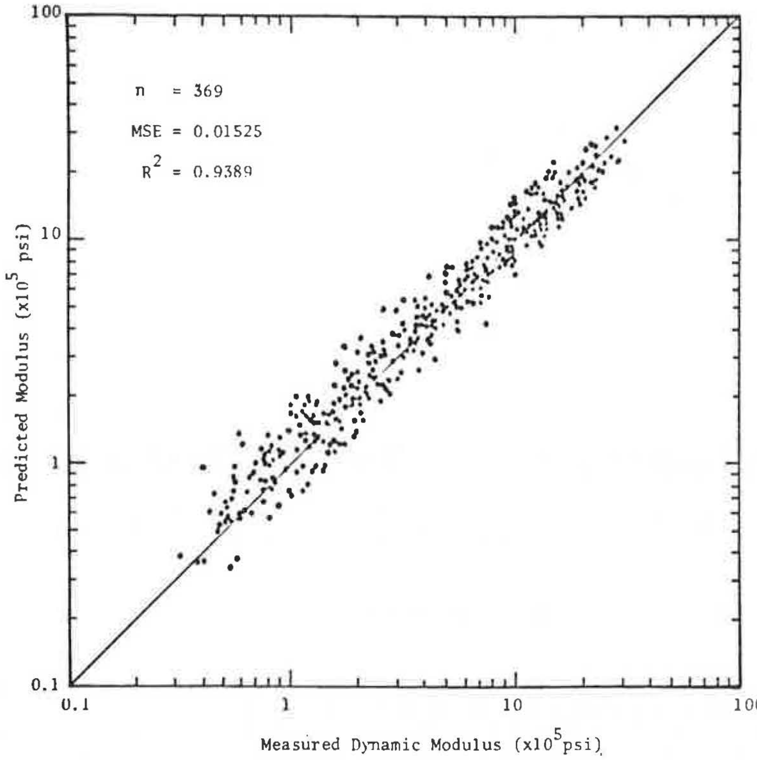


Figure 1. Results of original Asphalt Institute predictive equation.

Table 1. Summary of parameters for asphalt concrete mixes.

Mix Type	No. of Data Points	Source	Asphalt Content (%)	Volume of Voids	P_{200} (%)	$\eta(10^6, 70)$	Penetration at 77°F
Crushed stone, including two sand mixes	369	Asphalt Institute	3.5-6.2[a]	0.6-14.1	3.0-10.0	1.31-4.43	59-90
Crushed stone	162	University of Maryland	4.1-5.6	2.791-8.701	5.9	1.4	76
Bank run gravel	162	University of Maryland	3.9-4.9	2.848-8.818	5.0	1.4	76
Slag	162	University of Maryland	7.3-8.8	0.0-9.630	2.3-4.5	1.4	76
Hot mix sand asphalt							
Low	162	University of Maryland	8.2-10.2	6.168-15.90	2.0	1.4	76
High	162	University of Maryland	6.8-8.8	2.538-15.092	10.0	1.4	76

Note: Viscosities in the University of Maryland study were measured on fresh, unused asphalt cement.
[a]Single values of 3.0, 8.2, and 8.6 are eliminated due to negligible influence.

Shook and Kallas (11) in 1969 using a limited number of laboratory (dynamic modulus) test results. It was based on the relation between direct measurements of dynamic modulus and certain properties of the mixture. The equation was subsequently refined from an expanded data base and then further modified by Witczak (10) for use in calculating design curves currently used in the Asphalt Institute MS-1 design guide (12). The predictive equation is as follows:

$$\begin{aligned}\log_{10}|E^*| = {} & 0.553833 + 0.028829(P_{200}/f^{0.17033}) - 0.03476V_v \\ & + 0.070377\eta(10^6, 70) \\ & + 0.000005\,T\exp(1.3 + 0.49825\log_{10}f)Pac^{0.5} \\ & - 0.00189\,T\exp(1.3 + 0.49825\log_{10}f)(Pac^{0.5}/f^{1.1}) \\ & + (0.931757/f^{0.02774})\end{aligned} \qquad (3)$$

where

$|E^*|$ = dynamic modulus (10^5 psi),
P_{200} = percentage passing the No. 200 sieve,
f = loading frequency (Hz),
V_v = volume of voids (%),
$\eta(10^6,70)$ = viscosity of asphalt cement at 70°F (megapoises),
T = temperature of pavement (°F), and
Pac = percentage of asphalt cement by weight of mix.

It is important to note that the data used to develop the equation were almost exclusively derived from mixtures of crushed stone and gravel. This predictive equation was based on 41 different asphaltic mixtures prepared and tested at the Asphalt Institute over 10 years of laboratory research work. Each mix was tested for dynamic modulus at a full factorial of three test temperatures (40°, 70°, and 100°F) and three load frequencies (1, 4, and 16 Hz). Thus, the equation was based on 369 data points.

For the mixes studied, Equation 3 had an excellent correlation coefficient ($R^2 = 0.939$) and mean square error (MSE = 0.01525) and therefore was used with confidence for the material type and ranges of physical properties used in its development. This is clearly demonstrated in Figure 1, which compares the predicted and measured moduli for the original 369 data points tested by the Asphalt Institute. The parameters used in the equation are given in Table 1.

