CRASH Revisited: Additions to Its Clarity, Generality, and Utility

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A complete rederivation of the computer program CRASH is presented, with confirmation of its theoretical basis, elimination of many of its actual or supposed restrictions, and additions to its useful outputs. The physics and algebra, although clarified, are for the most part unchanged. However, in the trajectory solution, a closed-loop iteration replaces a best-fit form of solution. In the impulse solution, the physical basis of the common velocity check is clarified, the check is revised so that more cases can be treated, and the coefficient of restitution is found. In the damage solution, delta-V accuracy is improved by better fits to crash test data, corrected treatment of oblique impact, and inclusion of the energies of restitution and intervehicular sliding. Yaw rates and impact forces are found from the impulse solution and again from the damage solution; these and other paired output comparisons indicate the quality of the reconstruction and facilitate its refinement.

In this paper the program Calspan Reconstruction of Accident Speeds on the Highway (CRASH) is reviewed, rederived, and extended, with further commentary on published criticisms of CRASH and on alternative assumptions in published reconstruction treatments.

CRASH was developed under funding from the U.S. Department of Transportation (DOT) and was based by McHenry on the spin analysis of Marquard *(1)* and the deformation energy analysis of Campbell *(2)*. Initially it was a simulation setup routine for SMAC *(3)*, but later was an independent, user-friendly digital computer program for accident reconstruction *(4, 5)* specifically for evaluation of speeds at impact and speed changes during impact from the postimpact information for two vehicles colliding on a flat surface.

DOT accident reconstruction has emphasized the systematic, standardized evaluation of the speed changes of each vehicle in impact as a measure of occupant injury exposure, with the objective of evaluating the effectiveness of various safety measures for which DOT is responsible *(6)*. For these purposes, CRASH has become a highly respected functional standard.

However, the published derivations were unduly lengthy, restrictive, and obscure. Some reservations have been expressed as to their validity *(7)*, and the necessary programmed solution has been readable only as more than 130 pages of FORTRAN. As a result, many experts have avoided using CRASH as a basis for testimony in lawsuits, and litigants have been deprived of a useful analytic tool.

Precisely such reservations were the impetus for this paper. It will be shown that the CRASH equations are fundamentally correct, as far as they go, within (and sometimes beyond) their stated restrictions. Certain assumptions will be avoided; others will be clarified. All the CRASH equations will be derived, with some revisions; then the derivations will be extended to obtain results beyond those offered by CRASH. Thus in this paper CRASH will be confirmed and its clarity, generality, and utility will be enlarged.

As shown in a companion paper *(8)*, the results have been programmed in BASIC and for use in desktop and hand-held computers, making the improvements widely available and facilitating forensic use.

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DATA INPUTS

CRASH uses data gathered at the site of the accident and from both of the vehicles involved in each impact. In some cases, the tire marks or the vehicles, or both, no longer exist and the immediate sources must be photographs and police reports. General vehicle data and tire-road friction data may be obtained from published tables rather than from the vehicles, tires, and highway involved.

Vehicle speed changes (as desired by DOT) can only be determined from knowledge of the damage to the vehicles, but evaluation of the speeds before impact requires knowledge of their travel after impact. Axial impacts are indeterminate without damage data, but intersection impacts, despite the possible complication of vehicle spin after impact, can be fully reconstructed without knowing the severity of vehicle damage. This analysis is considered first.

SPINOUT ANALYSIS

For uniformly decelerated motion, CRASH utilizes the usual expression for initial speed, from the double integral with respect to time of $F = ma$,

$$U = \sqrt{2\theta\mu g s}] \tag{1.1}$$

where] in this paper closes any expression opened by $\sqrt{}$ or Σ, and the rate of deceleration is expressed as $\theta\mu g$, where θ is either the fraction of the available friction-limited lateral force applied as braking force as the vehicle slides endwise, or unity for the laterally sliding vehicle. Fonda *(8)* furnishes auxiliary equations (from both CRASH and SMAC) for traversing differing surfaces, for speed-dependent friction, and for nonseparation of the vehicles.

For the considerably more complex case of the spinning vehicle, the deceleration rate is nonuniform but may be treated by the approach of Marquard *(1)*, which considers alternating periods of predominantly angular and predominantly translational deceleration, with partial or full braking. This method was augmented by McHenry and incorporated in the START, CRASH, CRASH2, and CRASH3 programs *(3–5)*.

Consider a vehicle initially translating at rate U_s and spinning at rate $\dot{\psi}_s$, losing all of its angular momentum in time T

while losing only part of its linear momentum. The speed U_f before final translation for a distance s_f after the end of rotation can be found from Equation 1.1. The angle and the distance traveled while spinning are

$$\psi_s = \dot{\psi}_s T/\alpha_1 \quad \text{and} \quad s_s = (U_s + U_f)T/\alpha_5 \qquad (1.2,1.3)$$

where α_1 and α_5 are empirical coefficients that would each have the value 2 if the decelerations either were uniform or fluctuated symmetrically about fixed average values.

The actual declerations alternate between periods of predominantly angular deceleration while endwise for a total time t_1, with a total angular impulse

$$mk^2\ \dot{\psi}_s = \alpha_2\,(0.5l\mu mg)t_1 \qquad (1.4)$$

where $0.5l$ ($\cong\sqrt{ab}$]) is the numerical (or geometric) average of the lever arms of the tangential forces and α_2 is nominally unity, and periods of predominantly translational deceleration while broadside for the remaining time t_2 plus some deceleration during t_1 due to braking, with a total linear impulse

$$m(U_s - U_f) = (\alpha_3\theta t_1 + \alpha_4 t_2)\mu mg \qquad (1.5)$$

where α_1 and α_4 account at least for the angularity between the instantaneous force and the average direction of motion.

Accordingly, the time duration of the spin motion is given by

$$T = t_1 + t_2 = t_1\,(1 - \alpha_3\theta/\alpha_4) + (\alpha_3\theta t_1 + \alpha_4 t_2)/\alpha_4$$
$$= [k^2\ (\alpha_1\psi_s/T)/0.5\alpha_2 l\mu g][1 - \alpha_3\theta/\alpha_4][(\alpha_5 s_s/T) - 2U_f]/\alpha_4\mu g$$

whence $T^2 + 2BT - C = 0$ for a quadratic solution for T (Equation 1.6c).

From Equation 1.3, the translatory and angular speeds at separation are

$$U_s = (\alpha_5 s_s/T) - U_f; \quad \dot{\psi}_s = \alpha_1\ \psi_s/T \qquad (1.6a,b)$$

where

$$T = \sqrt{C + B^2]} - B; \quad B = U_f/\alpha_4\ \mu g \qquad (1.6c,d)$$
$$C = \{[2k^2\alpha_1\psi_s\,(1 - \alpha_1\theta/\alpha_4)/\alpha_2 l] + \alpha_5 s_s/\alpha_4\}/\mu g \qquad (1.6e)$$

These results can be verified against the spinout equations of CRASH (*5*, Section 9.2.a), noting the following:

1. The CRASH treatment first derives the equations for spin without braking per Marquard, second rederives with the partial braking effect per Marquard, third rederives with McHenry's contribution of the residual velocity and generalized empirical coefficients, and fourth recapitulates while specifying polynomial coefficients and certain computation routines. The present work instead proceeds directly to the final solution.

2. The present solution embodies the solution of a quadratic in T that is the separately stated CRASH quadratic in $\dot{\psi}_s$ transformed by use of Equation 1.2 after a publishing error in the CRASH equations has been corrected by reversing the sign of its unity coefficient. The quadratic coefficients become simpler because a lengthy expression in the denominators occurs now in only one term of the numerator.

3. Once the quadratic has been solved, U_f is found from the sum $U_s + U_f$ of Equation 1.3 rather than from the more complex difference $U_s - U_f$ of Equation 1.5.

