Characteristics of Metro Networks and Methodology for Their Evaluation

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Presented in this paper are the results of research on metro (rapid transit) networks, focusing on their geometric characteristics. The object is to define the most important measures, indicators, and characteristics of geometric forms that can improve the present predominantly empirical methods used in metro network planning and analysis. Several measures of metro network size and form, including length, number of lines, and stations, which express the extensiveness of the system, are selected; they are also needed for derivations of various indicators. A number of selected indicators are then presented. These represent the most effective tool for network comparison because most of them are independent of network size. Several indicators relating metro network to the city size and population express the degree of adequacy of the network to meet the city's needs. Based on experiences from a number of metro systems, characteristics of different types of lines (radial, diametrical, circumferential, and other) are defined. These allow evaluation of network types, such as radial-circumferential versus grid networks. An analysis of metro networks in 10 different cities is presented to illustrate the applications of theoretical and empirical materials; several basic measures and indicators of these networks have been computed and analyzed. The scope of quantitative elements, analyses of types of lines, and descriptions of networks are limited because of space constraints, but they illustrate the methodology that can be used, in a more comprehensive way, for a number of different analyses of metro networks, their designs, or their extensions.

The planning of metro (rapid transit) networks is usually predominantly empirical. Consideration of local conditions, such as demand characteristics, existence of transportation corridors, requirements for certain station locations, and so on, tend to suppress the analyses of network topology and geometric characteristics. However, even though designers of metro lines and networks may want to use some design guidelines, to perform comparative analyses or to make use of experiences from network operations in other cities, very few of these can be found in the professional literature.

Because of the high cost of metro construction and the permanence of its facilities, it is important to design optimal networks with respect to service for passengers, efficiency of operation, and relationship of the metro system to the city. This is a complex task and it deserves more attention than it has received until now. An attempt is made in this study to provide materials that may assist in the planning and design of metro system networks, lines, and stations.

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PURPOSE, ORGANIZATION, AND SCOPE

Presented in this paper is a systematic set of quantitative elements that defines the network characteristics of metro systems that can be used for their description, evaluation, and comparative analysis. Examples of such evaluations include planning of new or analysis of existing networks, their comparison with networks of other cities, and comparison of alternative network extensions.

The quantitative elements are grouped into five general categories, as follows:

1. Measures of network size and form,
2. Indicators of network topology,
3. Measures of relationship between network and city,
4. Quantity and quality of offered service, and
5. Measures of service use.

The sections covering these five categories are followed by a description of the basic forms of metro lines and their characteristics. Finally, an application of these measures to metro networks in 10 cities is given using many of the elements defined previously.

Many different aspects of metro networks can be analyzed; the emphasis here is on the geometric form and the method of operation of transit lines and networks; that is, on categories 1–3 listed. Categories 4 and 5 are given only as a guideline for analysis focusing on service quality and efficiency of operations.

In this paper, theoretical concepts are selectively used, particularly from graph theory (*1*), focusing on those with practical relevance to actual metro network planning. The paper is also based on experiences from many cities on metro network design, service, and operating characteristics.

As already mentioned, the data needed for network planning and analysis vary with the purpose of the analysis and local conditions in the city studied. The data used for the indicators and analyses presented in this paper are listed in Table 1.

MEASURES AND INDICATORS OF METRO NETWORKS

Network Size and Form

The main elements that express the size and form of a metro network are defined as follows. Corresponding terms from graph theory are given in parentheses.

TABLE 1 BASIC INFORMATION NEEDED FOR METRO NETWORK ANALYSIS

Number	Item	Symbol	Dimension
1	Population of the served area	P	Persons
2	Surface of the served area	S_u	km^2
3	Vehicle-km operated per day	W	veh-km
4	Vehicle or car capacity	C_v	Spaces/veh
5	Vehicles per train [transit unit (TU)]	n_{TU}	veh/TU
6	Operating speed	V_o	km/h
7	Frequency of service	f	TU/h
8	Number of metro passengers per day	P_{ad}	Persons/day
9	Number of metro passengers per year	P_{ay}	Persons/year
10	Average trip length	$\bar{l}_P$	km
11	Number of total transit passengers per day	P_t	Persons/day
12	Number of stations on the network	N	Stations
13	Number of stations with park-and-ride	N_p	Stations
14	Schematic map of network with stations	–	–
15	Number of lines and their lengths	n_l, l_i	–, km

a-1 *Number of stations (nodes) on a line i,* n_i.

a-2 *Number of interstation spacings (arcs) on a line i,* a_i, is related to n as follows:

$$a_i = n_i - 1 \tag{1}$$

a-3 *Length of line i,* l_i.

a-4 *Number of multiple stations (jointly used by two or more lines),* n_m^k, *and lengths of double sections,* l_m^k, in which k designates the number of lines using a station or a spacing. These superscripts must be introduced to avoid double counting in computations of the number of stations, spacings, and origin-destination (OD) pairs. For these computations the number of stations, n_m^k, must be computed as 1 less than the number of lines it serves (i.e., as $k - 1$, while the number of multiple spacings, a_m^k, and their lengths, l_m^k, are used as given). This will be illustrated on cases following *a*-6, *a*-7, and *a*-8.

a-5 *Number of lines in a network,* n_l.

