

Three-Dimensional Relationships Among Traffic Flow Theory Variables

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This paper is an investigation of the relationships among speed, flow, and occupancy, representing the three variables of traditional theory for uninterrupted traffic flow. The variables were examined in three-dimensional space, rather than two at a time as has previously been the case. Scatter plots showing connected data points were positioned in space using a three-dimensional rectangular coordinate system. Oblique views of the data were projected as two-dimensional plots for presentation purposes. The resulting pictures were evaluated for points of agreement with traditional traffic flow theory and with a possible new approach based on the cusp catastrophe theory. The results suggest that conventional theory is insufficient to explain the data and that the plotted data are visually consistent with the catastrophe theory model of uninterrupted traffic flow.

Traditional traffic flow theory presents speed-flow-concentration relationships as shown in Figure 1. This diagram (1) portrays the theoretical relationship between two variables. This representation implies that the underlying data can be adequately represented as three line functions using two-dimensional relationships and that the underlying data also represent a line function in three dimensions. This function has been represented as a horseshoe shape, set at an angle to each of the three projections (2, p. 50).

This underlying three-dimensional representation has not recently been tested, although the data collection capabilities of current freeway traffic management systems (FTMSs) make this feasible now. In particular, several systems now collect speed as well as volume and occupancy data, making the use of three independent variables possible. Earlier systems could obtain speed data only through a calculation that relied on the relationship investigated in this research.

The purposes of this investigation were as follows:

- To investigate empirically the three-dimensional relationship among speed, flow, and occupancy;
- To determine whether speed, flow, and occupancy relationships are best described using two-dimensional functions, a surface, or some other approach; and
- To identify the points of agreement between observed data and traditional theory, and between the data and a recently proposed catastrophe theory representation of traffic flow.

The first section of this paper briefly describes the difficulty researchers have encountered in matching traffic flow theory with data. The second provides a description of the data and its location source, collection method, and preparation. Fol-

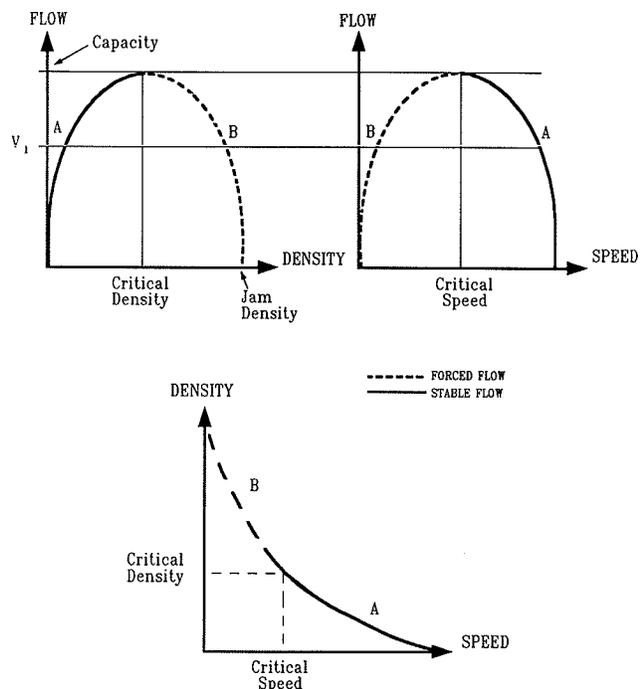


FIGURE 1 Relationships among speed, density, and rate of flow on uninterrupted flow facilities (1).

lowing that, projections of the data are presented and described. Next, there is a discussion of the relationship of the data to both traditional theory and catastrophe theory. Finally, a number of conclusions are presented.

BACKGROUND

Improved data collection methods have often led to serious questioning of traditional theory. This has happened for uninterrupted traffic flow condition theories as well as other fields, such as particle physics. The problem is the inability of traditional theory to explain the data obtained with new equipment or techniques.

Three examples of this in the development of traffic flow theory represent this type of concern. In 1965, use of newly acquired data from Chicago freeways was made in an effort to calibrate the standard two-variable models (3). The results showed that the current theory did not match the data well. However, an adaptation of the conventional theory was found that allowed data and theory to coexist peacefully.

In another study (4-6), the data did not match the conventional theory well, but modifications were found to salvage the conventional theory. One such modification is the concept of two-regime models. Unfortunately, even with this modification, it was found that two sets of data collected at the same location required different sets of parameters. This remains a concern in the modified theory.

