

Vehicle-Miles for a Freight Carrier with Two Capacity Constraints

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ABSTRACT

The amount of freight that can be fit on a vehicle depends on the vehicle's weight capacity and volume capacity. In this paper mathematical equations are developed for evaluating the impact of weight capacity and volume capacity on total vehicle-miles. It is shown that the number of vehicle loads needed to carry a large amount of material is minimized when all vehicles are filled to the same capacity constraint. This is accomplished by mixing light items with heavy items in vehicle loads. Following this policy can reduce the number of vehicle loads and vehicle-miles. Under ideal circumstances, the reduction can be as large as 50 percent. Simple equations are provided for estimating the potential reduction in vehicle loads and vehicle-miles to be realized.

The cost of transporting a large quantity of items from one location to another depends on the number of vehicle loads required to carry the material and the distance traveled per vehicle load. Decreasing either the number of loads or the distance traveled per load reduces total vehicle-miles (the total distance traveled by all vehicles) and the cost of transporting the material.

The number of vehicle loads depends on the quantity of items that can be fit on a vehicle. Typically, this quantity is determined by dividing the "capacity" of the vehicle by the "size" of each item. However, vehicle capacity and item size can be measured in more than one way. Most vehicles have both a weight capacity and a volume capacity. The vehicle is full when either capacity is reached. Depending on the type of items carried, some vehicles might be filled to the weight capacity, and others might be filled to the volume capacity (Figure 1).

In this paper equations are developed that readily show how the number of vehicle loads depends on the weight capacity and the volume capacity. These equations are used to prove that the number of vehicle loads is minimized when all vehicles are filled to the same capacity constraint (that is, all loads are filled to the weight capacity, or all loads are filled to the volume capacity). To minimize the number of loads, items that have a low density (pounds per cubic foot) must be mixed with items that have a high density in vehicle loads (Figure 2). There are several ways to mix low-density with high-density items in a vehicle load. If a supplier produces both low-density and high-density items, the different items can be loaded in the same vehicle on the loading dock. Alternatively, if different suppliers located in the same area produce low-density and high-density items, the different items can be mixed by routing vehicles by both types of suppliers. Low-density items can also be mixed with high-density items at a transportation terminal.

It is also demonstrated that standard vehicle routing methods do not minimize total vehicle-miles when some locations produce (or receive) items that have a low density and other locations produce (or receive) items that have a high density. Equations are provided to show when it is important to design

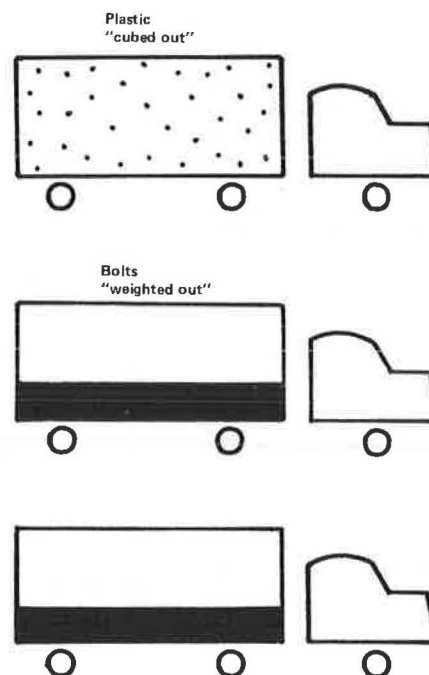


FIGURE 1 Light and heavy items shipped in separate vehicles.

modified vehicle routes that result in all vehicles being filled to the same capacity constraint.

Vehicle routing has been studied extensively during the last 25 years (1-3). For example, the vehicle routing problem (4) concerns routing a fleet of vehicles from a single terminal to a number of destinations so that travel distance is minimized and vehicle capacity constraints are not violated.

Although the vehicle routing problem is neper complete (5-7) and difficult to optimize, many heuristics identify close to optimal solutions. For example, simple heuristics for solving the closely related traveling salesman problem, such as the Clarke-Wright method (8) locate solutions within about 7 percent of the optimal cost (9).

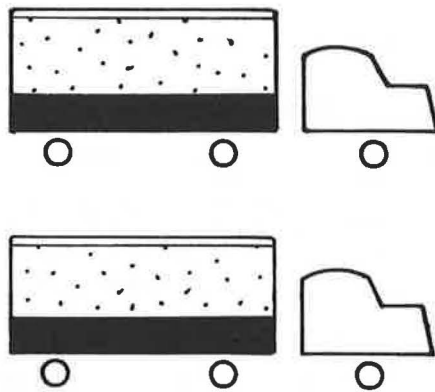


FIGURE 2 Light and heavy items mixed in vehicles.