Equations 1.6 provide a complete solution for the linear and angular speeds at the start of the spinout, given appropriate expressions for the empirical coefficients. The CRASH expressions were based on an analysis of a set of 18 SMAC runs [see User's Guide *(5)*, Section 10.4.d], not known to have been published, so neither the data nor the fitting technique is known. [Fonda *(8)* shows a revaluation technique; for brevity those two equations (1.7, 1.8) are omitted here.] CRASH uses the polynomials

$$\alpha_1 = 2.6 - 7.5\rho^* + 15\rho^{*2} \qquad (1.9a)$$

$$\alpha_2 = 0.9 - 4\rho^* + 8\rho^{*2} \qquad (1.9b)$$

$$\alpha_3 = 0.23 + 5\rho^* - 10\rho^{*2} \qquad (1.9c)$$

$$\alpha_4 = 0.67 + 1.6\rho^* - 5\rho^{*2} + 6\rho^{*3} \qquad (1.9d)$$

$$\alpha_5 = 1.2 + 17\rho^* - 99\rho^{*2} + 181\rho^{*3} \qquad (1.9e)$$

as functions of the initial radius to the instant center of rotation, $\rho = U_s/\dot{\psi}_s$, divided here for convenience by 1,000; ρ^* is the radius in kinches (thousands of inches).

The CRASH routine evaluates the five polynomials from a trial value of ρ flanked by trial variations of plus and minus 15 and 30 percent, the best value then being selected by a test for a minimum value of the radius error $|\rho\dot{\psi}_s/U_s| - 1$. To the same effect, setting that error algebraically to zero and iterating may be done for simplification. From Equations 1.2 and 1.3, the radius to the instant center is

$$\rho^* = 0.001\ (\alpha_5 s_s - TU_f)/\alpha_1\psi_s \qquad (1.10)$$

in thousands of inches, where α_1, α_5, and T are initialized at 2.0 and iterated. Convergence is strong; in the programmed version, with reasonable error criteria, the residual errors of iteration and the errors of the optional rounding in Equation 1.9 (which shortens the hand-held program) are insignificant, as are those of CRASH where the residual error in ρ varies randomly from zero to 7.5 percent.

The empirical curve fit of the CRASH polynomials extends to ρ = 250 in. This represents separation at instantaneous velocities, which, if continued in the same proportion, would give 180 degrees of rotation of the vehicle in $250\pi/12 = 65.4$ ft. This is a particular value of the distance between the readily identifiable cusps seen in typical spinout tire tracks, the successive points at which the vehicle is broadside to its new course and one axle has much more velocity than the other. CRASH locks the five polynomials at the respective values that they achieve (1.66, 0.40, 0.85, 0.85, 2.08) at ρ = 250; to the same effect ρ^* can simply be limited to 0.25.

A lightly braked vehicle with a small initial yaw rate will stop spinning and begin to roll endwise when it first rotates into alignment with its course, so large values of ρ will not persist. However, a heavily or fully braked vehicle can maintain a small yaw rate, giving slow spin to rest. So for large θ and zero s_f there is some interest in the case of large ρ. At and above

ρ = 250, with s_f = 0, $\alpha_3 = \alpha_4 = 0.85$, and either $\psi_s = 0$ or θ = 1, from Equations 1.6 the whole solution reduces to

$$U_s = \alpha_5 s_s/\sqrt{C}] = \sqrt{\alpha_4\alpha_5\mu g s_s}] = \sqrt{0.85(2.08)\mu g s_s}]$$
$$= 0.94\sqrt{2\mu g s_s}]$$

This is 6 percent less than the correct result for this case. CRASH handles this anomaly by means of a discontinuity, switching to the skid solution, $\sqrt{2\mu g s}]$, for $\rho \geq 500$ in. or $\psi_s \leq$ 20 degrees. To much the same effect, but avoiding the discontinuity, more simply the ρ^* limit is raised to 0.30 (for these polynomial coefficients), allowing U_s to reach $\sqrt{2\mu g s}]$ at ρ = 300 in.

ANGULAR IMPACT ANALYSIS

So far as possible, the aircraft notation introduced to the automotive industry by Calspan personnel in 1956 *(9)* is used, as formalized in the vehicle dynamics terminology of SAE J670 (1967ff). As in the spin analysis, the heading of a vehicle is designated as ψ (psi) and its sideslip (attitude) angle as β (beta); as shown in Figure 1a, their sum, the course angle (direction of motion) is ν (nu). With various subscripts, U will denote the course speed of either vehicle before or after impact and differences thereof, and ξ (xi) will denote various angles in the horizontal plane. The magnitude and direction of a vector U may be found from its orthogonal components as shown in Figure 1b.

The slide or spin analyses provide two speeds at separation, each in a known direction, hence two mass center velocity vectors at separation. For angular impact only, if further the vehicle weights and directions of approach are known, and horizontal tire forces for the period of the impact (even those due to underride or override) are neglected, their impact speeds can be found by impulse analysis, that is, by using the principle of conservation of momentum.

Conservation of momentum requires only that the intervehicular force act equally and oppositely on both masses, so that one mass gains as much momentum (the time integral of the force) as the other loses. Because this does not assume any commonality of velocity between the vehicles, contrary to statements in the CRASH literature, sideswipes can be treated correctly by impulse analysis, as can underride or override impacts, provided horizontal tire forces remain trivial.

With normalization to the mass of Vehicle 1 ($R = M_2/M_1$), Figure 1c shows the equivalence of the momentum at separation (from O to A via S) to the momentum at impact (from O to A via I). The direction of the mutual speed change (I to S) is necessarily the direction of the "principal" intervehicle force (the DOPF or, interchangeably, PDOF). The inset shows the vectors for a case of sideswipe; the total velocities are continuingly opposed, but their differential normal to their sides is reversed because of structural rebound.

From the linear momentum along the normal to the initial course of the respectively opposite vehicle, the vehicle speeds at impact are

$$U_{o1} = [U_{s1}\sin(\nu_{s1} - \nu_{o2}) + RU_{s2}\sin(\nu_{s2} - \nu_{o2})]/\sin(\nu_{o1} - \nu_{o2}) \quad (2.1a)$$

$$U_{o2} = [U_{s1}\sin(\nu_{s1} - \nu_{o1}) + RU_{s2}\sin(\nu_{s2} - \nu_{o1})]/\,R\sin(\nu_{o2} - \nu_{o1}) \quad (2.1b)$$

The inputs are the two separation speeds from Equation 1.1 or (with spin) Equation 1.6, the four vector directions, and the mass ratio R. Each equation expresses a rear view of the impact vectors as seen along one of the approach paths.

The same vectors should be found (given the same data) by any method of reconstruction; in CRASH (as published only in 1974 in the START routine of SMAC) and in the various publications by or based on Brach, such as CARR1 *(10*, Equations 72 and 73), they are written as the two unknowns in two equations, algebraically soluble for the individual speeds. Such indirect solutions are correct, but confusing, hindering insight in use. The matrix methods of Brach are, as he notes *(11*, p. 33), neither necessary nor preferable when sufficient data are available. But as shown by the original CRASH treatment by McHenry, and contrary to Brach *(11*, p. 33), zero rebound is no bar to the solution; up to this point impact has not even been assumed to be the source of the intervehicle force, much less impact with rebound.

