

Strategic Model for Operator Work-Force Planning in the Transit Industry

MARK D. HICKMAN, HARIS N. KOUTSOPOULOS, AND NIGEL H. M. WILSON

A model for analyzing and providing input to the determination of manpower levels within the transportation department of a transit agency is presented in this paper. Specifically, the model determines the hiring patterns, vacation allocation over the year, and staffing levels that minimize expected transportation manpower costs including wages, overtime pay, guarantee pay, and fringe benefits. A minimum cost solution is obtained subject to a set of constraints that may reflect transit authority policies such as minimum and maximum allocation of vacations to any period, minimum hiring frequencies, minimum and maximum hiring levels per period, and maximum permitted levels of overtime per period. The estimate of expected overtime recognizes inevitable inefficiency in matching available cover manpower with required open work each day, as well as the overtime required because inadequate manpower may be available. A case study, loosely based on the bus component of the Massachusetts Bay Transportation Authority system, is presented to show how the model works, and is used to explore the sensitivity of the total costs to the various constraints. It is concluded that the model can be a useful tool in the determination of efficient decisions with respect to hiring and vacation scheduling as well as the overall level of transportation manpower.

In this paper several interrelated strategic questions for the transit industry are addressed. Specifically a methodology for determining optimal manpower levels for transportation operations is proposed that also deals with the related problems of hiring frequency and levels and the allocation of vacation liability across the year. While the focus of this paper is the strategic manpower planning process, it is necessary to place this problem in the overall context of the entire operator work-force planning problem since the effectiveness of a particular work-force size will depend on the techniques used for managing manpower on a day-to-day basis. After a review of prior work on operator work-force planning, the overall framework used in this analysis is presented. This is followed by the problem description and methodology used for its solution. A detailed case study, which is loosely based on the bus operations of the Massachusetts Bay Transportation Authority (MBTA), is presented that shows the application of the methodology to a large bus system.

PRIOR WORK

In the past five years increasing attention has been paid to the transit work-force planning problem because of the recognition

that labor productivity is a key to improving the economic prospects of the industry. In terms of the overall staffing levels, Porter et al. (1) provided a detailed description of how three U.S. transit authorities managed their manpower levels. Typically, rules of thumb are used for determining total operator requirements based on scheduled requirements. A factor is applied to schedule requirements to determine total staff levels reflecting the effects of operator absenteeism, uncertainty in service requirements, attrition, promotion, and vacation requirements. Perin (2) showed that total operating costs were quite sensitive to this factor and Perry and Long (3) confirmed that management of the available work force is governed largely by judgment and experience with little analytic basis.

MacDorman (4-7) has done very significant analytic research on the problem of extraboard sizing and work-force planning. MacDorman's initial work on extraboard sizing provided the impetus for this work by identifying the major cost factors and showing how extraboard size could be determined. The procedures developed by MacDorman have been implemented on a microcomputer-based model (7) to be used by transit agencies to address vehicle-operator work-force requirements. The model takes into consideration the amount of scheduled work, the magnitude and daily variability of open work, operator compensation, and work rules.

Koutsopoulos and Wilson (8) presented a general analytic framework for the transit industry operator work-force planning problem and this framework will be summarized in the next section since it is the basis for the strategic model developed in this paper.

FRAMEWORK

Koutsopoulos and Wilson (8) proposed a work-force planning framework that decomposes the overall problem into three levels, defined as follows:

- *Strategic.* At this level, decisions are made on overall work-force size, hiring frequency and levels, and vacation allocation across the planning period. A logical period for analysis at this level would be 1 year, although longer or shorter periods may also be appropriate.

- *Tactical.* At this level the extra manpower available beyond the schedule requirements is allocated by garage and assigned to specific days of the week. This then establishes the appropriate size of extraboard for each day of the week for each garage for a particular timetable (the period within which these decisions cannot be easily changed).

- *Operational.* At this level the available extraboard personnel on any given day are assigned specific times of day to start their work duties.

While these three levels can be distinguished in terms of the decisions required and the scope of the problem, they are also clearly, and strongly, interrelated as shown in Figure 1. Consider first the arrows going down the hierarchy from the strategic level to the tactical level and from there to the operational level. At each level the solution is constrained by the decisions made at the higher levels. So, for instance, at the tactical level the extraboard allocation is constrained by the total manpower available during that period, which has been determined at the strategic level.

