Precision of the Maximum-Density Estimate in Control-Strip Specifications

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One method of controlling embankment densification refers to the maximum density observed in a local control strip. The local maximum is the standard against which all mainline densities are compared, yet research to date has not addressed the precision of what is truly an *estimated* maximum density. The precision of the maximum-density estimate is quantified by simulating typical rolling procedures and density-growth curves with pertinent parameters, such as the decision rule and sample size, controlled in a factorial design. Analysis of the simulation results leads to the following general findings: (a) The sampling plan with the same locations and correlated comparisons is the most efficient of the four plans investigated, and (b) by employing this plan, the true relative density will be greater than 90 percent about 95 percent of the time.

The embankment-densification process is sufficiently indeterminate to warrant flexible requirements in compaction specifications. These specifications require that maximum density be achieved during the construction process, not under operational loads. The precise value of the maximum density, however, is an elusive quantity. Maximum-density values vary between soils, as does the compactive effort necessary to achieve them.

Control-strip specifications afford the flexibility required by variable soil response characteristics. In these specifications, a pilot section, or control strip, is constructed and closely monitored during its densification. Successive passes with a roller are made, and when the incremental density change between any two passes drops to some predetermined small difference, compaction stops. The resultant density is declared the maximum. Other density measurements taken elsewhere in the project are evaluated in light of this relative maximum until conditions change, in which case a new control strip is created.

Another consideration, essential to the performance of control-strip specifications, has apparently been overlooked. The reference density accepted as the maximum is really an estimate. If this estimate were to be low, all other relative comparisons could be adversely affected. The principal objective of this analysis was to determine the precision of these maximum-density estimates. This was done primarily through computer simulation.

A second objective was to determine the optimum sampling strategy. Several specific questions addressed include the following:

1. Should the same locations be repeatedly sampled between successive passes of a roller or should different locations be randomly selected?
2. What effect would reuse of a sampled density observation have on the sampling plan's overall effectiveness? This situation would occur if a density value used to gauge the effect of a roller pass were to be recycled into the assessment of the effect of the next pass.
3. To what degree does the sample size affect the precision of the maximum-density estimate?

A final objective was to investigate the impact of various decision rules. The decision rule establishes the largest density change for which rolling can stop and is not necessarily zero. Previous field applications have been ambiguous in this regard, leaving the actual decision rule to the inspector's discretion.

DENSITY-GROWTH CURVE AND ROLLING DISTRIBUTION

Field experience indicates that the sampled density increases are large at first and then gradually become smaller. In actual practice, the final density change may be negative. This apparent density decrease may be attributable to two causes: (a) a true density increase did occur but was not detected due to sampling and testing error, or (b) tightly interlocked particles were actually loosened by the rolling process, which increased the volume and decreased the density. Additional passes would reconsolidate the material in the latter case. Points on an actual density-growth curve (1) have been plotted in Figure 1.

For this curve, the maximum average density of 138.5 pcf would be achieved after eight roller passes. Of course, this would not be known to field engineers; they would have to infer true points on the growth curve from sample estimates. One of the possible sampling plans used to estimate maximum density might read as follows:

> Select three random locations within the control-strip boundaries. By using a nuclear gauge, measure the density at each location before and after each pass of the roller. If the average density after the roller's pass is greater than the average density before, select another three random locations and repeat this procedure. Rolling should stop only when a decrease in the average density is observed for the current three locations. The maximum average density is then defined as the largest average density achieved by this procedure.

The effectiveness of this sampling plan may be assessed through computer simulation. With the growth curve shown in Figure 1, 1,000 applications of this procedure were simulated. The number of passes required to estimate maximum density was recorded for each simulation, which produced the frequency histogram shown in Figure 2. The average number of passes made was approximately 7 in this simulation. The overall average density when compaction stopped was 137.0 pcf, which yielded a relative density of approximately 98 percent.

There was some dispersion about this average value, as can be inferred from Figure 2. For example, 13 percent of the simulations stopped on or before the fourth pass, which corresponds to a relative density of about 93 percent. At the other extreme, 11 percent of the applications required nine or more passes. Thus, for this combination of growth curve and sampling plan, there is a 13 percent risk of stopping compaction at 93 percent relative density or less and a similar risk of requiring an excessive amount of compaction.

Although this type of information would be of great value to specification designers, this specific information is meaningful only to those dealing with similar growth curves and sampling plans. Density-growth curves are highly variable between embankment materials, which raises the question of this sampling plan's more general operating characteristics. A sensitivity analysis of these characteristics will be presented after the simulation

procedure and its underlying theory have been developed.

SIMULATION MODEL

An exponential equation was selected to model the density-growth curve. Although the simulation is relatively insensitive to the precise mathematical function used, provided its shape is reasonably correct, the exponential curve affords certain conveniences. Density increases behave similarly to those observed in the field, the curve is easily fitted to specific points, and the average maximum-density plateau is reflected in the theoretical asymptote.

Three sources of variability contribute to any measured density value. These are the variability of the virgin material, the testing variability, and the variability introduced by the rolling process. Of these, only the first two have been experimentally identified; the roller variability must be inferred from empirical observations. Fortunately, this is easily accomplished by a soil-variance anal-

Figure 1. Typical density-growth curve.