The approach directions must differ appreciably, preferably by more than 20 degrees, lest the vector components viewable from the rear become too small; at lesser angles the solution becomes a ratio of small quantities, unduly sensitive to usual

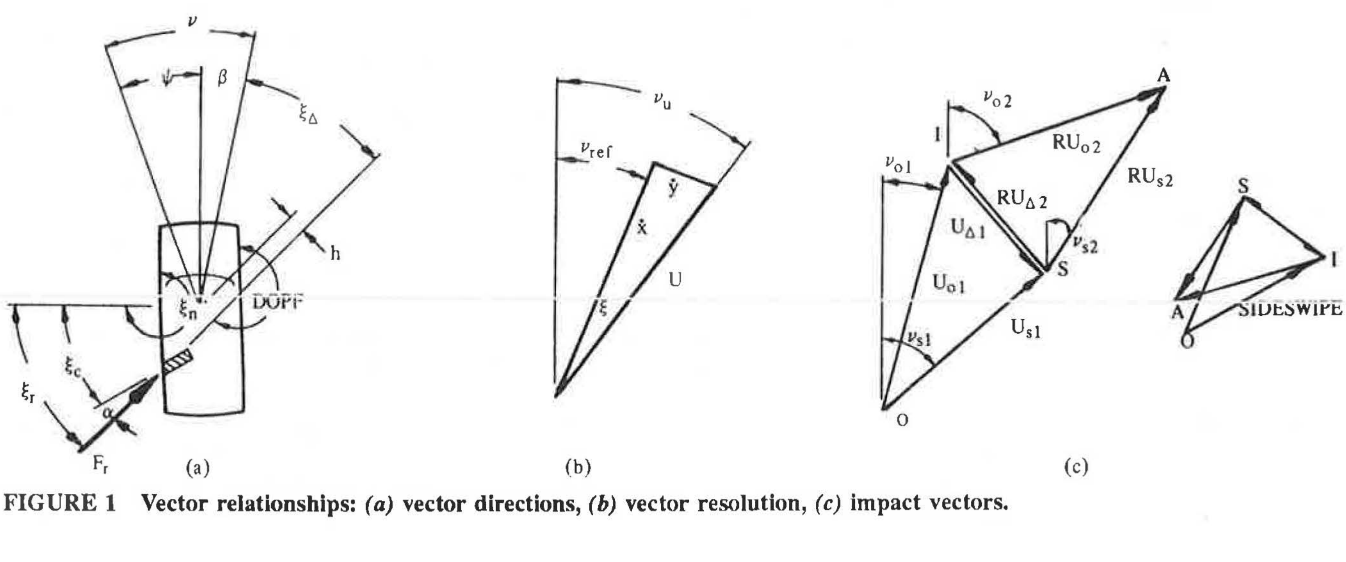

FIGURE 1 Vector relationships: *(a)* vector directions, *(b)* vector resolution, *(c)* impact vectors.

errors, and becomes indeterminate for axial impact. [Although Wooley et al. *(7)* appear to dispute this, Wooley *(12)* presents only the damage-based solution for the axial case, as in CRASH.

Once both approach velocities have been found, the velocity difference at impact, the closing velocity, may be found as a polar vector from its components,

$$U_{\Delta o}, \xi_{\Delta o} = Pol[U_{o2} \cos(\nu_{o1} - \nu_{o2}) - U_{o1}, U_{o2} \sin(\nu_{o2} - \nu_{o1})] \quad (2.2)$$

referred angularly to the original course heading of Vehicle 1. This is the apparent velocity of Vehicle 2 as seen from Vehicle 1 before impact. Similarly, the vector change of velocity of each vehicle during impact, from its components, is

$$U_{\Delta i}, \xi_{\Delta i} = Pol\ [U_{si} \cos(\nu_{si} - \nu_{oi}) - U_{oi}, U_{si} \sin(\nu_{si} - \nu_{oi})] \quad (2.3)$$

($i = 1, 2$). This gives the magnitude of the speed change of each vehicle and its direction relative to the original course heading of that vehicle. As demonstrated in Figure 1c, inherently these vectors will be 180 degrees apart in space.

The angle $\beta_i + \xi_{\Delta i}$ (with β_i usually zero) is the body-axis direction of the force of impact, hence the direction in which the vehicle is moved toward the unrestrained occupant or any other free object. It is also the direction in which the vehicle structure is deformed if isotropic ($\xi_c = \xi_r$), although allowance is made for reduced compliance in shear ($\xi_c < \xi_r$). The computed angle should be checked against the aforementioned physical evidence, often recited as a PDOF, a clock direction, which should equal $(\beta_i + \xi_{\Delta i} + 180)/30$.

The direction of the speed change and (hence) the intervehicle force relative to the normal to the surface along which sliding occurs is

$$\begin{aligned} \xi_{ri} &= \xi_{ni} - (\beta_i + \xi_{\Delta i} + 180) \\ &= \xi_{ni} - 30\ (\text{DOPF}) \end{aligned} \quad (2.4)$$

which as shown by Figure 1a is positive when the shear force exerts a clockwise moment on the vehicle. By Coulomb's law this angle must not, for either vehicle, exceed in magnitude a reasonable intervehicle friction angle, on the order of arctan $0.55 \cong 30$ degrees. This limit becomes a probable value for the larger of the two angles if there is visible evidence of "scraping" (a convenient term to distinguish intervehicular sliding from tire-to-road sliding). Absent scraping, any angle between the friction limits is reasonable.

Pocketing, snagging (which is extreme pocketing, possibly with shear failure), and corner impact can change the surface orientation as the deformation proceeds. Therefore the inputted value of ξ_{ni} may not be the orientation of the original surface but that of the developed surface; this requires careful vehicle examination and visualization of the impact process. At a corner there is initially a 90 degree range of possible normals until the corner flattens to a new surface with a new normal.

Unreasonably large angles between the computed DOPF and the developed normal require reconsideration of the input data. This is not emphasized in CRASH, which is weighted toward damage-only evaluation and will not override the user choice of PDOF (except for an adjustment of not more than 7.5 degrees to obtain colinearity of the two PDOFs). Contrary to SMAC, the frictional limit on shear force has been consistently overlooked in CRASH from CRASH1 to the present. There has been no admonition against excessive angles, the user estimate of the DOPF has been checked against neither the trajectory/impulse-based DOPF nor Coulomb's law before its use in the damage calculations, and gross violations of Coulomb's law have been specifically permitted (*5*, Section 9.1.f, arctan $\mu = 75$ degrees, $\mu = 3.73$) and algebraically "corrected for" in damage evaluation (*5*, Section 9.1.f). Such angles can never occur in practice; whatever the method of reconstruction, no input data should be accepted that imply that they have occurred. Similarly there is a danger of misapplication of the matrix methods of Brach *(10, 13)*, apparently also used in CARR1 *(11)*, in which the relative shear force μ may be naively assigned its positive limit value even when it should be negative or small.

It is preferable to adopt the PDOF computed from the velocity vectors in the case of angular impact, if credible, or to adjust the input data to obtain a credible value. If this fails, and always in axial impact, the PDOF indicated by the physical evidence, subject to the limitations imposed by Coulomb's law, is used in the further calculations.

It is implied in the CRASH treatment (*5*, Section 10.5) that the condition of common velocity is fundamental to the principle of conservation of momentum; it is not. A bullet passing a mutually magnetic target interacts without impact and never a common velocity; yet equal impulses occur, so momentum is exchanged. It might better have been stated that when bodies interact by means of a structural collision, there will be both impulse and impact, with an instant of common velocity; in that instance, the subsequent rebound velocity evaluated from the trajectory data should not be excessive. This is not a simplifying assumption, it is a physical observation, and it is not imposed on the dynamics involved in impulse but deduced from the structural mechanics involved in impact.

It is the rebound velocity that is found in and limited by the common velocity check—which might better have been called the rebound velocity check. Unfortunately, the limit adopted is overly severe. The common velocity check of CRASH3 will abort the trajectory/momentum solution if the speeds of rebound from the mutual mass center differ by more than about 4 mph. In a moderately severe impact, with between 1 and 2 ft of crush, this condition is usually not met. For this reason, many reasonable solutions are aborted, frustrating the intent of CRASH and discouraging the user. If the limit had been set at about 12 to 15 mph, the common velocity check might have served its intended purpose of excluding unreasonable inputs. As it is, CRASH3 gives no momentum solution in many instances of reasonable data inputs. Either its elimination (at line 860 of SPIN2) or revision of its limit value (at Line 350 of VELCHK) is recommended. More useful calculations to the same end will be suggested.

DAMAGE ASSESSMENT

It is often necessary to infer the speed changes from the damage done to the vehicles. This can be done from measurements of the location and depths of the vertically averaged residual deformation of both vehicles combined with empirically assigned structural parameters of the vehicles.

