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Edge-Stiffening Effect of New Jersey Barrier Walls on Cantilever Slabs

C. SADLER and M. HOLOWKA

ABSTRACT

The policy of using New Jersey type barrier walls along all major highways has been endorsed by the Ontario Ministry of Transportation and Communications. Consequently, new bridges and deck rehabilitations have massive barrier walls along outside edges of bridge decks. These barriers act as edge stiffening for cantilever slabs and have a significant effect on the distribution of live load on cantilever slabs. The current code specifications for the design of concrete cantilever slabs were established for slabs with rigid supports and with no edge stiffening. These specifications are conservative when edge stiffening is present. The load distribution of a typical cantilever slab supported by an exterior longitudinal girder was investigated by using three-dimensional finite elements. The study considered various edge-stiffening conditions and varying flexibility of longitudinal deck support. The results are compared with the methods given in the Ontario Highway Bridge Design Code and with other simplified methods. As a result of the enhanced load distribution, a significant potential saving in the quantity of cantilever reinforcing steel and in the cost of deck rehabilitation can be realized.

During the past few decades the nature of highways and vehicular traffic has changed rapidly. The highway system has developed with the objectives of providing for increased traffic volumes, increased truck loads, faster speeds, and greater safety. The vehicular traffic, in particular truck traffic, has dramatically changed in size and weight. As the heavy trucks have become more numerous and traffic has become more congested, a greater need to confine out-of-control trucks has arisen. Consequently, the nature of the restraining elements, which are designed to keep trucks within their right-of-way, has also changed.

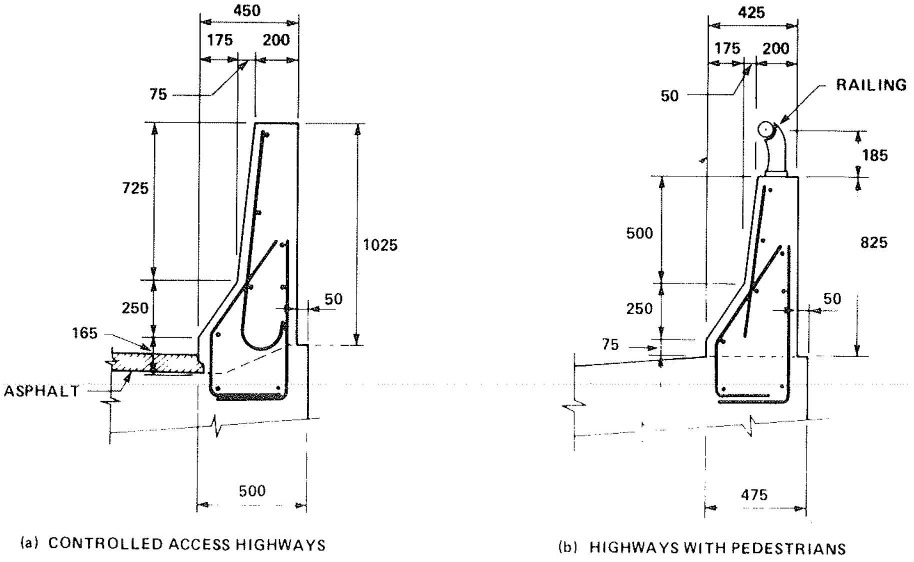
Initially, the railings, parapet walls, or barrier walls were of simple form, consisting of a post and railing type. The initial use of wood gave way to the stronger materials of steel and concrete. However, as truck size increased, the post and railing type were not sufficient to resist collision loads and could not redirect out-of-control trucks back onto the highway. Consequently, the province of Ontario adopted a standard barrier wall that consisted of a continuous reinforced-concrete barrier wall. The typical barrier wall used for controlled-access highways is shown in Figure 1. This barrier wall is 450 mm wide at the base and just more than 1 m in height, with a total mass of 760 kg/m. Also shown is a barrier wall with railing that is used for roads with pedestrians. These massive barrier walls are considered a restraint mechanism and are used to redirect traffic, but not in a structural sense.

In Ontario a popular form of bridge construction is the concrete slab on longitudinal concrete or steel girders. Economically, it is advantageous to minimize the number of girders; consequently, the use of a cantilever slab is common. A typical cross section of a recently designed continuous steel box-girder bridge is shown in Figure 2. The design of the cantilever is governed by (a) dead loads, (b) vertical live loads, and (c) horizontal collision loads. The dead-load effects are secondary compared to the live-load effects. The ratio of factored live-load effect to factored dead-load effect is approximately 2.5 to 3.5 for a cantilever span of 1.5 m.

Current design specifications do not take into account the presence of these massive barrier walls. The design specifications have not kept pace with the development of the barrier walls and their structural effect on the design of the supporting slab. The effect of the presence of continuous concrete barrier walls on the design of the supporting cantilever slab is investigated. The presence of barrier walls affects only the distribution of vertical live loads and collision loads. Dead-load effects are not altered by the presence of barrier walls.

CODE SPECIFICATIONS FOR CANTILEVER SLABS

Current codes have been developed so that the canti-



(a) CONTROLLED ACCESS HIGHWAYS (b) HIGHWAYS WITH PEDESTRIANS

FIGURE 1 Typical barrier walls, Ontario Ministry of Transportation and Communications.