The third example comes from the 1985 *Highway Capacity Manual* (HCM) (1). Although not explicitly recognized as a problem caused by new data acquisition techniques, the HCM does contain a discussion of the mismatch between data and theory in its coverage of the speed-flow relationship. There is explicit mention of previous attempts to fit a curve to data and to test a variety of theoretical curves (7). There is also implicit recognition of this ongoing concern since one section discusses work with discontinuous, or two-regime, models while other sections ignore this and rely on representations (see Figure 1).

These approaches to matching data with theory have all attempted to model the data using some form of a curve in two-dimensional space, so three sets of curves must be developed to explain traffic flow theory fully. It has been suggested that speed-flow-concentration relationships are better explained using a three-dimensional surface defined by the cusp catastrophe theory (8-10). This model of traffic flow indicates that data points representing speed, flow, and concentration lie on a surface similar to a sheet of paper with a tear in one section (see Figure 2). The portion of the sheet to one side of the tear is raised relative to the portion on the other side. The raised portion represents higher speeds (uncongested data), while the lower portion depicts lower speeds (occurring during congestion). Transitions between uncongested and congested conditions can occur either by crossing the tear or going around the end of it. The former movement produces a sudden jump in one of the three variables at the same time that the other two undergo continuous change. If the transition occurs by traversing the portion of the surface that has no tear, all three variables undergo continuous change.

However, the work undertaken so far to test this theory has relied on two-dimensional projections of the information, even though the underlying theory is explicitly three-dimensional. Inspection of the three variables concurrently is as necessary to validate this new model as it is to provide a more complete test of conventional theory.

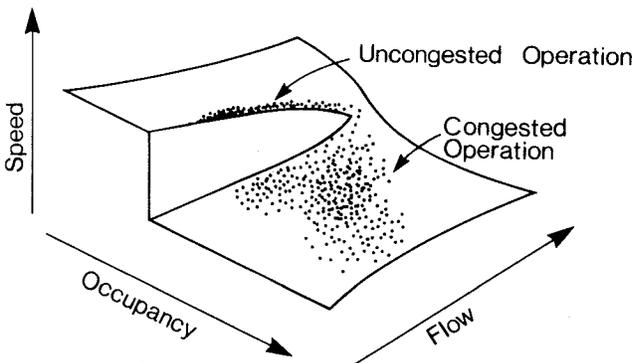


FIGURE 2 Hall's perception of catastrophe theory surface (8-10).

DATA ACQUISITION

The data for this analysis were obtained from the Mississauga FTMS on the Queen Elizabeth Way (QEW), just west of Toronto. The area of the QEW studied is a suburban freeway linking a sizable commuting population in Mississauga, Oakville, and points west with major employment centers in metropolitan Toronto. The six-lane freeway is situated in generally level terrain. The data presented in this paper are obtained from Station 16, on the east side of the Credit River, between the Mississauga Road entrance ramp and the exit ramp at Highway 10 (see Figure 3).

Eastbound travel into Toronto generates recurrent congestion between about 7:00 a.m. and 9:00 a.m. Interchange locations at Highway 10 and Cawthra Road contribute to congestion as traffic attempts to enter the main line from the ramps when the freeway is operating at or close to capacity. Ramp metering is in effect, but the ability of the metering process to serve mainline traffic is limited by the availability of queue space for the ramp traffic. Mainline queuing extends well west of Station 16.

Data used in this examination were taken from the FTMS computer tapes for Friday, September 30, 1987. All data are from the median, or left-most, side of the eastbound lanes. A single lane was chosen to reduce the amount of data to be examined, given that a large number of different views might be desired for analysis. The median lane was selected because it has been indicated (10) that median-lane traffic operations provide a sensitive and reliable indicator of freeway operations.