If, following Campbell *(2)*, it is assumed that the test speed involved in perpendicular barrier crash tests varies linearly with the resulting final crush depth while the force during impact increases as the crush depth, each from a threshold value, when the kinetic energy of approach is equated to the potential energy of crush (conservation of energy), it is found that

$$\begin{aligned} E_k &= 0.5M(V_o + CdV/dC)^2 = E_c \\ &= w_t \int (A + BC)dC + \text{constant of integration} \\ &= M[0.5V_o^2 + V_o\, CdV/dC + 0.5C^2\,(dV/dC)^2] \\ &= w_t(G + AC + 0.5BC^2) \end{aligned} \quad (3.1a)$$

whence

$$G = 0.5V_o^2\, M/w_t \text{ (inferred structural damping energy per unit width)} \quad (3.1b)$$

$$A = V_o\,(dV/dC)M/w_t \text{ (inferred threshold force per unit width)} \quad (3.1c)$$

$$B = (dV/dC)^2 M/w_t \text{ (inferred structural stiffness per unit width)} \quad (3.1d)$$

for a vehicle of mass M and involved width w_t. This establishes a data-fitting technique whereby, for given M/w_t, the test velocity intercept V_o solely determines G, the test velocity slope dV/dC solely determines B, and the threshold force $A = \sqrt{2BG}]$ is a jointly dependent parameter completing the square of the binomial (that is, sized for linearity of V with crush depth).

These relationships, adapted from Campbell *(2)* but not shown in the User's Manual, are the source of the CRASH3 table of structural data *(5)* and the earlier CRASH data *(14)*. Campbell's 1974 data *(2)* did show linearity of crush with test velocity for tests between 15 and 60 mph (24 and 97 km/hr). When the process is reversed, the intercept and slope data corresponding to the current CRASH3 structural tables are as shown in Table 1 for the first five vehicle classes.

Any velocity intercept and slope data, including those of Campbell (2, Table 1) and Wooley *(12)*, may be recast into the form used in both CRASH and SMAC. This can be especially useful if specific full-width barrier test data or partial-width data, proportioned up to full-width data (because they will be proportioned back down for a partial-width impact), are known for the vehicle in question. Piecewise fits may be useful, as shown by Strother et al. *(14)*. It is not necessary to maintain $A = \sqrt{2BG}]$ if another assumption fits the data better. For instance, setting B to zero leaves A as the constant force when the kinetic energy (instead of the test velocity) is seen to vary linearly with the crush depth from an intercept G.

In the CRASH3 data tables, anomalous values may be noted: large, opposed variations of B and G from the norm, most notably for the rear of Classes 4 and 5/6, less so for the rear of Classes 7 and 8 (B = 13, 70, 55, 25; G = 4986, 628, 818, 2373). Such opposed variations suggest chance rotation about a clump of data at a single test speed, with oppositely varying slope and intercept. For Classes 4 and 5 this can also be seen by inspection of Table 1; the intercept is high and the slope low for Class 4, conversely for Class 5.

This suggests that the data are based largely on 30-mph barrier crashes with a scattering of other data insufficient to well define the intercept. If Campbell's intercept data for 1974 GM large cars are a better guide for Classes 4–8, those intercepts might be changed to V' = 7.5, 7.5, 8.0, and 8.5 mph, giving G' = 1243, 1374, 1405, and 1586; $B' = B(30 - V_o')^2/(30 - V_o)^2$ = 28.9, 57.1, 46.6, and 30.1; and $A' = \sqrt{2BG}]$ = 268, 396, 362, and 291. This rotates the data fit about the data at 30 mph, altering the data only for the much less and much more severe cases. A similar adjustment for the front of vans is left to the reader.

In the process the anomalies have been reduced in the zero-velocity intercept of the same test data, $A/B = V_o/(dV/dC)$. A/B is also (as shown elsewhere by McHenry) the prestress distance required to establish a threshold level of A pounds for a nonreturning spring of rate B lb/in., with prestress energy expenditure of G. (This is a limited analogy, because the energy is actually lost to hysteresis at the time of the impact, not at the time of manufacture.) A/B is generally well behaved at 2.1 to 3.6 in. for the sides and 6.0 to 10.5 in. for the front of passenger vehicles, and 9.4 to 9.6 in. for the rear in Classes 1, 2, and 3. The foregoing adjustment has modified the anomalous values of A/B = 27.9(!), 4.2, 5.5, and 13.7 in. for the rear of Classes 4–8 to 9.27, 6.93, 7.76, and 9.66 in., which are credible horizontal intercepts or prestress distances.

Monk and Guenther *(15)* updated the 1983 CRASH damage tables, but although they list the sources and the data analysis program, there are no source data. The final data are both tabulated and plotted, but the two presentations fail to agree by substantial amounts (Table 2). Obviously the user should consider both versions until the uncertainty can be resolved; the amounts listed in Table 2 are possible upward or downward corrections applicable respectively to the speed changes found for light and for heavy damage.

All this is of little help to the CRASH user with no access to modify the present tables programmed into CRASH; the most he can do is to choose another vehicle classification with table entries closer to his known data.

Following from Equation 3.1, the further assumption that energy per unit width is a constant allows evaluation of the

TABLE 1 CRASH3 DATA EXPRESSED AS BARRIER IMPACT DATA

Class	1*	2	3	4	5	1*	2	3	4	5
	Velocity Intercept (mph)					Slope (mph/inch)				
Front	7.7	6.5	6.8	9.2	7.6	1.20	1.09	1.19	0.87	0.87
Side	2.2	2.8	3.7	3.0	3.7	1.06	1.35	1.21	1.07	0.98
Rear	10.4	10.1	9.9	15.0	5.1	1.08	1.06	1.06	0.54	1.20

* W=2173+296=2469 (was 2469+296) so M=6.39 (was 5.70)

TABLE 2 MONK APPENDIX E EXCESS OVER CRASH3 DATA

Class	1*	2	3	4	5	1*	2	3	4	5
	At zero crush (mph)					At crush for 30 mph (mph)				
Front	<1	<1	<1	-1.5	<1	1	<1	<1	<1	<1
Side	<1	<1	<1	<1	1.5	8	-4	<1	-2.5	<1
Rear	<1	-5	<1	9	1	-1.5	-6	-3	-3	-2.5

* W=2173+296=2469 (was 2469+296) so M=6.39 (was 5.70)

energy for any crush contour by using the integral across the width,

$$E_c = \int (G + AC + 0.5BC^2)dw \qquad (3.2a)$$

or by using a piecewise linear fit ($C = mx + b$) as suggested by Wooley et al. *(7)*:

$$E_c = \Sigma w_j\{G + (A/2)(C_{j-1} + C_j) + (B/6)(C_{j-1}^2 + C_{j-1}C_j + C_j^2)\}] \qquad (3.2b)$$

(j = 1 to n) for n trapezoidal segments of independent widths w_j.

This permits any number of arbitrarily sized segments to fit any profile; this seems preferable to the specific trapezoidal integrals for two, four, and six equispaced points used in CRASH. As in the CRASH3 User's Manual (*5*, Section 11.3), further equations can be written for the depth and offset from the midpoint of the geometric center of the crush area to refine the crush centroid definition.

Equation 3.2b defines the work done in crush only for axial impact. If the direction of deformation is oblique and if the structure is isotropic (homogeneous), then it appears obvious that the measurements to be used above must be taken along and normal to the direction of deformation or corrected to those axes. But because this is not the treatment provided in CRASH, and there are related considerations to discuss, oblique deformation will be considered later.

