Model Simulations of Winchendon Freeze-Thaw Field Data

LEWIS EDGERS AND LAURINDA BEDINGFIELD

This paper describes theoretical studies of the Winchendon field performance data using a computer model, FROST1, developed by the U.S. Army Corps of Engineers Cold Regions Research and Engineering Laboratory (CRREL). The Winchendon field test site was constructed by the Massachusetts Department of Public Works (MDPW) during the fall of 1977. Data on frost heave, frost depth, and thaw weakening were then obtained during the next three winter seasons. FROST1 assumes one-dimensional vertical heat and moisture flux, and is intended for use on problems of seasonal freezing and thawing of nonplastic soils that range from silts to silty sands and gravels above the water table. These simulations have shown that the computations are sensitive to the input thermal and hydraulic soil parameters, porosities, and boundary temperatures and pressures. Nevertheless, they provide guidance in the selection of input parameters for FROST1. Parametric studies were made to provide design curves that show, for two water-table depths, the reduction in maximum heave with increasing amounts of frost protection. These curves will assist designers in evaluating the required depth of frost protection and in particular the effects of only partial frost protection in situations where factors such as buried utilities or economics preclude the use of non-frost-susceptible (NFS) materials to the full frost depth.

This paper describes theoretical studies of the Winchendon field performance data using a computer model, FROST1, developed by the U.S. Army Corps of Engineers Cold Regions Research and Engineering Laboratory (CRREL). One objective of these studies was to develop relationships between physical properties of the soil and model input parameters so that the theoretical model could then be used as a design tool to predict frost heave and frost penetration versus time for trial design pavement cross sections. Computations of pore pressures and settlements during the thaw period might then be incorporated into mechanistic design approaches such as those described by AASHTO (*1*).

A second objective of these theoretical studies was to first calibrate the model to the Winchendon field performance data and then compute the effects on frost heave of variations in the depths to frost-susceptible (F) materials and the groundwater table. These studies will assist designers in evaluating the required depth of frost protection and in particular in evaluating the effects of only partial frost protection in situations where factors such as buried utilities or economics preclude the use of non-frost-susceptible (NFS) materials to the full frost depth.

The following sections of this paper include discussions of the Winchendon field test site, the computer model, theoretical studies of three of the Winchendon soils, parametric studies of idealized highway cross sections, and a summary, including design curves.

DESCRIPTION OF WINCHENDON FIELD TEST SITE

The Winchendon field test site, located in north-central Massachusetts, was constructed by the Massachusetts Department of Public Works (MDPW) during the fall of 1977. The site consists of 12 test cells, each a minimum of 28 ft wide (by 8 ft long in plan view) and consisting of a lower roadway and an upper roadway (Figure 1). The base of the test soils extended to a minimum of 6 in. below the groundwater level. The paved surface of the lower roadway of the cell was approximately 3 ft above the groundwater level and of the upper roadway, approximately 5 ft above the groundwater level. A bituminous concrete paved surface 8 ft wide and 3 in. thick was placed on both the upper and lower roadways of the test cells.

Test soils were selected to represent a wide range of soils with varying degrees of frost susceptibility. Table 1 gives the laboratory index property data. Edgers and Bono (*2*) have assembled the complete data in a separate data report.

Freezing data were obtained by the MDPW during three consecutive winter seasons, 1977–1978, 1978–1979, and 1979–1980 (*3*). Pavement surface deflections due to frost heave were measured at nine control points on both the upper and lower roadways over each soil by means of an engineer's transit. Frost penetrations were measured using a frost-depth indicator consisting of a transparent pipe containing a dye that turns colorless upon freezing at 32°F. Figure 2 shows typical plots of the frost heave, frost penetration, and groundwater observation data.

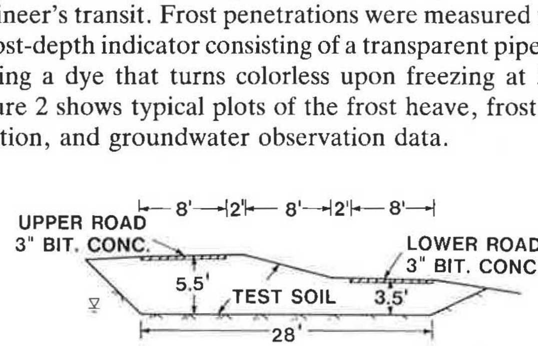

FIGURE 1 Transverse profile of typical test cell.

L. Edgers, Department of Civil Engineering, Tufts University, Medford, Mass. 02155. L. Bedingfield, Massachusetts Department of Public Works, 10 Park Plaza, Boston, Mass. 02116.