Despite the many applications of this problem, and the research invested in developing efficient routing algorithms, many industries continue to route vehicles manually. There are many reasons for this including lack of data and inability of available algorithms to account for all the important factors that influence the cost of operating vehicles.

The existence of two vehicle capacities (weight and volume) is one factor that routing heuristics do not normally consider (although computationally impractical, a second capacity can be used in some of the optimization algorithms). Most vehicle routing heuristics group stops into routes according to geographic proximity (Figure 3). Although this ap-

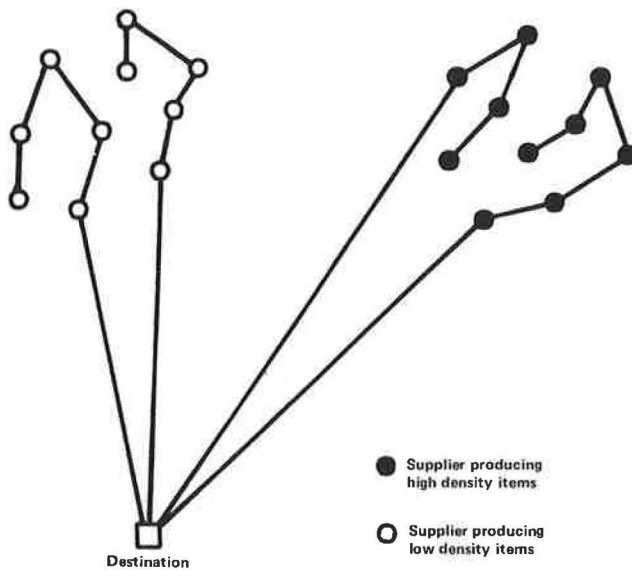


FIGURE 3 Possible vehicle routes when accounting for one capacity constraint.

proach may minimize the vehicle-miles traveled per load, it does not minimize the total number of loads and total vehicle-miles. Vehicles may have to travel "out of their way" to ensure that each load carries a mixture of low-density and high-density items (Figure 4).

Although this paper is written in the context of vehicles picking up items from many different origins, the results also apply to delivering items to many destinations. The equations developed in the first section can also be used to analyze transport-

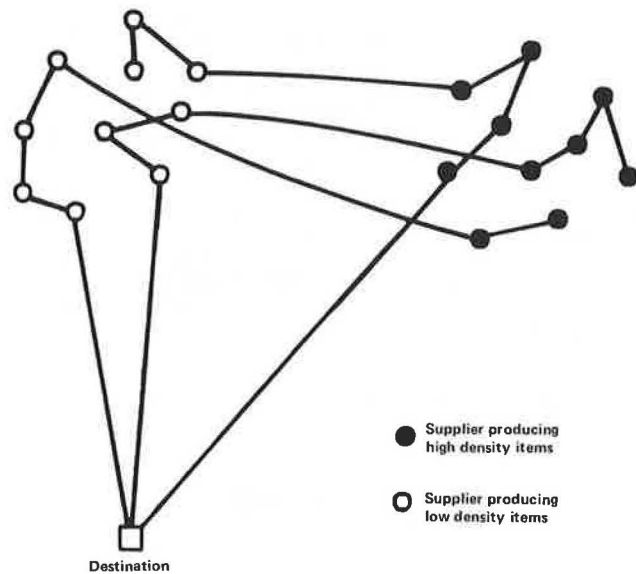


FIGURE 4 Possible vehicle routes when accounting for two capacity constraints.

ing different types of items from a supplier or transportation terminal.

NUMBER OF VEHICLE LOADS

Consider a region in which vehicles pick up different items from many different locations. Items differ in weight, volume, and production rate, where

W_i = weight of item i (pounds),
 V_i = volume of item i (cubic feet),
 F_i = total production rate of item i in the region (items per week), and
 d_i = material density of item i (W_i/V_i , pounds per cubic foot).

The quantity of material that can be loaded onto a vehicle depends on the vehicle's weight capacity and volume capacity, where

C_w = vehicle's weight capacity (pounds) and
 C_v = vehicle's volume capacity (cubic feet).

The weight and volume capacity are dictated by vehicle design, risk of damage to cargo or to other vehicles, and ability of the guideway (road or tracks) to sustain the load. A weight capacity of 80,000 lb and volume capacity of 4,200 ft³ are common for large trucks operating on U.S. highways.