a-6 *Number of stations in a network, N,* is computed as the sum of stations on individual lines minus the multiple stations as follows:

$$N = \sum_{i=1}^{n_i} n_i - \sum_{k=2}^{k_{max}} (k-1) \times n_m^k \tag{2}$$

For example, the network in Figure 1*a* consists of three lines with 8, 9, and 4 stations, respectively:

$$n_m^2 = 4 \text{ and } n_m^3 = 2$$

thus, the total number of stations (single and multiple) in the network is

$$N = 8 + 9 + 4 - (1 \times 4 + 2 \times 2) = 13 \text{ stations}$$

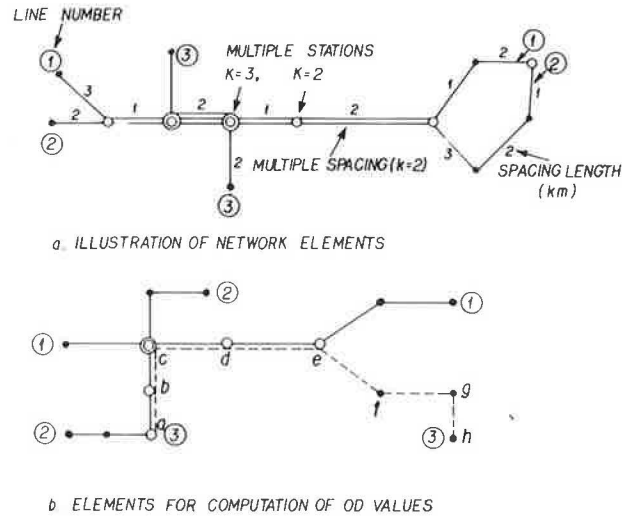


FIGURE 1 Network schematics.

a-7 *Number of station spacings in a network, A,* is computed as the sum of spacings on individual lines minus the multiple spacings, as explained under *a*-4:

$$A = \sum_{i=1}^{n_l} a_i - \sum_{k=2}^{k_{max}} k \times a_m^k \tag{3}$$

This number can also be computed via N:

$$A = N - 1 + C_r \tag{4}$$

where C_r is the number of closed circles in the network. In the case illustrated in Figure 1*a*, the number of spacings (including single and multiple) computed by line is

$$A = 7 + 8 + 3 - (1 \times 3 + 2 \times 1) = 13 \text{ spacings}$$

Alternatively, using the number of stations (Equation 4), the number of spacings is

$$A = 13 - 1 + 1 = 13 \text{ spacings}$$

a-8 *Length of network, L,* is the sum of line lengths minus the multiple spacings:

$$L = \sum_{i=1}^{n_l} l_i - \sum_{k=2}^{k_{max}} (k-1) \times l_m^k \tag{5}$$

Using the example from Figure 1*a*:

$$L = 12 + 14 + 5 - (1 \times 1 + 2 \times 2 + 1 \times 1 + 1 \times 2) = 23 \text{ km}$$

a-9 *Number of circles in a network,* C_r. The number of circles in a network can be computed by Equation 4 as:

$$C_r = A - N + 1 \tag{6}$$

The range of C_r values is $0 < C_r < 2N - 5$.

a-10 *Number of station-to-station travel paths, OD,* consists of direct paths, OD_d, and paths that include one or more transfers, OD_t. In a network with N stations, the number of all possible OD paths is

$$OD = N(N - 1)/2 \tag{7}$$

For a line with i stations, the number of OD paths is

$$OD_i = n_i(n_i - 1)/2 \tag{8}$$

and all of them are direct. The total number of direct paths in a network is equal to the sum of OD paths along each line and along each single branch plus the paths on overlapping lines (i.e., those serving jointly sections of different lines):

$$OD_d = \tfrac{1}{2} \sum_{i=1}^{n_l} n_i(n_i - 1) + \sum_{j=1}^{n_t} n_{mj} \times n_{bj} \tag{9}$$

where

n_m = the number of joint stations that the "double" line shares with the already counted "basic" line,
n_b = the number of stations on the branches (single sections) of that line, and
n_d = the number of lines with joint sections with other lines.

These additional direct OD paths must actually be computed among all line sections that such lines connect. The number of paths that require transfers, OD_t, is computed as the difference between all paths and direct paths:

$$OD_t = OD - OD_d = \tfrac{1}{2}\left[N(N-1) - \sum_{i=1}^{n_l} n_i(n_i - 1)\right] - \sum_{j=1}^{n_t} n_{mj} \times n_{bj} \tag{10}$$

An example for illustration of the computations of OD values is given in Figure 1*b*. The network shown by solid lines consists of two lines with one joint (transfer) station. Line 1 has 6, and Line 2 has 7 stations. To compute OD values, Equations 2, 7, and the first member of Equation 9 are used:

$N = 6 + 7 - 1 \times 1 = 12$ stations

$OD = \tfrac{1}{2} \times 12(12 - 1) = 66$ paths

$OD_d = \tfrac{1}{2}[6(6 - 1) + 7(7 - 1)] = 36$ paths

$OD_t = 66 - 36 = 30$ paths

If Line 3, with 8 stations (shown by dashes), is added to this network, the numbers change as follows:

$N = 6 + 7 + 8 - (4 \times 1 + 1 \times 2) = 15$ stations

$OD = \tfrac{1}{2} \times 15(15 - 1) = 105$ paths

The computation of OD_d now becomes more complex. The first sum in Equation 9 is:

$$(\tfrac{1}{2}) \sum_{i=1}^{n_l} n_i(n_i - 1) = (\tfrac{1}{2}) \times (6 \times 5 + 7 \times 6 + 4 \times 3) = 42 \text{ paths}$$

the last member (4×3) representing the branch section *e–h*. The second sum from Equation 9 must include the additional paths between stations: *a–b* and *d–e* (2×2), *a–d* and *f–h* (4×3); thus,

$$\sum_{j=1}^{n_d} n_{mj} \times n_{bj} = 2 \times 2 + 4 \times 3 = 16 \text{ paths}$$

$OD_d = 42 + 16 = 58$ paths

$OD_t = OD - OD_d = 105 - 58 = 47$ paths

The 10 elements expressing size and form of metro network are listed in Table 2.