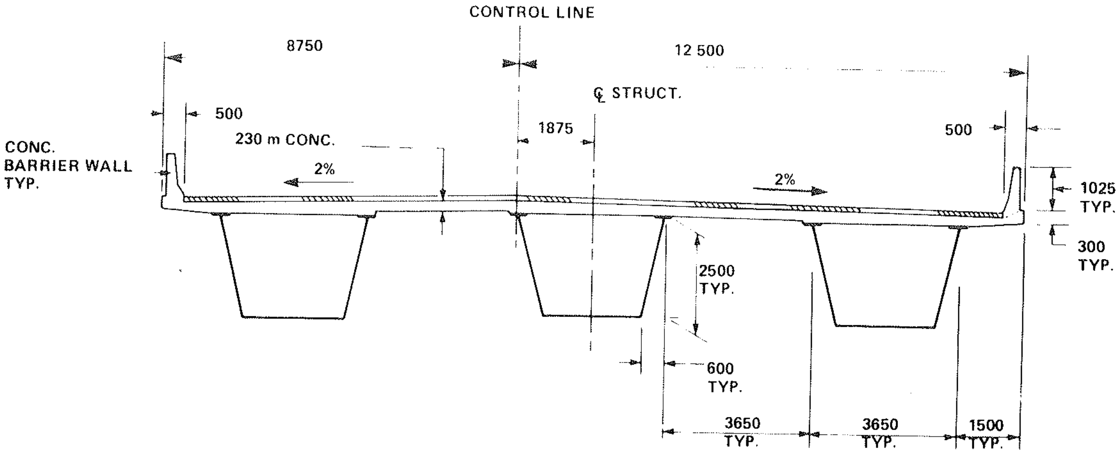


FIGURE 2 Typical section of a steel box-girder bridge.

lever slab can be designed on a unit length basis. The unit length is parallel to the support line of the cantilever. The AASHTO specifications (1) for the distribution of live load have not changed over the past 35 years. The formulas for determining the effective length of slab-resisting live load were developed so that the slab is designed to support the load independent of the end support along the cantilever. For railing or collision loads, the provisions do attempt to take into account the presence of a parapet; however, they do not take into account the length of the cantilever. The design moments are calculated by distributing the moment caused by a concentrated vertical wheel load or a collision load over a prescribed effective length (E):

$$M \text{ (per unit length of slab)} = (P \cdot X)/E \qquad (1)$$

where

M = moment resultant;
P = concentrated vehicle load caused by wheel or collision load;
X = moment arm, distance from wheel load to supported end, or distance from location of collision load to base of barrier wall; and
E = effective length of cantilever slab resisting live-load moments (see Table 1).

The recently published Ontario Highway Bridge Design Code (OHBDC) (2) bases its formulas for cantilever slabs partly on Westergaard's (3) elastic method (AASHTO) and partly on a semigraphical elastic method (4), as shown in Table 1. In the cases of (a) cantilever slabs spanning less than 1.2 m and supporting wheel loads and (b) barrier wall collision loads, the applicable equations are identical to those of AASHTO. In the case of wheel loads applied to cantilever slabs greater than 1.2 m and where the cantilever span is small in comparison to the length of the supported end, the semigraphical elastic method proposed by Bakht and Holland (4) is specified. The design moment for live load is given in the form:

$$M \text{ (per unit length of slab)} = (PA'/\pi)\left\{1/\cosh[A'y/(C-x)]\right\} \qquad (2)$$

where

A' = coefficient (see Figures 3 and 10);
y = distance measured along supported end from the wheel load;

C = distance of concentrated wheel load from support end, measured along the x-axis; and

x = distance measured perpendicular to and from the supported end.

Equation 2 is applicable to slabs with linearly varying thickness, but it does not account for the presence of edge stiffening. The coefficient A' is dependent on the slab thickness ratio, the position of the load on the slab, and the location of the reference point, as shown in Figure 3, which is taken from the OHBDC. For the majority of bridges, the cantilever span is less than 1.2 m in length; consequently, the AASHTO format was retained in lieu of the more general form of Equation 2 because for small spans the variation between the two equations is minimal. It should be noted that for both codes the cantilever span for slab-on-girder bridges is limited in length to 1.8 m.

SCOPE OF INVESTIGATION

The present study was undertaken to determine the effect of the presence of edge stiffening or barrier walls on the distribution of live-load effects in typical cantilever slabs. A typical cantilever deck section (Figure 2) was considered in the formulation of the mathematical models.

The distribution of live-load effects in cantilever slabs is affected by the following structural parameters:

1. Length of slab in the direction of the overhang,
2. Thickness of the slab overhang,
3. Material properties of the slab (modulus of elasticity),
4. Presence of edge stiffening,

TABLE 1 Moment and Distribution Length Formulas

	Direction of Force Effect [distribution length (E), m]	
Load Case	Transverse Moment	Longitudinal Moment
AASHTO		
Wheel loads	$0.8X + 1.143$	$0.35X + 0.98 \leq 2.134$
Collision loads		
With barrier wall	$0.8X + 1.524$	–
Without barrier wall	$0.8X + 1.143$	–
OHBDC		
Wheel loads		
Cantilever span $\leq$ 1.2 m	$0.8X + 1.15$	
Cantilever span > 1.2 m	$M = (PA'/\pi)\{1/\cosh[A'y/(C - x)]\}$	$0.35X + 1.0 \leq 2.1$
Collision loads		
With barrier wall	$0.8X + 1.15$	–
Without barrier wall	$0.8X + 1.15$	–