In 1978, a laboratory study was initiated at the University of Maryland for the Maryland State High-

way Administration (MSHA) to develop dynamic modulus characterizations for typical base and subbase materials used in pavement systems (13). This comprehensive study resulted in the dynamic modulus evaluation of 45 additional mixes containing crushed stone, gravel, slag, and sand aggregates. By using the same temperature and frequency factorials as used by the Asphalt Institute, along with variations in asphalt content and aggregate gradation, 810 additional dynamic modulus test results were obtained. This number, combined with the 369 data points from the previous Asphalt Institute work, allowed for a combined data pool of 1179 sets of data to be used in the study.

The major objective of this paper is to use the new data base generated by the University of Maryland study to assess the accuracy (verify or modify) of the Asphalt Institute predictive equation for the broader range of material types (asphaltic mixtures) investigated.

SOURCES OF DATA

General

Table 1 summarizes the range of aggregate types and associated mix parameters of the laboratory specimens for which the dynamic modulus was measured and then predicted by using Equation 3.

Asphalt Institute

Laboratory studies conducted for use in the work of Witczak (10) and Kallas and Shook (11) used a total of 41 different bituminous mixtures. These specimens were tested at full factorial levels of 1, 4, and 16 Hz and 40°, 70°, and 100°F, which yielded 369 values of dynamic modulus. The mixtures used in the Asphalt Institute studies consisted of crushed stone and gravel aggregates (except for two sand mixes) with asphalt content ranging from 3.0 (only one specimen) to 8.6 (again, only one specimen) and averaging approximately 5.2 percent. Viscosity of asphalt cements ranged from 1.31 to 4.43 megapoises. In general, almost all of the mixes tested were of a dense-graded crushed aggregate variety.

University of Maryland Study

The specimens used in the laboratory study were made from combinations of five aggregates. The crushed stone, bank run gravel, and slag were each graded to conform to MSHA specifications pertinent to BF, BI, and BC mixes. In addition, each gradation was represented by six specimens, two for each of three levels of compaction (100, 98, and 95 percent of maximum density determined by the Marshall design method). The two sand mixes were made with three levels of asphalt content (Marshall optimum, +1 percent, and -1 percent) instead of the gradations. There were six specimens for each asphalt content consisting of two specimens for each of three compaction levels (98, 96, and 92 percent of maximum). This yielded 90 specimens of 45 different aggregate-gradation (asphalt content) combinations. When the specimens were tested at full factorial levels of 1, 4, and 16 Hz and 40°, 70°, and 100°F, the laboratory study yielded 810 values of dynamic modulus. A much broader range of aggregate types and asphalt contents was investigated in this study than in the Asphalt Institute work.

For the crushed stone and gravel in the Maryland study, the asphalt content ranged from 3.9 to 5.6 percent, which was reasonably close to that used in the original (Asphalt Institute) model development. However, the other three aggregates investigated had average asphalt contents very different from that of the original study [slag, 7.8 percent; sand, high P_{200}, 7.8 percent; and sand, low P_{200}, 9.2 percent]. Equation 1 was used to compute predicted modulus values for all 810 combinations with the aid of the University of Maryland UNIVAC 1100/82. All data were plotted to compare the measured dynamic moduli with the predicted values for each data set.

RESULTS

Comparative Study

Figure 2 shows a comparison between the measured dynamic modulus results for the 810 test points developed in the University of Maryland (UM) study and the Asphalt Institute predictive equation. It is obvious that the general agreement is not nearly as good as that shown in Figure 1 with the original 369 Asphalt Institute data results. These results necessitated an in-depth study of the original Asphalt Institute predictive equation by material type.

Figure 3 shows the excellent agreement achieved for the UM-MSHA crushed-stone mixes. Although this material type exhibited an accurate predictive response correlation, Figures 4-7 show the poor correlations for the aggregate types not included in the model development. The R^2 values ranged from 0.09 to 0.70 for the four aggregates other than the crushed stone whereas the University of Maryland crushed-stone values produced an R^2 of 0.948. This compares favorably with the R^2 of 0.939 achieved with the original 41 mixes (10,11). Although the correlation values for the slag, the gravel, and both sands were not satisfactory, the trend toward linearity shown in Figures 4-7 strongly indicates that a material-dependent (mix-dependent) adjustment factor can be used to improve the original predictive equation.

The lack of fit for mixtures other than crushed stone is probably related to the fact that the ranges of mix properties differ significantly from those used in the original model development. Table 1 indicates that the percentage of asphalt in the slag and sand mixtures is much greater than the upper limit of the range used in the predictive equation. It is interesting to note that the mixture properties of the gravel material are all in the range of values of the original prediction and that measured moduli are only slightly higher than predicted (by a factor of 1 to 3). The above results appear to indicate that the high percentage of asphalt is a primary cause of the lack of fit shown in Figures 4-7.

This led to the further investigation of the possible causes of the lack of fit for non-crushed-stone mixtures. A sensitivity study was conducted to determine the magnitude of change in the predicted modulus as a consequence of varying a particular parameter value. An examination of the change in $\log_{10}|E^*|$ versus P_{200}, V_v, and Pac showed that, for the range of values in the University of Maryland study, the greatest change in dynamic modulus occurred as the percentage of asphalt increased. The effect of the percentage passing the No. 200 sieve and the volume of voids is relatively small, and they both fall within the range of values used in developing the original equation. As a result, a procedure was investigated to correct the predicted modulus equation by using percentage of asphalt as the primary variable to be corrected.