TABLE 1 LABORATORY INDEX PROPERTIES OF WINCHENDON TEST SOILS

	Test Soil	% Finer By Weight .02 mm	% Finer By Weight .075 mm	Uniform Coeff.	Plasticity Index
Many Fines	Moulton Pit Silt	52-69	96-99	3.4-4.2	N.P.
	Graves Silt Sand	8-12	44-47	5.2-6.2	N.P.
	Morin Clay	77-82	95-98	--	10.8
	Hyannis Sand	4-20	32-90	2.7-5.1	N.P.
	Ikalanian Silt-Sand	6-10	38-57	4.0-4.2	N.P.
	Sibley Till	17-21	32-37	Over 100	4.2
Intermed. Fines	Worcester Till	7-8	14-21	Over 100	N.P.
	Keating Stone Dust	~12	17-20	~100	N.P.
	Hart Brothers Sand	4-5	15-18	6.0	N.P.
Few Fines	Mason Pit Sand	1-3	5-9	5.8-6.7	N.P.
	Keating Dense Graded	~6	6-10	25-100	N.P.
	Corbosiero Sand	2-4	12-16	4.0-4.3	N.P.

N.P. = Non-plastic

CRREL MODEL OF FROST HEAVE AND THAW SETTLEMENT

The CRREL theoretical model is described in detail by Berg et al. (*4*), Guymon et al. (*5, 6*), and Johnson et al. (*7*). This model was developed as part of a cooperative research program begun in 1975 and involving the U.S. Army Corps of Engineers, the Federal Highway Administration, and the Federal Aviation Administration. The theoretical model serves as a basis for the computer program, FROST1, made available to the project by CRREL.

The CRREL model assumes one-dimensional vertical heat and moisture flux and is intended for use on problems of seasonal freezing and thawing of nonplastic soils that range from silts to silty sands and gravels above the water table. The model assumes that moisture transport in the unfrozen zone is governed by the unsaturated-flow equation based on Darcy's law and that moisture flow in the frozen zone is negligible. Heat transport in the entire soil column is governed by the sensible heat transport equation, which includes a convective term, and freezing or thawing is approximated as an isothermal phase change process. Detailed features and assumptions embodied in the model are described by Johnson et al. (*7*).

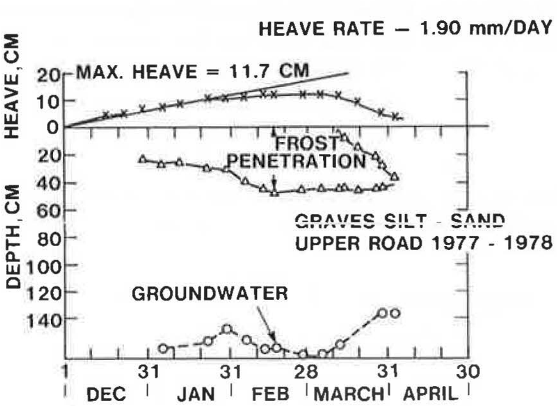

FIGURE 2 Typical frost depth, groundwater depth, and heave data.

Figure 3 shows a typical model simulation at a given time and defines the major variables. These include

- Head—water pressure (h_p) and overburden (σ_o)
- Temperature—T, T_u, and T_L
- Porosity
  - —Initial (n)
  - —Unfrozen water (θ_r)
  - —Variable with pressure (θ_u)
  - —Ice (θ_i)
  - —Segregated ice (θ_s)

FROST1 assumes that the variations of porosities (θ_u) and unsaturated permeabilities (K_u) with pressure are accurately described by Gardner functions (*8*). The unfrozen permeability K_u is reduced because of ice formation in soil pores in accordance with the following equation:

$$K_f = K_u \cdot 10^{-E\theta_i} \qquad (1)$$

where K_u and θ_i have already been defined, K_f is the frozen permeability, and E is an empirically determined factor. No rigorous theoretical principles or laboratory tests are available for determining E. At present, this parameter must be determined by calibrating, or tuning, FROST1 with either laboratory freezing-column data (*9*) or field data.

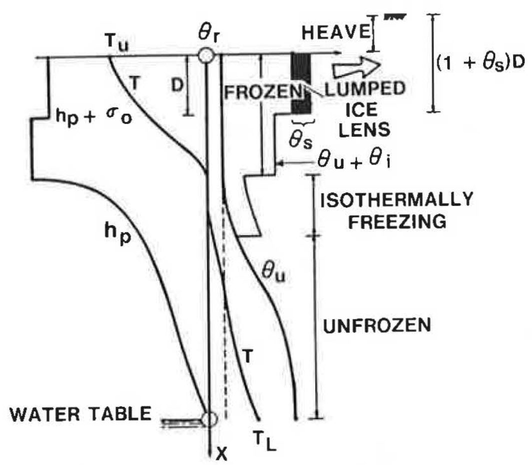

FIGURE 3 Typical model simulation result at a given time (*7*).