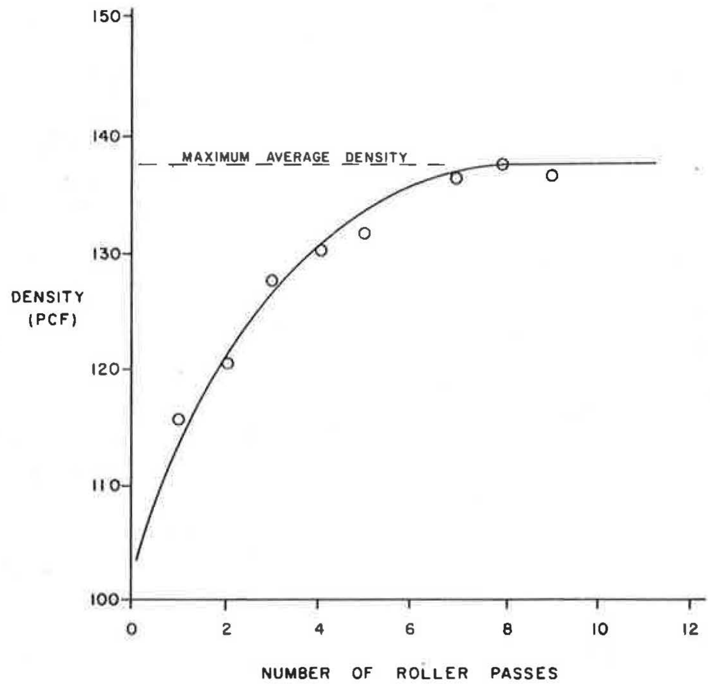
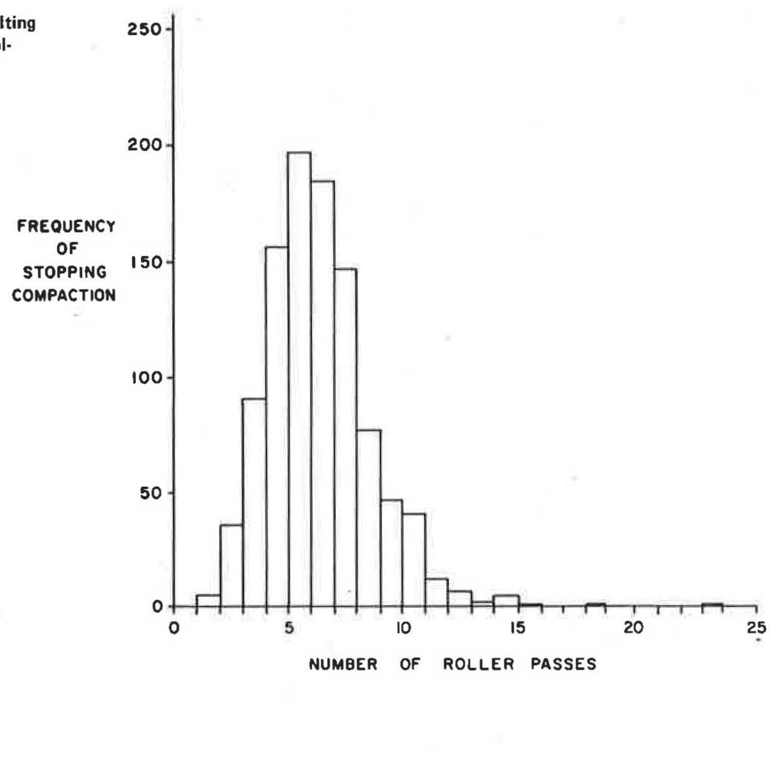


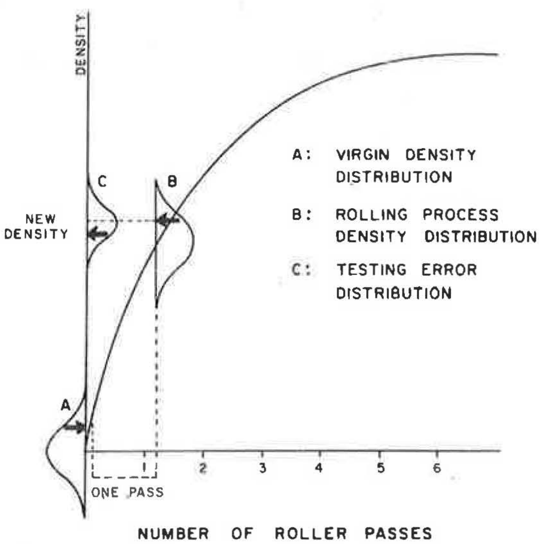
Figure 2. Frequency histogram resulting from computer simulation of control-strip procedure.



ysis. The total variability after maximum density has been reached is known, as is the shape of the growth curve. A trial-and-error process may be used to determine the magnitude of roller variability that must be used in the model to match the final dispersion observed in the field. Typical standard deviations found for the virgin material, roller variability, and testing error are 3.0, 2.5, and 2.0 pcf, respectively.

Figure 3 shows the algorithm used to simulate each successive roller pass. Individual density observations were randomly selected from a normal distribution on the Y-axis and used to find the corresponding X-values. Each X-value, representing the theoretical number of roller passes, was incremented by one unit to simulate the pass of a roller. New Y-values were then computed to determine the resultant densities. These new densities were then perturbed with a scaled random normal deviate to model the variability of the rolling procedure. The resultant densities now represent the true distribution after X passes but must be perturbed once more to model the testing error incurred in their interpretation. Finally, these transformed densities are averaged and either stored for later review or disposed of in accordance with the procedure under test.

Figure 3. Simulated compaction process.



The above procedure was repeated 1,000 times for each X-value, and the entire process was repeated for as many as 20 roller passes. Figure 4 shows several typical density distributions as the control strip is transformed along the growth curve. The distributions remain essentially normal although there is a subtle negative skewness. This skewness parallels the real-world tendency in which distributions are skewed away from a natural boundary (maximum density in this case), but the magnitude of this skewness is negligible for practical purposes. Note that the total dispersion decreases slightly with additional passes, a result of the nonlinear transformation in which low density values are increased at a faster rate than high values.

POSSIBLE SAMPLING SCHEMES

Maximum-density estimation using the control-strip technique is essentially a form of a statistical hypothesis test. Two hypotheses are made, one that maximum density has been reached and the other that further densification is possible. Only if a small density increase is observed can it be inferred that the soil is at or near its maximum density.

Three important distinctions make this particular hypothesis test unlike most of its statistical counterparts. First, the test itself ignores dispersion, because only mean values (i.e., the average densities) are computed. Second, the test is intended to be performed iteratively. Thus, due simply to chance, it is unlikely that even an unrealistically long growth curve will survive many of the

Figure 4. Typical density distributions obtained by simulation procedure.