Network Topology

Various ratios of the previously defined measures of network size can be used as quantitative indicators of network topology. Those particularly useful in metro network planning and analysis are selected here.

b-1 *Average interstation spacing, S,* can be computed for a network as

$$\bar{S} = \frac{L}{N - n_l + \sum_{k=2}^{k_{max}} n_m^k} = \frac{L}{A} \tag{11}$$

Spacings between stations, $\bar{S}$, are usually selected as a compromise between good area coverage (short spacings) and high operating speed (long spacings). Therefore, longer lines tend to have longer spacings: regional rail networks [e.g., San Francisco Bay Area Rapid Transit (BART), and the Munich S-Bahn] have average spacings of 1000–2500 m, compared with the spacings of 500–800 m on typical urban metro systems such as those in Paris, Philadelphia, and Mexico (*2*).

b-2 *Line overlapping index, λ, is computed as*

$$\lambda = \frac{\sum_{i=1}^{n_l} l_i}{L} = 1 + \frac{\sum_{k=2}^{k_{max}} l_m^k}{L} \tag{12}$$

TABLE 2 METRO NETWORK MEASURES AND INDICATORS

Item code	Definition	Symbol	Equation	Range
Network Size and Form				
a-1	Number of stations on line i	n_i		
a-2	Number of interstation spacings (arcs) on line i	a_i	$n_i - i$	
a-3	Length of line i	ℓ_i		
a-4	Number of multiple stations, spacings and their lengths	n_m^k, a_m^k, ℓ_m^k		
a-5	Number of lines in network	n_ℓ		
a-6	Number of stations in network	N	$\sum_{i=1}^{n_\ell} n_i - \sum_{k=2}^{k_{max}} (k-1) n_m^k$	
a-7	Number of interstation spacings	A	$\sum_{i=1}^{n_\ell} a_i - \sum_{k=2}^{k_{max}} k \cdot a_m^k$	
a-8	Length of network	L	$\sum_{i=1}^{n_\ell} \ell_i - \sum_{k=2}^{k_{max}} (k-1) \ell_m^k$	
a-9	Number of circles	C_r	A-N+1	$0 \le C_r \le 2N-5$
a-10	Number of station-to-station	OD	$(1/2) \cdot N(N-1)$	
	travel paths: total - OD, direct	OD_d	$(1/2) \sum_{i=1}^{n_\ell} n_i \cdot (n_i - 1) + \sum_{j=1}^{n_d} n$	
	OD_d, with transfer - OD_t	OD_t	$OD - OD_d$	
Network Topology				
b-1	Average interstation spacing	$\bar{S}$	$\frac{L}{A}$	
b-2	Line overlapping	λ	$1 + \frac{\sum_{k=2}^{k_{max}} \ell_m^k}{L}$;	$\lambda \ge 1$
b-3	Circle availability	α_c	$\frac{C_r}{2N - 5}$	$0 \le \alpha_c \le 1$
b-4	Network complexity	β	$\frac{A}{N}$	$\beta \ge 0.5$
b-5	Network connectivity	γ	$\frac{A}{3(N-2)}$;	$0.33 \le \gamma \le 1$ for $N < 2$
b-6	Directness of service	δ	$\frac{OD_d}{OD}$;	$\frac{2}{A+1} \le \delta \le 1$

Descriptively, this index is the ratio of the sum of line lengths to the network length. Therefore, a network consisting of independently operated lines (e.g., Leningrad, Mexico, or Toronto) has a value of 1.00. The more the lines are interconnected, sharing common sections with other lines and then branching, the greater is the value of λ.

Independent operation of lines is the simplest method from the operational point of view because there is no diverging or merging of trains and no mutual influence among lines (no transfer of delays). Operation of interconnected services (e.g., trunk lines with branches, overlapping routings among line sections, and so on), although more sensitive operationally, has the advantage that it offers more direct trips (without transfers) and, in some cases, allows a better "fitting" of offered capacity to demand, resulting in higher use of transportation work (train- or space-km).

Frequency is another element of service that is affected by the type of line operation. For a given required capacity of service, determined by passenger demand and level of offered service on a particular line section, an independently operated line has an *n* times greater frequency than that existing if the same section were served by *n* lines branching in different directions so that each passenger traveling beyond the joint section could take only every *n*th train.

b-3 *Circle availability*, α_c, represents the ratio of the number of circles (sections of lines making up the closed loops in the network) to the maximum number of circles that, theoretically, the network with the given number of nodes could have:

$$\alpha_c = \frac{C_r}{2N+5} = \frac{A-N+1}{2N-5} \tag{13}$$

The theoretical range of this indicator is $0 \leq \alpha_c \leq 1$. Open networks with lines radiating from the central trunk lines (e.g., Atlanta or Rome) have $\alpha_c = 0$. The greater α_c is, the more options passengers have to travel through the metro network: the complex Paris Metro network has the largest α_c indicator of all cities, 0.11.

b-4 Network complexity indicator, β, is the ratio of the number of spacings (arcs) and stations (nodes):

$$\beta = A/N;\ \beta \leq 0.5 \tag{14}$$

This indicator reflects complexity in terms of the number of interstations spacings (arcs) related to the stations (nodes) of the network. Its minimum value of 0.5 is obtained on an elementary line with two stations; as the line is extended, adding more stations and spacings, β asymptotically approaches 1. On closed networks with cross connections, β can exceed the value of 1.

b-5 Network connectivity, γ, represents the ratio of the number of arcs existing in a network and the maximum number that could exist for the available number of nodes:

$$\gamma = \frac{A}{3\,(N-2)};\ 0.33\ \gamma \geq 1, \text{ for } N \geq 2 \tag{15}$$

Similarly to the indicator β, the more connections among nodes in the network there are, the greater is the value of γ.

b-6 Directness of service, δ, is the indicator that reflects the proportion of OD paths that can be traveled without transfer:

$$\delta = \frac{\mathrm{OD}_d}{\mathrm{OD}_d + \mathrm{OD}_t} = \frac{OD_d}{\mathrm{OD}};\ 0 \geq \delta \geq 1 \tag{16}$$

For a single line, $\delta = 1$; for more complex networks, δ tends to decrease, but operation of interconnected lines increases it.