The thaw settlement portion of the model is an extension of early work by Morgenstern and Nixon (*10*). FROST1 uses probabilistic concepts to consider the effects of uncertainties in the input soil parameters and the nodal domain integration method. The soil profile is represented by a sequence of elements and nodes as shown in Figure 4, and the time-domain solution is by the well-known Crank Nicolson or fully implicit method.

FROST1 requires the following input:

1. Volumetric parameters
   a. Porosity n and Gardner moisture parameters A_w and α for porosity
   b. Unfrozen water content factor, θ_r
   c. Soil density
2. Hydraulic parameters
   a. Unsaturated permeability K_u
   b. Gardner's parameters A_k and β for permeability
   c. A multiplier factor for permeability (usually 1.0)
   d. Permeability correction factor for freezing soil E

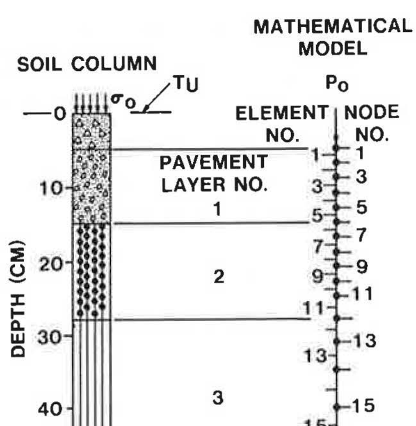

FIGURE 4 Example soil profile divided into finite elements (*6*).

3. Thermal parameters
   a. Volumetric heat capacity of soil C_s
   b. Thermal conductivity of soil K_s
   c. Freezing-point depression of soil water T_f

In addition, the following boundary and initial conditions must be specified:

1. Initial pore pressure, ice content, and temperature;
2. Soil surface boundary conditions for temperature (T_u), determined from air temperature multiplied by a factor to represent soil surface temperature (*11*); and
3. Lower boundary pressure (h_{pL}) and temperature (T_L)—if the bottom of the profile is at the water table, h_{pL} equals zero.

Output from the model includes frost heave at the surface, frost depth, thaw depth, subsurface temperature, and pore pressure. The computed frost heave can be used directly to aid in selection of an appropriate pavement design by relating it to pavement roughness criteria. Pore-water pressure may be used in empirical equations developed from laboratory tests to estimate resilient modulus values of layers within the pavement system at various times of the year. The resilient modulus data may then be used in a pavement structural response model, where output can be related to pavement performance criteria.

In the last half-dozen years, development efforts have included simulation studies of Fairbanks silt (*5*); evaluations of model uncertainties, parameter errors, and boundary condition effects (*6*); and initial model calibrations with laboratory soil column and field data on a number of soils, including six of the Winchendon soils (*6*). There are still, however, major uncertainties regarding selection of soil input parameters, especially E, the calibration factor that accounts for the effects of freezing on soil permeability.

Johnson et al. (*7*) discuss the shortcomings of the model and, in particular, the inability to derive some of the necessary input parameters from basic concepts of soil physics. The modeling requires calibration of the hydraulic conductivity correction factor E and estimates moisture tension in the freezing zone from laboratory relationships between moisture tension and unfrozen water content. The studies described in the following sections of the paper will assist in the evaluation of these input parameters.

FROST1 STUDIES OF THREE WINCHENDON SOILS

Initial calibrations of FROST1 described by Guymon et al. (*6*) included simulations of the field performance during the winter of 1978–1979 of six of the Winchendon soils. Calculations were made for the upper roadway only, and computed frost heave and frost penetration agreed well with measured values once the CRREL model had been tuned by using the frozen hydraulic conductivity factor E.

CRREL provided Tufts University with a version of FROST1 that has been modified to make it compatible with the university's Vax 11/780 computer system. Preliminary FROST1 runs were then made (*12*) to verify that the Tufts version of the program was operating correctly; to familiarize the researchers with the operation of FROST1, especially the preparation of input; and finally, to evaluate the sensitivity of the computations to variations in mesh formulation and the input parameters. These runs established that FROST1 computations are sensitive to the volumetric and hydraulic soil parameters, including the permeability correction factor E. Initial pore pressure and temperature do not strongly influence the computed frost heave and frost penetration, provided the soil is initially unfrozen. The lower boundary pressures (h_{pL}) and temperatures (T_L) strongly influence the computed frost heave if the frost penetrates more than halfway to three-fourths of the way into the finite-element mesh.