DAMAGE DYNAMICS

Consider two vehicles in central impact along the X-axis separated by a distance $z = x_1 - x_2$ and acted on mutually by the impact force F_r, which causes crush of each vehicle. For now, neglect possible slippage between the vehicles and assume that the kinetic energy expended in impact equals the total crush energy

$$\Sigma E_k = \Sigma E_c \qquad (4.1)$$

Now the relative acceleration due to the force will be

$$\ddot{z} = \ddot{x}_2 - \ddot{x}_1 + F_r/M_2 + F_r/M_1 = F_r\Sigma M^{-1} \qquad (4.2a)$$

where $\Sigma M^{-1} = M_1^{-1} + M_2^{-1} = (M_1 + M_2)/M_1M_2 = (1 + R^{-1})/M_1$. Solving for the force and integrating over the distance gives

$$\Sigma E_k = \int F_r dz = \int \ddot{z}dz/\Sigma M^{-1} = \int (d\dot{z}/dt)dz/\Sigma M^{-1}$$
$$= \int \dot{z}d\dot{z}/\Sigma M^{-1} = 0.5(\dot{z}_o^2 - \dot{z}_f^2)/\Sigma M^{-1}$$
$$= 0.5\dot{z}_0(1 - \varepsilon^2)/\Sigma M^{-1} \qquad (4.2b)$$

which is the loss in kinetic energy during the impact, the kinetic energy of approach (E_k) less the kinetic energy of separation. The latter was assumed to be zero in CRASH; the more general treatment here will always reduce to the CRASH treatment by the assumption that $\varepsilon = 0$. It is so readily included that it will be done at this point rather than later.

The quantity $\dot{z}_o$ is the closing speed, and $\dot{z}_f = \varepsilon\dot{z}_o$ is the speed of separation (in the opposite direction except in the case of perforation—the total penetration of a target by a bullet—when ε is negative). Their sum is the change in differential speed along the X-axis during the entire impact,

$$\Delta\dot{z} = \dot{z}_o + \dot{z}_f = (1 + \varepsilon)\dot{z}_o$$

or, by substituting Equation 4.2b solved for $\dot{z}_o$ and then Equation 4.1,

$$\Delta\dot{z} = (1 + \varepsilon)\sqrt{2\Sigma M^{-1}\Sigma E_c/(1 - \varepsilon^2)]} = \sqrt{2E^*\Sigma M^{-1}]} \qquad (4.3a)$$

For $\varepsilon = 0$ this reduces to the CRASH equation (*5*, Section 9.1b, Equation 12) obtained by using the further assumption of simple linear stiffness of the structure, which restriction evidently is superfluous. In Equation 4.3a, two corrections for rebound have been consolidated into the definition of an equivalent energy of deformation,

$$E^* = \Sigma E_c(1 + \varepsilon)^2/(1 - \varepsilon^2) \qquad (4.3b)$$

For each vehicle the divisor $(1 - \varepsilon^2)$ gives, from the actual crush energy E_c, the kinetic energy of approach along the crush axis, the quantity evaluated as E_k in the usual barrier test, so that part of the correction is built into the available data. The multiplier $(1 + \varepsilon)^2$ enables the determination of the speed change of the mass from approach to departure, as if more damage but no rebound had been found, or else (in the case of perforation) less damage but no exit velocity.

In Equation 4.3a, the time integral of Equation 4.2a, the individual speed changes for Vehicles 1 and 2 occur in inverse proportion to the affected mass (which is the principle of conservation of momentum), giving

$$
\begin{aligned}
U'_{\Delta 1} &= \Delta\dot{x}_1 = (M_2/M_t)\sqrt{2E^*\Sigma M^{-1}]} \\
&= (1 + R^{-1})^{-1}\sqrt{2E^*(1 + R^{-1})/M_1]} \\
&= \sqrt{2E^*/M_1(1 + R^{-1})]}
\end{aligned} \tag{4.4a}
$$

$$
\begin{aligned}
U'_{\Delta 2} &= \Delta\dot{x}_2 = (M_1/M_t)\sqrt{2E^*\Sigma M^{-1}]} \\
&= U'_{\Delta 1}/R
\end{aligned} \tag{4.4b}
$$

where $M_t = M_1 + M_2$ is the total mass and $R = M_2/M_1$ as before.

Now, although the angular energy involved in any accompanying rotation is neither considered nor restricted, the effect of offset of the mass center from the impact force F_r studied earlier may be found. Assume a fixed offset,

$$h_i = y_i \cos(\beta_i + \xi_{\Delta i}) - x_i \sin(\beta_i + \xi_{\Delta i}) \tag{4.5a}$$

(i = 1, 2) where x_i and y_i are the body coordinates of the crush centroid and $\beta_i + \xi_{\Delta i}$ (with β_i usually zero) is the direction of the impact force in body axes. Then the acceleration at the point of impact is

$$
\begin{aligned}
\ddot{x}_{pi} &= \ddot{x}_i - h_i\ddot{\psi}_i = (-F_r/M_i) - h_i(F_r h_i/M_i k_i^2) \\
&= -F_r/\gamma_i M_i = \ddot{x}_i/\gamma_i
\end{aligned} \tag{4.5b}
$$

(i = 1, 2) where

$$\gamma = k^2/(h^2 + k^2) \tag{4.5c}$$

So at the crush centroid the ratio of force applied to the vehicle to resulting linear acceleration $(F_r/\ddot{x}_{pi})$, which may be called the effective mass of each vehicle, is $\gamma_i M_i$. The force is still found from the deformation by conservation of energy; when that force is offset from the mass center, the acceleration at the crush centroid is $\ddot{x}_{pi}$ and at the mass center is $\dot{U}_{\Delta i} = \ddot{x}_i \gamma_i \ddot{x}_{pi}$.

Substituting effective masses in Equations 4.4 and letting M' denote their total, the speed change at each mass center in offset impact is obtained:

$$
\begin{aligned}
U'_{\Delta 1} &= \gamma_1(\gamma_2 M_2/M')\sqrt{2E^*\Sigma(\gamma_i M_i)]} \\
&= \sqrt{2E^*/M_1(\gamma_1^{-1} + R^{-1}\gamma_2^{-1})]}
\end{aligned} \tag{4.6a}
$$

$$U'_{\Delta 2} = \gamma_2(\gamma_1 M_1/M')\sqrt{2E^*\Sigma(\gamma_i M_i)]} = U'_{\Delta 1}/R \tag{4.6b}$$

Evidently linear momentum continues to be conserved. Exclusive of oblique impact, the CRASH derivation (*5*, Section 9.1) covered the same physics as Equations 3.1 through 4.6 for the special case of linear crush stiffnesses and zero rebound, with no errors due to the omission of angular momentum or energy terms for that case, contrary to the reservations expressed by Wooley et al. (*7*).

CRASH correctly incorporates no moment coefficient of restitution as proposed by Brach (*10, 13, 16*), who remarks (*16*) that "the point of application of the (collision force) is never known precisely, (so) the resultant must consist of both a force (at some arbitrary point) and a moment." Means of obtaining that moment are then postulated, but this postulates knowledge of the unknown. If the error is known, the correction can be made; if it is not known, the moment coefficient of restitution is not known. All experimental values of that coefficient are no more and no less than discoveries of the investigational error in centroid location. Incorrect results can issue if the moment coefficient of restitution is assigned a generalized value other than unity, which applies zero correction to the centroid location.

APPROACH VELOCITIES BASED ON DAMAGE ANALYSIS

Whether for angular or for axial impact, to find the approach velocity of each vehicle by using the damage data, the components along the approach course are taken and the speed change is subtracted from speed at separation, giving

$$U'_{oi} = U_{si}\cos(\nu_{si} - \nu_{oi}) - U'_{\Delta i}\cos\xi_{\Delta i} \tag{5.1}$$

(i = 1, 2). For an exactly in-lane, centered, northbound rear impact, for the overtaken vehicle the angles are zero, the cosines are +1, and the separation velocity exceeds the approach velocity; for the overtaking vehicle $\xi_{\Delta i}$ is 180 degrees, its cosine is −1, and the approach velocity exceeds the separation velocity.

The cosines remain close to unity for impacts close to axial, so zero and 180 degree inputs could used for impacts within about 10 degrees of axial. For large angles, the exact form should be used. At about 20 degrees the angular solution based on location data becomes credible. In comparing the result from Equation 5.1 with the result from Equation 2.1, because the separation velocities are the same for both, these results will differ only to the extent that the speed changes during impact differ.