Suppose initially that each vehicle carries only one type of item. Then T , the minimum number of vehicle loads per week required to transport a large amount of material, is

$$T = \lceil F_i \max [(V_i/C_v), (W_i/C_w)] \rceil \quad (1)$$

Equation 1 is simplified by introducing the symbol d^* to represent the material density that simultaneously fills the vehicle to both the weight capacity and the volume capacity. That is,

$$d^* = C_w/C_v \quad (2)$$

Also substituting W_i/d_i for V_i , Equation 1 can be rewritten as

$$T = \lceil (F_i W_i / C_w) \max [(d^*/d_i), 1] \rceil \quad (3)$$

If the density of an item is less than d^* , the load reaches the volume capacity before the weight capacity. Otherwise, the load reaches the weight capacity first. The ratio d^*/d_i is an adjustment factor to account for the actual weight of material that can be fit onto the vehicle, taking into account both the weight and volume capacities.

Equation 3 can be expressed as a function of a few parameters that represent average item weights and densities. First, let F be the total number of items produced per week (the summation of F_i). Let a "light" item be an item with a density less than d^* and a "heavy" item be an item with a density greater than d^* . Also let L be the set of light items, H be the set of heavy items, and

$$p = \text{proportion of items that are light} \\ = \sum_{i \in L} F_i / F$$

$$W_L = \text{average weight of the light items} \\ = \sum_{i \in L} F_i W_i / \sum_{i \in L} F_i$$

$$d_L = \text{average density of the light items} \\ = \sum_{i \in L} (F_i W_i) / \sum_{i \in L} (F_i V_i)$$

$$W_H = \text{average weight of the heavy items} \\ = \sum_{i \in H} F_i W_i / \sum_{i \in H} F_i$$

Equation 3 can now be written as

$$T = (F/C_w) [(W_L p d^*/d_L) + W_H (1-p)] \quad (4)$$

Letting W be the average weight of all items [$W = W_L p + W_H (1-p)$], Equation 4 becomes

$$T = (FW/C_w) \{1 + [W_L p (d^* - d_L) / W d_L]\} \quad (5)$$

Equation 5 can be reduced further by introducing two new composite variables. Let

$$P = \text{proportion of weight produced per week that is composed of light items} = W_L p / W \text{ and} \\ r = \text{ratio of the average material density of the light items to } d^* = d_L / d^*.$$

The minimum number of vehicle loads required per week can now be expressed as a function of just five parameters:

$$T = (FW/C_w) [1 + P(1-r)/r] \quad (6)$$

P and r must both be less than one and greater than zero. They must also satisfy the following inequality:

$$d = \{W / [V_L p + V_H (1-p)]\} < W / V_L p = (W_L / V_L) (W / W_L p)$$

for $d < d_L / P$. When $d > d^*$, $d^* < d < d_L / P$. In terms of r and P ,

$$P < r \quad \text{if } d > d^* \quad (7)$$

If $d < d^*$, P and r are only constrained to be between zero and one.

Returning to Equation 6, the first term gives the number of vehicle loads when accounting for the weight capacity alone. The second term is an adjustment factor that specifies the additional number of loads when accounting for both weight capacity and volume capacity. Notice that the adjustment factor must always be greater than one, and that it increases as the proportion of weight composed of light

items (P) increases, and increases as the average density of light items (rd^*) decreases.

LOADS CONTAINING DIFFERENT ITEMS

Suppose now that vehicles carry different types of items with different weights and densities. Then the number of vehicle loads (T) is minimized when all loads are filled to the same capacity constraint. That is, all loads are filled to the weight capacity, or all loads are filled to the volume capacity.

This statement can be proved by contradiction. Suppose that one load contains light items and is filled to the volume capacity and another load contains heavy items and is filled to the weight capacity. Then any arbitrary proportion of material can be exchanged between the two loads without violating a capacity constraint.

Let w_1, w_2, v_1 , and v_2 be the respective weights and volumes of the light and heavy loads, where $w_1 < C_w, v_1 = C_v, w_2 = C_w$, and $v_2 < C_v$. Let the circumflex ($\hat{}$) denote the weight or volume of a load after a proportion (q) of material is exchanged between loads. Then

$$\begin{aligned} \hat{w}_1 &= w_1(1-q) + w_2q & \hat{v}_1 &= v_1(1-q) + v_2q \\ \hat{w}_2 &= w_2(1-q) + w_1q & \hat{v}_2 &= v_2(1-q) + v_1q \end{aligned} \quad (8)$$

which can also be written as

$$\begin{aligned} \hat{w}_1 &= w_2 - (w_2 - w_1)(1-q) < C_w \\ \hat{v}_1 &= v_1 - (v_1 - v_2)q < C_v \\ \hat{w}_2 &= w_2 - (w_2 - w_1)q < C_w \\ \hat{v}_2 &= v_1 - (v_1 - v_2)(1-q) < C_v \end{aligned} \quad (9)$$

Notice that exchanging any proportion (q) of material between the two loads reduces the weight and volume of both loads below the respective capacities. Therefore, a necessary condition for minimizing T is that all loads be filled to the same capacity constraint.