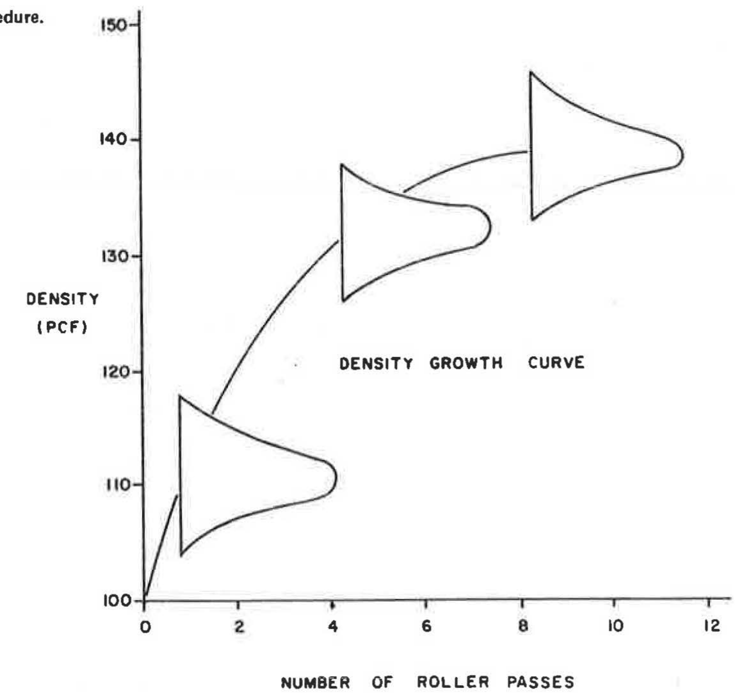
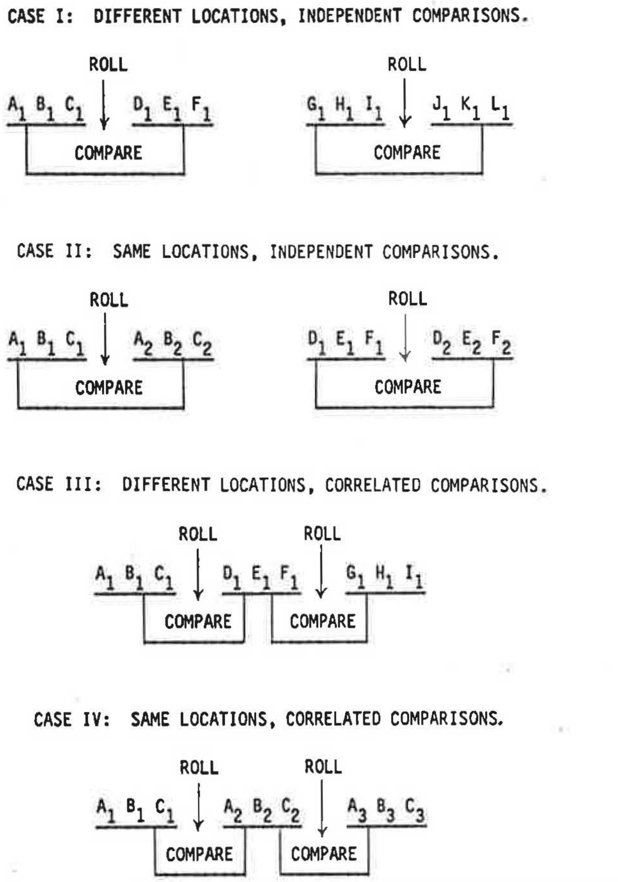


Figure 5. Four alternative sampling plans.



NOTE: LETTERS REPRESENT LOCATIONS, SUBSCRIPTS REPRESENT SUCCESSIVE TEST MEASUREMENTS AT A GIVEN LOCATION.

repeated stop-continue decisions. Finally, the sampling procedure itself may inadvertently influence the outcome. This would occur if independence were lost between any two successive comparisons. This last point is subtle and will be explained below in the analysis of alternative sampling strategies.

Three decisions must be made by an observer who wants to make sequential inferences about a density-growth curve. The first is simply the number of density measurements that will be averaged together. The second is whether the measured densities are to be paired by location before and after a pass or whether entirely different locations are to be measured. Finally, a decision must be made whether the sample averages will be used more than once in successive comparisons.

For a sample size of 3, Figure 5 shows the four possible sampling plans that result from these decisions. Each of these plans may be distinguished by the manner in which the test locations are selected and the comparison process is repeated.

Consider case I. Densities are measured at random locations A, B, and C before a pass and at random locations D, E, and F afterwards. Thus, the locations are independent from each other before and after the pass. A new set of random measurements is then made, and the entire test procedure is repeated for the next pass.

In case II the same random locations are monitored before and after each pass. The comparisons remain independent, however, because the entire procedure is replicated for each rolling sequence.

A subtle variation is introduced by the sampling plan in case III. Here locations A, B, and C are measured before the first pass, and locations D, E, and F are measured afterwards. Then, without an additional three density measurements as in case I, the second roller pass is made. Densities at new locations G, H, and I are subsequently compared with densities at previous locations D, E, and F. Although both the first and the second comparisons are individually independent, they result in a correlated test procedure because the measurements at locations D, E, and F were used twice. Thus, the outcome of the first comparison may have some influence on the outcome of the second.

The final sampling plan, case IV, simply remeasures the densities at the same locations after every pass. Although this may be a practical alternative, it most certainly compounds the correlation problem cited in case III.

One criterion by which the relative merits of these four plans may be evaluated is the sampling effort required. Note that 12 density measurements are required for two roller passes in cases I and II, but only 9 measurements are required for cases III and IV. Thus, the latter plans require a lesser sampling effort.