After these preliminary FROST1 runs, three of the Winchendon soils were selected for detailed study. Graves silt sand was selected as representative of highly frost-susceptible materials; Keating dense graded stone was selected as representative of low-frost-susceptible materials; and Hart Brothers sand was selected as representative of materials of intermediate frost susceptibility. In situ, Hart Brothers sand was strongly affected by the water-table location, with about three times greater heave on the lower roadway (high water table) than on the upper roadway (low water table). One objective of this series of runs was to first simulate the field performance of these three soils at the upper roadway and then test FROST1 by using the same thermal, volumetric, and hydraulic parameters to simulate the performance of these three soils at the lower roadway.

Initial ground temperature, pore-water pressure, and lower boundary temperature were determined from the field measurements. Initial ice content was assumed to equal zero. Ground surface temperature was estimated from mean daily air temperatures using the Corps of Engineers n-factor method (*11*). The grid depth was taken to the average groundwater table depth; 150 cm (upper) and 100 cm (lower) for Graves silt sand and Hart Brothers sand; and 120 cm (upper) and 60 cm (lower) for dense graded stone. Thus the lower boundary pressure head (h_{pL}) was approximated as zero, and fluctuations in groundwater table during the simulation period were neglected. The grids were divided into 43 to 61 variable-length segments, from 0.5 cm at the top to 5 cm at the column bottom. Table 2 gives the soil parameters used in these analyses. These parameters were taken from the laboratory measurements or are based on the initial CRREL analyses (*6*).

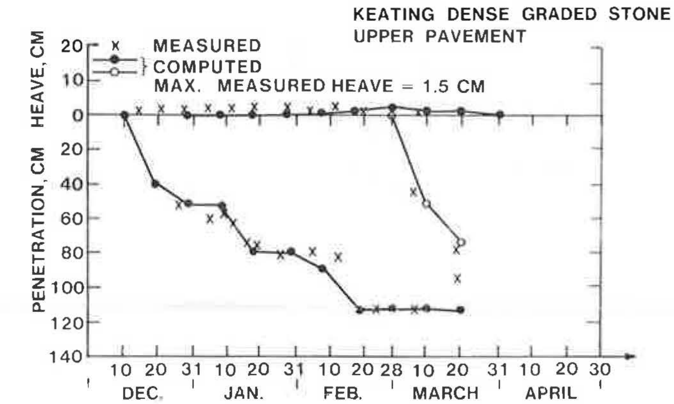

FIGURE 5 Computed frost heave and frost penetration for Keating dense graded stone, upper roadway, 1978–1979.

The results of these computations are shown in Figures 5 through 11. For the dense graded stone (Figures 5 and 6) and Hart Brothers sand (Figures 7 and 8), computed values of frost heave, frost penetration, and thaw penetration show excellent agreement with measured values. Some differences in detail between measured and computed values occur because the lower boundary ground-

TABLE 2 SOIL PARAMETERS FOR REMOLDED WINCHENDON TEST SITE SOILS USED IN TUFTS UNIVERSITY STUDIES

Parameter	Graves Silty Sand	Hart Bros. Sand	Dense Graded Stone
Soil density (g/cm^3)	1.49	1.69	1.87
Soil porosity (cm^3/cm^3)	0.460	0.282	0.334
Soil-water freezing point dep. (^{o}C)	0	0	0
Vol. heat cap. of soil (cal/cm^3 ^{o}C)	0.2	0.2	0.2
Thermal cond. of soil (cal/cm hr ^{o}C)	17.0	17.0	17.0
Unfrozen water cont. factor (cm^3/cm^3)	0.12	0.04	0.15
Soil water characteristics [A_w,(α)]	.00560 (.900)	.022 (.867)	.053 (.462)
Permeability characteristcis [A_k,(β)]	.00081 (2.536)	2.681 (1.2)	2.681 (1.3)
Saturated hydraulic cond. (cm/hr)	1.92	4.08	5.54
Frozen soil hydraulic cond. factor-E	6 (upper road) 9 (lower road)	10.0	23.0