Development of Correction Factors

As stated previously, examination of the effect of each predictor variable on the resulting dynamic

modulus predicted by Equation 1 indicated that the asphalt content of the mix caused the greatest change in modulus. This, along with the large differences in optimum asphalt content determined by Marshall method procedures for each aggregate in relation to the original range of this parameter used to develop the equation, tended to confirm the suspicion that asphalt content was the primary cause of the lack of fit. Figures 4-7 show that the magnitude of the lack of fit (and, hence, the magnitude of any correction factor) is apparently material dependent.

The method used to define the correction assumed that it would be constant for a given aggregate type. This was highly desirable because the original mathematical form of the equation could be used

Figure 2. Overall comparison of predicted versus measured moduli: University of Maryland study, uncorrected.

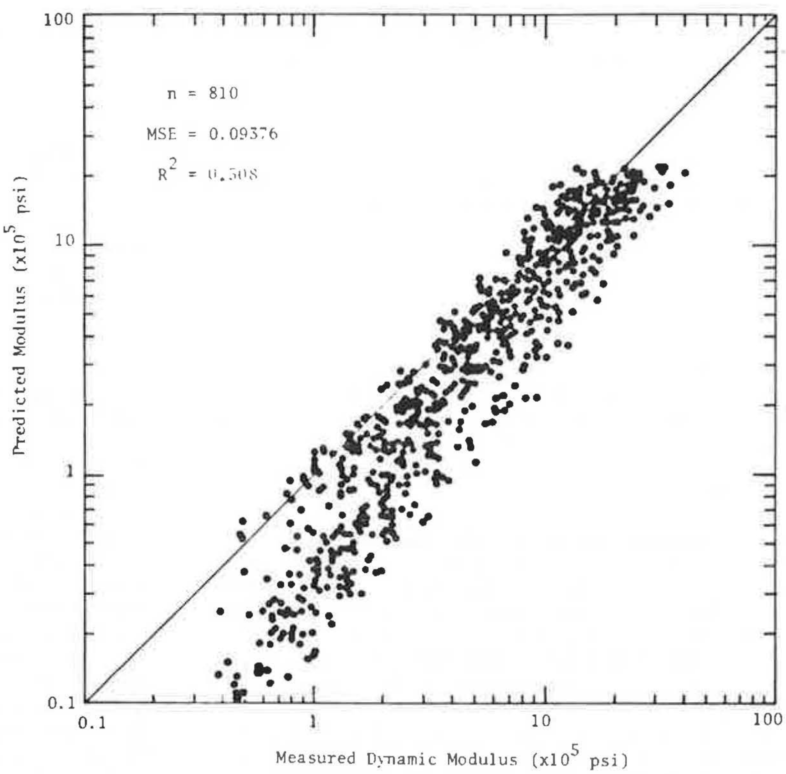
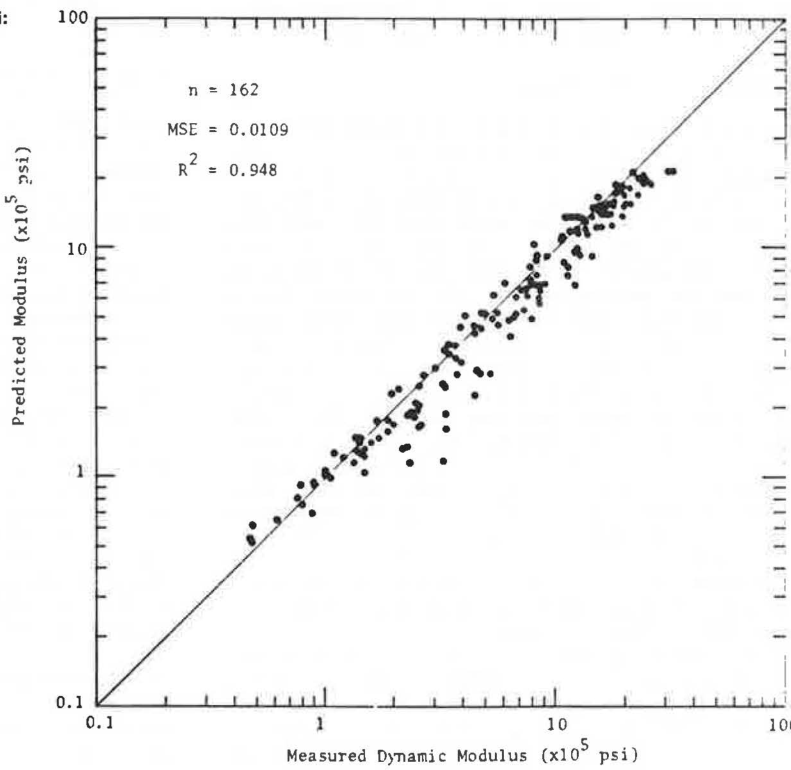


Figure 3. Comparison of predicted versus measured moduli: University of Maryland crushed stone, uncorrected.



instead of having to develop a different equation. The simplest and most desirable correction would be to adjust directly (add or subtract) the percentage-of-asphalt parameter used in the model. The reasoning for this lies in the comparison between the range of asphalt contents used in the original model development and the UM-MSHA crushed-stone data and the ranges of optimum asphalt contents determined for the four additional UM-MSHA mixes.