The inferred closing velocity will be the result of substitution of the values from Equation 5.1 in Equation 2.2, which in axial impact reduces to a simple subtraction.

OBLIQUE IMPACT

As previously mentioned, by using the Campbell structural model the impact force and energy can be inferred from the depth of crush, based on their observed interdependence in barrier crashes. If the force (F_r in Figure 1a) is along the normal to the surface, the crush of the jth segment (C_j in Figure 2a) is normal to the surface and the width w_j is along the surface. If the force is oblique at the angle ξ_r, logically the crush (C_j' in Figure 2b) is also oblique at the angle ξ_c in general and can be so measured, with the associated width w_j' measured normal thereto. There is no further correction for oblique impact for that structural assumption and those directions of measurement; the treatment is complete.

But field measurements along a field-selected direction of crush are not only inconvenient but presumptuous, because a new direction might be assigned later. It is more prudent to take measurements along and normal to a major axis of the vehicle and subsequently convert to the oblique measurements. The oblique depth and width are then given by

$$C_j' = C_j/\cos\xi_c; \quad w_j' = w_j\cos\xi_c + (C_{j-1} - C_j)\sin\xi_c \tag{3.2c,d}$$

In programming these can be used directly in Equation 3.2b. Neglecting the component due to change of depth, from Equation 3.2a the crush energy becomes

$$E_c = \int [G + A(C/\cos\xi_c) + 0.5B(C/\cos\xi_c)^2] (\cos\xi_c)dw$$
$$= \int (G\cos\xi_c + AC + 0.5BC^2/\cos\xi_c)dw$$

showing multiplication of G by the cosine of the angle and division of B by the same quantity. If G were large and C were small (light oblique end impact), this could reduce, rather than increase, the energy for a given contour. (Less measured width, despite more measured crush, can mean less energy.)

Monk and Guenther (*15*, p. 48) began a similar departure from CRASH3 but omitted the cosine effect in depth, the sine effect in width, and G; the result was zero sensitivity to ξ_c. The experimental evidence discussed was so subject to uncertainty with regard to the structural resistance as to be inconclusive with regard to angularity effects, except that it did appear to confirm the absence of a $1 + \tan^2\alpha$ effect.

The nonisotropic assumption ($\xi_c \neq \xi_r$; $\alpha \neq 0$; crush not in the direction of the applied force) is discussed at length by Fonda (*8*) and appears to account for the different treatment of oblique impact in CRASH (with inconsistencies). However, even though the latter have been eliminated in the present treatment, it is preferred to assume the structure to be isotropic, as in SMAC. Then both skins (not just one) must buckle in corner impact, and, much as in SMAC, the structural characteristics in the deformed corner are conveniently divided along the corner trace, shown in Figure 2c. Each part is evaluated independently, by using the respective values of G, A, and B, and the results are summed. This weights the structural properties according to the involved width, varying smoothly from fully frontal to fully side deformation (not possible in CRASH).

Proceeding to the question of damage dynamics in oblique impact, the possibility of intervehicle sliding (scraping), which often occurs in oblique impact, is included. Using the force and motion vectors shown in Figure 2d, allowing for a nonisotropic structure ($\alpha \neq 0$), given each of the crush forces F_{ci}, the intervehicle force is

$$F_r = (F_c/\cos\alpha)_1 = (F_c/\cos\alpha)_2$$

which acts through a distance that is the cosine component of the crush distance plus the sine component of the scrape length,

$$dz = dz_1 + dz_2 = \Sigma(dC\ \cos\alpha + ds_v\ \sin\xi_r)_i$$

So the total work done, the product of force and distance, is

$$\Sigma E_p = \int F_r dz = \int (F_c/\cos\alpha)\ (dC\ \cos\alpha)_i + \int F_r(ds_v\ \sin\xi_r)]_i$$
$$= \Sigma E_c + \int F_r(\sin\xi_r)ds_v]_i \qquad (4.1a)$$

This expresses all of the potential energy expended as work done, scraping included. The scraping component will be discussed later.

Absent scraping, the total work done is no more and no less than the total crush energy; there is no correction for oblique impact. This is reasonable, because no other work is done; and this should have been the result reached in CRASH2 and 3. Instead, by using an inconsistently nonisotropic assumption, the force F_r was assumed to act through an excessive distance $dC/\cos\alpha$ shown in Figure 2d, increasing by the factor $\tan^2\alpha$ the distance traversed and hence the work done. There was no such extra work done; the derivation was and is incorrect. For the case of oblique impact with no rebound and no scrape, CRASH3 will overvalue the impact energy. (A method of adjusting the CRASH3 PDOF and damage midpoint was found, but was rather cumbersome and is not offered.)

THE GENERALITY AND VALIDITY OF CRASH

This completes the rederivation of the CRASH equations; it is hoped that the treatment has been clarified while the algebraic length has been reduced and its generality has been illuminated. The following points have been shown:

1. In CRASH there are no violations of the principles of conservation of momentum and conservation of energy. Mutual forces acting between two bodies inherently result in a conservation of linear momentum, in offset as in central impact, whatever the effect of offset on the intervehicle forces. When those forces are found from the damage assessment, the resulting moment due to offset determines the change in angular momentum. Lacking two flywheels interacting on a common shaft, angular momentum is not conserved. The principle of conservation of energy is applied. Criticisms of CRASH for failure to consider angular momentum, conservation of angular momentum, and conservation of energy are ill founded.

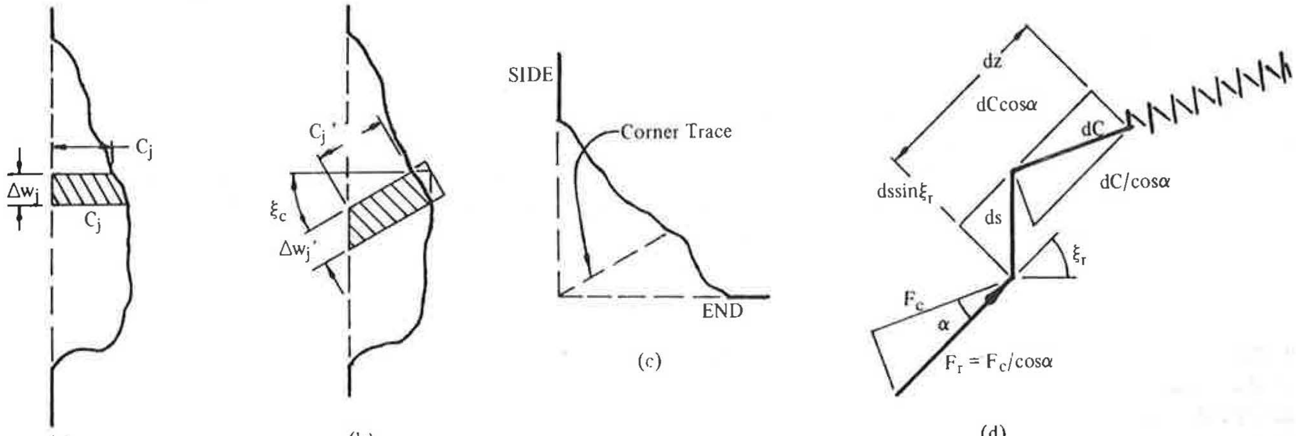

FIGURE 2 Crush motion relationship: *(a)* normal deformation, *(b)* oblique deformation, *(c)* corner deformation, *(d)* slide and crush.

2. The assumption in the damage dynamics equations of CRASH of linear crush characteristics of the vehicle structures can be seen as merely illustrative, a convenience in derivation. The rederivation here avoids any assumption as to the distance or time pattern of the intervehicle force, so that only the damage assessment so much as implies any particular force pattern for crush of the structure. Further research could alter that model whereas the remainder of CRASH would remain fully applicable.

3. CRASH3 overvalues damage and speed change for oblique impact without scraping and rebound, which are never considered.