This station has paired detectors in each lane, from which speed, flow, and occupancy are transmitted to the central

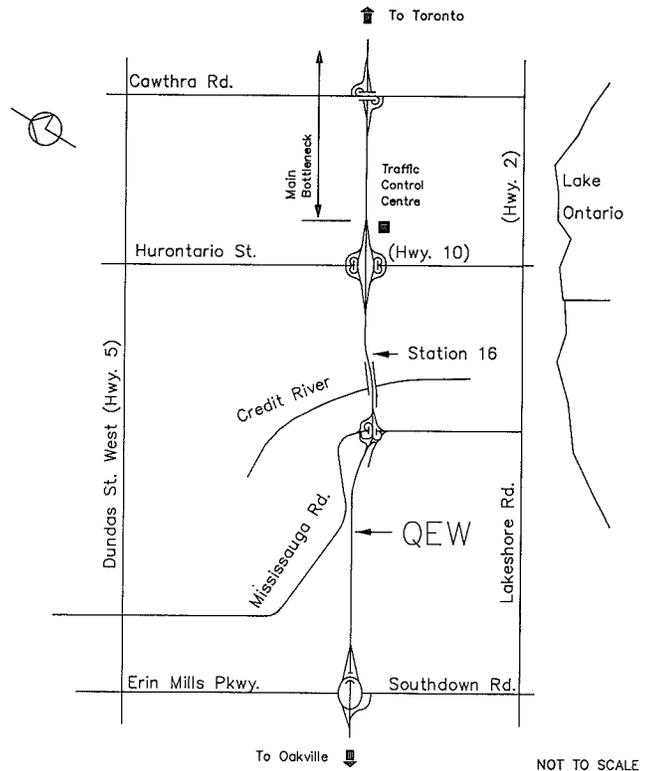


FIGURE 3 QEW Freeway in Mississauga, Ontario.

computer every 30 sec. The data used for plotting purposes included 410 30-sec intervals. The first point plotted was 6:00:30 a.m.; the last was 9:25:30. Between Interval 1 and Interval 75 (6:37:00), speeds were consistently over 80 km/hr. At about Interval 75, there was a sudden drop in speed as traffic operations became congested. At Interval 397 (about 9:16:30), speeds again climbed above 80 km/hr. From about that interval on, the freeway again operated under uncongested conditions.

A review of the data showed that speeds were missing during 15 of the 410 intervals. However, the upstream detector was able to report flows and occupancies in all these cases, and occupancies exceeded 40 percent. These 15 observations were simply deleted from the data file. Some potentially important information may be missing because of this, particularly since the diagrams rely on connections between successive points to help make sense of the pattern. However, in view of the low number of intervals for which speeds were missing, and the fact that consecutive intervals with missing speeds exceeded two only once, it was assumed that missing data would not significantly affect this investigation.

ANALYSIS

In the following subsections, data are first presented as views in the plane of the axis system (Figure 4) to obtain the traditional view, as in Figure 1. Following that are parallel oblique projections of the data where one of the planes in the xyz-

coordinate system is placed in the plane of the page. (Figure 5 shows this with the flow-occupancy plane in the plane of the page.) The third dimension is viewed obliquely (speed in Figure 5). Projection lines in the third dimension are plotted parallel to each other (i.e., there is no apparent perspective to the view as there would be if the third dimension lines tended toward a "vanishing point").

The three-dimensional plotting is left-handed; hence, with this type of reference system, the initial origin is located at the bottom left corner. The positive x-axis extends horizontally to the right. The positive y-axis extends vertically "upward" on the page. The positive z-axis extends into the plane of the page, "away" from the reader.

All of the plots show data points connected with lines. The original computer work and the photography for the original report (11) were in color, with five different colors representing different speed ranges. For this paper, black and white were alternated for the five speed ranges, which allows each of them to stand out clearly in many figures. Figure 5 is a good example; in Figure 5a, only three ranges are clear, but all five stand out in Figure 5b. Area A contains the data with speeds above 80 km/hr. Area B (light lines) covers the range 70 to 80 km/hr; area C (dark lines) the range 60 to 70 km/hr; area D (light lines) the range 50 to 60 km/hr; and area E (dark lines) the range below 50 km/hr. (Interpretation of these figures is discussed later in this paper.)