In the development of the predictive model, a relatively narrow range of asphalt contents was used and excellent results were achieved. In contrast, the University of Maryland study resulted in the use of widely different ranges of optimum asphalt con-

Figure 4. Comparison of predicted versus measured moduli: gravel, uncorrected.

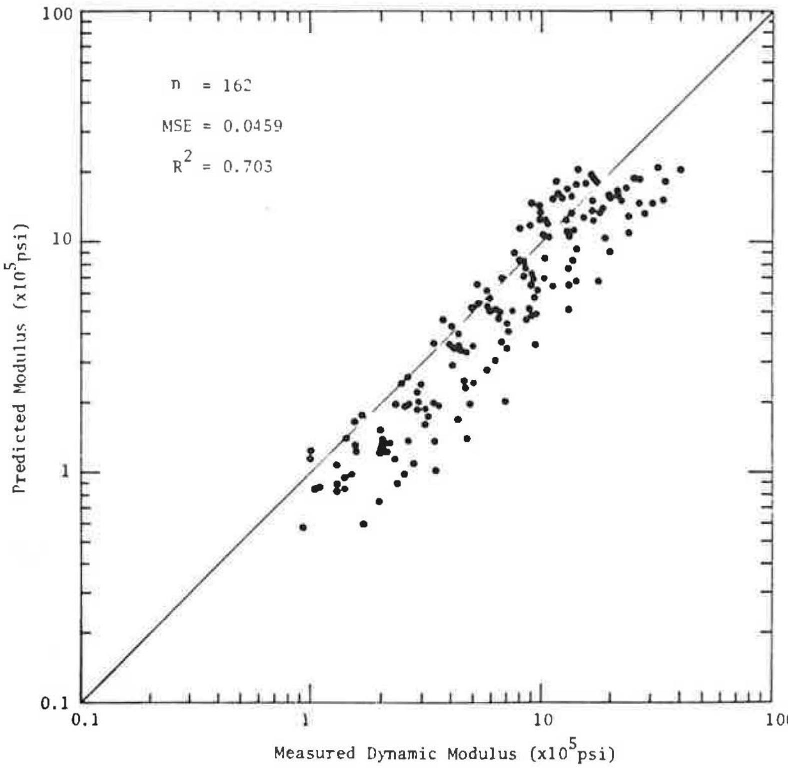
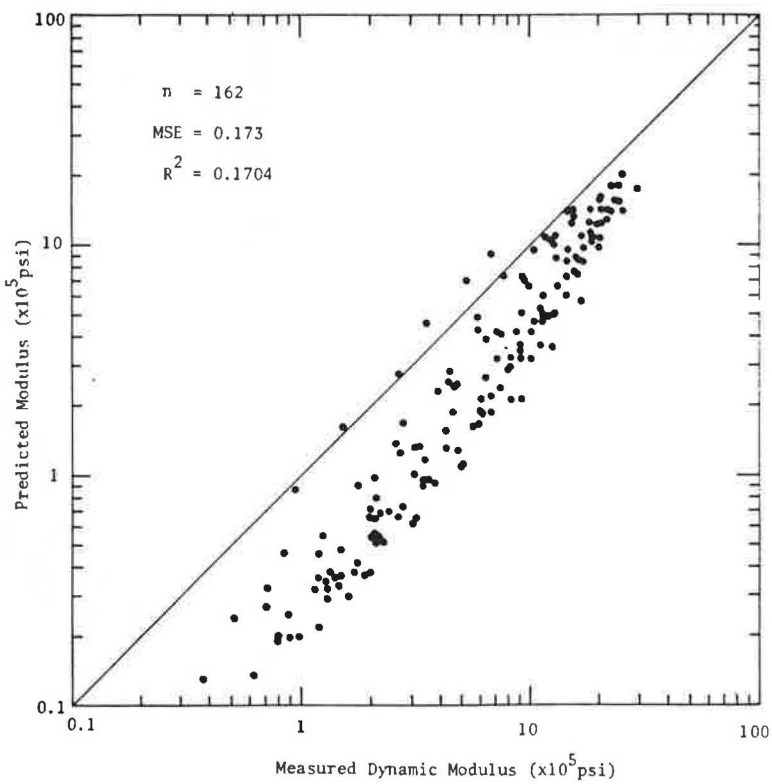


Figure 5. Comparison of predicted versus measured moduli: slag, uncorrected.



tents according to the material used as aggregate and achieved only fair to poor agreement. In addition, an examination of Equation 3 shows that the terms containing percentage of asphalt tend to be negative and their magnitude tends to increase as higher values of temperature, frequency, and asphalt content are used. Because temperature and frequency values were the same in both sets of data, it is likely that the original model can only tolerate minor deviations in asphalt content and still predict reasonable modulus values.

To determine the specific correction to be ap-

Figure 6. Comparison of predicted versus measured moduli: sand, low P_{200}, uncorrected.