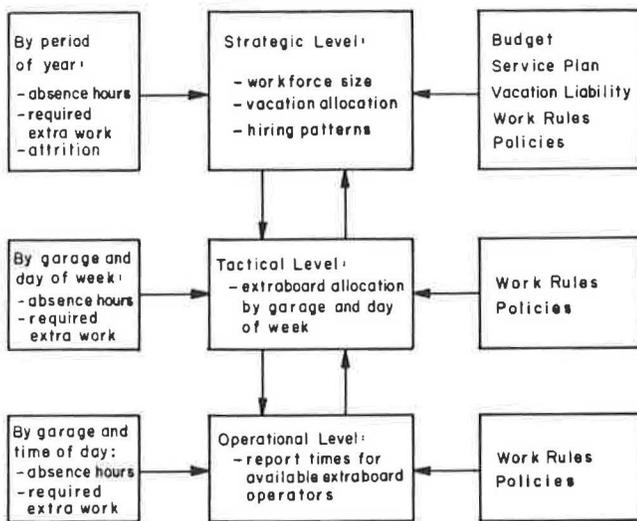


FIGURE 1 Framework for work-force planning.

Conversely, the arrows passing up the hierarchy carry information on the lower-level process which affects the higher-level decisions. For example, in order to make sound decisions on the work-force size at the strategic level, it is necessary to estimate the total cost, including the overtime cost, associated with a particular manpower level. This can only be determined by understanding the processes at the tactical and operational levels. Thus these arrows represent overtime relationships that are derived from analysis of the lower-level processes.

It is important to note that at each level either an existing process can be represented or a model can be proposed to improve decision making at that level. In this paper, the strategic level will be analyzed with the objective of minimizing total cost but assuming that existing decision-making processes are maintained at the tactical and operational levels. To the extent that existing tactical and operational level processes are not fully effective, this strategic level model will not produce a true minimum-cost solution. The strategic level optimization is conditional on the lower-level decision-making process being incorporated. For a fully effective work-force planning and management process, optimization models must be incorporated at each level (8).

PROBLEM DESCRIPTION

At the strategic level of the work-force planning process, the most critical decisions are made, including overall staffing levels, hiring patterns, and vacation scheduling over the planning period. These decisions are interrelated; so they are analyzed simultaneously in the proposed model taking into account the uncertainty with respect to operator availability caused by absenteeism and attrition, and with respect to work requirements caused by service adjustments and extra work assignments.

The obvious, direct application of the model is in assisting in planning decisions, such as budgeting, sizing the work force, setting hiring levels, and allocating vacations. However, the model, because of its generality, can also be valuable in analyzing various policy questions that may be of interest to management. For example, an agency may be used to hiring on a quarterly basis but is considering moving to a monthly hiring cycle. What will the implications be? Another agency may be under conflicting pressures either to maintain a constant hiring level per hiring cycle or to maintain a constant work-force size. The model could be used to predict the impacts of these alternative policies and what each policy would cost above the most cost-effective strategy.

Various alternatives of how the work of vacationing employees is covered can also be evaluated. Frequently, vacation relief assignments require each employee selecting vacation relief to fill in for the scheduled activities of different vacationing employees throughout the timetable. In this case a constant number of vacation spots will be available each week of a timetable. In other agencies, the work open due to vacations is covered directly from the extraboard so that the number of vacation slots can vary from week to week. The effect on operating costs of allowing a portion of vacation liability to be taken as single days, rather than as one-week blocks, can also be assessed.

Finally, the model would be used before each hiring decision so that the decision maker may include up-to-date information on attrition and refine the remaining inputs to reduce the uncertainty in the model. This use of the strategic model also emphasizes the need for work-force planning to be a continuous process, in which forecasts are checked against reality and the models and their assumptions are continually evaluated and updated.

While the formulation of the model assumes a 1-year planning horizon, it could easily be extended to accommodate a longer planning horizon and so could be used in developing 5-year service plans and budgets. In this case the random variable representing work requirements becomes the dominant one.

Critical input to the strategic level model includes

- Schedule requirements by timetable,
- Total vacation requirements by job classification,
- Expected attrition rates,
- Movement between job classifications and promotion policies,
- Frequency of hiring,
- Absence by week of year,

- Mean required extra service by type and level of predictability,
- Overtime premiums and availability of employees to work overtime,
- Fringe benefits by job classification, and
- Manpower-related service reliability objectives.