The data are drawn in a reference cube; horizontal and vertical scales become ineffective when the data are rotated in three-dimensional space and where relative reference points

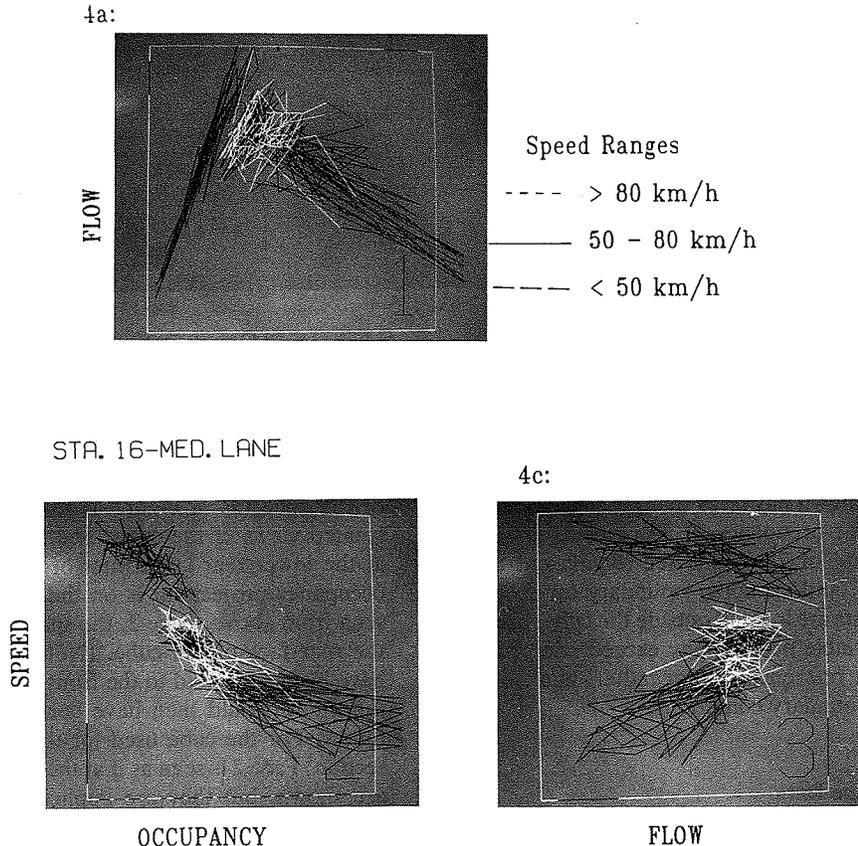
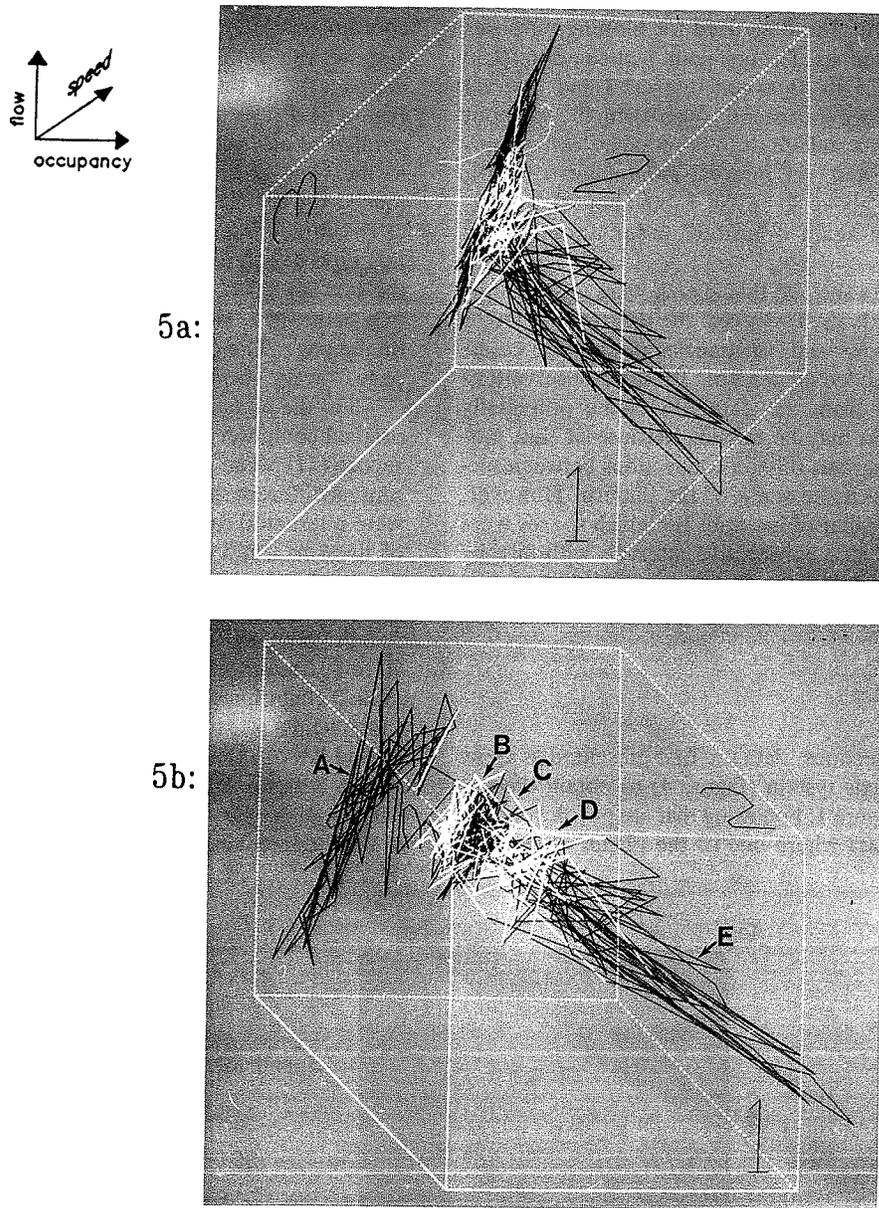