OPTIMUM SAMPLING STRATEGY

Figure 6 shows the operating characteristics of four distinct sampling strategies. Density increases are plotted on the X-axis and the probability of stopping compaction is plotted on the Y-axis. Note that the probability of stopping compaction, which is equivalent to the risk of a false maximum-density indication whenever a true density increase does occur, becomes larger with progressively smaller density changes. In other words, it becomes more difficult to detect density increases as the true density approaches the maximum.

It is desirable to minimize the risk of false maximum-density indications. This is done if for any given density increase, a particular sampling plan is associated with the smallest probability of stopping compaction. Figure 6 indicates that of the four plans investigated, the one with the same locations and independent comparisons is most powerful because it has the lowest operating-characteristic (OC) curve. The plan with different locations and independent comparisons is the next most powerful for small density increases.

Note the distinct impact of intercomparison correlation: The risk of prematurely stopping compaction is substantially increased near the point of maximum density. Within the subclassification of comparison type, however, the same-location sampling plan is still more powerful. (Curve IV is lower than curve III, as curve II is lower than curve I.)

A trade-off must be considered in deciding which same-location sampling plan is most efficient, the one with independent comparisons or another in which the comparisons are correlated. Plans with independent comparisons are clearly more discriminating, but they also require a larger effective sample size. For the same effective sample size, i.e., the same number of total measurements between two passes, the independent-comparison OC curve and the correlated-comparison OC curve cross so that neither is consistently lower. (The effect is similar to that of curve I crossing curve IV.) The independent-comparison OC curve is lower for small density changes, and the correlated-comparison OC curve is lower for large density changes. Under these circumstances, the net effect of the two sampling plans must be evaluated directly from the density distributions when rolling stops.

Figure 6. Comparison of sampling plans.

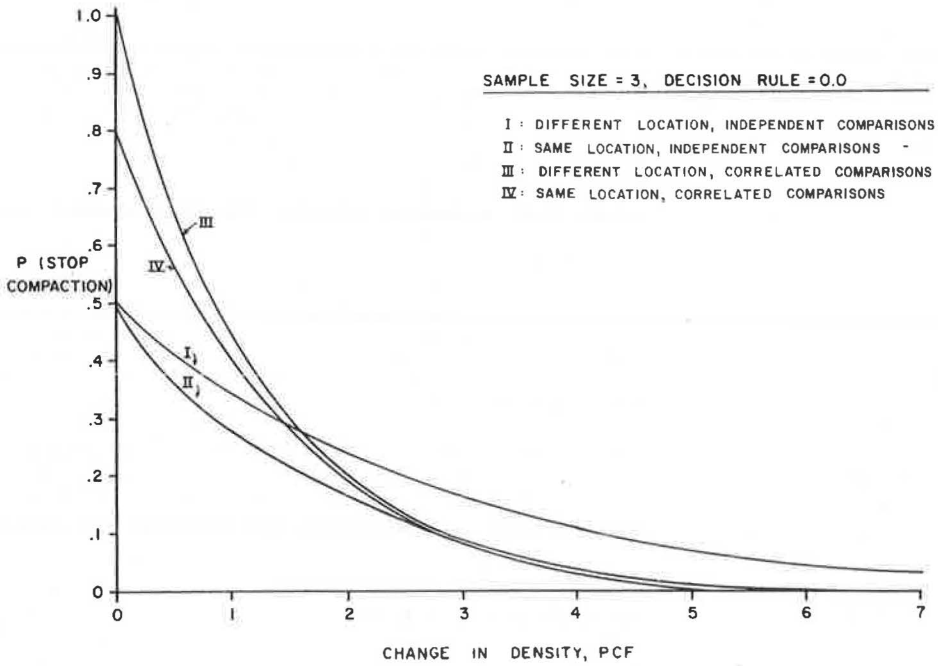
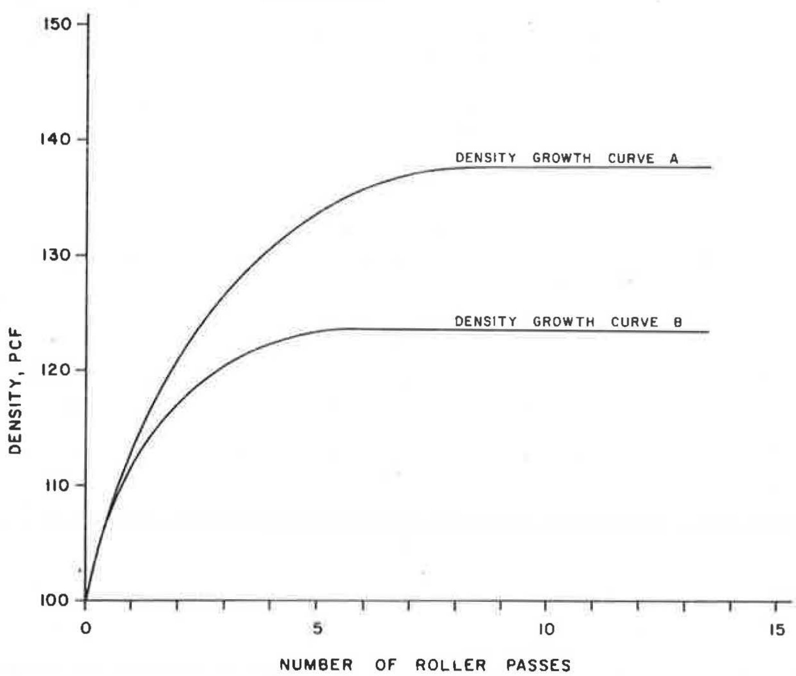


Figure 7. Two possible density-growth curves.