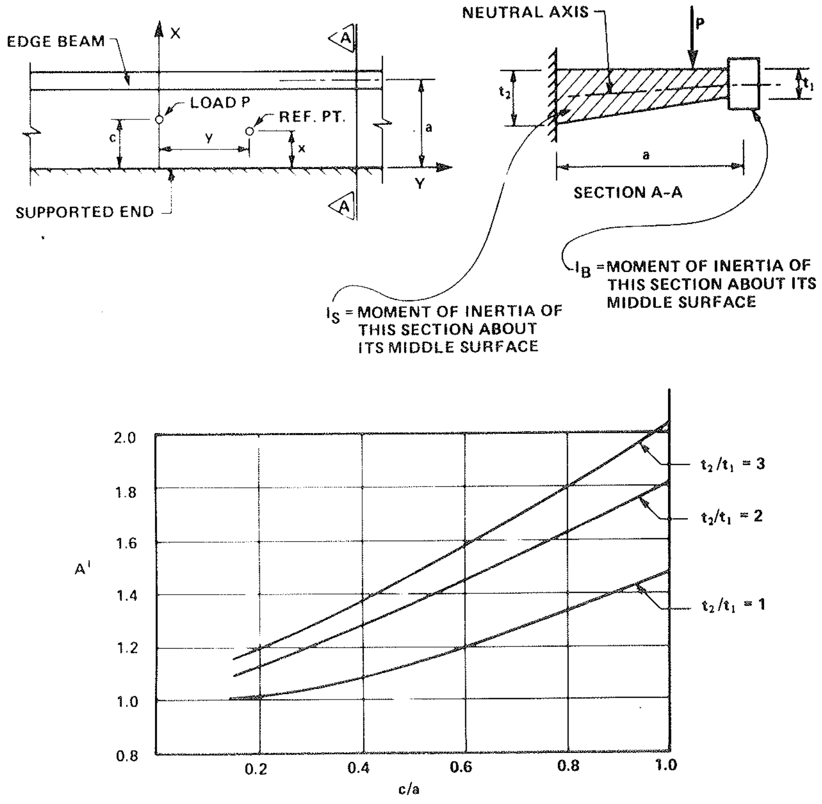


FIGURE 3 Value of coefficient A' at Y = 0.

5. Longitudinal stiffness of supporting girders, and
6. Location and type of load.

Several of these parameters are kept constant or specified in order to simplify the analysis. The first three items are established as follows: slab span = 1.5 m, slab thickness = 230 mm, and material property of the concrete slab (E_{slab}) = 27 700 MPa. (Note that E_{slab} = Young's modulus of elasticity of a concrete deck.)

The parameters studied are 4, 5, and 6. The degree of edge stiffening is varied by assigning appropriate values of the modulus of elasticity to the barrier wall. The longitudinal stiffness of the supporting girder is varied by assigning different boundary conditions to the supports. The location and magnitude of the applied vertical and horizontal live loads are as specified in the OHBDC.

The results from the mathematical model were then compared with both code methods and with other existing methods of analysis (4,5).

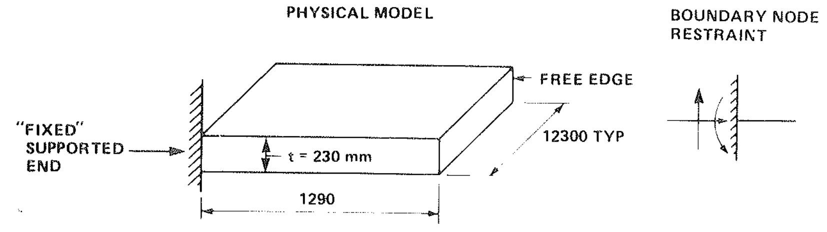
FINITE-ELEMENT ANALYSIS

Analysis of cantilever slab configurations was carried out by using the finite-element program QUEST (6). The computer program was used to perform a linear elastic analysis of a bridge by representing structural elements with quadrilateral thin shell finite elements capable of simulating both membrane and flexure behavior. The program is based on the displacement formulation of the finite-element method and considers all six degrees of freedom at each element node. The method allowed any one or more of the six degrees of freedom (three translatory and three rotational) to be constrained at a nodal point.

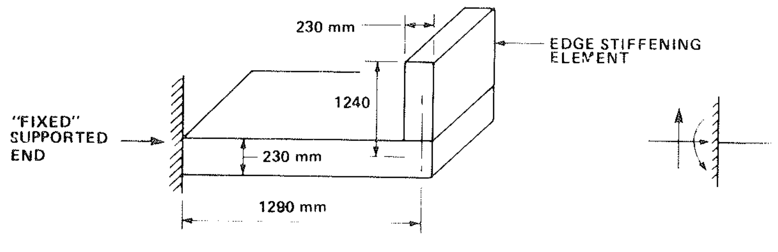
Three different cantilever slab configurations were investigated. The cases chosen represent extreme structural or boundary states. In the first case a cantilever slab fixed at the supported end against all translations and rotations and has no edge stiffening along the free edge is modeled. In the second case a cantilever slab, again, is modeled; it is fixed at the supported end but has edge stiffening along the free edge. In the third case a cantilever slab that has the supported end resting on a flexible media is modeled; for example, a steel plate girder parallel to the direction of traffic. The free edge is edge stiffened. These three conditions are shown in Figure 4.

The load cases for each of the three cases are shown in Figure 5 and are as follows:

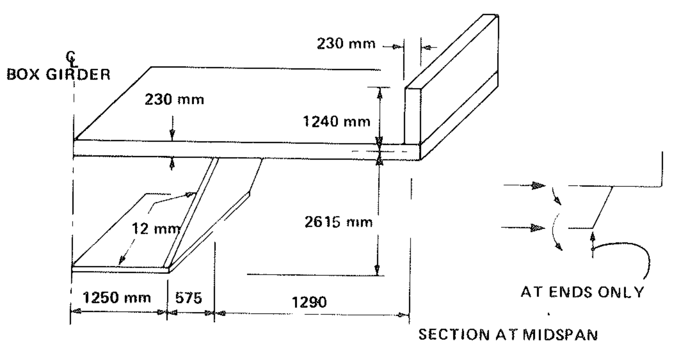
1. An 80-kN horizontal collision load applied at the top of the barrier wall,



a) FIXED SUPPORTED END – NO EDGE STIFFENING

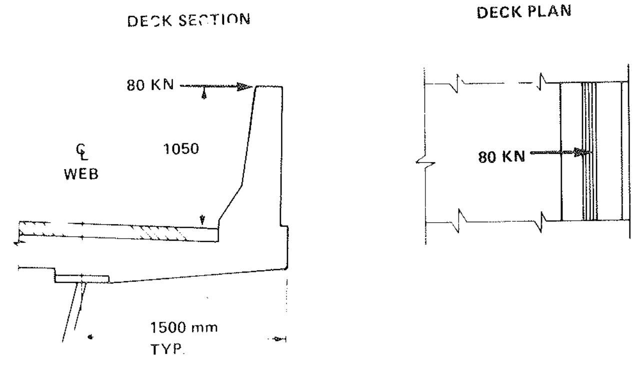


b) FIXED SUPPORTED END – EDGE STIFFENING PRESENT

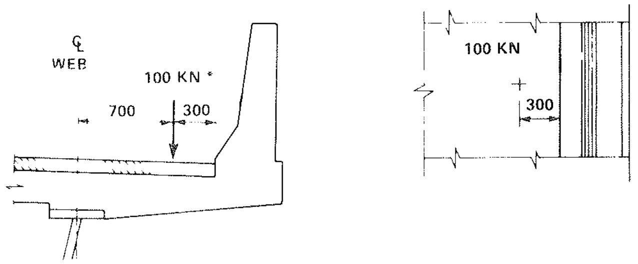


c) FLEXIBLE SUPPORTED END – EDGE STIFFENING PRESENT

FIGURE 4 Cantilever slab configurations used to model structural response for finite-element analysis.