For an easy review, these indicators of topology are listed in Table 2 with their symbols, equations, and, where applicable, ranges of value.

In Table 3, eight different types of networks are given with their sizes and indicator values to illustrate their computations and the relative magnitudes of different network topologies. In all eight cases, network length is the same but other elements are changed one by one to show how each one of the changes in measure influences different indicators.

Case 1 is a single line with variable interstation spacing lengths and a longer $\bar{S}$ than the others. Case 2 is similar to Case 1, a single line, but with 12 equal spacings. The difference in *b*-1 also causes the changes in *b*-4 and *b*-5. In Case 3, the same network length is reorganized into three lines with one joint (transfer) station. Compared with Case 2, of all the indicators only *b*-6 changes. Case 4 is of the same network as Case 3, but with lines staggered to divide a single triple station into three double stations; also, a circle is created. Case 5, with four lines intersecting at four points, changes only *b*-6. Cases 6 and 7 have the same network topology but have different line operations: in Case 6 there is one line with a "feeder;" in Case 7 there are 2 lines, each connecting the trunk with a branch. The differences are reflected in indicator λ (*b*-2) and in δ (*b*-6). Finally, Case 8 is of the same network as Case 4 but has line overlaps on the entire network; λ (*b*-2) is doubled and δ (*b*-6) is increased, reflecting a greater portion of travel paths without transfers. It should be noted, however, that this increase in direct service is traded for decreased frequency of service on each line. If one branch requires a service frequency of 12 trains/hr, or headways of 5 min, for example, operation from Case 4 will offer such headways; whereas Case 8 will offer only a 15-min headway for each line.

These cases provide a clear illustration of various influences on indicators. In real-world cases much larger differences exist, so that greater variations among network indicators can be found.

Application to Network Analysis

An example of an application of the measures previously presented to an actual metro system is given as follows. In 1980, the Washington Metropolitan Area Transit Authority (Metro) network consisted of three lines: red, blue, and orange, the latter two with substantial joint section; when completed, the network will consist of five lines.

Using the first three lines (in their final full length) as an example, the question can be asked: What additions to the network will be obtained by the construction of the fourth and fifth lines, which will be operated as the yellow and green lines, with a joint section between the two and a joint section between the blue and yellow lines?

Shown in Figure 2 is the initial network of red, blue, and orange lines, with the additional yellow and green lines indicated by dashes. The elements and indicators (from *a* and *b* categories) that change with this addition (given in Table 4) show that the number of stations increases by 30 percent and the network length increases by 41 percent. Because of the interconnections among lines, however, network complexity increases even more substantially: whereas the old network had no circles, the new one has three and the number of OD pairs increases by 70 percent. Most topology indicators, expressing network complexity (*b*-2 through *b*-5), also increase, reflecting a more complete network with more diversified services.

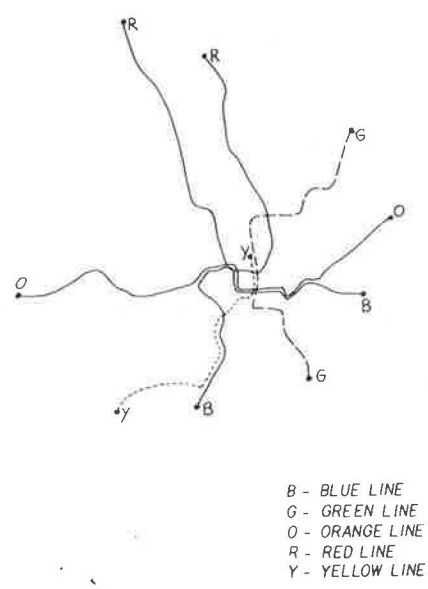


FIGURE 2 Washington Metro network without and with Yellow and Green lines.

TABLE 3 DIFFERENT NETWORK TOPOLOGIES, SHOWING THEIR MEASURES AND INDICATORS

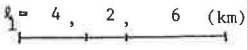
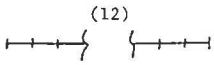
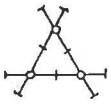
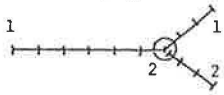
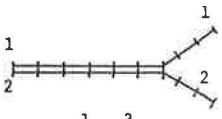
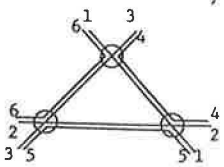
No.	Topology	a-1 n_i	a-2 a_i	a-3 ℓ_i	a-4 n_m^k, a_m^k, ℓ_m^k	a-5 n_ℓ	a-6 N	a-7 A	a-8 L	a-9 C_r	a-10 OD, OD_d, OD_t	b-1 $\bar{S}$	b-2 λ	b-3 α	b-4 β	b-5 γ	b-6 δ
1		4	3	12	0 0 0	1	4	3	12	0	6 6 0	4	1.0	0	0.75	0.50	1.00
2		13	12	12	0 0 0	3	13	12	12	0	78 78 0	1	1.0	0	0.92	0.36	1.00
3		5 5 5	4 4 4	4 4 4	1 0 0	3	13	12	12	0	78 30 48	1	1.0	0	0.92	0.36	0.38
4		5 5 5	4 4 4	4 4 4	3 0 0	3	12	12	12	1	66 30 36	1	1.0	0.05	1.00	0.40	0.45
5		4 4 4 4	3 3 3 3	3 3 3 3	4 0 0	4	12	12	12	1	66 24 42	1	1.0	0.05	1.00	0.40	0.36
6		10 4	9 3	9 3	1 0 0	2	13	12	12	0	78 51 27	1	1.0	0	0.92	0.36	0.65
7		10 10	9 9	9 9	7 6 6	2	13	12	12	0	78 69 9	1	1.5	0	0.92	0.36	0.88
8		5 5 5 5 5 5	4 4 4 4 4 4	4 4 4 4 4 4	18 12 12	6	12	12	12	1	66 45 21	1	2.0	0.05	1.00	0.40	0.68