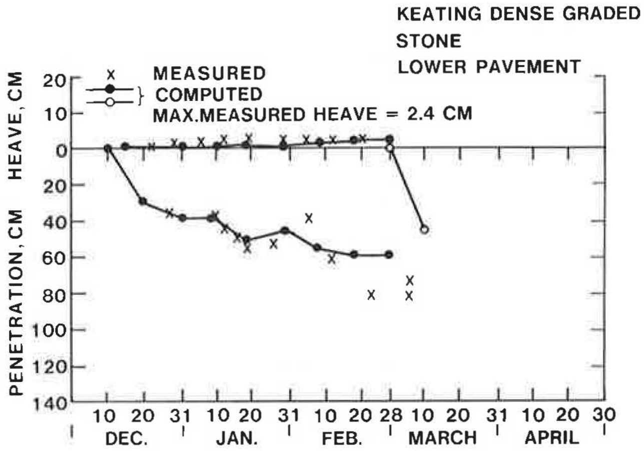

FIGURE 6 Computed frost heave and frost penetration for Keating dense graded stone, lower roadway, 1978–1979.

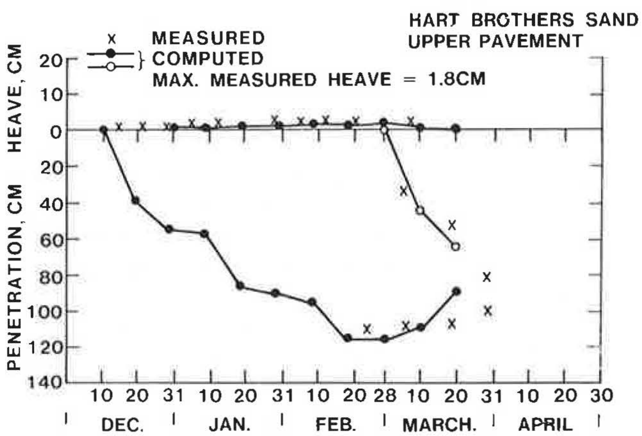

FIGURE 7 Computed frost heave and frost penetration for Hart Brothers sand, upper roadway, 1978–1979.

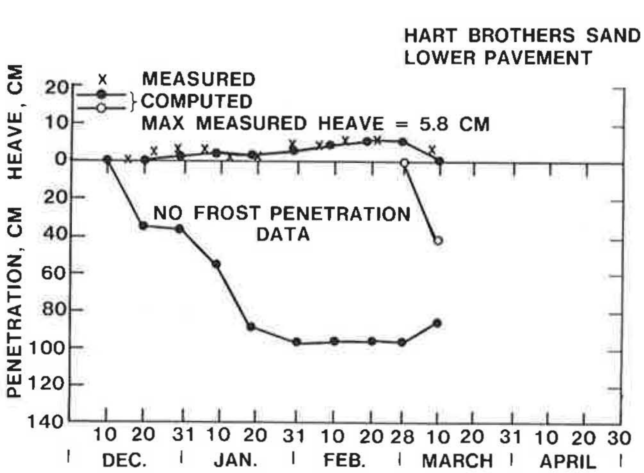

FIGURE 8 Computed frost heave and frost penetration for Hart Brothers sand, lower roadway, 1978–1979.

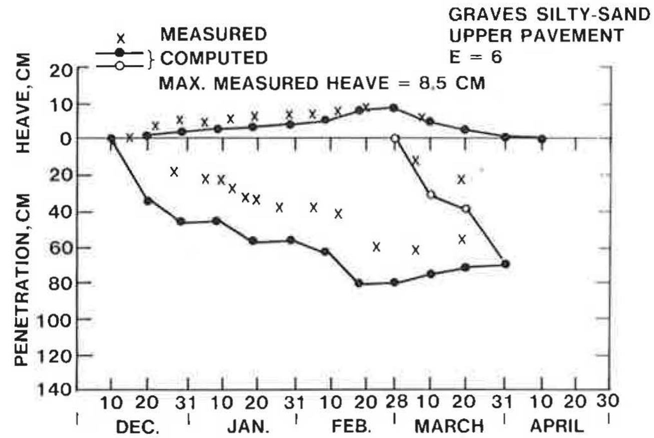

FIGURE 9 Computed frost heave and frost penetration for Graves silty sand, upper roadway, 1978–1979 (E = 6).