Given the above input, the model simultaneously determines the least cost values for

- Expected work-force size and, consequently, number of extraboard operators, for each job classification for each subperiod in the planning period. The extraboard is the basic mechanism within transit agencies for maintaining service reliability in the face of absenteeism and uncertain work requirements and thus is the heart of the model. Extraboard design involves determining how many employees are appropriate for this cover function for each subperiod and by job classification for each garage (or rail line) in the system.
- Hiring levels for each hiring cycle and job classification. The model can be used to evaluate the various hiring alternatives the agency may have (e.g., direct hiring and promotion from part-time to full-time status). An interesting feedback relationship which could be incorporated into the model is the increased work required immediately after hiring due to the training function.
- Allocation of vacation liability over the planning period. The objective is to exploit any seasonal variation in both the service requirements and absenteeism and to provide operators with their preferred vacation times. More operators will be given vacations during the low-requirement weeks of the year than in the peak periods. The summer timetable, for example, may require fewer operators because of reduced ridership associated with vacations among the general public and lack of school trips, and thus more operators can be scheduled for vacations in the summer. The summer is also the preferred time for many operators to take vacations.

PROBLEM FORMULATION

The strategic model has been formulated as a constrained optimization problem with minimization of total expected work-force costs as the objective. Total work-force costs consist of the following components: overtime pay, regular wages, and fringe benefit costs. While the determination of regular operator costs (both wages and fringe benefits) is straightforward, the estimation of expected overtime is a difficult task. The difficulty arises from the fact that a certain percentage of open work is completely unpredictable. Consequently overtime is not only a function of extraboard size and the distribution of open work but also of the rate of use for extraboard operators, as indicated by the feedback effects from the lower-level models in Figure 1.

Therefore, an accurate estimate of expected overtime should include the feedback effects due to unpredictability of open work and its impact on the use of extraboard operators. Since direct incorporation of all the factors that affect overtime in a single analytical model is very difficult, a semiempirical model has been developed that adequately accounts for these considerations. The proposed model consists of two terms: a term analytically developed which will be referred to as “regular

overtime,” and an empirical term which will be referred to as “slop overtime.”

Regular overtime is the overtime which would be expected under ideal conditions (i.e., the extraboard is fully used to meet the work requirements, and only the excess work, if any, is covered by regular overtime). Under these conditions on any given day, either

- There is more cover available than work required, in which case there is no overtime and there is some unassigned cover time (sometimes referred to as “guarantee”), or
- There is more required work than cover available, in which case there is overtime (or missed trips if no manpower is available to work the overtime) and no unassigned cover (or guarantee) time.

The above situation is illustrated in Figure 2. When the amount of cover available (X) is less than the amount of required work (E), then overtime will be required as indicated by the straight line AO . Overtime is zero when $X - E \geq 0$. Observed that under these conditions only overtime or unassigned cover can occur on any given day, not both.

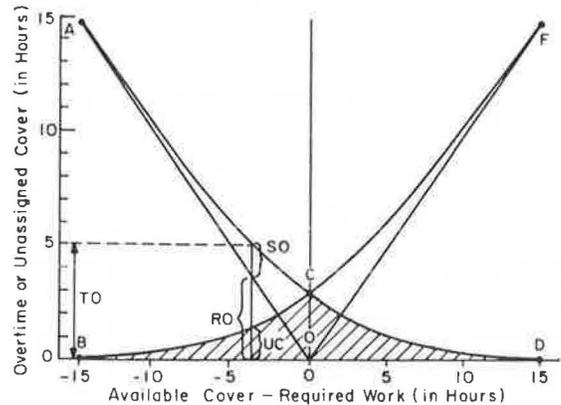


FIGURE 2 Overtime and unassigned cover.

Defining $f(E)$ as the probability density function of the amount of open work per day, the expected regular overtime, O_r , is given by

$$E(O_r) = \int_X^\infty (E - X)f(E)dE$$

Assuming that daily open work is normally distributed with mean μ and standard deviation σ , the expected daily overtime becomes

$$E(O_r) = \sigma \phi\left(\frac{\mu - X}{\sigma}\right) + (\mu - X)\Phi\left(\frac{\mu - X}{\sigma}\right)$$

where ϕ and Φ are the standard normal probability density function and cumulative density function, respectively.

Over a 4-week period, consisting of M days, the expected overtime is simply $ME(O_r)$ or, using period mean μ_i , and standard deviation σ_i ,

$$\mu_i = M\mu \quad \sigma_i = (M)^{1/2} \sigma \quad X_i = MX$$

and the expected overtime for a period is

$$E(O_r) = (M)^{1/2} \sigma_i \Phi \left[\frac{\mu_i - X_i}{(M)^{1/2} \sigma_i} \right] + (\mu_i - X_i) \Phi \left[\frac{\mu_i - X_i}{(M)^{1/2} \sigma_i} \right]$$

Since some open work is completely unpredictable, overtime may exist even if the total number of open runs is equal to the number of available extraboard operators; consequently, the above model will underestimate the actual overtime. For unexpected absences, the report times will match the starting times of open runs only by chance. Even if the available cover is optimally allocated over the day, because the actual times of occurrence of required work are uncertain, it is quite likely that on the same day both overtime will be needed to cover some open work and extraboard operators will not be fully used. Slop overtime captures this inevitable imperfection in assigning cover personnel to open work and approximates the difference between the actual and regular overtime. Clearly, the amount of slop overtime will depend on the work rules applying to the extraboard and on the effectiveness of the report time-setting process. However, in general, one would expect slop overtime to be present and its importance needs to be assessed.