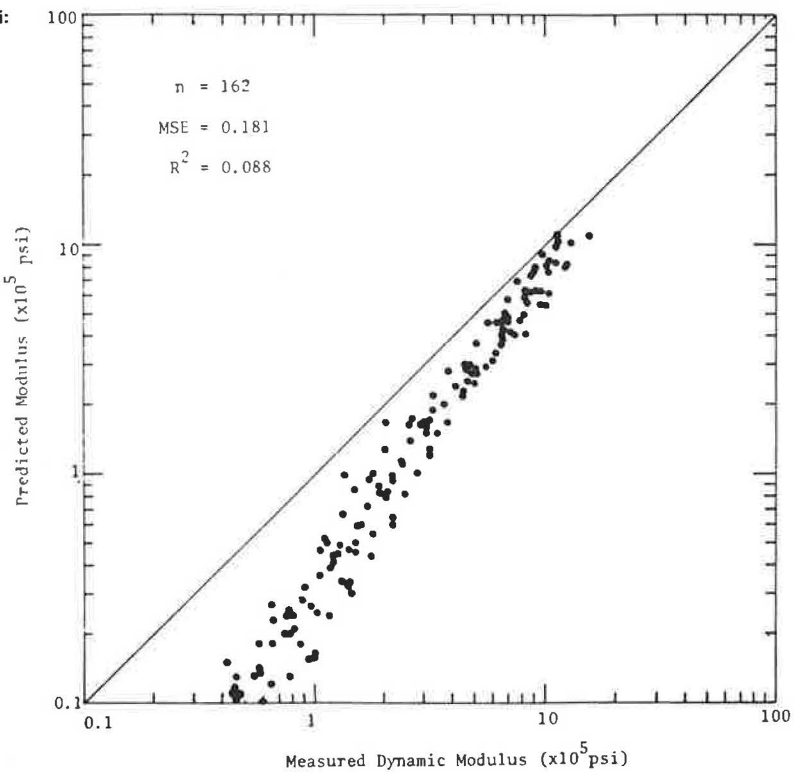
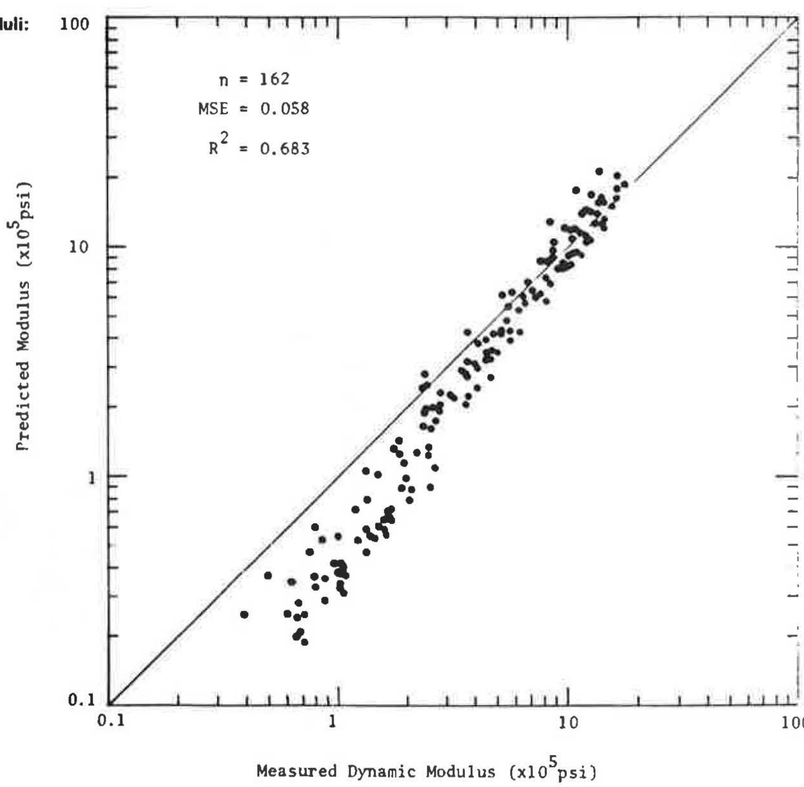


Figure 7. Comparison of predicted versus measured moduli: sand, high P_{200}, uncorrected.



plied to asphalt content, Equation 3 was rewritten to isolate the terms containing percentage of asphalt content:

$$\log_{10}|E^*| = C1 + C2\,(Pac - \alpha)^{0.5} \quad (4)$$

where α is a correction constant and

$$C1 = 0.553833 + 0.028829(P_{200}/f^{0.17033}) - 0.03476V_v + 0.070377\eta_{(10^6,70)} + (0.931757/f^{0.02774})$$

$$C2 = 0.000005T\exp(1.3 + 0.49825\log_{10}f) - [0.00189T\exp(1.3 + 0.49825\log_{10}f)/f^{1.1}]$$

Because the correction constant, α, should apply to the entire range of parameters for a given mix type, a statistical optimization procedure was used to achieve the best fit. Draper and Smith (14) present the method of optimizing a nonlinear function (which describes Equation 4). The results can be achieved by an iterative process designed to minimize the mean square error (MSE). Essentially, a correction constant is chosen and the sum of the squared differences between the measured dynamic modulus and the predicted modulus (calculated by using the chosen constant) is computed from

$$MSE_i = (1/n)\sum_{j=1}^{n}[\log_{10}|E^*|_{meas_j} - C1 + C2(Pac - \alpha_i)_j^{0.5}]^2 \quad (5)$$