The following results are obtained in this paper from the site data alone in all equations through 2.4 and from the damage data alone in the remaining equations, except that Equation 5.1 uses both:

Variable	*Equation*
Speed at separation	1.1 or 1.6
Speed at impact	2.1, 5.1
Closing velocity	2.2, 5.1
Velocity change in impact for Vehicle 1	2.3, 4.6a (i=1)
Velocity change in impact for Vehicle 2	2.3, 4.6b (i=2)
Direction of force from normal	2.4, examination
Damage energy	3.2
Effective damage energy	4.3b

For axial impacts, Equations 2.1 through 2.4 do not apply, but for angular impacts the separately obtained magnitudes and directions of the speed changes should be compared and reasonable input data revisions should be adopted when they result in greater compatibility of the independent results.

CRASH likewise furnishes two pairs of speeds for angular impacts, but only gives components inferring the computed PDOF and outlines no technique of refinement of the reconstruction. If the CRASH equations are as represented, results identical to those of CRASH could be obtained if the CRASH assumptions were reinstated. Although site data, cartesian-to-polar data reduction, crush centroid determination, trajectory simulation, and SMAC setup have not been attempted, all of the accident reconstruction results of CRASH have been duplicated or refined.

EXTENDED CRASH

In the process of showing the generality of the established CRASH equations, CRASH has already been extended by refining the common velocity check, by allowing irregular crush contour segmentation, by replacing the $1 + \tan^2\alpha$ correction with a proper consideration of oblique crush (including subdivision of corner crush along the corner trace), and by introducing the effects of rebound.

In the spin analysis, the yaw rate and spin time in Equations 1.6b and 1.6c (which were found internally in CRASH) and the post-spin speed (U_f) and time ($U_f/\theta\mu g$) can be furnished as program outputs.

The reconstruction can be further extended by means of certain further computations. These are informative in themselves and help to refine the reconstruction by providing additional pairs of values for the same quantity evaluated independently from different data. This proceeds according to logical equation number.

By extending the angular impact analysis, the peak force of impact may be approximated by assuming the $A = G = 0$, $B_1 = B_2 = B$ case of the structure and $M_1 = M_2 = M$—in effect, the barrier impact case with no velocity intercept. Then the affected structure will undergo harmonic motion of frequency $\sqrt{K/M}] = \sqrt{Bw/M}] = 17.6(dV/dC)$ rad/sec. The peak acceleration is then the frequency times the initial velocity, giving the peak intervehicle force as

$$F = 17.6(dV/dC)MU_{\Delta i} = 0.80(dV/dC)WU \tag{2.5}$$

where $17.6/g = 17.6/22 = 0.80$. As dV/dC is typically somewhat more than 1 mph/in., with good reason the peak impact acceleration can be approximated as a little under 1 g/mph of speed change. This is consistent with the 12.5 g/ft and 0.9 in./mph cited by Mason and Whitcomb *(17)*; 0.9 (12.5/12) = 0.9375 g/mph. In the metric system this is $(17.6/35.3)dV/dC = 0.50dV/dC$, or a little over 1/2 g/kph. A less approximate treatment (revoking the simplifying assumptions) could be developed from these principles.

In either axial or angular impact, with location data, by combining the mass center speed at separation in the (confirmed) direction of the principal force with the velocity at the crush centroid location induced by the yaw rate, the speed of separation at the crash centroid in the direction of the force may be found:

$$U_{\Delta s} = \Sigma U_{si} \cos\{\nu_{si} - (\nu_{oi} + \xi_{\Delta i})\} - h_i\dot{\psi}_{si}] \tag{2.6}$$

(i = 1, 2), with the centroid offset h found from Equation 4.5a. As the vehicles separate in the direction of the forces, this is inherently positive and there is inherent subtraction of respective velocity components.

This is use of the damage location data without regard to damage severity in impulse analysis. It neglects the speed loss of each vehicle due to tire forces during impact up to the instant of actual separation; for side impacts this loss can be 1 or 2 mph, but is likely to be in much the same direction for both vehicles and will not significantly affect their speed of separation.

The coefficient of restitution is evaluated by dividing by the corresponding closing speed (inherently negative):

$$\varepsilon = -U_{\Delta s}/\Sigma U_{oi} \cos\xi_{\Delta i} - h_i\dot{\psi}_{oi}] \tag{2.7}$$

where the initial yaw rate $\dot{\xi}_{oi}$ normally is zero.

The same result could be obtained from CARR1 *(10)* if those equations (74ff) were used strictly to solve for the coefficient of friction and the coefficient of linear restitution, with a 1.0 value assumed for the coefficient of moment restitution.

As part of the damage analysis, it will be useful to evaluate for each vehicle the mean final crush depth in the direction of crush:

$$\begin{aligned} C_i^* &= \Sigma w_j'(C_{j-1} + C_j)/2]_i/\Sigma w_j']_i \quad (i = 1,2) \\ &\cong \Sigma[(w_j \cos\xi_c)(C_{j-1} + C_j)/2 \cos\xi_c]/\Sigma w_j \cos\xi_c \\ &= \Sigma[w_j(C_{j-1} + C_j)]/2w_t \cos\xi_c \end{aligned} \tag{3.3}$$

and thence the average of the mean final crush depths, $C^{**} = 0.5(C_1^* + C_2^*)$. This will allow the independent estimation of the coefficient of restitution from the damage data by the following method, from SMAC *(3)*.

SMAC finds all velocities as the results of structural (and tire) forces applied to inertias, and stops the elastic recovery according to the coefficient of recovery $c = (\delta_{max} - \delta_f)/\delta_{max}$, as distinct from the coefficient of restitution $\varepsilon = \dot{\delta}_f/\dot{\delta}_0$. The values of ε that were the basis of the published values of c will be reconstituted.

Writing the ratio of the net work done on the structure to the gross work done before rebound (with the spring rate a constant, strictly a SMAC assumption) equated to the ratio of the respective kinetic energy losses by the impacting mass results in

$$\begin{aligned}\text{Net } E/\text{gross } E &= 0.5K\delta_f^2/0.5K\delta^2_{max}\\ &= 0.5M(\dot{\delta}_0^2 - \dot{\delta}_f^2)/0.5M\dot{\delta}_0\\ &= (1 - c)^2 = 1 - 2c + c^2\\ &= 1 - \varepsilon^2;\ \text{whence } \varepsilon = \sqrt{2c - c^2}\end{aligned} \tag{3.4a}$$

It is desired to find ε from the data available for the SMAC equation,

$$c = C_0 - C_1\delta + C_2\delta^2 = C_0\,(1 - \delta/\delta_l)^2 \quad (\delta \le \delta_l)$$

giving c as a function of crush depth with $C_1 = 2\sqrt{C_0C_2}$, as is imposed for the SMAC data. Because typically $C_0 = 0.064$ in SMAC, c^2 is negligible in Equation 3.4a and $\varepsilon = \varepsilon_0(1 - \delta/\delta_l)$ (for $\delta \le \delta_l$), or in the present notation,

$$\varepsilon' = \varepsilon'_0 = (1 - C^{**}/C_f) \quad (C^{**} \le C_f) \tag{3.4b}$$

which is a straight line between the intercepts $\varepsilon'_0 = \sqrt{2C_0}$ and $C_f = \sqrt{C_2/C_0}$. The currently standard SMAC inputs give $\varepsilon'_0 = 0.358$ and $C_f = 36.8$ in., or essentially $\varepsilon = 0.36 - 0.01C^{**}$. Of course, other expressions might be used [Smith and Tsongas *(18*, p. 47)].