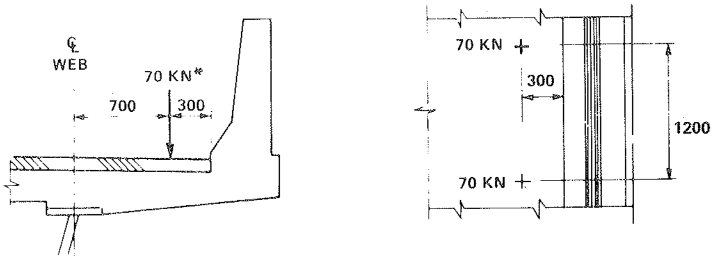


a) COLLISION LOAD



NOTE: (*) WHEEL LOADS ACT AS CONCENTRATED POINT LOADS

b) 100 KN VERTICAL LOAD



c) 2 – 70 KN VERTICAL LOADS

FIGURE 5 Load cases: magnitudes and positions.

2. A single 100-kN vertical wheel load acting on the slab, and

3. Two 70-kN wheel loads that act on the slab and are 1.2 m apart in the direction normal to the span of the slab.

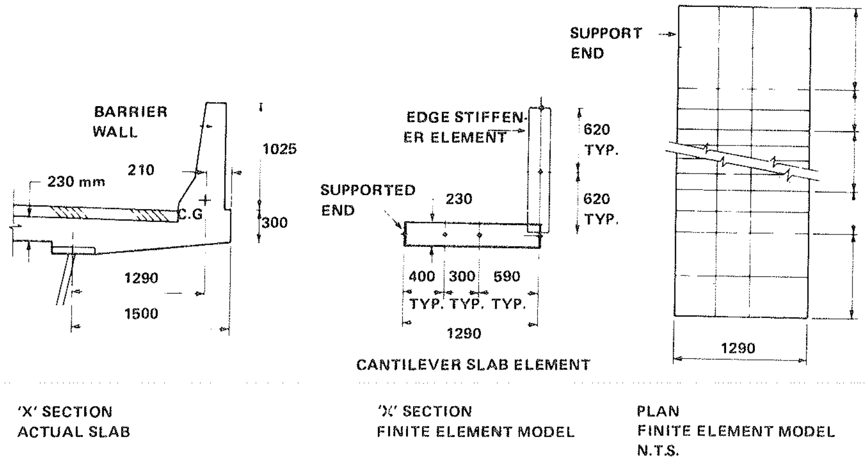
The vertical loads were applied at 300 mm from the face of the barrier wall. Dynamic load effects were not included in this investigation because the effect simply affects the magnitude and not the distribution of the live load.

The process of modeling the structural behavior of a cantilever slab requires an arrangement of quadrilateral elements so as to closely approximate the load-distribution characteristics of the real structure. Figure 6 illustrates the subdivision of the idealized structure into finite elements. The arrangement shown provides reasonable element aspect ratios and provides for an element layout that results in a reasonable resolution of the resulting force effects. To avoid local effects caused by slab discontinuities, a width 8 times the cantilever slab span is used. In this way the condition of an infinitely wide cantilever slab is modeled. Structural components that comprise the barrier wall, slab, and support beam are all modeled. In modeling the barrier wall it was decided to replace the actual barrier wall shape with elements of a rectangular cross section to simplify the modeling for the finite-element analysis. The only requirement is that the overall height and moment of inertia of the barrier wall about the base of both the model and actual barrier wall be the same. This facilitated the correct application of the horizontal collision load and at the same time closely approximated the vertical stiffness of the edge-stiffening element.

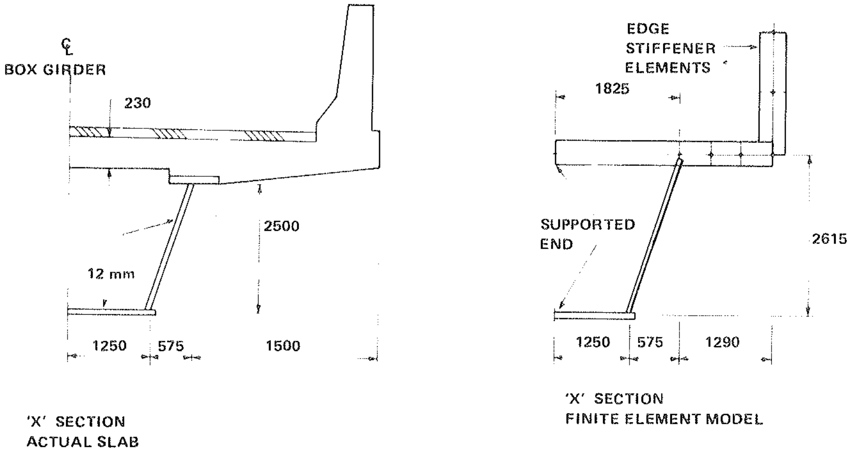
OBSERVATION AND DISCUSSION OF RESULTS

A total of three different idealizations, each with the same three load cases, were investigated. The results of the analysis are given in Figures 7-9. The bending moments in the direction of the cantilever span are shown as contours on a plan of the mathematical model.