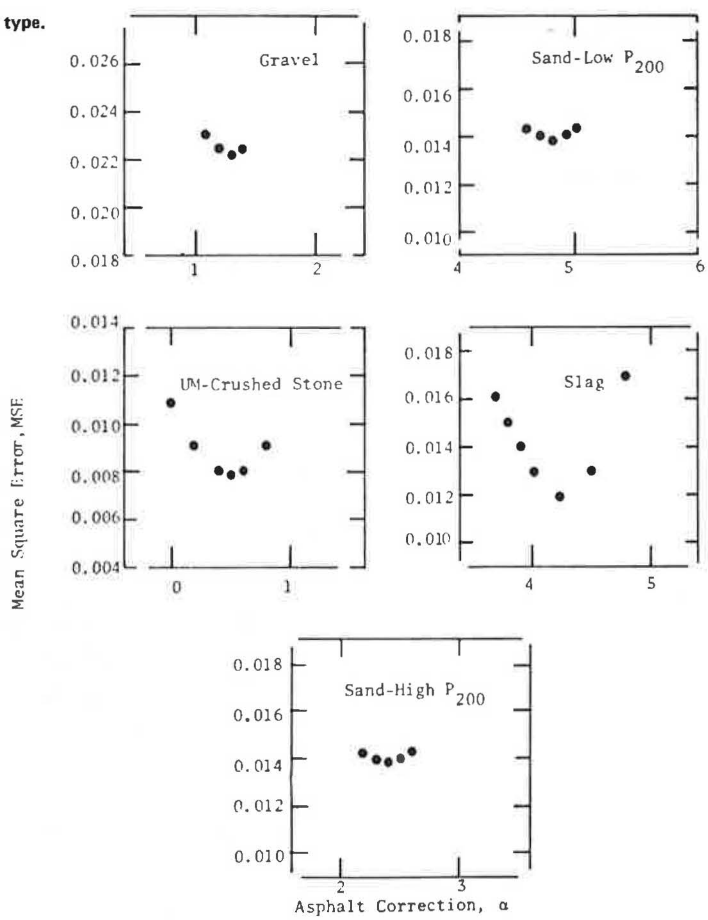
where n is the number of values of modulus. By iteration, the value of the MSE is determined for a range of correction values. This procedure results in a curve with a unique minimum MSE for a single α. Figure 8 shows the graphical results of MSE_i versus α_i for each of the aggregates in the UM-MSHA study. All display clearly defined minimum values on the convex functions.

Analysis of Effects of Correction Factors

Table 2 gives the corrections developed for each type of aggregate in the University of Maryland study. The value listed for crushed stone approaches 0.0 when the Asphalt Institute data are included and therefore is not included in the analysis. The corrected values achieved for slag and both sands reduce the asphalt content to values within the limits of the range used in developing Equation 3. The bituminous mix with bank run gravel achieved the poorest fit due to both inherent scatter (Figure 4) and the fact that the corrected asphalt content was slightly below the range used in the model development.

Table 3 summarizes the MSE and R^2 values determined with and without the correction factor applied to the original Asphalt Institute equation. It can be seen that a very significant increase in R^2 and decrease in MSE are obtained with the correction factor for all asphalt mixes investigated. The correlation coefficient obtained for the combined data when no asphalt corrections were used was $R^2 = 0.669$. When each material correction factor was applied, the correlation improved dramatically to $R^2 = 0.928$ and MSE = 0.01438. For the combined 1179 data points, this translates into a standard error of 20.6 percent (of expected modulus). Figure 9 shows the results of the corrected UM-MSHA results (810 points). Figure 1 shows the original comparison of the 369 data points used in the Asphalt Institute analysis.

Figure 8. Optimization of corrections to asphalt content by mix type.



Suggested Implementation of Results

The analysis presented in this study clearly shows that the original Asphalt Institute modulus equation achieved excellent results for both the Asphalt Institute and UM-MSHA crushed-stone mixes. However, the study also clearly identified the need to modify further the original Asphalt Institute equation in order to predict accurately dynamic modulus for a broad range of bituminous mixtures. In the methodology developed, it was shown that a unique correction factor could be calculated for each mix type studied. It was also shown that implementation of the derived correction factors substantially improved the correlation between predicted and measured moduli for mixes that otherwise would show poor predicted values (Table 3). Because the different correction factors corresponded to different mix types, it is obvious that the correction factor was material (mix and type) dependent. Because of this, a modification scheme for the original Asphalt Institute equation was developed that could provide accurate predicted moduli for mixes not included in the UM-MSHA study.

It is a widely accepted fact tha Marshall optimum asphalt content is material dependent and can be taken as a global variable to describe material behavior. Logically, if the two material variables--the correction factor and Marshall optimum asphalt content--vary in the same way and are strongly correlated, a simple relation can be developed to express the unknown variable (α) in terms of the readily available optimum asphalt content. The relation between α and optimum asphalt content is shown in Figure 10. The results indicate a linear relation between the two variables. From a practical viewpoint, the equation can be approximated by

$$\alpha \cong P_{opt} - 4.0 \quad (6)$$

Substitution of this equation into the original Asphalt Institute equation (Equation 4) yields

$$\log_{10}|E^*| = C1 + C2(Pac - P_{opt} + 4.0)^{0.5} \quad (7)$$

Table 2. Summary of computed correction factors by mix type.