To minimize T it is not necessary that all loads carry exactly the same mix of different items or carry exactly the same weight and volume of material. For example, if all loads are filled to the volume capacity, it does not matter how much weight of material is loaded onto each vehicle. Thus the statement that all loads are filled to the same capacity constraint is both a necessary and a sufficient condition for minimizing T .

SAVINGS FROM COMBINING DIFFERENT ITEMS IN VEHICLE LOADS

Whenever light items ($d_i < d^*$) are shipped in separate vehicles than heavy items ($d_i > d^*$), as is the case when vehicles contain only one type of item, the number of loads is given by Equation 6. Combining light with heavy items in vehicle loads always results in decreased loads. Let T^* denote the number of loads when all vehicles are filled to the same capacity constraint (that is, when T is minimized). Then

$$T^* = F \{ \max [(V/C_v), (W/C_w)] \} \\ = (FW/C_w) \{ \max [(d^*/d), 1] \} \quad (10)$$

where V is average volume of all items and d is average density of all items (W/V).

The first term of Equation 10 gives the number of loads when accounting for weight capacity alone, and the second term is an adjustment factor that specifies the additional number of loads when accounting for both capacities. If d^*/d is greater than one, all vehicles are filled to the volume capacity and the adjustment factor equals d^*/d . Otherwise, all vehicles are filled to the weight capacity and the adjustment factor equals one. Therefore the adjustment factor is greater than or equal to one.

T/T^* is the ratio of the number of loads when light items are not mixed with heavy items to the number of loads when light and heavy items are mixed, and equals the ratio of Equation 10 to Equation 6:

$$T/T^* = \begin{cases} 1 + P(1-r)/r & \text{for } d > d^* \\ (d/d^*) [1 + P(1-r)/r] & \text{otherwise} \end{cases} \quad (11a)$$

$$(11b)$$

Equation 11 can be used to estimate quickly the maximum reduction in vehicle loads from filling all vehicles to the same capacity constraint.

Recall that P must be less than r when d/d^* is greater than one. Equation 11a is maximized when P equals r . Therefore substituting P for r in Equation 11a,

$$T/T^* < 2 - P \quad \text{for } d > d^* \quad (12)$$

As a function of P , T/T^* approaches two as P approaches zero, and approaches one as P approaches one. Figure 5 plots Equation 12 as a function of P and plots Equation 11a as a function of P and r . Notice that T/T^* increases both when P increases and when r decreases. Therefore, when $d > d^*$, it is most important to combine light and heavy items in vehicle loads when a large proportion of the weight produced per week is composed of light items, and the average density of light items is much less than d^* .

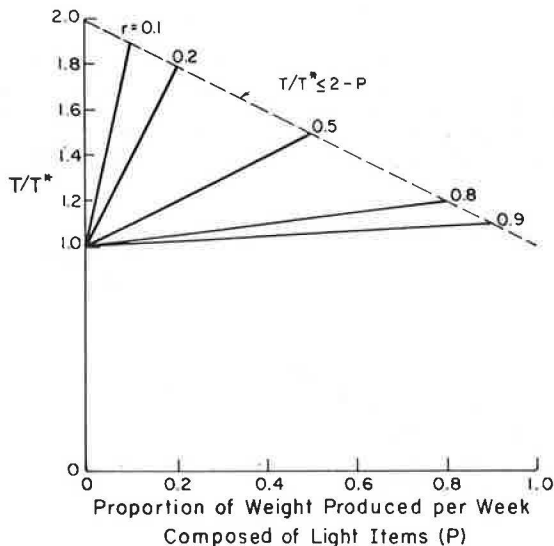


FIGURE 5 Ratio of loads per week without mixed loads to loads per week with mixed loads.