Simulation results indicate that for the same effective sample size, plans with correlated comparisons produce higher average relative densities. They also tend to produce density distributions with a smaller degree of dispersion. Consider two plans with an effective sample size of 4, for example. A same-location, independent-comparison plan would have a sample size of 2, and a same-location, correlated-comparison plan would have a sample size of 4. One simulation analysis revealed that the average relative densities were 97.4 percent for the correlated-comparison plan and 96.4 percent for the independent-comparison plan. The corresponding threshold densities at the 5 percent level of risk, i.e., the lower relative-density limits that are exceeded by 95 percent of the observations, were 94.0 and 91.6 percent, respectively. Clearly the correlated-comparison plan is more powerful. Further discussion will concentrate on the same-location, correlated-comparison plan because it is more efficient for the small effective sample sizes commonly used (case IV in Figure 5).

SENSITIVITY ANALYSIS

Alternative control-strip simulations were investigated in which the effect of the growth curve, the variability of individual density values about the growth curve, the sample size, and the decision rule were all controlled parameters. For each combination of these parameters, 1,000 replications were simulated. Although the individual final density distributions were fairly sensitive to these parameters, reflecting the variable nature of soil compaction characteristics, the density-change OC curves were not. This sensitivity analysis focuses on these OC curves because they are most general, but control-strip-specific results are also presented in a summary format.

Two distinct density-growth curves were considered. Figure 7 shows that growth curve A reaches a higher average maximum density than growth curve B but requires additional roller passes to do so. Although individual density values frequently exceeded these maximum average values, neither growth curve

Figure 8. Effect of density-growth function on density-change OC curve.

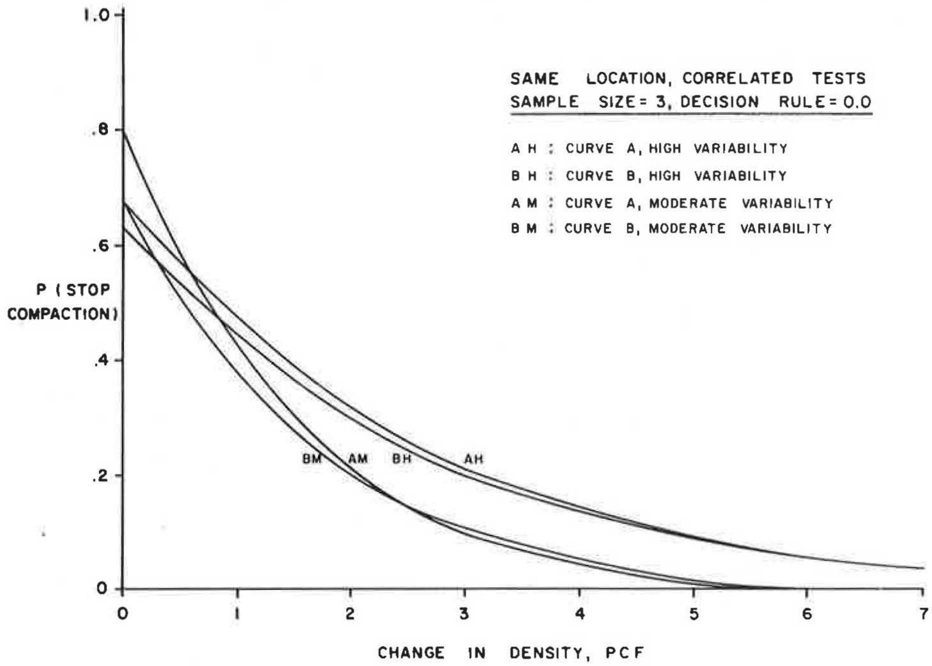
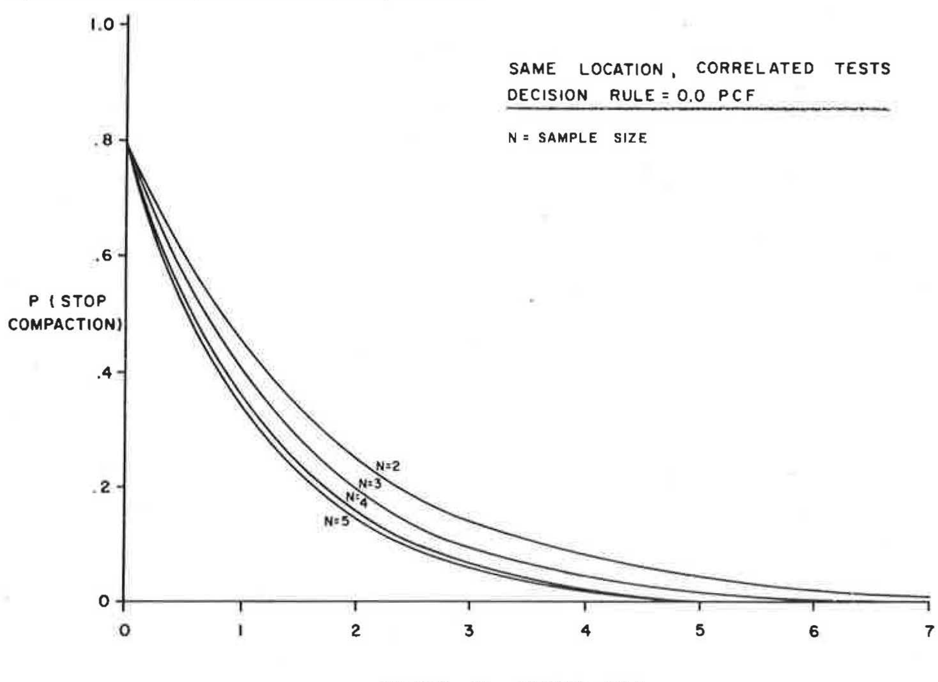


Figure 9. Effect of sample size on density-change OC curve.



exhibited a substantial increase in the average density once the theoretical asymptote was reached.

Moderate- and high-variability components were incorporated into each of the growth curves. The total initial standard deviation of 4.4 pcf, which has been used in the preceding examples, was increased to 7.3 pcf for the high-variability simulations. (The contributory standard deviations were 2.0, 5.0, and 5.0 pcf for the testing error, virgin material, and roller variability.)