Mix Type	Asphalt Range (%)	Correction Factor: Asphalt	Correction Factor: Asphalt Content Range
Crushed stone			
Asphalt Institute	3.5-6.2	0.0	3.5-6.2
University of Maryland	4.1-5.6	0.5[a]	4.1-5.6
Bank run gravel	3.9-4.9	1.3	2.6-3.6
Slag	7.3-8.8	4.2	3.1-4.6
Sand			
Low P_{200}	8.2-10.2	4.8	3.4-5.4
High P_{200}	6.8-8.8	2.4	4.4-6.4

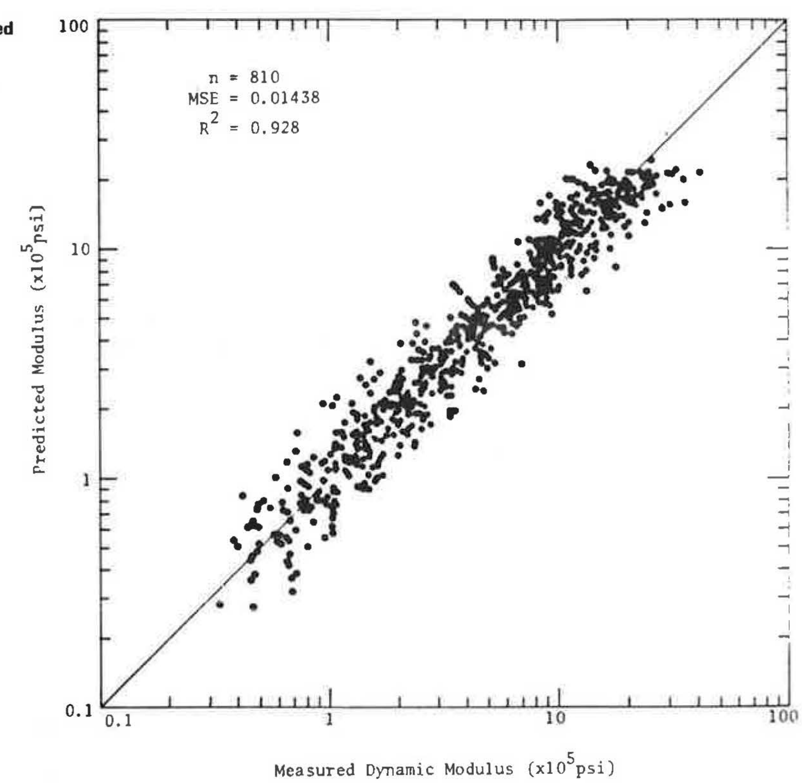
[a]Because the University of Maryland crushed-stone correction approaches 0.0 when combined with Asphalt Institute data, it is not considered in further analyses.

Table 3. Summary of error analyses.

Mix Type	No Correction		Correction Applied		
	MSE	R^2	MSE	R^2	SE (%)
Crushed stone					
Asphalt Institute	0.01525	0.9389			
University of Maryland	0.0109	0.9480	0.01392[a]	0.9412	20.3
Gravel	0.0459	0.7033	0.0223	0.856	28.9
Slag	0.173	0.1704	0.0123	0.941	17.8
Sand					
Low P_{200}	0.181	0.0880	0.0105	0.947	13.8
High P_{200}	0.058	0.6828	0.01389	0.924	22.8
Average	0.06935	0.669	0.01438	0.928	20.6

[a]No correction applied to University of Maryland crushed stone; values reflect Asphalt Institute and University of Maryland data taken together.

Figure 9. Overall comparison of predicted versus measured moduli corrected by mix type.



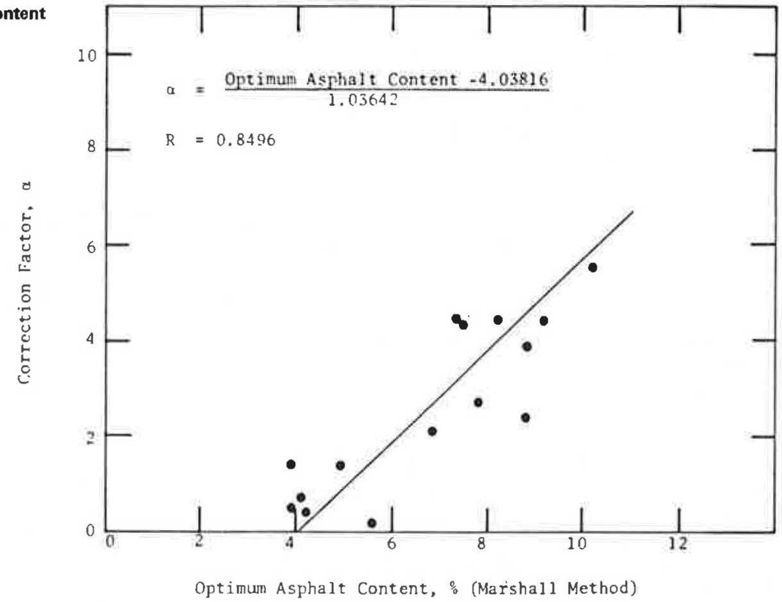


Figure 10. Suggested relation between optimum asphalt content and correction factor.

Table 4. Results of correction methodology applied to University of Maryland study.

	Asphalt Content (%)		Analysis of Correction	
Mix Type	Actual	Corrected[a]	R^2	MSE
Crushed stone	4.1-5.6	4.0	0.917	0.0173
Gravel	3.9-4.9	4.0	0.739	0.0404
Slag	7.3-8.8	4.0	0.887	0.0235
Sand[b]				
Low P_{200}	8.2-10.2	3.0-5.0	0.939	0.0122
High P_{200}	6.8-8.8	3.0-5.0	0.796	0.0373

Note: Number of data points for all mix types = 162.