Similar results occur when $d < d^*$. T/T^* also ranges between one and two, depending on P , r , and d . Tables 1 and 2 give sample data for Equation 11a. Five suppliers located in the same city produce 11 different parts. Parts are transported in 3,800-ft³, 70,000-lb capacity trucks. Hence, d^* equals 70,000/3,800 = 18.42 lb/ft³. The parts produced by Sup-

TABLE 1 Example Part Data

	Weight (lb)	Volume (ft ³)	Production Rate (parts/week)
Supplier 1			
Part A	1.0	1.0	1,000
Part B	0.5	0.4	2,000
Part C	0.8	0.2	1,000
Supplier 2			
Part D	10.0	0.2	5,000
Supplier 3			
Part E	0.1	0.01	50,000
Part F	0.2	0.005	50,000
Part G	5.0	0.1	50,000
Part H	0.4	0.01	50,000
Supplier 4			
Part I	5.0	10.0	1,500
Part J	5.0	10.0	500
Supplier 5			
Part K	2.0	0.1	10,000

TABLE 2 Summary Data

Supplier	Production Rate		Average Density (lb/ft ³)	Trucks per Day
	lb/week	ft ³ /week		
1	2,800	2,000	2.8	0.53
2	50,000	1,000	50.0	0.71
3	285,000	6,250	45.6	4.07
4	10,000	20,000	0.5	5.26
5	20,000	1,000	20.0	0.29
Total	367,800	30,250	12.2 < d^*	

Note: $P = (2,800 + 10,000)/367,800 = 0.0348$; $d_1 = (2,800 + 10,000)/(2,000 + 20,000) = 0.582$; $r = d_1/d^* = 0.0316$; and $T/T^* = 1.36$. Vehicle load comparison: one route per supplier, 10.86 trucks per week; one route for all suppliers, 7.96 trucks per week; saving, 2.90 trucks per week (27 percent).

pliers 1 and 3 have small densities and fill the vehicle to volume capacity. The parts produced by the other three suppliers have large densities and fill the vehicle to weight capacity.

If each supplier shipped independently of the others, 10.86 truckloads, on average, would be needed per week. However, if the different parts were combined in the same vehicles, so that all vehicles were filled to the volume capacity, the number of truckloads would drop to only 7.96 per week (a reduction of 27 percent). Equation 11a predicts that the ratio of T to T^* should equal 1.36 for this example, which exactly matches the ratio of 10.86 to 7.96.

MODIFYING ROUTES TO REDUCE NUMBER OF LOADS

Most vehicle routing heuristics group stops into routes according to geographic proximity and do not necessarily minimize total vehicle-miles (8,10,11). It is not unusual for geographic regions to contain many different companies engaged in the same industry. For instance, one region may contain a large concentration of plastic companies, and another may contain a large concentration of fastener (nuts and bolts) manufacturers. If vehicles are routed on the basis of geographic proximity alone, all the vehicles in the plastics region would be filled to volume capacity, and all the vehicles in the fastener region would be filled to weight capacity, resulting in as many as twice as many loads as necessary.

Figure 6 shows a situation in which manufacturers in one city produce light items and manufacturers in another city produce heavy items. A fleet of vehicles picks up items at these two cities and delivers them to a common destination. If routed on proximity alone, each vehicle would visit only one of the two cities. However, to minimize the total number of loads (and

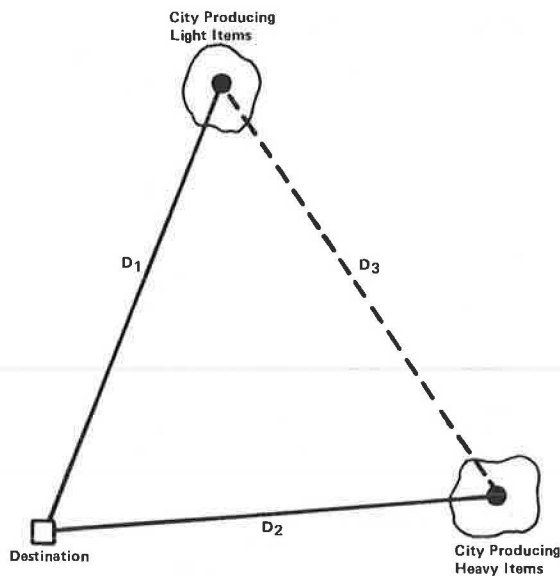


FIGURE 6 Locations of cities and destination.

ensure that all vehicles are filled to the same capacity constraint) vehicles must be routed through both cities.

The total distance traveled on a vehicle route includes the local distance traveled to pick up items within the two cities and the line-haul distance between the cities and the destination. The local distance depends on the total number of stops and the stop density [e.g., stops per square mile (12,13)]. Assuming that the two cities do not overlap, the local distance is nearly independent of whether all vehicles visit both cities or just one. Therefore only the line-haul distance is considered in the following analysis.