FIGURE 4 Traditional views of the Station 16 data.



STA. 16-ME. . LANE

FIGURE 5 Oblique view of Station 16 flow-occupancy data.

change from view to view; hence, scale and labels are not displayed. The cube shown has faces that range from zero to an upper boundary for occupancy of 70 percent, an upper boundary for flow rate of 22 veh/30 sec (equivalent to an hourly rate of 2,640), and an upper boundary for speed of 120 km/hr. A few points are located outside the cube, because actual values of a variable occasionally exceed one of these values. The cube is a reference system rather than a set of maximum values or boundary conditions.

Traditional Views of the Data

Views of the speed-flow-occupancy relationship are obtained by observing the three-dimensional data plots from different positions in space. The traditional views are shown in Figure

6. The three numbers are reference points to label the faces of the cube (as in Figure 4) for assistance in following the rotated views of the cube. The labeled axes in Figure 6 are at the lower rear left corner of the cube. Face 1, the flow-occupancy projection, is on the bottom of the cube, and is seen from above. Face 2, the speed-occupancy projection, is the rear face of the cube. (Note that this curve could also have been projected to the front face of this box, except that the figure would then have been more difficult to draw or follow. For the cube used for the oblique views in the next section, Face 2 is seen as if at the front.) Only the projections are shown, but they are implicitly from a data set located within the cube.

Figure 4 shows these traditional views for the actual data (with the replacement of density by occupancy). It is worth noting several characteristics of the data in these figures, to

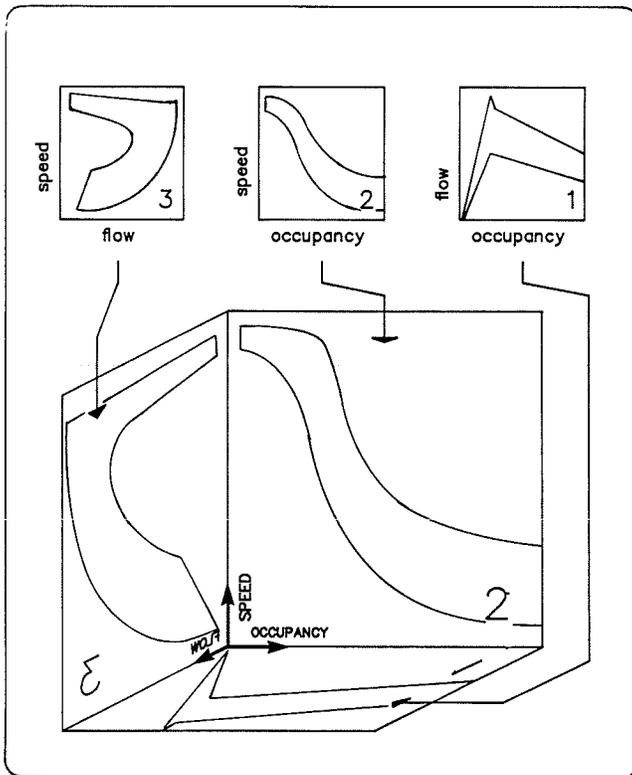


FIGURE 6 Projections of data used to obtain traditional views.