Figure 8 shows the OC curves for growth curves A and B at both moderate and high levels of variability. The OC curves appear to be relatively insensitive to the shape of the density-growth curve but not to its level of variability. This will be used to advantage in an analytical approximation that is briefly discussed in the following section. Note here that growth curves with high variability tend to have a greater risk of false maximum-density estimates.

The impact of varying the sample size for a single combination of growth curve and acceptance plan

Figure 10. Effect of decision rule on density-change OC curve.

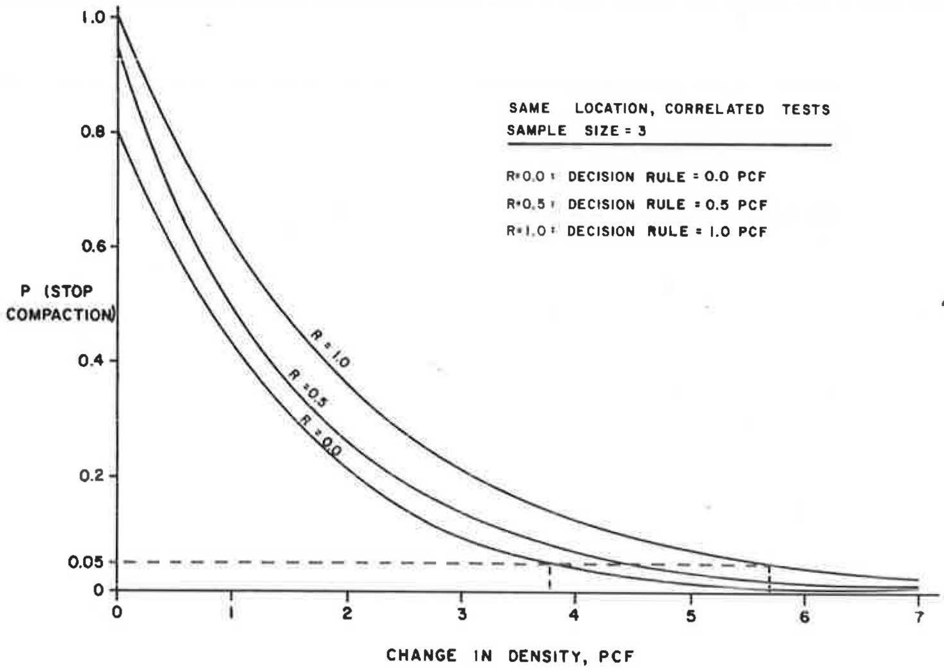


Table 1. Simulation results: paired-location, correlated-test sampling plan.

			Moderate Variability[b]			High Variability[c]		
Curve[a]	Sample Size	Decision Rule (pcf)	Expected Relative Density (%)	Relative Density at 95 Percent Confidence Limit (%)	Maximum No. of Passes at 95 Percent Confidence Limit	Expected Relative Density (%)	Relative Density at 95 Percent Confidence Limit (%)	Maximum No. of Passes at 95 Percent Confidence Limit
A	2	0.0	96.63	91.09	11	97.11	89.37	10
A	2	0.5	96.62	90.54	10	96.81	88.81	9
A	2	1.0	95.72	90.14	10	96.44	88.80	9
A	3	0.0	97.03	93.09	11	97.78	92.49	10
A	3	0.5	96.71	92.13	11	97.52	91.18	10
A	3	1.0	96.27	91.44	10	97.23	90.86	9
A	4	0.0	97.41	93.97	12	98.16	92.91	10
A	4	0.5	96.97	93.19	11	97.78	92.49	10
A	4	1.0	96.54	92.50	10	97.39	90.95	9
A	5	0.0	97.48	94.04	12	98.34	93.58	10
A	5	0.5	97.17	93.56	11	98.09	93.18	10
A	5	1.0	96.66	92.90	10	97.61	92.51	9
B	2	0.0	99.09	95.47	8	98.92	93.31	7
B	2	0.5	98.88	95.10	8	98.78	92.58	7
B	2	1.0	98.74	94.97	7	98.60	92.00	7
B	3	0.0	99.26	96.19	9	99.10	95.62	8
B	3	0.5	99.05	95.55	8	98.99	94.55	7
B	3	1.0	98.84	95.39	8	98.85	93.60	7
B	4	0.0	99.35	96.86	9	99.28	96.21	8
B	4	0.5	99.20	96.23	8	99.16	95.98	7
B	4	1.0	99.04	95.64	7	99.01	94.89	7

[a] Curves A and B in Figure 7. [b] Total initial standard deviation = 4.4 pcf. [c] Total initial standard deviation = 7.3 pcf.

Table 2. Regression coefficients for empirical OC curves.

Sample Size	Decision Rule (pcf)	Moderate Variability[a] B_1	Moderate Variability[a] B_2	High Variability[b] B_1	High Variability[b] B_2
2	0.0	0.80	-0.57	0.70	-0.32
	0.5	0.90	-0.51	0.77	-0.31
	1.0	1.00	-0.45	0.85	-0.29
3	0.0	0.80	-0.68	0.69	-0.39
	0.5	0.95	-0.62	0.77	-0.38
	1.0	1.00	-0.51	0.89	-0.38
4	0.0	0.83	-0.81	0.69	-0.48
	0.5	0.91	-0.64	0.77	-0.43
	1.0	1.00	-0.55	0.88	-0.40
5	0.0	0.76	-0.82	0.73	-0.53
	0.5	0.99	-0.74	0.80	-0.47
	1.0	1.00	-0.63	0.90	-0.42

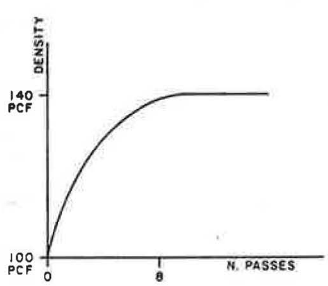
[a]Total initial standard deviation = 4.4 pcf.
[b]Total initial standard deviation = 7.3 pcf.

is shown in Figure 9. Increasing the sample size does effect an improvement, although a point of diminishing returns is soon reached. It is thought unlikely that a sample size greater than 5 would be considered practical in this application.