The following equations apply to the situation in which $d > d^*$. Following the same approach, it is not difficult to derive similar equations for the case where $d < d^*$. Let

- D_1 = distance from city producing light items to the destination (miles),
- D_2 = distance from city producing heavy items to the destination (miles), and
- D_3 = distance between the two cities (miles).

Assume initially that all vehicles return empty from the destination. Then, if all vehicles visit only one city, the number of vehicle-miles traveled per week (L_1) is

$$L_1 = (FW/C_w) 2 \{ [D_1(P/r)] + [D_2(1-P)] \} \quad (13)$$

Alternatively, if all vehicles visit both cities, the number of vehicle-miles is

$$L_2 = (FW/C_w) (D_1 + D_2 + D_3) \quad \text{if } d > d^* \quad (14)$$

It is not necessary for all vehicles to visit both cities to minimize T . However, if $d > d^*$, all vehicles visiting the "light" city must also visit the "heavy" city, and, if $d < d^*$, all vehicles visiting the "heavy" city must also visit the "light" city. Equation 14 is an upper bound on total vehicle-miles with this type of coordination. Exact calculation of total vehicle-miles is not complicated, but it does require detailed information on the densities

of all items. Therefore this calculation will not be performed.

The ratio of L_1 to L_2 is

$$L_1/L_2 = \{ 2[D_1(P/r) + D_2(1-P)] \} / (D_1 + D_2 + D_3) \quad \text{if } d > d^* \quad (15)$$

If L_1/L_2 is greater than one, it is better to route all vehicles through both cities than through just one. This ratio ranges from zero (e.g., when $D_2 = 0$ and $P = 0$) to two (when $D_3 = 0$, $P = r$, and $r = 0$). Therefore routing all vehicles through both cities can reduce total vehicle-miles by as much as 50 percent.

SPECIAL CASE: $D_2 = D_1 + D_3$

To facilitate interpreting Equation 15, two special cases will be examined. This section examines the case in which $D_2 = D_1 + D_3$ (that is, the destination and the two cities fall on a line and the "heavy" city is farther from the destination than the "light" city). For this special case, the number of vehicle loads is minimized when all vehicles visit the "heavy" city, and Equation 15 is exact. In the following section the case in which $D_1 = D_2$ (that is, the two cities are the same distance from the destination) will be examined. All of the following equations assume that $d > d^*$.

When $D_2 = D_1 + D_3$, Equation 15 can be reduced by substituting $D_2 - D_1$ for D_3 :

$$L_1/L_2 = [2D_1(P/r) + 2D_2(1-P)] / 2D_2 \quad \text{if } D_2 = D_1 + D_3 \quad (16)$$

Because the 2s cancel out in Equation 16, L_1/L_2 is the same whether or not vehicles must return empty from the destination.

Let K equal the ratio of D_1 to D_2 ($K = D_1/D_2$). K must be between zero and one. Then

$$L_1/L_2 = K(P/r) + (1-P) \quad \text{if } D_2 = D_1 + D_3 \quad (17)$$

Because P must be smaller than r , and K must be less than one, L_1/L_2 must be less than or equal to two. Figure 7 shows plots of L_1/L_2 as a function of P and r , for a value of $r = 0.5$. Notice that this ratio increases as K increases. When K is greater than r ,

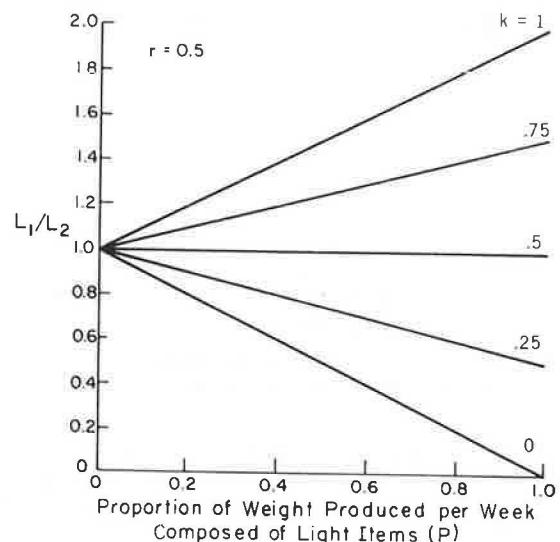


FIGURE 7 Ratio of vehicle-miles when visiting both cities to vehicle-miles when visiting one city.