help find the speed ranges in the oblique parallel views of the next subsection. Particularly in the flow-occupancy view (Figure 4a), uncongested data are grouped within a narrow band, extending from a point close to the origin upward at an angle of about 70°. Speeds are consistently in excess of 80 km/hr, and this area of data is characterized by larger longitudinal oscillations over a relatively narrow band width. The congested data are shown by a wide band extending downward and to the right, at about 40° (from the horizontal). Data at the left end of the band are characterized by substantial vertical movement perpendicular to the longitudinal axis of the band. Speeds in this area of the band are greater than 70 km/hr. At speeds less than 50 km/hr, there appears to be a more prominent movement along the longitudinal axis of the band. At speeds between 50 km/hr and 70 km/hr, there seems to be a transition in the data from the vertical oscillations of higher speeds to the longitudinal oscillations of lower speeds. There is a small data gap between the uncongested and congested data. This gap is crossed only twice, and is more obvious in the speed-flow diagram (Figure 4c) than in the flow-occupancy figure.

The other main point to note, in these “conventional” views of the data, is the “error” pattern, i.e., the scatter of the data about what is thought to be the functional relationship (as in Figure 1). In the flow-occupancy plot (Figure 4a), there is little scatter about the left-hand branch of the curve but a considerable amount about the right-hand branch. In the other two figures, there is a very wide scatter of data about the entire curve, perhaps even sufficient to cast doubt on the narrow linear representation. It is worth considering how this error pattern affects the interpretation of statistical inference. Random variation with constant variance is expected but does

not seem to be present. Further, the error for each of the three curves is not independent of the other two.

Oblique Views of the Station 16 Data

For the oblique views, the center of the view is the center of the cube, and the cube is rotated about this center point. In all of the views, the rotation of the cube is the same relative to the plane of the page. The horizontal rotation of the cube is about 28° to the left or right of a center line through the cube, and about 28° either upward or downward. View projections are effectively taken from near the corners of the cube looking toward the center of the cube. The three axes passing through the origin ($x = 0, y = 0, z = 0$) are shown as solid lines, while all other lines are dashed. Labels have been replaced by the three numbers on three faces of the cube, as displayed in Figure 4.

Figure 5 shows the data set as an oblique view of the flow-occupancy projection. (For this view, the revised cube of Figure 6 has been rotated 90° about the occupancy, or horizontal, axis.) The upper picture (Figure 5a) is a view taken from the upper right corner of the cube. The true origin is located at the lower left front corner of the view, and the cube surface with the number 1 is closest to the reader. The origin is located at the front of the cube, so points corresponding to high speeds are positioned at the “rear” of the cube (and are on the left side of the data). Low-speed points are located toward the “front” of the cube (on the right side of the data). The data in the 60 to 70 km/hr range are situated in the center, almost obscured behind other data. (These data are much more visible in Figure 5b).

In fact, all of the data for speeds above 60 km/hr seem to form a narrow band, clustered even more tightly than the uncongested data in the normal flow-occupancy view. Oscillations within the band seem to be longitudinal. All of the data for speeds greater than 50 km/hr seem to lie in a tilted plane. The view looks toward the edge of the plane, which may or may not have a slight twist to it. (Subsequent views seem to confirm the presence of this twist.)

The data for speeds of less than 50 km/hr are not as well defined, at least in this view. These data, again, form a band extending from about the middle of the cube downward and to the right. Movement between data points appears to exhibit a pattern of oscillation along the axis of the band. This suggests that very low speeds (and high occupancies) do not often occur for two consecutive intervals within this data set.

Figure 5b shows the flow-occupancy data set as an oblique view taken from the left upper corner, rather than the right. The surface with the number “1” is still closest to the reader. High-speed uncongested data are again located near the back of the cube (A). In this view, the high-speed uncongested data appear to be well grouped at higher speeds but spread out at the lower speeds and higher occupancies. The band width of these data is much wider than in the upper photograph, and more lateral movement within the band is visible. The gap in the data, which is not evident in Figure 5a, is evident in this picture.