TABLE 4 WASHINGTON METRO NETWORK MEASURES AND INDICATORS WITHOUT AND WITH THE YELLOW AND GREEN LINES

Item Code	Item	Symbol	Without Yellow and Green Lines	With Yellow and Green Lines	Percent Change
a-5	Number of lines in network	n	3	5	+67
a-6	Number of stations in network	N	63	82	+30
a-7	Number of station spacings in network	A	62	84	+35
a-8	Length of network (km)	L	115	162	+41
a-9	Number of circles	C	0	3	+∞
a-10	Number of station-to-station travel paths (origin-destination)	OD	1,953	3,321	+70
b-1	Average interstation spacing (km)	$\bar{S}$	1.85	1.93	+4
b-2	Line overlapping	λ	1.19	1.24	+4
b-3	Circle availability	α_c	0	0.02	+∞
b-4	Network complexity	β	0.99	1.02	+3
b-5	Network connectivity	γ	0.34	0.35	+3

METRO NETWORK AND THE CITY

Relationship of Metro Network to City

An important aspect of metro network evaluation is its relationship to the city: its size and the number of stations in relation to the city size, population, and the role of the metro among other transportation modes.

Among the numerous measures and indicators, some of which most directly reflect the relationship between a metro network (with the primary emphasis on its geometric form) and the city it serves, several are selected and defined; these are listed in Table 5.

c-1 *Density of Metro Network,* L_a, is the ratio of the network length to the area of the city. This indicator reflects the extensiveness of a network with respect to the area it serves, primarily center city; for regional networks this indicator is sometimes imprecise because of the difficulty in delineating the "served area" of the region. The indicator is defined as

$$L_a = \frac{L}{S_u} \text{ (km/km}^2\text{)} \tag{17}$$

where S_u is the area of the city or of the served area, as applicable.

c-2 *Network extensiveness per population,* L_p, expresses the ratio of network length to the population of the served area:

$$L_p = \frac{L}{P} \text{ (km/persons)} \tag{18}$$

Comparing cities with similar populations, the greater value of L_p indicates a more extensive network and, generally, a more important role in the metro system.

The service that a metro network offers to the urban area and the various forms of access to its stations are more conveniently measured by the three indicators defined as follows; the first one affects access by walking (pedestrian access); the latter two measure the convenience of access to the metro network by street transit and automobile, respectively.

TABLE 5 METRO NETWORK CHARACTERISTICS INDICATORS

Item code	Definition	Symbol	Equation
	RELATIONSHIP TO THE CITY		
c-1	Density of metro network	L_a	$\frac{L}{S_u}$
c-2	Network extensiveness per population (10^6)	L_p	$\frac{L}{P}$
c-3	Area coverage	N_a	$\frac{N\ S_i}{S_u}$
c-4	Street transit integration ratio	η_t	$\frac{n_s^t}{n_s}$
c-5	Auto across integration ratio	n_a	$\frac{Np}{N}$
	SERVICE MEASURES AND UTILIZATION INDICATORS		
d-1	Operating speed weighted by veh-km per time (day)	V_{cv}	$\frac{\sum_{i=1}^{n\ell} W_i \cdot V_{oi}}{\sum_{i=1}^{n\ell} W_i}$
d-2	Frequency of service during peaks weighted by stations	f_w	$\frac{\sum_{i=1}^{n\ell} f_i\ n_i}{\sum_{i=1}^{n\ell} n_i}$
d-3	Highest design line capacity in network	C	$f_{i\ max} \cdot n_{TU}\ C_v$
d-4	Max scheduled line capacity	C_s	Max $f_i \cdot n_{TU} \cdot C_v$
d-5	Space-km offered per day	S_d	$W \cdot C_v$
e-1	Line capacity utilization coef.	η_c	$\frac{C_s}{C}$
e-2	Riding habit (annual trips per capita)	R	$\frac{P_{ay}}{P}$
e-3	Passengers per year per network length	R_L	$\frac{P_{ay}}{L}$
e-4	Passenger-km per day	P_w	$P_{ad} \cdot \bar{\ell}_p$
e-5	Passenger-km per day over space-km per day	$\bar{\alpha}$	$\frac{P_{ad}}{S_d}$
e-6	Metro daily passengers as % of transit daily passengers	P_m	$\frac{P_{ad}}{P_t}$

c-3 Area coverage, N_a, is the percentage of the urban area (S_u) that is within walking distance of metro stations:

$$N_a = \frac{n\,S_i}{S_u} \times 100\ (\%) \tag{19}$$

where S_i is the area around the metro station with a radius of 400 m (sometimes a 500-m radius is used as the standard).

Area coverage is the most important measure of the availability of metro services within the entire served area; this indicator is therefore used extensively in the planning of metro lines and networks.

c-4 Street transit integration ratio, n_t, is the ratio of street transit lines that have transfers to metro network, n_s^t, to all street transit lines, n_s:

$$n^t_s = \frac{n_s^t}{n_s^t} \times 100\ (\%) \tag{20}$$

This indicator expresses the relative geometric and functional role of the metro network within the city's total public transport network.

c-5 Auto access integration ratio, n_a, is the percentage of stations that have park-and-ride (P+R) facilities (N_p as a percentage of N):

$$n_a = \frac{N_p}{N} \times 100\ (\%) \tag{21}$$

Measures of Service and Use

Most analyses of metro networks, even those that focus on geometric characteristics, involve some consideration of overall service offered by the metro system and its use. These analyses are usually rather general and are based on a few global measures and indicators such as those given in Table 5. For more detailed analyses of transit system performance, there is extensive literature available (*3–6*).

Service measures, designated as items *d*-1 to *d*-5 in Table 5, include the most important components of level of service (speed and frequency) and of system performance (design and scheduled capacities, and performed work) (*7*). It is important to use speed weighted by vehicle-km per time, to reflect the service on the entire network. For the same reason, frequency is weighted by stations.