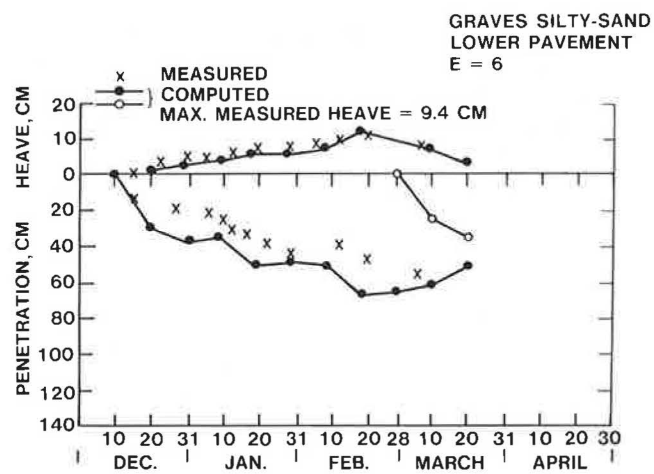

FIGURE 10 Computed frost heave and frost penetration for Graves silty sand, lower roadway, 1978–1979 (E = 6).

water condition was assumed constant with time. Comparison of Figure 5 with Figure 6 and Figure 7 with Figure 8 shows that the same soil parameters accurately simulate the field performance of both the upper and lower roadways.

The analyses of Graves silt sand (Figures 9 through 11) show poorer agreement with field performance. The computed frost penetration depths are larger than measured values, especially for the upper roadway. Also, a value of 6 for E best simulated the performance of the upper roadway (Figure 9); this value was used to compute a heave-versus-time curve for the lower roadway (Figure 10), which agreed in general with the measured heaves but resulted in a maximum computed heave for the lower roadway that is slightly too large (Figure 10). Figure 11 shows a FROST1 simulation of the lower roadway of Graves silt sand that uses a value of 9 for E. The computed maximum heave shows good agreement with the measured maximum heave.

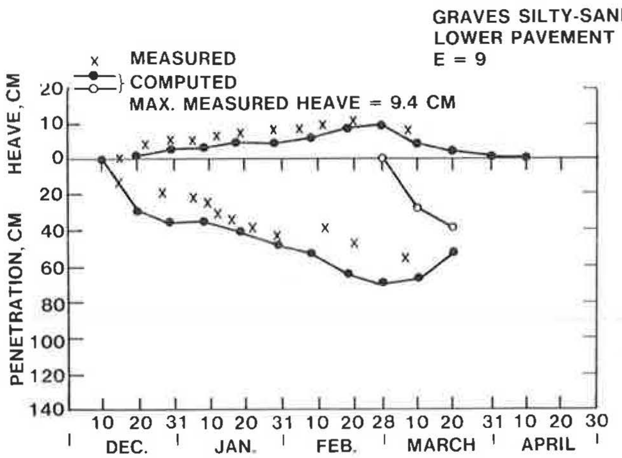

FIGURE 11 Computed frost heave and frost penetration for Graves silty sand, lower roadway, 1978–1979 (E = 9).

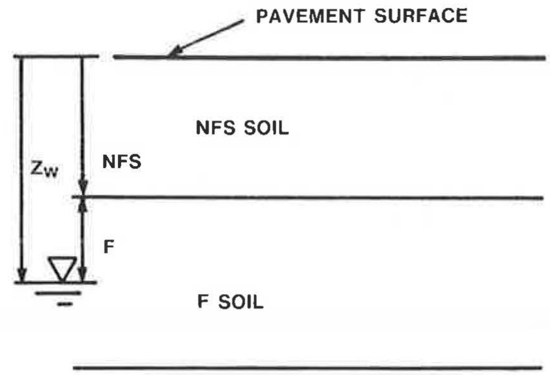

FIGURE 12 Geometry of two-layer roadway system for FROST1 parametric study.

Guymon et al. (*6*) further discuss difficulties in using FROST1 to model the Winchendon field performance data. They identify the possibility of errors due to incorrect thermal conductivity, incorrect surface moisture flux boundary condition (assumed zero), and variations in soil parameters due to freeze-thaw cycles. They conclude that "the most likely problem with the Winchendon soils simulation is that the pavement surface temperature was used as a boundary condition. More accurate results will have been possible if soil surface temperatures below the pavement were used."

It is believed that in addition to these, errors in estimating the input soil parameters, especially the unfrozen water content factor, θ_r, and the unsaturated permeabilities and porosities may also have contributed to differences between the computed values and measured performance data. Nevertheless, these analyses show reasonable agreement between the computer simulations and the Winchendon field performance.

PARAMETRIC STUDIES OF TWO-LAYER SYSTEM

Parametric studies were made to evaluate the effects on frost heave of variations in the depth to frost-susceptible materials and the groundwater table. These studies will assist designers in evaluating the required depth of frost protection and, in particular, the effects of only partial frost protection in situations where factors such as buried utilities or economics preclude the use of NFS materials to the full frost depth.