L_1/L_2 also increases as P increases, but when K is less than r , L_1/L_2 decreases as P increases. Therefore it is most important to route vehicles through both cities when K is large and a large portion of weight produced per week is composed of light items (provided that $K > r$). The ratio also increases as r decreases. Therefore it is more important to route vehicles through both cities when the average density of light items is small than when it is large.

The breakeven point between routing vehicles through both cities and routing through just one occurs when $L_1/L_2 = 1$.

$$1 = K(P/r) + (1-P) \quad \text{if } D_2 = D_1 + D_3$$

or, more simply, when

$$K = r \quad \text{if } D_2 = D_1 + D_3. \quad (18)$$

When r is less than K , it is better to route all vehicles through both cities than through just one. For example, if the average density of items produced in the "light" city is one-half d^* ($r = 0.5$), vehicles should be routed through both cities when K is greater than 0.5; that is, the "heavy" city is less than twice as far from the destination as the "light" city. Because K approaches zero when r approaches zero, it can still be worthwhile to route all vehicles through both cities when the two cities are far apart.

SPECIAL CASE: $D_1 = D_2$

For the special case in which $D_1 = D_2$, the two cities are the same distance from the destination. For this special case, it may not be necessary to route all vehicles through both cities to minimize the number of vehicle loads. Therefore L_2 gives an upper bound on total vehicle-miles when the number of loads is minimized. Let $D = D_1 = D_2$. Then Equation 15 becomes

$$L_1/L_2 = 2D[P/r + (1-P)]/(2D + D_3) \quad (19)$$

The breakeven point between routing all vehicles through both cities and routing all vehicles through just one city occurs when Equation 19 equals one. That is, when

$$(2D + D_3)/2D = 1 + P(1-r)/r \quad (20)$$

Notice that the left side of Equation 20 is the ratio of vehicle-miles per route when vehicles visit both cities, to vehicle-miles per route when vehicle visit just one city. The right side of Equation 20 is the ratio of the number of vehicle loads when vehicles visit both cities, to the number of vehicle loads when vehicles visit just one city (T/T^*). Let D_b be the vehicle-miles per route when vehicles visit both cities and D_o be the vehicle-miles per route when vehicles visit just one city. Then Equation 20 can be rewritten as

$$D_b/D_o = 1 + P(1-r)/r \quad (21)$$

When D_b/D_o is less than the right side of Equation 21, fewer total vehicle-miles are required when vehicles visit both cities than when they visit just one. Otherwise, total vehicle-miles are minimized when vehicles visit just one city.

Notice that the right side of Equation 21 is identical to the right side of Equation 11a, which is plotted in Figure 3. Therefore the breakeven point between routing vehicles through both cities and

routing vehicles through just one increases as P increases and r decreases.

Equation 21 also applies when vehicles do not return empty from the destination. D_b would then be $D + D_3$ (the length of two legs of the route) and D_o would then be D . For any given D and D_3 , this ratio is larger when vehicles do not return empty than when vehicles do return empty. Therefore it is less advantageous to route vehicles through both cities when they do not have to return empty from the destination than when they do have to return empty from the destination.

Equation 19, which gives the ratio L_1 to L_2 , can also be expressed as a function of D_b/D_o :

$$L_1/L_2 = (D_o/D_b) [1 + P(1-r)/r] \quad (22)$$

Equation 22 is identical to the right side of Equation 11a (plotted in Figure 3), except that Equation 22 is multiplied by the factor D_o/D_b . If the two cities are in opposite directions from the destination, D_o/D_b can be as small as 0.5, and L_1/L_2 would range between 0.5 and one (i.e., it would always be better to route vehicles through one city than two). D_o/D_b can be as large as one if the two cities are located at the same place, in which case L_1/L_2 would range from one to two (i.e., it would always be better to route vehicles through both cities than through just one).

SUMMARY

The impact of weight capacity and volume capacity on total vehicle-miles has been discussed. It has been shown that the number of vehicle loads is minimized when all vehicles are filled to the same capacity constraint (that is, all vehicles are filled to the weight capacity or all vehicles are filled to the volume capacity). This may be accomplished by mixing heavy items and light items in vehicle loads.

Combining light items with heavy items in vehicle loads was shown to reduce the number of loads and vehicle-miles by as much as 50 percent. The exact reduction depends on two parameters. When the number of loads is minimized by filling all vehicles to the weight capacity, these parameters are (a) the ratio of d_1 , the density of light items, to d^* , the density of material that simultaneously fills vehicles to both the weight and volume capacities and (b) the proportion of weight produced per week that is composed of light items (P). A similar equation results when the number of loads is minimized by filling all vehicles to the volume capacity.