Figure 7 shows the distribution of moments for the collision load of 80 kN. As can be expected, the maximum moment occurs at the free edge of the cantilever slab. The first contour is representative of the condition where no barrier wall stiffening is considered. The second and third contours are representative for the cases where barrier wall stiffening is considered. The peak moment decreases by almost 30 percent when barrier walls are consid-

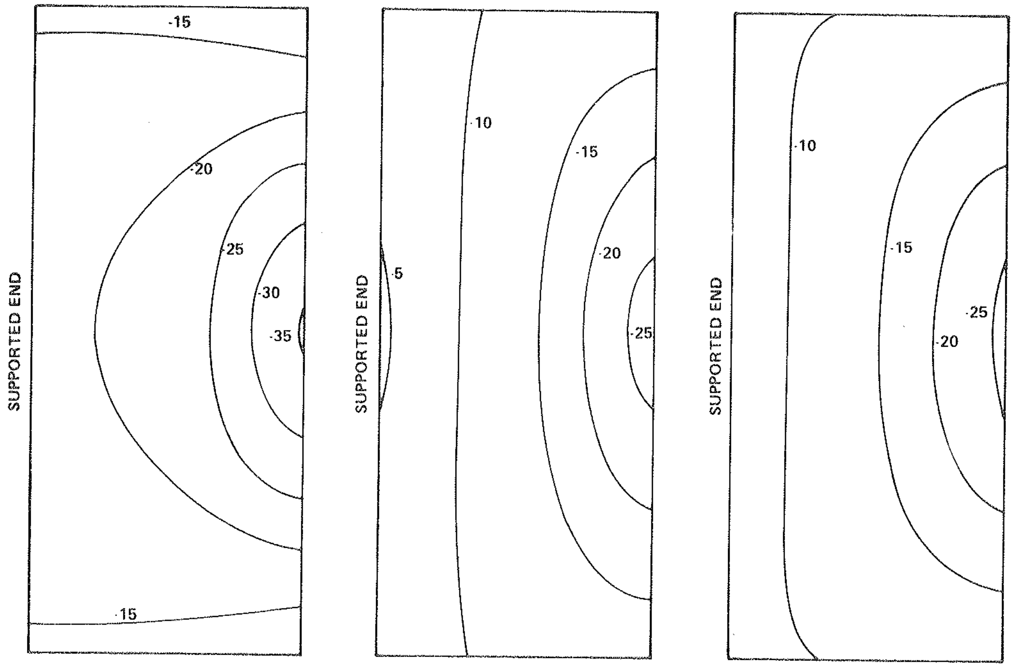


a) STRUCTURE IDEALIZATION FOR QUEST RUN No. 1 & 2



b) STRUCTURE IDEALIZATION FOR QUEST RUN No. 3

FIGURE 6 Finite-element arrangement.



a) QUEST RUN No. 1 b) QUEST RUN No. 2 c) QUEST RUN No. 3

FIGURE 7 Moment diagram, 80-kN collision (kN·m/m).

TABLE 2 Comparison of Results of Analysis (moments at centerline web)

	Maximum Moment Results (kN·m/m)							
	AASHTO		OHBDC		QUEST 1: Fixed-End Support Without Barrier Wall	Bakht(5): Fixed-End Support With Barrier Wall	QUEST 2: Fixed-End Support With Barrier Wall	QUEST 3: Flexible-End Support With Barrier Wall
Load Case	With Barrier Wall	Without Barrier Wall	With Barrier Wall	Without Barrier Wall				
80-kN collision	39.2 66.1[a]	45.5 86.3[a]	39.2 66.1[a]	45.5 86.3[a]	18.8 35.0[a]	– –	10.3 28.0[a]	9.0 25.0[a]
100-kN wheels	–	41.2	–	36.3	35.4	25.5	24.0	11.6
2- to 70-kN wheels	–	–	–	32.5	32.5	26.3	21.8	11.9

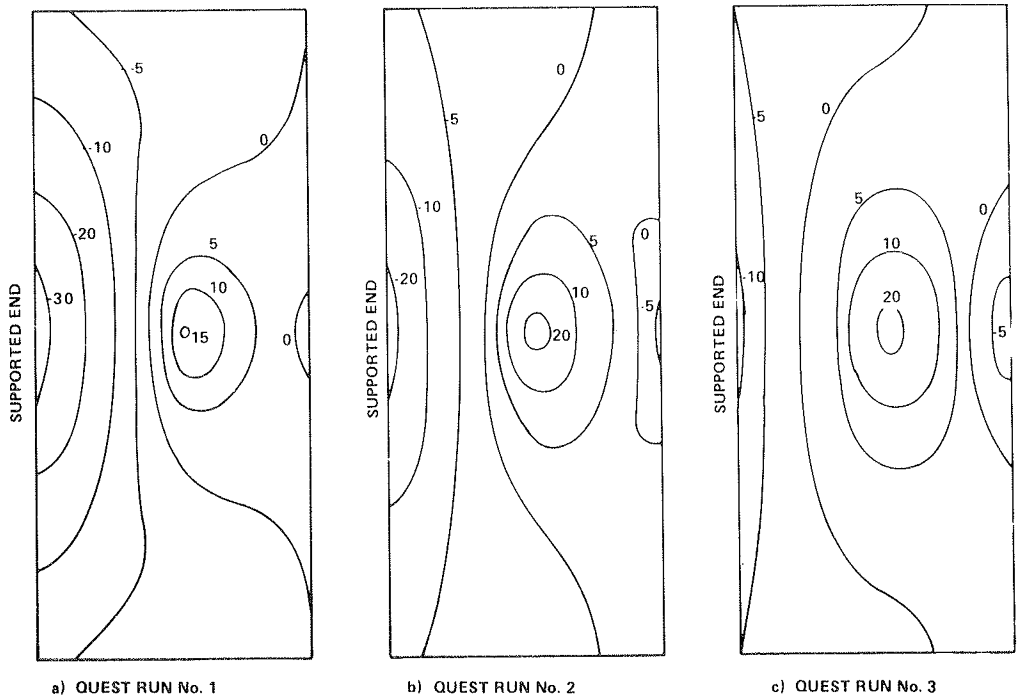
[a] Moment at face of barrier wall.

ered. The boundary condition of the supported end has little effect on the moment distribution at the free edge.