For either vehicle the total crush force in the direction of crush, assuming that structural damping forces have subsided during the impact, is

$$\begin{aligned}F_c' &= \Sigma[(w_j \cos\xi_c)\,(A + B(C_{j-1} + C_j)/2\,\cos\xi_c)]\\ &= w_t\,(A\,\cos\xi_c + BC^*)\end{aligned}$$

Incorporating the possible angularity α due to nonisotropic structure, the total intervehicle force is

$$F'_{ri} = F'_{ci}/\cos\alpha_i = w_{ti}(A\,\cos\xi_{ci} + BC_i^*)/\cos\alpha_i \tag{3.5}$$

($i = 1, 2$). The damage-based values for the two vehicles should be in reasonable agreement with each other and with the location-based result from Equation 2.2c. In unusually light or heavy impacts, structural property adjustment by rotation about the 30-mph case as previously noted might substantially improve agreement between the forces. If crush data exist for only one of the vehicles, it is reasonable to reconstruct crush data for the missing vehicle by assuming a matching contour and peak force. The present approximations are not expected to closely evaluate the actual force of impact but rather to assist in refinement of the reconstruction.

The work done in intervehicular scraping will now be considered. Neither CRASH nor (to the author's knowledge) any other reconstruction treatment has considered the work done in scraping (intervehicular sliding), but it is entirely practicable.

Referring to the target vehicle, if the shear force has reached its friction limit ($\xi_r = \arctan\mu$) and scraping has occurred, additional work has been done on that vehicle by the shear component of the intervehicle force ($F_r \sin\xi_r$) moving along the shear surface through a distance s_v. Equivalently, it may be stated that the intervehicle force acts through the sine component of the shear motion plus the cosine component of its crush depth, as already expressed in Equation 4.1a. This expresses all of the potential energy as work done, scraping included.

The kinetic energy already found in Equation 4.2 does all of this work. If the same treatment is applied to the potential energy and if the direction of the force and the ratio of sliding to crushing are constant during the impact, substitution of the force from Equation 4.2a into 4.1a gives

$$\begin{aligned}\Sigma E_p &= \Sigma[E_c + (\sin\xi_r)\textstyle\int \ddot{z}ds_v/\Sigma M^{-1}]\\ &= \Sigma[E_c + (\sin\xi_r)\textstyle\int (d\dot{z}/dt)\,(ds_v/dz)\,dz/\Sigma M^{-1}]\\ &= \Sigma E_c] + (z_s/z_t)\Sigma E_k] = \Sigma E_k]\end{aligned}$$

so that the work done only in crush is

$$\Sigma E_c = (1 - z_s/z_t)\Sigma E_k = (z_c/z_t)\Sigma E_k = \Sigma E_k/R_s \tag{4.1b}$$

with

$$R_s = 1/(1 - z_s/z_t) = z_t/z_c = 1 + z_s/z_c \tag{4.1c}$$

where $z_t = z_s + z_c$; $z_c = \Sigma C' \cos\alpha = \Sigma C'$ if $\alpha = 0$; $z_s = \Sigma s_v \sin|\xi_r| = \Sigma s_v \sin|\xi_c + \alpha| = \Sigma s_v \sin(\arctan\mu) = \mu\Sigma s_v/(1 + \mu^2) \cong \mu\Sigma s_v$ so that R_s is the ratio of the total intervehicle motion to the component due to crush, or 1 plus approximately μ times the ratio of slide distance to crush depth. Equations 4.3 become

$$\begin{aligned}\Delta\dot{z} &= (1 + \varepsilon)\sqrt{2\Sigma R_s M^{-1}\Sigma E_c/(1 - \varepsilon^2)\cos^2\alpha}\\ &= \sqrt{2E^*\Sigma M^{-1}}\end{aligned} \tag{4.3a}$$

where

$$E^* = \Sigma E_c/\cos^2\alpha]R_s(1 + \varepsilon)^2/(1 - \varepsilon^2) \tag{4.3b}$$

is the equivalent energy of deformation, incorporating the multiplying factor for scraping; the obsolescent CRASH2-CRASH3 correction for oblique impact, if desired; and the two rebound correction factors previously discussed.

There must be appreciable vehicle crush to provide a measure of the normal and hence the shear forces. Data collection will now include observation, identification, and measurement of the scrape marks on the surfaces of the vehicles. For each set of simultaneous scrape marks, lest the same distance be counted twice, it is necessary to consider one vehicle as the target vehicle, which provides a relatively flat surface traced by a limited area of the bullet vehicle. The marks on the bullet

vehicle, made at the same time, are not informational. However, there could be another set of marks that occurred before or after the first, in which the vehicle roles are reversed; the scrape process must be visualized carefully.

With damage data from the yaw acceleration found in Equation 4.5b, the yaw rate for each vehicle is inferred:

$$\dot{\psi}_{si} = \int (Fh_i/M_i k_i)dt = -U'_{\Delta i}(h_i/k_i^2) + \dot{\psi}_{oi} \qquad (4.6c)$$

(i = 1, 2), with $\dot{\psi}_{oi}$ normally zero. For each vehicle this provides a second separation yaw rate to compare with the first.

The speed of separation at the crush centroid along the DOPF based on the deformation data is

$$U'_{\Delta s} = \varepsilon U'_{\Delta 1}(\gamma_1^{-1} + R^{-1}\gamma_2^{-1})/(1 + \varepsilon) \qquad (4.6d)$$

This gives values that can exceed 10 mph, much in excess of the hard limit of under 5 mph for an "acceptable" trajectory solution in CRASH3; yet it can also give small values that $U_{\Delta s}$ from Equation 2.6 should not greatly exceed.

Because $U'_{\Delta 1}$ will increase but ε will decrease as the crush increases, $U'_{\Delta s}$ will not vary rapidly with assumed crush depth. It is therefore a fairly reliable value to use in correcting the trajectory data, which obtain the separation velocity only from the difference in the trajectories to rest and could be considerably in error in individual cases.

The following equations provide six additional pairs of quantities of interest and of value in refining the reconstruction:

Variable	*Equation*
Separation yaw rate for Vehicle 1	1.6d, 4.6c (i=1)
Separation yaw rate for Vehicle 2	1.6d, 4.6c (i=2)
Force of impact for Vehicle 1	2.5, 3.5 (i=1)
Force of impact for Vehicle 2	2.5, 3.5 (i=2)
Separation speed at crush centroid	2.6, 4.6d
Coefficient of restitution	2.7, 3.3b

The degree of correlation to be expected between these sets of values will have to be found by experience. But inherently these all serve as validity checks whereby the reconstruction is checked for internal consistency and the bracketing is narrowed.

For purposes of statistical accident data collection, coefficients of restitution for angular (intersection) impacts between two vehicles are available for the first time by using Equation 2.8. The method requires no instrumentation; the result is reliable within some range according to uncertainties in the location data. This suggests application of effort in the statistical collection of empirical coefficients of restitution from real accidents, subject to avoidance of systematic errors in the site exam and the intervehicle coefficient of friction. DOT's continuing interest in occupant injury exposure data requires only a good damage exam, which may be all that is possible by the time DOT investigators arrive, but full reconstructions of intersection impacts, when possible, will give empirical coefficients of restitution and also improve the injury exposure data.

SUMMARY

The validity of CRASH in general has been confirmed, although some details have been revised and some limitations avoided. CRASH has never contained any errors due to omission of angular motion considerations and has never had a true limitation to the assumption of linearity of crash force with deformation.

The original treatment of CRASH gave damage-basis speed change overvaluation in the case of diagonal deformation and undervaluation by omission of rebound velocity and scraping. Overall, CRASH generally undervalues impact speed changes.

The new equations in this paper extend CRASH to give new results: peak impact force, individual speed changes including rebound, individual directions of speed change, individual yaw rates, joint speed of separation at the crush centroid, and joint coefficient of restitution in impact, all (at least in the case of intersection impacts) in pairs of values independently derived from different input data. These are of interest in themselves and allow input data refinement and increased accuracy of reconstruction.

As with the original CRASH programs, these solutions become practicable only when programmed for automated solution, as shown elsewhere *(8)*. Programming in BASIC allows full user review of the programming as well as the physics and algebra of the treatment. Whereas CRASH incomprehensibly treated or invisibly programmed is precarious for forensic or other critical purposes, the present paper provides CRASH techniques in a form acceptable for demanding applications.

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