Utilization indicators, given as items *e*-1 to *e*-6, include use of offered services, which influences the economic efficiency of operations and can be strongly linked to the design of metro lines and the topology of its network. Other utilization indicators express intensity of metro system use in absolute terms in relation to total transit use in the city. Both groups are also related to the extensiveness and topology of a metro network and the role it plays in the city.

GEOMETRY OF METRO LINES AND NETWORKS

Types of Lines

Geometric forms and their location in the city give transit lines certain functional and operational characteristics. Although some lines have irregular form, many can be classified into several basic types. The most common types are defined in the following paragraphs; their characteristics, based on theoretical analyses and experiences from many cities, are also briefly outlined [see also further review (*8, 9*)].

- Radial lines, following alignments from center city outward, usually trace the directions of heavy passenger demand, which gradually decreases toward the suburbs. This decreasing demand can be matched either by turning some trains back at an intermediate station or by branching the line into several directions and thus distributing its capacity and increasing area coverage in the suburbs. Because they serve many commuters, radial lines often have sharp peaking of ridership volumes.

The main advantage of radial lines, and the dominant reason for their extensive use in many cities, is that they tend to serve the heaviest travel corridors in the city; their disadvantages are that they often have limited distribution in center city and that their inner terminals may be constrained in space (expensive construction), making their operations difficult.

- Diametrical lines connect two different suburbs and pass through city center. They are often equivalent to two radial lines connected in the center. Because they are connected, diametrical lines do not have the two disadvantages of radial lines previously listed—their terminal operations take place in suburbs.

Diametrical lines should be planned with two major considerations in mind. First, their two parts from center city should have similar maximum passenger volumes to ensure good use of offered capacity; and second, it is desirable that they connect suburbs between which there is demand for travel.

- Tangential lines serve noncentrally oriented travel, usually in very active areas of the "ring" around center city.
- Circumferential lines are similarly located, serving tangential trips but in a circular form. When such lines are closed in a circle, they represent ring or circle lines; these exist in several metro networks with some variation in the methods of train routing: metros in London, Moscow, and Tokyo [Japanese Railways (JR)] have circle lines, whereas those in Paris and Hamburg have two circumferential lines that form a circle.

Circle lines usually play an important role in the metro network. In addition to serving tangential trips they connect radial lines, shortening trips among them; their trips are thus distributed to various points in the city. Because of their multiple purpose, circle lines often have rather even passenger loadings along their length and during different periods of day. This results in high use of capacity and makes their operations economical.

Circle lines have some operational problems. First and most serious is the absence of terminal times, which prevents recovery of delays and reduces their reliability; and second, their speed can be changed only in certain increments because of the fixed ratio between headway and cycle time. For these reasons, some transit operators avoid using ring lines.

- Trunk lines with branches are often used in metro systems and even more commonly in regional rail networks. This type of line, which is functionally effective, has the problem of handling short headways when the branches merge into the joint section. Capacity and service frequency are therefore limited by the operation on the trunk line.

Many metro networks have two branches, which do not have the major problems of short headways, but some have three (San Francisco BART) and four branches (Oslo); whereas regional rail networks have up to six or seven branches, but have longer headways than are typical of metro systems (Munich, Philadelphia).

- Irregular lines are those that do not have any regular geometric form. The most common geometric forms of transit lines, including those already described, are illustrated in Figure 3.

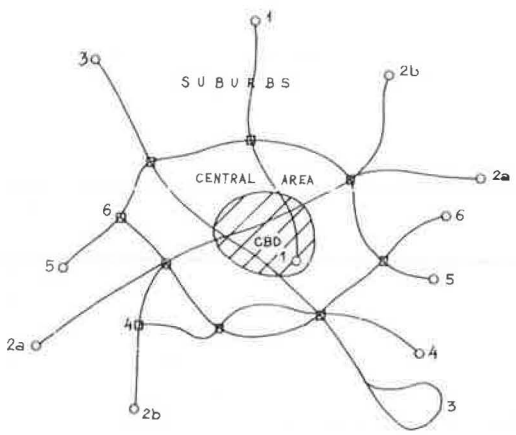


FIGURE 3 Types of transit lines (*8*).

Types of Networks

A number of metro networks can be classified into different geometric forms. These are defined as follows (*8, 9*):

- Radial networks consist of radial and diametrical lines meeting or intersecting in city center. They are sometimes supplemented by a circumferential or ring line. These networks concentrate on center city and tend to have high peaks because of the large number of commuters they carry. Examples of radial rail transit networks are found in Moscow, San Francisco (BART and Muni Metro), and Munich (S-Bahn).
- Rectangular or grid networks consist of parallel and rectangular lines, usually following a grid street pattern (Toronto, Mexico). These networks provide better area coverage and less focus on a single point than radial networks; on the other hand, much of the radial travel is rather indirect.
- Modified grid is the network form in which lines of different types (radial, tangential, circumferential, branches, and so on) are used in irregular form to obtain an evenly distributed network providing extensive central city area coverage. The most typical examples of this network are the Paris Metro, Tokyo rapid transit, and the Munich U-Bahn networks.

EXAMPLES OF NETWORK EVALUATIONS

Comparison of Quantitative Measures for Different Cities

Metro networks in several cities are briefly analyzed here to illustrate the use of the quantitative measures and characteristics of geometric forms presented.

Ten rather complete metro networks have been selected and the items from categories *a*, *b*, and *c* most relevant for their comparison have been computed. These values, presented in Table 6, must be considered with caution because a number of relevant local conditions could not be included in it. For example, in London the Underground is the only network in the central area, whereas in Paris the Metro is supplemented by three major regional lines; in Hamburg the U-Bahn is nearly duplicated by an urban S-Bahn system, and in Tokyo the case is similar: although the rapid transit (metro) system in Tokyo carries close to 6 million passengers a day, total ridership on all rail systems in the region is about 26 million.