The marginal use of each additional cover operator is a decreasing function of the number of operators. Therefore, referring back to Figure 2, overtime and unassigned cover are represented not by *AOD* and *BOF*, but rather by *ACD* and *BCF*, and the total overtime is equal to the regular overtime *RO* increased by the amount *SO* (slop overtime). When *X* is less than *E* the amount of overtime is greater than the unassigned cover, with the opposite true when *X* is greater than *E*. The two curves (overtime and unassigned cover) intersect at *C*, where *X* = *E*. The following two relationships hold, between slop overtime (*SO*), unassigned cover (*UC*), and total overtime (*TO*).

1. When *X* ≤ *E*: From Figure 2 it is clear that *TO* = *RO* + *SO*, but the total overtime is also given by *TO* = *E* - *X* + *UC*. Therefore, *RO* + *SO* = *E* - *X* + *UC*. Since by definition *RO* = *E* - *X*, then, *UC* = *SO*. This result also demonstrates that the two curves intersect at *E* = *X*.

2. When *X* > *E*: In this case *RO* = 0 and *TO* = *SO*. Combining the two cases, *SO* = min {*UC*, *TO*}.

Furthermore, in Figure 2 (since *SO* = *UC* when *X* < *E*), total slop overtime is represented by the shaded area *BCD*, which is approximately triangular.

This concept of slop overtime is illustrated by the following example. In a given day there are 72 hr of cover available at a garage but absences and extra work require 77 hr of cover time. The regular overtime associated with this situation is 5 hr, based on the simple assumption that the available cover is fully used. However, in practice the actual overtime may be 9 hr and there may be 4 hr of unassigned cover. The associated slop overtime is (9 - 5) = 4 hr and is equal to the amount of unassigned cover which could have been used, in an ideal situation, to reduce the observed overtime. Now suppose on another day when there are 67 hr of available cover and 60 hr of required work that the actual overtime is 2 hr and there are 9 hr of unassigned cover. In this case the regular overtime is 0 while the slop overtime is equal to 2 hr. Thus, slop overtime is defined as the minimum of unassigned cover and overtime, and

it measures the inefficiency (or slop) in the assignment of report times to extraboard operators.

The slop overtime function can be derived either theoretically or empirically, depending on whether the desire is to model an optimal operational-level decision-making process or the existing one. In this case, the concern is with the existing operational-level process and so actual data are used. Empirical studies on the MBTA bus system confirm the qualitative result obtained above—that the amount of slop follows an approximately triangular distribution around the point where the available cover (*X*) is equal to the required extra work (*E*) with the greatest amount of slop occurring when *X* = *E*.

Accordingly, the following general model for predicting slop overtime is proposed:

$$O_s = \begin{cases} K[b - |E - X|] & \text{if } |E - X| \leq b \\ 0 & \text{otherwise} \end{cases}$$

Parameters *b* and *K* are empirically determined for the agency (and each garage) being studied. For a triangular distribution centered around the point *X* = *E*, the slop function goes from -*b* to +*b*, and has slope *K* on the left-hand side of the distribution. Thus, at *X* - *E* = -*b* and *X* - *E* = +*b*, the slop overtime is 0, while the maximum slop overtime is *K* · *b* at the point *X* - *E* = 0.

The advantages of the above model for predicting slop overtime, besides its simplicity, are that it can be calibrated for each agency and it takes into account the use of extraboard operators. The factors affecting overtime, which are specific to each agency, are incorporated in the calibrated values of the parameters *K* and *b*.