Considering the two parts of Figure 5, there is a strong indication that the uncongested data, and, indeed, the rest of the data above perhaps 60 km/hr, fall on a surface. From the right side of the cube (Figure 5a), the lines are very compact, suggesting the surface is viewed on edge. From the left side

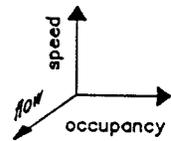
(Figure 5b), the surface is viewed from a much less acute angle and gives the impression that the data are much more dispersed. The lines connecting data points suggest that, while significant variation in uncongested data is present, variation appears to arise from oscillations taking place on a planar surface.

In the lower speed range during congested operation, the pattern is not as evident. It is clear, however, that these data are not on the same plane as the higher speed data. There may also be an indication of a surface for the low-speed data. At speeds close to 70 km/hr (B and C), the data exhibit a pattern similar to higher speeds. At the lowest speeds of congested data (E in Figure 5b), there is a gradual tapering; thus, the band width at lower speeds is quite narrow—not unlike the narrow band width of the uncongested data in Figure 5a. An examination of the very-low-speed data (e.g., much less

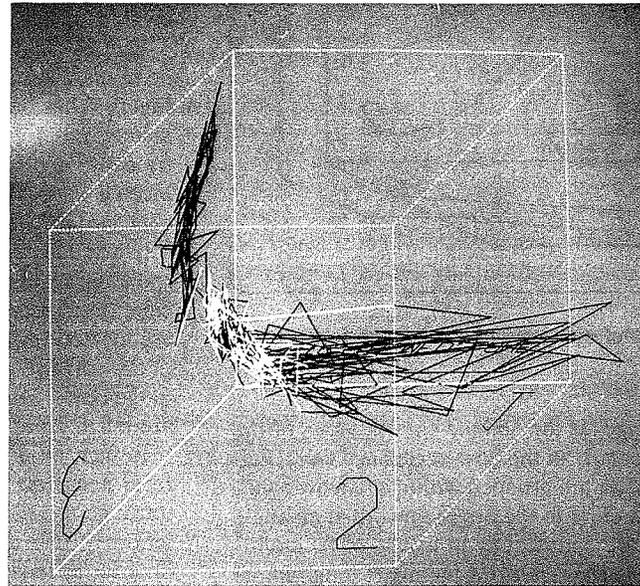
than 50 km/hr) in Figure 5a indicates a wider band width. In other words, the upper view appears to “disperse” the very-low-speed congested data relative to the lower view. Hence, there is some evidence that these data may also lie on a surface, although the nature of that is less clearly defined than the surface for the uncongested data.

Figure 5 then suggests that the data may lie on a pair of planes or on a surface. More specifically, the data appear to take the form of a U or a V, with fairly broad arms, with the plane of one of the arms having been rotated away from the plane of the other arm. The result is a twisted U.

Figure 7 shows the speed-occupancy plane parallel to the page. Figure 7a is a view taken from the upper right corner of the cube looking down at the data set. High speeds are near the top of the cube, and low flows are located at the rear of the cube. The number 2 is located on the front of the



7a:



7b:

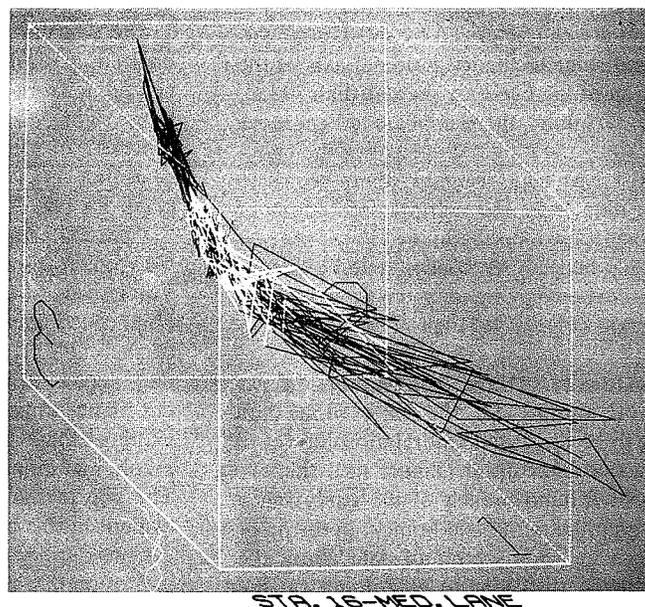


FIGURE 7 Oblique view of Station 16 speed-occupancy data.

cube (i.e., the high-flow side). The cube is positioned so the true origin is located at the back of the cube.