Some arbitrary decisions had to be made in defining the networks and line lengths. In Hamburg, for example, the two branches on the north are not considered to be separate lines, whereas in San Francisco all branches are considered to be separate lines. In London the definition of lines and branches is particularly complex (e.g., Should the Metropolitan Line be considered a set of five lines? In this study it is considered one).

In spite of these difficulties, Table 6 yields some interesting observations about the relationship among these quantitative values and the characteristics of different networks. A few are discussed in the following paragraphs.

Network size is expressed most directly by the number of lines (*a*-5) and network length (*a*-8): Paris, Tokyo, and London are leading examples. Extensiveness and diversity of trip opportunities are mostly related to the number of stations (*a*-6), whereas the density of area coverage can be observed through the density and area coverage indices (*c*-2 and *c*-3): Paris has the fourth largest network, but by density it stands well ahead of all the others.

Diversity of connections in the network is expressed by several items: number of circles (*a*-9) and circle availability indicator α_c (*b*-3), and by complexity and connectivity indicators (*b*-4 and *b*-5): The Paris network leads again, followed by the London network. For smaller and simpler networks (e.g., Baltimore, Lisbon, and Rome) these indicators would be smaller.

The San Francisco network, although rather extensive, has no circles (*a*-9, *b*-3). This is typical for regional networks that offer primarily long trips into the city. Line overlapping coefficient λ (*b*-2), however, shows that the San Francisco network offers the most direct trips by extensive overlapping; the Munich and Washington, D.C., networks, with branching diametrical lines, follow in this respect.

Average station spacings (*b*-1), showing the urban or regional character of the networks, vary from 0.66 km in Paris to 3.47 km in San Francisco. They tend to be greater for regional rail and for U.S. cities because of their large spread and automobile access.

TABLE 6 MEASURES AND INDICATORS OF SELECTED METRO NETWORKS

Item code	Symbol	CITY									
		Chicago	Hamburg	London	Milan	Munich	Osaka	Paris	S.Franc.	Tokyo	Washington
a-5	n_ℓ	7	3	9	2	5	6	15	4	11	4
a-6	N	144	80	247	51	48	77	240	34	173	61
a-7	A	144	85	274	51	49	84	292	33	181	62
a-8	L	161	89	386	40	41	94	158	114	197	113
a-9	C_r	1	6	28	1	2	8	53	0	9	2
a-10	OD	10296	3160	30381	1275	1128	2926	28680	561	14878	1830
b-1	$\bar{S}$	1.12	1.05	1.41	0.78	0.83	1.22	0.66	3.47	1.09	1.82
b-2	λ	1.11	1.00	1.01	1.00	1.48	1.00	1.03	1.86	1.10	1.24
b-3	α_c	0.01	0.04	0.10	0.01	0.02	0.05	0.11	0	0.03	0.02
b-4	β	1.00	1.06	1.11	1.00	1.02	1.09	1.22	0.97	1.05	1.02
b-5	γ	0.34	0.36	0.37	0.35	0.35	0.37	0.41	0.34	0.35	0.39
c-1	L_a	0.28	0.12	0.24	0.13	0.13	0.45	1.50	0.06	0.34	0.71
c-2	L_p	53.60	54.60	57.60	24.90	31.70	37.20	68.70	35.90	23.60	45.20
c-3	N_a	0.13	0.05	0.08	0.16	0.08	0.18	1.15	0.01	0.15	0.19

The New York City rapid transit system is conspicuously missing from this analysis. The main reason for this absence is because of its great complexity and the difficulty in analyzing its numerous interconnections, overlapping lines, treatment of express-local services, multiple track lines, and so on. A special study of the city's network geometry would be interesting, but it could not be included in this work.

Overall Evaluations of Several Networks

Five of the cities listed in Table 6 and their types of lines and networks are discussed as follows:

1. London Underground: The oldest metro system in the world, it is also the most extensive. It has radial lines extending far into the suburbs, reaching areas served by the British Railways lines. Thus it has an urban or regional character.

The network consists of nine separate lines, but most of them have a number of branches. Duplication of some lines, with the Circle Line sharing tracks with two other lines on its entire length, further increases network connectivity. The Circle Line plays a major role in connecting all metro lines as well as most British Railways terminals in the city.

Because of its long interstation spacings, the London Underground does not provide a very dense area coverage even in the central city, leaving the considerable task of serving local travel to buses.

2. Munich U-Bahn: Like the Washington Metro, it is still under construction. When completed, it will serve a smaller area than that served by the Washington Metro because its region is already covered by the extensive S-Bahn network. Thus the U-Bahn is limited to central city but designed to give a rather complete area coverage. The network consists of three diametrical lines, each (with one exception) having two branches on each side so that there is a total of 11 branches. The interconnected operation of lines and the rather short interstation spacings are aimed at offering convenient routing for many OD paths.

3. Paris Metro: This network consists of 15 lines, which include diametrical, radial, tangential, and circumferential examples, all interconnected in a "modified grid"—a dense network with complete area coverage and good service for OD paths among virtually all points in the central city. One or two transfers are often required because most lines are operated independently.

4. San Francisco BART: Although considered to be rapid transit, this is a regional system. In addition to the indicators given, data not presented in this study show that BART has a relatively low number of passengers per km of line but a long average trip length, which brings up the issue of its passenger-km/space-km ratio.

The San Francisco BART, by its topology, represents a set of joint diametrical lines which, truncated on their west side when the plans for its two western branches (San Mateo and Marin Counties) were dropped from the plan in the 1950s, are unbalanced toward the east; there it has three branches. Covergence of these three lines and the superposition of the tangential Richmond–Fremont line require a sophisticated control of train operations—which BART has.