The comparison of results for the various methods of analysis is given in Table 2. For this load case, moment values at both the supported and free edge are tabulated. Both code methods (AASHTO and OHBDC) give the same results. The finite-element analysis gives moments that are significantly smaller than the code requirement. If the barrier wall is treated as a cantilever slab supported by the deck, then the moments at the base of the barrier wall can be calculated in the same fashion as for vertical loads and in accordance with the code methods. Consequently, E = 0.8X + 1.15 = 2.142 m, and the live-load moment = PX/E = 46.3 kN•m/m. This is 72 percent greater than that predicted by the finite-element analysis, but it is only 70 percent of the code predictions. At the supported end of the cantilever slab the finite-element results are at least 50 percent smaller than the code values. It is obvious that for the cantilever slab with this type of concrete barrier wall, the provisions of both codes are extremely conservative.

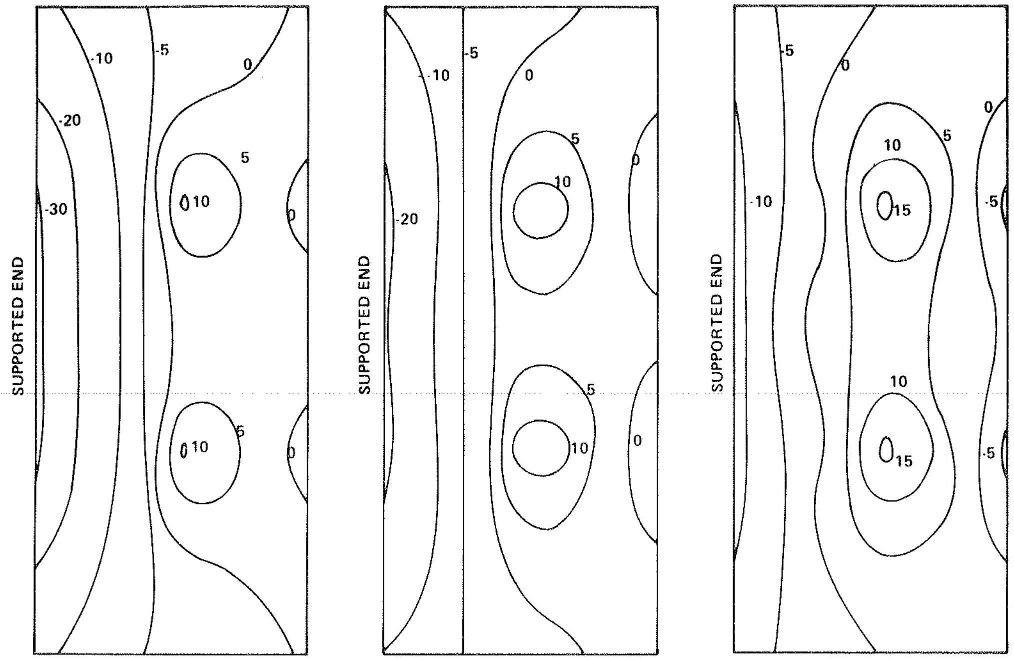
Figure 8 shows the distribution of bending moments for a 100-kN vertical load. Directly under the wheel loads and for all three conditions there are local positive bending moments. At the supported end of the cantilever slab the moments are negative. As expected, there is no moment at the free edge of the unstiffened case; however, there are small negative moments at the free edge for the stiffened cases. With no barrier wall, the finite-element result of 35.4 kN•m/m indicates satisfactory correlation to the OHBDC value of 36.6 kN•m/m and the AASHTO value of 41.2 kN•m/m (see Table 2). With the addition of a barrier wall, the moments decrease to 24.0 and 11.6 kN•m/m, depending on the support condition. For the case of a rigid support, this is a 33 percent reduction. It should be noted that local positive moments directly under the wheel point are of the same magnitude as the negative moment when the presence of barrier walls is taken into account. These are present for the extreme case of point loads. If a distributed load that represents the actual tire print is used, this moment would be much smaller.

Figure 9 shows the moment contours for the loading case of two adjacent wheel loads. This case represents typical dual-axle loads where there is an interaction of the two closely spaced loads. The AASHTO code does not address this condition. The



a) QUEST RUN No. 1 b) QUEST RUN No. 2 c) QUEST RUN No. 3

FIGURE 8 Moment diagram, 100-kN wheel (kN·m/m).



a) QUEST RUN No. 1 b) QUEST RUN No. 2 c) QUEST RUN No. 3

FIGURE 9 Moment diagram, 2- to 70-kN wheels (kN·m/m).

method outlined in the OHBDC for cantilever spans greater than 1.2 m can handle this condition. Figure 3 is used to compute the value for the coefficient A' (Equation 2). A comparison of Figures 8 and 9 clearly shows the difference between the two load conditions. Local effects are present at the two points of loading. At the support end the moments are more uniform because the loading has been spread into two discrete point loads. The various bending moments are given in Table 2. The OHBDC method and the finite-element method yield the same result (32.5 kN•m/m) for the condition of no barrier wall. The maximum negative moment decreases to 21.8 kN•m/m when the barrier wall effect is introduced, and it further decreases to 11.9 kN•m/m when a flexible support is introduced.

The results indicate that the OHBDC analysis, based on the method for spans greater than 1.2 m, shows a satisfactory comparison to the finite-element results, not including edge stiffening. The introduction of a continuous barrier wall causes a significant reduction in the maximum moment effect. A flexible support causes a further reduction in moment. None of the designer-oriented formulations given in Table 1 takes into account the significant effect that end support flexibility has on the load-distribution characteristics of the cantilever slab.