These parametric studies were performed on a two-layer system (Figure 12) consisting of NFS material underlain by a highly frost-susceptible material. The thickness of the NFS material, *NFS*, was varied from zero to the full groundwater table depth, Z_w. Computer runs were made for two groundwater table depths, 3.1 and 5 ft. The properties of the NFS material were represented by those of the Keating dense graded stone, and the frost-susceptible material by those of the Graves silt sand (Table 2). Thus, these parametric studies have been calibrated to the preceding Winchendon field simulations. The initial and boundary temperatures and pore pressures were idealized from the field measurements. The air temperatures correspond to the normal average daily temperatures recorded at Winchendon, providing a freezing index (FI) of 835 °F-days.

Figures 13 and 14 show for Z_w of 3.1 ft and 5 ft, respectively, computed frost heave and frost penetration versus time. The families of curves correspond to the different thicknesses of NFS material. These curves are summarized by Figure 15, which plots maximum heave versus thickness of NFS soil.

Figure 15 shows that at $NFS = 0$ (full thickness of Graves silty sand), the maximum heave computed in this parametric study, 10.5 and 11.9 cm for Z_w equal to 5 and 3.1 ft, respectively, is about 20 percent larger than the corresponding values computed for Graves silt sand in the simulations of Figures 9 and 11. This is because of differences in the initial and boundary conditions in these parametric studies.

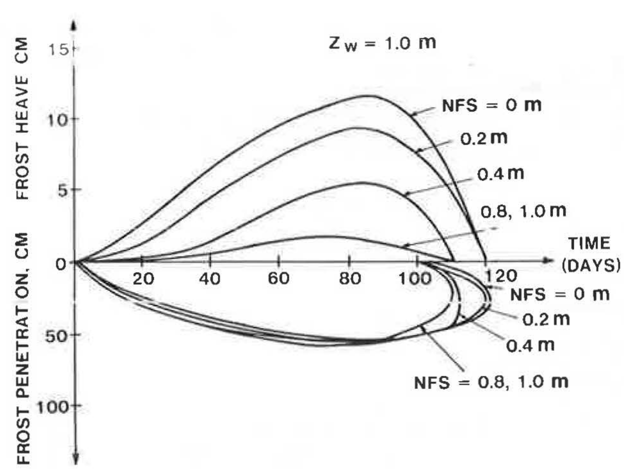

FIGURE 13 Computed frost heave and frost penetration versus time, two-layer parametric study (Z_w = 3.1 ft).

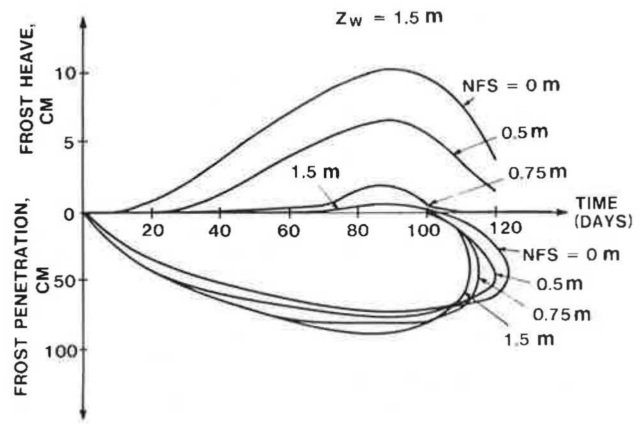

FIGURE 14 Computed frost heave and frost penetration versus time, two-layer parametric study (Z_w = 5 ft).

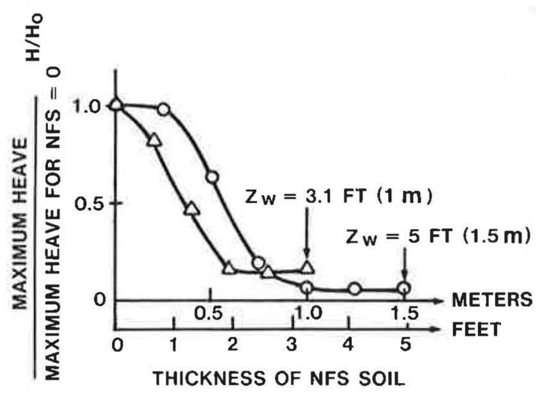

FIGURE 16 Normalized maximum heave versus thickness of NFS soil.

For Z_w = 5 ft, the maximum heave decreases gradually with increasing thickness of NFS material, until *NFS* = 3 ft. This depth is slightly greater than the computed frost penetration depth (Figure 14). NFS material of 2 ft reduces the maximum heave to about 5 cm, only about a 50 percent reduction. Little additional benefit is obtained by placing more than 3 ft of NFS material over the frost-susceptible material.