Thus the expected daily slop overtime is

$$E(O_s) = \int_{X-b}^X K(E - X + b)f(E)dE - \int_X^{X+b} K(E - X - b)f(E)dE$$

Assuming that *E* is normally distributed with mean μ and standard deviation σ , the expected daily slop overtime becomes

$$E(O_s) = K\sigma \left[\Phi \left(\frac{\mu - X - b}{\sigma} \right) + \Phi \left(\frac{\mu - X + b}{\sigma} \right) - 2\Phi \left(\frac{\mu - X}{\sigma} \right) \right] + K \left[(\mu - X - b)\Phi \left(\frac{\mu - X - b}{\sigma} \right) + (\mu - X + b)\Phi \left(\frac{\mu - X + b}{\sigma} \right) - 2(\mu - X)\Phi \left(\frac{\mu - X}{\sigma} \right) \right]$$

This can simply be multiplied by M to estimate the expected slop overtime for the period which can be rewritten using the period mean μ_i and standard deviation σ_i as

$$E(O_s) = K(M)^{1/2} \sigma_i \left\{ \phi \left[\frac{\mu_i - X_i - Mb}{(M)^{1/2} \sigma_i} \right] + \phi \left[\frac{\mu_i - X_i + Mb}{(M)^{1/2} \sigma_i} \right] - 2\phi \left[\frac{\mu_i - X_i}{(M)^{1/2} \sigma_i} \right] \right\} + K \left\{ (\mu_i - X_i - Mb) \Phi \left[\frac{\mu_i - x_i - Mb}{(M)^{1/2} \sigma_i} \right] + (\mu_i - X_i + Mb) \Phi \left[\frac{\mu_i - X_i + Mb}{(M)^{1/2} \sigma_i} \right] - 2(\mu_i - x_i) \Phi \left[\frac{\mu_i - x_i}{(M)^{1/2} \sigma_i} \right] \right\}$$

The relationships of regular and slop overtime to the total expected overtime are illustrated in Figure 3, which assumes that E is normally distributed with a standard deviation of 1,400 hr per 4-week period. For this example, the values of b and K were estimated as 296 and 0.4, respectively, based on MBTA data. The expected slop overtime is described by the bell-shaped curve centered around the point where manpower surplus equals 0 (i.e., $X = E$).

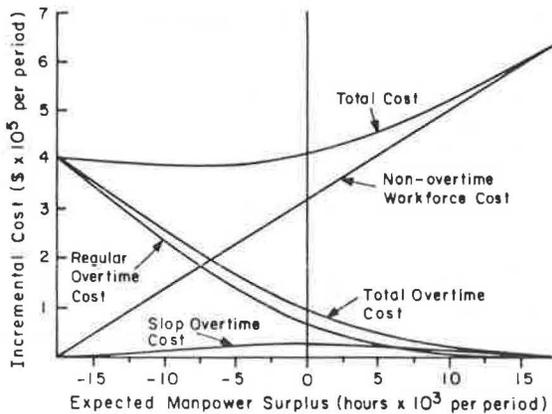


FIGURE 3 Total work-force costs.

Figure 3 also shows total work-force costs, in the vicinity of $X = E$. While the total cost curve may have more than one local minimum, depending on the values of the various parameters, it is expected that the global minimum will almost always lie to the left of E (i.e., $X_{opt} < E$) since the cost of overtime is generally not much larger than the average hourly work-force cost (wage plus fringe benefits) and the marginal use of each additional extraboard operator is declining.

Using the above models for predicting regular and slop overtime, the following formulation of the strategic model is proposed:

Minimize

$$N \cdot cf \cdot yf + N \cdot cp \cdot yp + \sum_{i=1}^N (N + 1 - i) \cdot (cf \cdot hirf_i + cp \cdot hirp_i) + co \sum_{i=1}^N [E(O_{ri}) + E(O_{si})]$$

subject to

$$xf_i = FT_0 + yf + \sum_{j=1}^i (hirf_j - rf_j) - vacf_i \quad \forall i$$

$$xp_i = PT_0 + yp + \sum_{j=1}^i (hirp_j - rp_j) - vacp_i \quad \forall i \quad (1)$$