Nonzero decision rules tend to degrade the precision of maximum-density estimates. The three OC curves shown in Figure 10 indicate that at the 5 percent level of risk, the threshold density change increased by about 2.0 pcf as a result of increasing the decision rule from zero to 1.0 pcf. The risk increase is intuitively logical: To state it simply, rolling stops at a lower point on the density-growth curve.

These same results are reflected in Table 1, where curve-specific results are listed for several of the combinations investigated. It can be seen that the expected relative densities were generally high and generally unaffected by the sample size. Although in some cases growth curves with high variability had slightly higher expected relative densities, they also had a greater dispersion about

Figure 11. Worksheet for empirical OC curve construction.



Sample Size 3
Decision Rule 0.5 PCF
Variability Moderate
B_1 0.95
B_2 -0.62

Column 1	Column 2	Column 3	Column 4	Column 5	Column 6	Column 7
PASS NUMBER (i)	ABSOLUTE DENSITY (PCF)	RELATIVE DENSITY (%)	DELTA, CHANGE IN ABSOLUTE DENSITY (PCF)	P(STOP) ON THIS PASS $P(STOP)=B_1e^{B_2 DELTA}$	REPLICATIONS REACHING THIS PASS	REPLICATIONS STOPPING ON THIS PASS
0	100.00	71.43	-	-	1000	0
1	112.00	80.00	12.00	0.00	1000	0
2	120.00	85.71	8.00	0.01	1000	10
3	126.00	90.00	6.00	0.02	990	20
4	130.00	92.86	4.00	0.08	970	78
5	133.00	95.00	3.00	0.15	892	134
6	135.00	96.43	2.00	0.27	758	205
7	136.50	97.50	1.50	0.37	553	205
8	137.50	98.21	1.00	0.51	348	177
9	138.50	98.93	1.00	0.51	171	87
10	139.25	99.46	0.75	0.60	84	50
11	139.75	99.82	0.50	0.70	34	24
12	140.00	100.00	0.25	0.81	10	8
13	140.00	100.00	0	0.95	2	2
					Check: Column 7 Total=1000	

EXPLANATION

Columns 1 and 2: From assumed density growth curve.

Column 3 : $(\text{Col. }3)_i = (\text{Col. }2)_i/(\text{Col. }2)_{i\,=\,\text{final}}$

Column 4 : $(\text{Col. }4)_i = (\text{Col. }2)_i - (\text{Col. }2)_{i-1}$

Column 5 : B_1, B_2 from Table 2, Delta = $(\text{Col. }4)_i$

Column 6 : 1000 initially, then $(\text{Col. }6)_i = (\text{Col. }6)_{i-1} - (\text{Col. }7)_{i-1}$

Column 7 : 0 initially, then $(\text{Col. }7)_i = (\text{Col. }5)_i \times (\text{Col. }6)_i$

Expected Relative Density = 96.7%
Average of (Col. 3) weighted by (Col. 7)

Threshold Relative Density, 95% confidence limit = 93.4%
(Interpolate value in Column 3 for an observation of 950 in Column 6.)

Maximum Number of Passes, 95% confidence limit = 11
((Col. 1) value for which (Col. 6) ≈ 50.)

this average. Their threshold relative densities at the 95 percent level of confidence for the high-variability growth curves were uniformly lower than those of their moderate-variability counterparts. These threshold densities are sensitive to the sample size and may be influential in determining the required sampling effort. Finally the maximum number of required passes at the 95 percent confidence limit is also listed. This enables the user to estimate the maximum number of passes that can be expected for any one control strip.

ANALYTICAL TECHNIQUES

Specification designers fortunate enough to recognize in Table 1 a growth curve modeling their regional soils may directly implement the results of these findings. The sample size and decision rule would be selected with reference to the relative densities, the anticipated measuring-time delay, and the maximum number of passes. [Of course, the specification that subsequently uses the maximum-density estimate would also be considered. These specifications typically allow a certain percentage of the density distribution to fall below the estimated maximum (see paper by Barros, Weed, and Willenbrock in this Record).] Other designers may generate tables similar to Table 1 by using the density-change OC curve.

The density-change OC curve underlies all relative-density estimates and is not sensitive to the shape of the density-growth curve. If this OC curve can be constructed and paired with a particular growth function, then specific probabilities may be computed. One nonlinear regression model that closely matches the form of the simulated density-change OC curve is given as follows:

$$P(STOP) = B_1 \exp(B_2 DELTA)$$

where P(STOP) is the probability of stopping compaction given a DELTA density increase and DELTA is the observed density increase in pounds per cubic foot. The regression coefficients that best estimate the simulated same-location, correlated-test OC curves are given in Table 2. The coefficients were derived specifically from growth curve A, but they are similar to and conservatively represent the coefficients associated with lower growth functions. Plots of these regression models are similar to those shown in Figures 6 and 8-10.

An analytical technique that empirically reconstructs the results of computer simulation analyses will now be discussed briefly. This technique uses the density-change OC curve to determine the shape of the rolling-frequency distribution for a given sample size, such as the one shown in Figure 2. As indicated by the worksheet shown in Figure 11, a rough approximation of the density-growth curve must be assumed. The relative densities at incremental stages of compaction are then computed, as is the incremental density increase after each pass. These incremental density increases are then substituted into the appropriate regression model of Table 2, and the probability of stopping after any pass may be estimated. The product of this probability and the number of replications surviving previous stop or continue decisions gives the number of replications that stop after the current pass.

The average relative density and the average number of passes made, both weighted by their frequency of occurrence, represent the expected relative density and expected number of passes, respectively. Threshold values are determined simply by proceeding from either tail of the frequency distribution until a sufficient tail area has been accumulated. An example of this procedure is presented in worksheet form in Figure 11.