ANALYSIS OF EDGE-STIFFENED CANTILEVER SLAB

The finite-element results for unstiffened cantilever slabs compared favorably with the OHBDC method for slabs with spans greater than 1.2 m. The OHBDC method is based on a manual method developed by Bakht and Holland (4). The method gives a simple procedure to analyze the problem of concentrated loads on elastic cantilever slabs of linearly varying thickness made of isotropic materials. However, the effect of edge stiffening is not included. A subsequent paper by Bakht (5) takes into account the effect of edge stiffening by elaborating on the method given by Bakht and Holland (4). Equation 2 can be used with a new series of curves for A' that take into account the effect of the edge stiffening as a parameter of the ratio of the moment of inertia of the edge stiffening to the moment of inertia of the section of slab about its middle surface. The introduction of an edge-stiffening beam in the cantilever slab does not change any of the essential conditions. Figure 10 shows the graphs for the new values of the coefficient as developed by Bakht (5).

The data in Table 2 give the results for these coefficients in the column titled Bakht. A comparison of the finite-element results for the second case to these methods indicates a satisfactory comparison, with the former results generally being smaller. For the single wheel load, the method by Bakht (5) underestimates the moment by 6 and 20 percent for the single and the dual wheel load, respectively. Consequently, the results indicate that the simplified methods outlined by Bakht and Holland (4) and by Bakht (5) can reasonably predict moments in cantilever slabs that have rigid support. These methods can be also used for cantilever slabs of semi-infinite length.

The flexibility of the cantilever slab support has a significant effect on the bending moment. However, only an elaborate method of analysis can reasonably predict the moment values. The flexibility of the support will vary with span; consequently, the cantilever slab should be designed for a variable moment along the supported end and consequently should have a varying amount of reinforcing steel along the length of the support end. From a practical point of view this may not be economical. The effect of nonrigid supports appears to be a difficult aspect to incorporate in a design office.

The effect of edge stiffening is shown in Figure 11. The coefficient A' is plotted as a function of the ratio I_B/I_S. (Note that I_B = moment of inertia of edge beam about the middle surface, and I_S = moment of inertia of longitudinal section of slab about its middle surface.) This graph shows that increasing the edge stiffness to cantilever slab stiffness ratio to beyond 15 results in a minimal

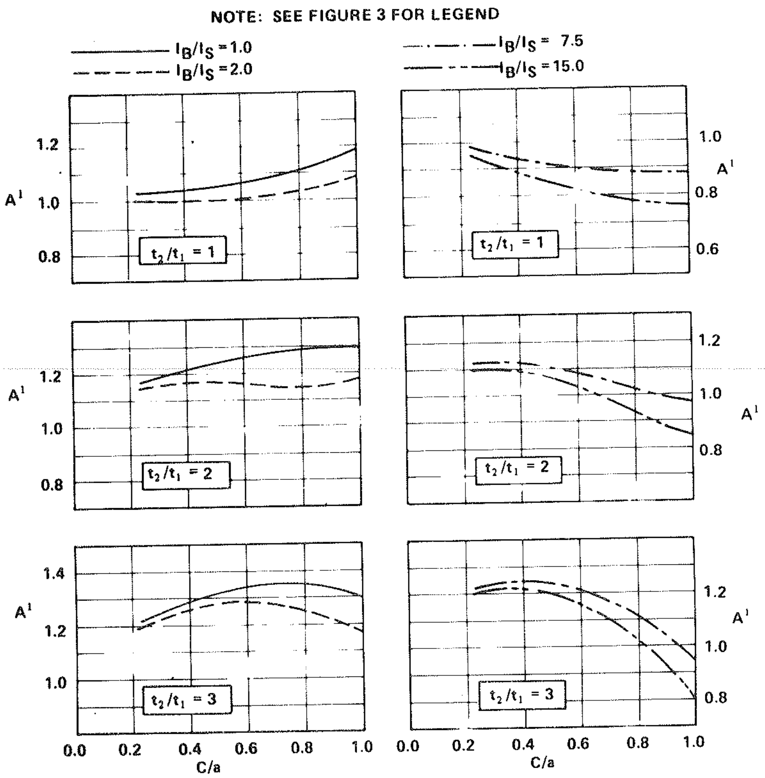


FIGURE 10 Value of coefficient A′ at Y = 0.

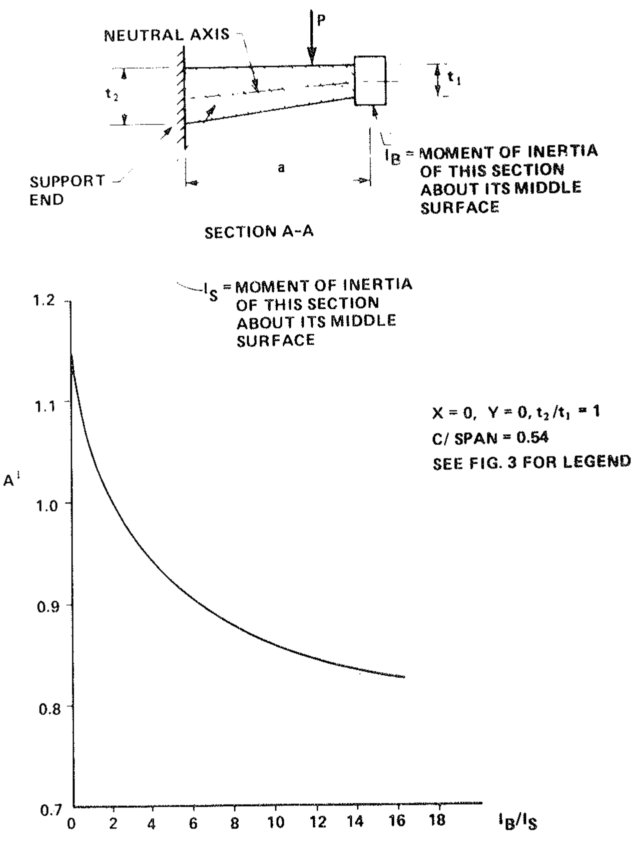


FIGURE 11 Variation of coefficient A′ with ratio I_B/I_S.