$$xp_i + vacp_i - \beta \cdot (xf_i + vacf_i) \leq 0 \quad \forall i$$

$$PT_0 + yp - \beta \cdot (FT_0 + yf) \leq 0 \quad (2)$$

$$\sum_{i=1}^N hirf_i = \sum_{i=1}^N rf_i$$

$$\sum_{i=1}^N hirp_i = \sum_{i=1}^N rp_i \quad (3)$$

$$\sum_{i=1}^N vacf_i = V_f$$

$$\sum_{i=1}^N vacp_i = V_p \quad (4)$$

$$E(O_{ri}) + E(O_{si}) \leq \gamma \cdot SH_i \quad \forall i \quad (5)$$

where

- yf = full-time employees added to (subtracted from) the initial work force;
- yp = part-time employees added to (subtracted from) the initial work force;
- $xf_i(xp_i)$ = full-time (part-time) operators available in period i , including cover operators;
- $hirf_i(hirp_i)$ = full-time (part-time) hires made in period i ;
- $rf_i(rp_i)$ = expected full-time (part-time) attrition in period i ;
- $vacf_i(vacp_i)$ = full-time (part-time) operators on vacation in each week of period i ;
- $V_f(V_p)$ = full-time (part-time) vacation liability;
- $E(O_{ri})$ = expected regular overtime in period i ;
- $E(O_{si})$ = expected slop overtime in period i ;
- $cf(cp)$ = cost per period for a full-time (part-time) operator, including benefits;
- co = cost for 1 hr of overtime;
- β = maximum ratio of part-time operators to full-time operators;
- γ = maximum ratio of overtime to scheduled work hours;
- N = number of periods per year;
- $FT_0(PT_0)$ = initial number of full-time (part-time) operators; and
- SH_i = scheduled service hours in period i .

While this formulation assumes that 100 percent service reliability is required (i.e., all open work is covered either on overtime or by extraboard operators), it can be extended to include more general treatment of service reliability. It is also assumed that the system is in steady state (i.e., work requirements immediately after the planning period are the same as at the start of the planning period). This is usually consistent with a 1-year analysis period; however, this assumption can also be relaxed if necessary.

The objective function represents total annual expected work-force costs. Constraint Set 1 defines the number of full-time and part-time operators available in each period, as a function of the hiring decisions and the vacation allocations. Constraint Set 2 represents the contract or other agreement on the maximum ratio of part-time operators to full-time operators. Constraint Set 3 requires that the total hirings during the period under study be equal to the total expected attrition, so that steady-state conditions are maintained. Constraint Set 4 guarantees that vacation allocation satisfies the vacation liability for both part-time and full-time operators. Vacation liabilities V_f and V_p are easily determined based on the average number of vacation weeks per year and average work-force size:

$$V_f = K_f \frac{\left[\sum_{i=1}^N (xf_i + vacf_i) \right]}{N}$$

and

$$V_p = K_p \frac{\left[\sum_{i=1}^N (xp_i + vacp_i) \right]}{N}$$

where K_f is average vacation weeks per full-time operator per year, and K_p is average vacation weeks per part-time operator per year.

Finally, Constraint Set 5 guarantees that the expected overtime hours used in period i do not exceed a certain percentage of the total scheduled hours in the same period. In practice, this constraint can be used to control service reliability since missed service results when more overtime is required than can be obtained from the work force.

CASE STUDY

The model described above was tested in a case study that was loosely based on the bus system of the MBTA. The intent of this case study is to test the applicability of the model and to investigate the sensitivity of the proposed solutions to the various constraints that may be of importance to a transit authority. It is not the intent to make specific recommendations for changes in MBTA management practice. Furthermore, the strategic level optimization is conditional on the continuance of existing practice at the tactical and operational levels. MINOS (9), an optimization package for linear and nonlinear programs developed at Stanford University, was used to solve the strate-

gic level problem formulation presented in the previous section.

Before presenting and discussing the results, it is necessary to summarize the key characteristics of the MBTA bus system which affect application of the model:

- The 1-year planning period was divided into thirteen 4-week periods. All hiring and vacation allocation decisions were made on this basis.
- The ratio of part-time operators (PTOs) to full-time operators (FTOs) was constrained to be more than 0.4, approximately the current ratio.
- Total costs (including both wages and fringe benefits) per 4-week period were estimated to be \$3,160 for an FTO (who works 40 hr/week) and \$1,695 for a PTO (who works 30 hr/week).
- Overtime cost (including fringe benefits) was assumed to be \$23.04/hr. This is only slightly higher than the effective hourly cost for an FTO, which strongly influences the results, as will be seen later.
- Vacation liability was 3 weeks per year for an FTO and 1 week per year for a PTO.
- Expected attrition for both FTOs and PTOs was assumed known for each 4-week period.
- A 16-week hiring and training lead time was assumed so that variability in attrition over 16 weeks was included in the standard deviation of required work hours.
- The remaining variation in work hours was based on a detailed analysis of absence hours and unscheduled work hours at one large bus garage (out of six bus garages in all) for one timetable (12 weeks in all). The same coefficient of variation was then applied to the average work hours for the system as a whole.
- The same data set was used to estimate the slop distribution parameters of $K = 0.4$ and $b = 296$ hr.
- The amount of scheduled service is similar in all timetables except the summer, which has less service because of fewer work and school trips.
- Table 1 summarizes the work hours and attrition data by period.