This is a practical, if empirical, analytical technique. It provides a means by which specification provisions may be linked to previously unquantified conditions. Iteration of the analysis for a range of possible growth functions should provide a reasonable estimate of the maximum-density estimate obtainable in control strips under field conditions.

SUMMARY AND CONCLUSIONS

Control-strip specifications monitor the magnitude of successive density changes to estimate relative densification. Small density changes, which occur with increased height on the density-growth curve, signal the approach of maximum density. The density-monitoring procedure is therefore critical to the precision of the maximum-density estimate.

Same-location sampling plans are more effective than their different-location counterparts. Apparently some location-to-location variability is screened from the inference-making process by these plans, thereby increasing the plan's efficiency.

Correlation of successive comparisons in a sampling plan adversely affects the maximum-density estimate. This consideration must be weighed against the sampling effort itself: Correlated-comparison sampling plans require a smaller effective sample size. In practice, it is anticipated that the same-location, correlated-comparison sampling plan will be most useful.

Two density-growth curves were investigated by using both moderate- and high-variability components. Although moderate variability should more realistically reflect true field conditions, high variability was included as part of a sensitivity analysis. Aspects of the sampling plan, such as the decision rule and sample size, were investigated at both levels of variability for the two density-growth curves.

Although nonzero decision rules tend to degrade the ability to achieve maximum density, two factors are in their favor. The marginal loss in precision is not great, and a nonzero decision rule may be more easily implemented. Both agency inspectors and the contractor's personnel may be more easily persuaded that a small density deficiency is critical if the decision rule is 0.5 pcf rather than 0.0.

Efficiency of the estimation procedure does improve with increased sample sizes, but a sample size of 3 may be sufficient in practice. In any event, these and other subjective decisions must be made by the specification designer.

A precision-gauging technique was presented that quantifies key aspects of the decision-making procedure. This technique led to three application-specific parameters: the expected relative density, the threshold relative density, and the maximum number of passes.

Finally, although maximum-density estimates may be influenced by the growth curve and its inherent variability, these are not within the designer's control. Specification provisions have been identified that will control the density-change OC curve, and the expected relative densities are consistently high. When these densities are evaluated in light of the relatively small sampling effort, it is evident that control-strip maximum density estimates may be exceptionally precise.

REFERENCE

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Software Package for Design and Analysis of Acceptance Procedures Based on Percent Defective

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The trend toward statistical end-result specifications has led to the development of construction specifications based on the concept of percent defective. To analyze the risks and determine the effectiveness of the acceptance procedures associated with these specifications, operating-characteristic curves must be constructed. However, many potential users do not have a working knowledge of the noncentral t and beta distributions necessary for this development. The underlying theory, several useful references, and a conversational computer program that greatly simplifies the design and analysis of specifications of this type are presented.

The current trend toward statistical end-result specifications has been a natural step in the evolution of the highway quality-assurance system. Whereas the earlier method-type specifications outlined in detail precisely how the work was to be accomplished, the more modern approach has been to define the characteristics and quality requirements of the finished product. Contractors are allowed considerable flexibility in meeting these requirements and the specifying agency is responsible primarily for the evaluation of the finished work.

The end-result approach offers several advantages over the earlier method-type specifications. First, by recognizing the existence of both inherent and testing variability, it deals with construction parameters in a more realistic manner. Highway engineers have begun to realize that it is not unusual, nor necessarily undesirable, for a small percentage of test values to fall outside realistic specification limits. Second, by defining the control of the construction process as the contractor's responsibility and the acceptance of the work (end result) as the agency's responsibility, the likelihood of contractual disputes can be reduced. Third, by clearly defining acceptance criteria and random-sampling procedures, the risks to both the contractor and the highway agency can be controlled and known in advance. Under the earlier method-type specifications, a contractor's bid was often influenced by the reputation of the highway inspector assigned to the project. Fourth, the development of adjusted-payment schedules provides a practical means to deal with work that is substandard but not so deficient that it warrants removal and replacement. Finally, because the random-sampling plans avoid the biases that are likely to occur when an inspector attempts to select a representative sample, reliable estimates of the as-built construction quality can be made. This information can also be used as feedback to determine whether further modifications of the specifications are desirable.

One of the most important steps in the design of an end-result specification is the development of the operating-characteristic (OC) curve describing its capabilities. Although most of the necessary theory is available in one form or another, much of it is not familiar or easily accessible to highway engineers. In this paper this theory is outlined, appropriate references are cited, and a conversational computer program that greatly simplifies the design or analysis of the type of statistical acceptance procedure normally used with end-result specifications is presented.

PERCENT DEFECTIVE AS A MEASURE OF QUALITY

Although several statistical measures of quality are available, highway engineers have exhibited a strong preference for the concept of percent defective, the estimated percentage of the work falling outside specification limits (or its complement, the percent within limits). This measure is particularly appealing, not only because the amount of material falling within limits is believed to be strongly related to actual performance, but because it can be applied to virtually any construction quality characteristic. This general philosophy is promulgated in Standard 214 (1) of the American Concrete Institute (ACI), for example, although the ACI acceptance criteria do not use a purely percent defective approach.

Two statistical parameters commonly used with these procedures are the process mean and standard deviation. In this paper the situation is addressed in which the values of these parameters are not known and must be estimated from sample observations. This development is appropriate for those situations in which these values may change during the course of a project.

Figure 1 illustrates three possible parent populations having identical percent defective levels and the sampling distribution associated with a sample size of 5. The sampling distribution is strongly skewed, but because the technique for estimating percent defective is unbiased, its mean is exactly at the true population percent defective. The significance of this is that although the quality of any single lot may be overestimated or underestimated, the long-term average of these estimates will be exactly equal to the true lot quality. This is of particular importance in developing fair and equitable construction specifications.

The theory associated with the development of specifications based on percent defective is somewhat involved and uses frequency distributions seldom encountered in introductory statistics courses.