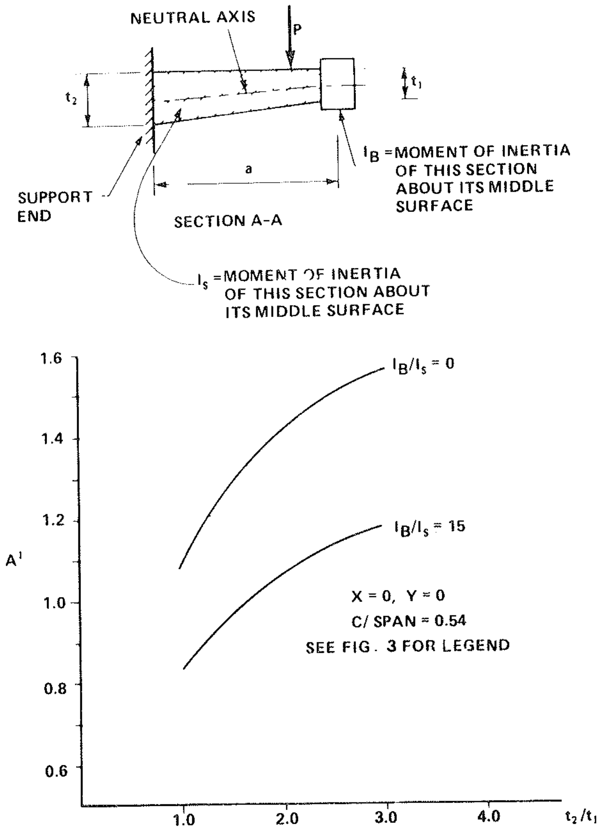


FIGURE 12 Variation of coefficient A′ with ratio t_2/t_1.

decrease to the design moment. For the case shown, the coefficient A', which is proportional to maximum moment, can decrease by as much as 30 percent.

The barrier walls shown in Figure 1 would generally result in a I_B/I_S ratio greater than 15. Sidewalks and curbs can also act as edge stiffening. The ratio of stiffness of a sidewalk compared to that of the supporting cantilever slab, which is typically between 1 and 3, is of a much lower magnitude than for barrier walls. This reduced degree of edge stiffening can still result in a reduction of the maximum moment on the order of 10 to 15 percent.

The effect of varying slab thickness is not considered by AASHTO and was not considered in the finite-element analysis. However, the methods given by Bakht and Holland (4) and Bakht (5) do consider slab thickness variations. For the geometry considered, increasing the slab thickness ratio (t_2/t_1) has the effect of increasing the peak support moments. (Note that t_1 = slab thickness at free edge, and t_2 = slab thickness at supported end.) Figure 12 shows that this increase can be as high as 30 to 40 percent. Consequently, varying thickness could effectively eliminate any beneficial effect of edge stiffening. Generally, the ratio t_2/t_1 does not vary significantly in the typical cantilever slab considered and thereby has little effect on its design.

ECONOMIC CONSIDERATION

By adopting the method of analysis given by Bakht (5), a cost saving can be achieved. By using the current version of the OHBDC for the design of the cantilever slab, the 100-kN wheel load will produce a factored maximum bending moment of 92.5 kN•m/m at the ultimate limit state. To resist this moment, the 230-mm slab with a nominal cover of 70 mm would require 2200 mm^2 of reinforcing steel per meter of slab. This translates into 20M bars at 135-mm centers. By considering the effect of the barrier wall, the maximum bending moment is only 71.2 kN•m/m. The reinforcing-steel requirement is 1600 mm^2, or 20M bars at 185 mm.

It can be seen that a savings of 27 percent, or 600 mm^2/m length of cantilever, can be achieved. If it is assumed that the cantilever steel is terminated approximately 1.5 m past the centerline of the girder (see Figure 1), the total length of the reinforcing-steel bars would be 3.0 m. With a cantilever slab along each side of the bridge, this translates into a mass savings of 28 kg/m of bridge length. For a bridge length of 60 m, the savings in mass would be 1700 kg and would result in a total savings of approximately $2,100 (assuming cost of epoxy-coated reinforcing steel to be $1,250/t). (Note that $1 Canadian = $0.810 U.S.)

Less reinforcing steel is necessary when the barrier wall is taken into account; therefore, smaller bars or a larger concrete cover can be used. These two features increase the durability of the concrete slab when it is exposed to deicing chemicals.

A potential savings is also possible for rehabilitation. During rehabilitation the existing substandard barrier walls of many bridges are upgraded to current standards. Often there is also a need to widen the existing bridge deck. A narrow widening could be achieved by simply adding to the cantilever length. By taking into account the better distribution of the live load caused by the presence of the barrier wall, the structural capacity of the existing cantilever slab may be sufficient; otherwise, the existing cantilever slab would have to be removed and replaced or somehow strengthened.

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER RESEARCH

The finite-element analysis confirms that standard barrier walls significantly enhance the distribution of live-load effects in cantilever deck slabs. The maximum live-load bending moments in the direction of the span decrease by 33 percent.

A general simplified method of analysis based on the extension of a method specified in the OHBDC (2) and developed by Bakht (5) can be used for the design of cantilever deck slabs with barrier walls. This method can be used for determining moments anywhere along the slabs, including areas of the slab near deck expansion joints.

Several parameters that affect the behavior of a cantilever slab subjected to live load have not been considered.

1. The effect of construction joints in the barrier walls was not considered. These construction joints have no reinforcement passing through them, and consequently a weak link exists. Because these joints are still effective in compression but not in tension, their presence needs to be considered. The local discontinuity should not have a significant effect on the deflection characteristic of the system because the overall stiffness of the barrier wall would not be affected. Consequently, these discontinuities should have little effect on the distribution of moments.
2. The investigation considered only point loads. In reality, the wheel loads are patch loads of finite area, thereby spreading the concentrated load over a larger area. This effect may result in an even better distribution and a decrease in the local moment effect directly under the wheel loads.
3. The effect of nonhomogeneity of the concrete slab (such as cracking, honeycombing, and so forth) on the load distribution was not considered.

These three items should be investigated. Prototype full-scale models should be constructed and monitored to ensure satisfactory behavior and agreement with the theoretical approach.

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