TABLE 1 CASE STUDY DATA

Period	Average Work Hours	Standard Deviation	Expected FT Attrition	Expected PT Attrition
1	263,480	1487	3	3
2	263,480	1484	1	3
3	270,870	1566	5	3
4	265,248	1531	8	5
5	262,553	1527	8	3
6	269,916	1596	9	3
7	251,663	1485	9	4
8	244,760	1391	7	4
9	253,914	1478	10	5
10	258,276	1445	10	5
11	258,276	1446	5	4
12	258,276	1442	6	5
13	267,936	1565	8	2
			89	49

An initial formulation was run using only the regular overtime (i.e., ignoring slop overtime), excluding the overtime constraint and with no additional constraints. This formulation was run primarily for comparison to the six slop overtime scenarios (as listed below), which all include both regular and slop overtime:

1. Excluding the overtime constraint and with no additional constraints.
2. With a constant level of hiring in each period.
3. With the following vacation allocation in each timetable (this approximates the MBTA vacation allocation for bus operators):
 - Winter: 19.7 percent for FTOs, 3.9 percent for PTOs,
 - Spring: 16.8 percent for FTOs, 27.8 percent for PTOs,
 - Summer: 36.0 percent for FTOs, 47.5 percent for PTOs, and
 - Fall: 27.5 percent for FTOs, 20.8 percent for PTOs.
4. With the overtime constraint set at 2.5 percent of total scheduled hours for each 4-week period.
5. With the overtime constraint and the fixed vacation allocation.
6. With the overtime constraint, fixed vacation allocation and constant level of hiring per period.

The full results of the six runs are shown in Tables 2 and 3. For each scenario Table 2 shows average work-force size, overtime percentage of scheduled work hours, and expected costs; while Table 3 shows the optimal hiring and vacation allocation patterns. In considering these results, it is important

to recognize that, for the problem parameters, the slope of the total curve to the left of the optimum is only very slightly negative. This is true because the effective overtime cost is only slightly higher than the cost per hour for an employee in the planned work force. When factored to take unassigned cover into account, the cost per hour for the planned work force is approximately \$22, or just under the overtime cost of \$23.04. Thus, on the left of the optimum, planned work force is only slightly more cost-effective than overtime. Furthermore multiple solutions may produce the same total cost. For these reasons, multiple solutions exist at, or very close to, the same lowest cost point, allowing considerable flexibility in imposing restrictions without significantly affecting the cost. This flexibility is particularly evident with respect to hiring and vacation assignments. The basic costs for meeting the schedule requirements are \$54.8 million.

The first formulation, which is used for comparison purposes, ignores slop overtime in the work assignment process. The cost of this solution, considering only regular overtime, is \$66.3 million, while the actual cost of the same work-force model with slop overtime included is \$66.7 million. Regular overtime alone understates the total overtime by almost 33 percent. The average work-force size under this scenario is 1,279 FTOs and 512 PTOs.

Under the first scenario, and in all subsequent scenarios, slop overtime is included in the objective function, which, as expected, shifts the optimal solution to the left. Average work-force size has been reduced to 1,257 FTOs and 503 PTOs, and overtime levels have increased substantially to an average of 4.9 percent of total scheduled hours in each period. The total

TABLE 2 CASE STUDY SUMMARY STATISTICS, WITH SLOP OVERTIME

Scenario	FTOs	PTOs	Overtime (%)	Total Cost (\$ millions)
1. Unconstrained	1,257	503	4.9	66.4
2. Constant hiring	1,262	504	4.6	66.4
3. Fixed vacation allocation	1,257	503	5.0	66.4
4. Overtime cap	1,297	519	2.5	66.7
5. Overtime cap and fixed vacation allocation	1,315	524	1.8	67.0
6. Overtime cap, fixed vacation allocation, and constant hiring	1,330	531	1.3	67.3

TABLE 3 TOTAL HIRING AND VACATION ALLOCATION BY PERIOD

Period	Total Hiring by Scenario						Vacation Allocation by Scenario					
	1	2	3	4	5	6	1	2	3	4	5	6
1	2	11	2	2	0	11	69	68	59	30	62	62
2	3	11	3	3	0	11	68	75	59	29	62	62
3	33	11	42	26	24	11	46	24	59	0	62	62
4	2	11	2	94	0	11	70	75	63	110	66	67
5	11	11	1	1	0	11	86	77	65	116	67	68
6	11	11	21	2	33	11	41	31	65	61	67	68
7	0	11	0	0	0	11	142	140	125	170	131	132
8	1	11	1	1	0	11	147	160	145	208	152	154
9	12	11	35	1	22	11	122	125	145	131	152	154
10	9	11	1	1	0	11	71	86	72	90	75	76
11	18	11	2	2	0	11	81	87	72	80	75	76
12	19	11	3	3	0	11	79	87	72	72	75	76
13	17	11	26	3	58	11	48	38	72	6	75	76

work-force cost in this scenario is \$66.4 million yielding an aggregate annual cost for vacations, holidays, absences, and extra service of \$11.6 million.