[a]Corrected according to Equation 7; i.e., asphalt content = actual asphalt content - optimum + 4.0.

[b]Sand asphalt mixes were designed by using 51 samples each at Marshall optimum, Marshall optimum + 1.0, and Marshall optimum - 1.0.

Table 5. Comparison of analysis for all data with and without corrections.

	Uncorrected		Corrected[a]	
Mix Type	R^2	MSE	R^2	MSE
Crushed stone[b]	0.9412	0.0139	0.933	0.0159
Gravel	0.7033	0.0459	0.739	0.0404
Slag	0.1704	0.1703	0.887	0.0235
Sand				
Low P_{200}	0.0880	0.1814	0.939	0.0122
High P_{200}	0.6828	0.0579	0.796	0.0373

[a]Corrected according to Equation 7.

[b]Includes both Asphalt Institute and University of Maryland data. Asphalt Institute data were not corrected due to lack of Marshall optimum asphalt content value.

Equation 7 implies the following:

1. All material types have similar behavior around their respective optimum asphalt contents. In essence, one may view the α correction parameter as a shifting factor to converge all modulus-Pac curves into a unique master curve.

2. For all mixes prepared at Pac = P_{opt}, the modulus can be computed when Pac = 4.0 is substituted in the original Asphalt Institute equation (Equation 3). This does not mean that the modulus is independent of the asphalt content, because the actual asphalt content must equal the optimum one. Because common practice requires that Pac ≅ P_{opt}, a separate study of the correlation of the predicted and measured moduli was conducted. The results are given in Table 4, in which it can be observed that the R^2 and MSE values are quite satisfactory for the broad range of materials used in the study. It should be noted that the 4.0 value corresponds to a fictitious material (practically the crushed-stone and gravel material) with a 4.0 percent optimum asphalt content.

However, it should be noted that Equation 7 is not limited to actual asphalt contents equal to the optimum ones. A study of the goodness of prediction was conducted by using all UM-MSHA samples at all asphalt contents used in the study. Table 5 gives the results of the statistical summary (R^2 and MSE) for the predicted values without and with the correction factor by using Equation 6 incorporated into the model. It can be observed that, although the corrected R^2 values (correction based on optimum asphalt content) are not as high as those given in Table 3 (correction based on specific mix type), they are nonetheless quite satisfactory from a practical engineering viewpoint. As Table 5 indicates, the greatest improvement with the correction is achieved for the slag and sand mixes. All of these bituminous mixtures had optimum asphalt contents much larger than the original Pac range used in the Asphalt Institute study on crushed-stone and gravel mixes.

The form of this recommended equation obviously indicates that certain conditions related to the range of Pac values that result in negative values within the brackets must be avoided. In fact, it is recommended that the range of Pac and P_{opt} combinations used should be such that

$$Pac - P_{opt} \geq -1.5 \text{ (minimum)}$$

$$Pac - P_{opt} \leq -2.5 \text{ (maximum)}$$

SUMMARY

The study described in this paper was based on com-

parisons between nearly 1200 dynamic modulus values measured in the laboratory and predicted values calculated by using the Asphalt Institute regression equation. The results of the comparisons showed a general lack of fit for aggregate mixes, other than crushed stone, whose optimum asphalt content (Marshall method) was outside the range of 3.5-6.2 percent used to develop the original equation. A procedure was developed to minimize the MSE between measured and predicted modulus values while reducing the asphalt content by a constant amount for a given aggregate type. This led to the development of unique correction factors applied directly to the percentage of asphalt variable in the original equation for specific mixes. A more generalized correction factor was also developed that relates the magnitude of this value to the optimum asphalt content of the mix.

Although the results indicated that modification of the original Asphalt Institute equation was necessary for a wide variety of bituminous mixes, it can also be stated unequivocally that the original equation is highly satisfactory for dense-graded crushed-stone and gravel mixes. It is only for other widely differing mixes such as sand asphalt and slag asphalt mixes that the correction (modification) is really necessary.

The proposed methods of correction presented in this paper now create a highly accurate predictive equation that is applicable for a wide range of mix types and physical properties. The modified equation using the mix-dependent correction factor resulted in an R^2 of 0.928, whereas the more generalized correction based on optimum asphalt content yielded an R^2 of 0.891. These R^2 values, based on a data base of 1179 laboratory tests conducted at the Asphalt Institute and University of Maryland laboratories over the past decade, point out the accuracy and wide range of applicability of the predictive equation.

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Discussion

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In the abstract, and in some places in the text, the authors use the term correlation coefficient in referring to R^2 values corresponding to the different regression equations developed in this study. In the terminology of statistics, correlation coefficient is referred to as R value (14, p. 46). In the case of multiple linear regression analysis, R is the correlation between Y (the response variable) and estimated Y and is also called the multiple correlation coefficient. R can take any value between -1 and +1 (14, p. 44). On the other hand, R^2 measures the "proportion of total variation about the mean, $\bar{Y}$, explained by the regression" (14, p. 33). R^2 can take a value between zero and 1 and is also called the coefficient of multiple determination, if the regression equation contains more than one independent variable.

It is suggested that the authors should be consistent with the proper terminology and use coefficient of multiple determination in referring to R^2 values.

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