5. Washington Metro: This system is being built relatively late with the task of serving both the city and its region. It has three diametrical trunk alignments in the center, used by 5 lines that then radiate as nine branches far into the suburbs. Although center-city coverage and radial lines serving the suburbs do not offer such extensive area coverage as do some older

systems (rapid transit and regional rail networks in New York or Chicago, or the regional rail system in Philadelphia, for example), the Washington Metro will provide (by 1993) a relatively extensive service with a total length of only 162 km serving a very large region.

The metro networks from 10 cities, including those discussed previously, are listed with their basic characteristics in Table 7.

This review of the basic characteristics of metro networks shows that (a) the network geometry, type of operation (independent versus interconnected lines), interstation spacings, and other design elements are related to the role the metro system plays in the city; (b) urban networks differ from regional ones; and (c) the backbone network relies heavily on street transit and represents the skeleton of the transit network, whereas metro as the basic system carries most of the trips itself without the extensive support of street transit lines.

Selection of Evaluation Items for Specific Analysis

To further illustrate the potential use of the materials presented for the planning and design of metro networks, several typical applications are defined as follows:

1. Evaluation of an existing network: comparison with other peer cities to estimate its adequacy;
2. Change from trunk-feeder into trunk-branch operation (or vice versa): evaluation of the impact on services and operations;
3. Addition of a new line to the network: estimation of its impact;
4. Selection from among several alternative network extensions;
5. Planning a new metro network.

Table 8 lists the most useful quantitative and other items for each one of these types of analyses.

SUMMARY AND CONCLUSIONS

The geometric form of metro (rapid transit) networks and lines can have a major impact on services for passengers and the efficiency of the system's operation (*10, 11*). It is therefore important to base network design on analyses of different geometric alternatives, to compare features with those of other (preferably similar) cities and to use their operational experiences.

Although there are almost 80 metro systems in the world today (*12, 13*), research and literature on the geometry of their networks is limited and planning of new systems is often performed empirically.

In an attempt to advance the knowledge and understanding of metro networks, a set of items is presented in this paper that can be used in network design. First, the measures of network size and form that are particularly relevant to its geometry are

TABLE 7 METRO NETWORKS OF FIVE CITIES AND THEIR SELECTED CHARACTERISTICS

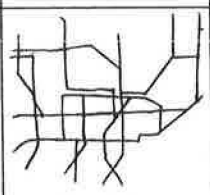
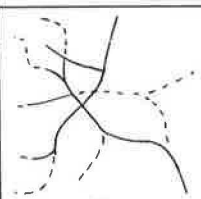
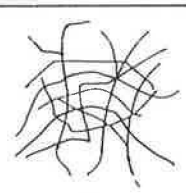
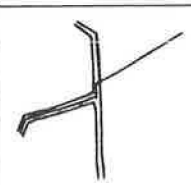
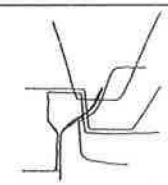
City / Characteristics	London	Munich	Paris	San Francisco	Washington
Network sketch (Not to scale)					
Urban/regional (U) (R)	U - U/R Also large regional	U Also large regional	U Also large regional	(U) R	U/R Also limited regional
Role in the city relation to other modes	All purpose, integrated with regional	All purpose, integrated with tram, bus and regional	All purpose, integrated with regional	Regional, commuting substantial integrated with LRT, bus, P + R	All purpose, integrated with bus, P + R
Extensiveness/ completeness	Very extensive, served area very large	Medium extensiveness, good coverage	Very extensive, total center city area coverage	Connects city to major towns in eastern suburbs	Moderate extensiveness, medium central city coverage
Geometric type	Diametrical lines, circle line	Diametrical lines in a modified grid	Modified grid with many different types of lines	Diametrical lines unbalanced to the east	Diametrical trunk lines with staggered intersections
Branches	Many	Two per line at each end	Very few	Three major branches	Two per line at each end typical
Indep./interconn. operation	A number of lines overlap	Trunk served by two lines	Nearly all independent	High degree of overlap	Trunk served by two lines

TABLE 8 ITEMS TO BE USED FOR DIFFERENT TYPES OF NETWORK ANALYSIS

Item	Symbol	Evaluation of an Existing Network	Change from Trunk-Feeder to Trunk-Branch	Adding a New Line to Network	Selecting from Among Several Alternatives	Planning a New Network
a-5	n	x		x	x	x
a-6	N	x		x	x	x
a-7	A	x		x	x	x
a-8	L	x		x	x	x
a-9	C	x		x	x	x
a-10	OD, OD_d, OD_t	x		x		x
b-1	$\overline{S}$	x		x	x	x
b-2	λ	x	x	x	x	x
b-3	α_c	x		x		x
b-4	β	x		x		x
b-5	λ	x		x	x	x
b-6	δ	x	x	x	x	x
c-1	L_a	x				x
c-2	L_p	x				x
c-3	N_a	x		x	x	x
c-4	η_r	x		x	x	x
c-5	η_a	x		x	x	x
–	Analyze topology	x	x	x	x	x
–	Line geometry	x		x	x	x

presented; then several concepts of graph theory are used to develop a set of network indicators that reflect its complexity, type of operation (relationship among lines), and form. The third group of items includes those relating a network to the city or area it serves; a set of performance and use indicators are also included.

Although it is important to perform quantitative analyses in planning networks and evaluating their alternatives, these must be complemented by other factors such as the designer's experience, knowledge of various network features, and creative imagination. To assist the designers in this respect, a brief review of the characteristics of several basic types of metro lines and networks is presented.

To illustrate the application of the selected theoretical and empirical materials to metro network analysis, indicators have been computed and reviewed for metro networks in several cities and their geometric characteristics briefly described.

The problem of metro network design is so complex that a single paper can only present the basic concepts, their characteristics, and examples of analyses. However, it is expected that the materials presented here will serve as helpful tools for planning and analysis, the content and scope of which may vary depending on specific purpose.

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