It has also been shown that commonly used heuristics for routing vehicles can obtain solutions that are far from optimal when light items and heavy items are produced in geographically separated regions. Most heuristics group stops into routes according to geographic proximity. Although this may minimize vehicle-miles traveled per route, it does not minimize total vehicle-miles. If vehicle routes are designed to ensure that all vehicles are filled to the same capacity constraint, the number of vehicle loads (and vehicle-miles) can be reduced by as much as 50 percent.

Considerable effort has been expended in the last 25 years to improve the efficiency and effectiveness of algorithms designed to solve the vehicle routing problem. However, even straightforward heuristic (such as the Clarke-Wright method), can generally obtain solutions within 7 percent of the optimum of the vehicle routing problem. The evidence provided in this paper indicates that the savings from accounting for two capacity constraints can well exceed 7 percent.

REFERENCES

1. S. Eilon, C.D.T. Watson-Gandy, and N. Christofides. *Distribution Management: Mathematical Modelling and Practical Analysis*. Hafner Publishing Company, New York, 1971.
2. T.L. Magnanti. *Combinatorial Optimization and Vehicle Fleet Planning: Perspectives and Prospects*. Networks, Vol. 11, 1981, pp. 179-213.
3. W.C. Turner, P.M. Ghare, and L.R. Fourds. *Transportation Routing Problem--A Survey*. AIIE Transactions, Vol. 6, 1974, pp. 288-301.
4. G.B. Dantzig and J.H. Ramser. *The Truck Dispatching Problem*. Management Science, Vol. 6, 1959, pp. 80-91.
5. S.A. Cook. *The Complexity of Theorem-Proving Procedures*. Proc., 3rd Annual ACM Symposium on Theory of Computing, Association for Computing Machinery, New York, 1971, pp. 151-158.
6. R.M. Karp. *Reducibility Among Combinatorial Problems*. In *Complexity of Computer Computations* (R.E. Miller and J.W. Thatcher, eds.), Plenum, New York, 1971, pp. 85-103.
7. J.L. Lenstra and A.H.J. Rinnoy Kan. *Complexity of Vehicle Routing and Scheduling Problems*. Networks, Vol. 11, 1981, pp. 221-227.
8. G. Clarke and J.W. Wright. *Scheduling of Vehicles from a Central Depot to a Number of Delivery Points*. Operations Research, Vol. 12, 1964, pp. 568-581.
9. B. Golden, L. Bodin, T. Doyle, and W. Stewart, Jr. *Approximate Traveling Salesman Algorithms*. Operations Research, Vol. 28, 1980, pp. 694-711.
10. R.M. Karp. *A Patching Algorithm for the Nonsymmetric Traveling-Salesman Problem*. SIAM Journal of Computing, Vol. 8, 1979, pp. 561-573.
11. D.J. Rosenkrantz, R.E. Stearns, and P.M. Lewis II. *An Analysis of Several Heuristics for the Traveling Salesman Problem*. SIAM Journal of Computing, Vol. 6, 1977, pp. 563-581.
12. J.J. Bearwood, H. Halton, and J.M. Hammersley. *The Shortest Path Through Many Points*. Proc., Cambridge Philosophical Society, Vol. 55, 1959, pp. 299-327.
13. C.F. Daganzo. *The Distance Traveled to Visit N Points with C Stops per Vehicle: An Analytic Model and Application*. Submitted to Transportation Science.

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Urban Freight Practice—An Evaluation of Selected Examples

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ABSTRACT

A diverse group of urban goods movement projects and actions taken by municipalities are documented and the principal lesson or lessons derived from each project are highlighted. The research used the literature, field visits, interviews, and independent research to formulate the presentation of the selected examples. The paper contains eight examples of municipalities that have implemented projects in curb space management, off-street facility planning, and zoning. Six examples are drawn from U.S. cities and two from Canada. An evaluation follows each example to highlight the positive and negative results of each as they might affect application elsewhere. This paper is drawn from research sponsored by the UMTA University Research Program and was conducted by the author while at the Polytechnic Institute of New York.

This paper provides a detailed review of a selected number of actions taken by various municipalities to address urban freight transportation. The documentation for several of these actions included field trips and interviews. The literature, plus the author's personal knowledge or involvement, provided the documentation on the other actions.

The urban transportation planner's or the engi-

neer's justifiable preoccupation with the need to optimize the transportation infrastructure to move people has, to date, left a wide gap in professional skills necessary to foster successful urban freight project development and evaluation. The ability to draw on the work and experiences of similar projects has not only markedly facilitated people-transportation project development but has also provided