In the second scenario a constraint has been imposed that requires that there be a constant level of hiring in each period—this may be imposed to smooth the hiring and training workload and to simplify the process. The total cost of this scenario is essentially unchanged from the unrestricted solution (Scenario 1) although the vacation allocation has changed somewhat. This result is not surprising considering the existence of multiple solutions as mentioned above.

Scenario 3 replaces the fixed hiring constraint with a set of constraints, which results in a vacation allocation approximating that currently employed by the MBTA. As indicated in Tables 2 and 3, this constraint can also be accommodated at a minor increase in total cost, but hiring becomes bunched at the end of each rating, and overtime fluctuates considerably from period to period, ranging from 3.5 to 7.3 percent of scheduled hours.

Under the fourth scenario, an overtime constraint of 2.5 percent is imposed without additional constraints. The 2.5 percent overtime cap has considerable effect on the work-force size, since part of the high overtime levels from the unconstrained scenario must now be covered by new extraboard operators. Average work-force size has increased by 40 FTOs and 9 PTOs. Hiring is heavily bunched in Periods 3 and 4 and vacations are again allocated more heavily in the summer rating when schedule requirements are lowest. The net work-force cost in this scenario is \$66.7 million, only a very modest increase over the unconstrained solution.

Thus, Scenarios 2–4 have shown that each of the three principal types of constraints, on hiring rates, vacation allocation, and overtime levels, can be introduced separately at reasonable levels without significant impact on the total cost, although, of course, the solution itself does change.

In Scenario 5 the fixed vacation constraint is applied in addition to the overtime cap. Under these conditions total costs increase to \$67.0 million, a more significant increase over the unconstrained optimal cost, but still under 1 percent of the total cost. Average work-force size has increased further and hiring is now very heavily focused on the periods immediately before each new timetable.

The last scenario, which combines all three constraints, on vacation allocation, hiring levels, and overtime, is, as expected, the most costly. The overtime constraint was only binding in the final period at the end of the fall rating but the total cost for this scenario was \$67.3 million, an increase of \$0.9 million over the unrestricted case (Scenario 1). Most significantly the average manpower levels have increased by 100 employees (73 FTOs and 27 PTOs). This final scenario emphasizes the importance of the simultaneous consideration of hiring, vacation, and overtime decisions at the strategic level.

The model results, summarized in Table 2, can be used in various ways. First, the actual manpower levels, vacation allocation, and hiring program can be compared with the unconstrained scenario results. This difference is the additional annual cost of the current decisions—in the case study this annual cost is about \$900,000. Second, the benefits of relaxing one or more of the existing constraints can be gauged. In this

case, for example, it may be essential to restrict overtime to no more than 2.5 percent of scheduled hours because of service reliability concerns, but it may still be possible to save more than \$500,000 annually by altering the hiring program and the allocation of vacations over the year.

CONCLUSIONS

A model for strategic operator work-force planning in the transit industry has been presented that determines appropriate work-force size and mix along with hiring and vacation allocation. To estimate the expected overtime, a semiempirical model has been developed that consists of two terms: regular overtime and slop overtime.

Application of the model in a case study, loosely based on the Massachusetts Bay Transportation Authority bus system, indicated that the methodology is applicable to transit systems. The results emphasize the importance of slop overtime in estimating the appropriate work-force levels and the simultaneous consideration of hiring, vacation scheduling, and overtime restrictions. It should be emphasized that the problem of work-force planning at the strategic level can have multiple solutions with costs very close to the cost of the optimal solution. This provides agencies with certain flexibility to incorporate secondary objectives with respect to hiring and vacation allocation without incurring significant additional costs.

A critical area for further work is to incorporate directly into the model the feedback relationships among overtime, absence, and reliability. Clearly, at some point a requirement for additional overtime may lead to higher absence rates, and if this effect can be measured it can readily be incorporated into the model. Similarly a higher requirement for overtime is likely to lead to reduced service reliability, and while this can be approximated by the overtime constraint, a more realistic model of this interaction would be helpful. The current model could easily be extended to incorporate reliability constraints directly.

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