For Z_w = 3.1 ft, the maximum heave decreases much more rapidly with increasing thickness of NFS material, until *NFS* = 2 ft. This depth is approximately the computed frost penetration depth (Figure 13). NFS material of 2 ft reduces the maximum heave to about 2 cm, about an 80 percent reduction. Little additional benefit is obtained by placing more than 2 ft of NFS material over the frost-susceptible material.

The comparison for the two water-table depths is shown also in Figure 16, which plots the reduction in normalized heave versus thickness of NFS soil. Figures 15 and 16 show that there is more heave for the shallow water table, as expected, for a large thickness of NFS soil, corresponding to full-section thicknesses. Figures 15 and 16 also show that as *NFS* increases from zero, that is, as the thickness of protective NFS soil placed over the frost-susceptible soil (F-soil) is increased, more relative benefit is obtained by an equal thickness of NFS material when the water table is at the shallower depth (3.1 ft) than when the water table is at the greater depth (5 ft). The more rapid decrease in heave with increasing thickness of NFS soil for the shallower water table occurs because for an equal NFS, less frost-susceptible material is left in the ground within the zone of frost penetration. In fact, Figure 17 plots the normalized heave versus the thickness of frost-susceptible soil penetrated by frost and shows almost the same normalized relationship for the two water-table depths analyzed.

SUMMARY AND CONCLUSIONS

The theoretical studies of the Winchendon field performance data using the CRREL computer model FROST1 have shown that the computations are sensitive to the input thermal and hydraulic soil parameters, porosities, and boundary temperatures and pressures. FROST1 computations were made to simulate the field performance of Graves silt sand, Keating dense graded stone, and Hart

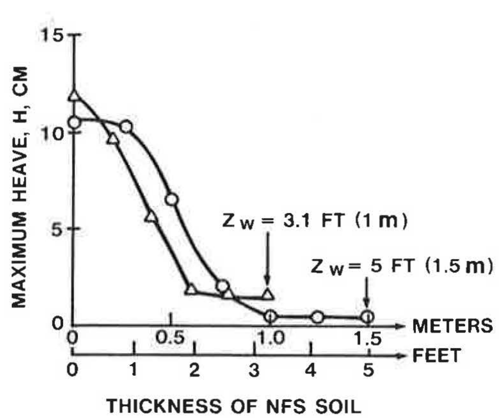

FIGURE 15 Maximum heave versus thickness of NFS soil.

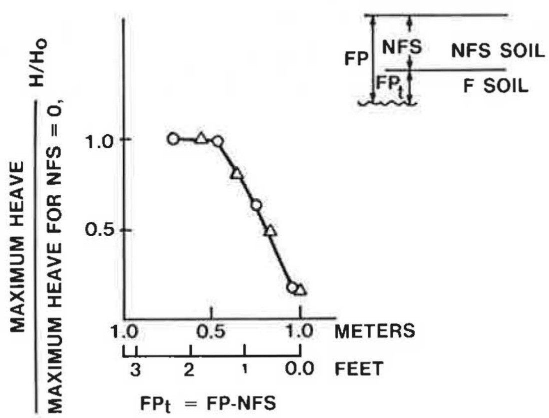

FIGURE 17 Normalized heave versus thickness of frost-susceptible (F) soil penetrated by frost.

Brothers sand. Computed and measured values of frost heave, frost penetration, and thaw penetration showed excellent agreement, especially for Keating dense graded stone and Hart Brothers sand. For Graves silt sand, the computed frost penetration depths are larger than measured values, and it was necessary to use different values of E to accurately simulate the performance of the upper ($E = 6$) and lower ($E = 9$) roadways. Table 2 gives the input thermal and hydraulic soil parameters used in these simulations. Except for the slight variations in E described above, the same parameters accurately simulated the field performance of both the upper and lower roadways of a particular soil.

Parametric studies were made to evaluate the effects on frost heave of variations in the depth to frost-susceptible materials and the groundwater table. The parametric studies were performed on a two-layer system consisting of non-frost-susceptible material underlain by a highly frost-susceptible material. The properties of these materials were represented by those found in the FROST1 simulations described above, and thus these have been calibrated to the Winchendon field performance data.

These parametric studies are summarized in Figure 16, which shows, for two water-table depths, the reduction in maximum heave with increasing amounts of frost protection. Figure 16 evaluates the required depth of frost protection and in particular the effects of only partial frost protection in situations where factors such as buried utilities or economics preclude the use of non-frost-susceptible materials to the full frost depth. It shows, for example, that for equal thicknesses of non-frost-susceptible soils, more relative benefit is derived for the shallow groundwater condition. No benefit is derived from placement of additional non-frost-susceptible material below the depth of frost penetration.

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