The most striking aspect of this illustration is that it presents a U shape in the data, as just suggested, and there are grounds for finding the twist in the U as well. Figure 5b showed considerable scatter in the high-speed data, but that is greatly reduced in this view. This means the plane is being viewed close to edge-on. This is not true for the low-speed data, however. Hence, the twist may well be visible somewhere in the vicinity of the 60 to 70 km/hr data or just beyond it in the 50 to 60 km/hr range.

Figure 7b shows the speed-occupancy view from the lower corner on the same side of the cube pictured in Figure 7a. This view looks up at the data set, and higher speeds are still at the top. High flows are located at the front of the cube, as is the face with the number 2. The true origin is located at the back of the cube on the left side.

This view of the data set appears to confirm the idea that the surface is rotating, or twisted. Data with speeds in excess of 60 km/hr are compact laterally but spread out linearly along the length of a narrow band. The general form of the data suggests the view is near the edge of a surface for the high-speed data range. The surface rotates in the lower speed range (to the right of the dark cluster) so the view is no longer along the edge of a surface but at an oblique angle instead.

Figure 8 presents the speed-flow plane parallel to the page. Figure 8a is a view taken from the lower left corner of the cube, looking up at the data. The number 3 appears reversed because it is on the back face of the cube. The direction of the U is backward from the traditional view because the view is from the "rear" of the typical projection. High occupancies lie closer to the reader. The U appears tilted into the page, and the view rotates the data so the arms of the "U" seem closer together. The two arms of the U would be expected to overlap if the front of the cube were raised slightly. The uncongested data trend upward to the left, while the low-speed congested data trend upward to the right. This appears to support the earlier suggestion of a twist in the surface.

Figure 8b is also taken from the "rear" of the typical speed-flow projection. The data are viewed from above, and the view is similar to that of the upper photograph, but the data have been rotated about 60° to the left and about 60° vertically. The front of the cube is defined by the large square in the lower left corner of the cube, which is the same face that was to the front in Figure 8a. The number 3 is obscured by the data but is located in the center of the photo, in reverse orientation. The true origin is at the rear of the cube on the right side.

Both of these views present the high-speed uncongested data in a very dispersed form compared with previous views. The uncongested data are at the top of the cube. The original photographs showed the congested data in the mid-speed ranges to be well stratified by color. Even in Figure 8, without color, the data in the mid-speed range (60 to 70 km/hr) are clearly visible. The view of the uncongested data in Figure 8b is such that the surface appears almost perpendicular to the line of vision. The area occupied by these data seems quite large from this angle.

The low-speed congested data have primarily vertical oscillations in Figure 8b, particularly in the lowest speed range (points at the bottom of the figure). The lines formed by these oscillations are parallel in nature, indicating short dwell times

for these low speeds. However, in this view, the very-low-speed congested data take place over about the same range as the higher-speed congested data; both band widths have approximately the same lateral dimension. While this presents a further consistency between all speed ranges in the congested data, it does not confirm the presence of a surface for low-speed congested data.

Figure 8c is a view taken from the upper right corner of the cube, looking down at the data. The view is from the "front" of the speed-flow projection so that the U is in the normal position. The front of the cube is defined by the large square in the lower left corner of the photograph and has the number 3 in its lower right corner. The uncongested data, showing a pronounced horizontal oscillation over a wide band width, are located at the top of that front face. This is similar to the positioning of the data for the traditional speed-flow plot. However, the congested data are very compact and complex. In this view, all of the congested data lie within a mass having similar horizontal and vertical dimensions. The view is effectively looking at the congested data along the axis of oscillation—at least for the low-speed congested data. Higher speed congested data appear to be grouped within a very narrow range closest to the reader, while lower speed congested data are the farthest away.

The compression of the very-low-speed data in the projections of Figures 8b and 8c illustrates a strong consistency with the higher speed congested data. Figure 8c is an end view of the vertical oscillations for the very-low-speed congested data indicated in Figure 8b. It is not clear from the projections whether the low-speed congested data lie on a surface.

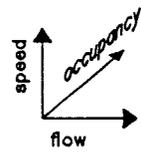
DISCUSSION

Three items are discussed in this section. The first is one possible physical description of the Station 16 data. The second deals with the match between these results and conventional theory, and the third looks at how these results match the model of traffic flow suggested by the cusp catastrophe theory surface.

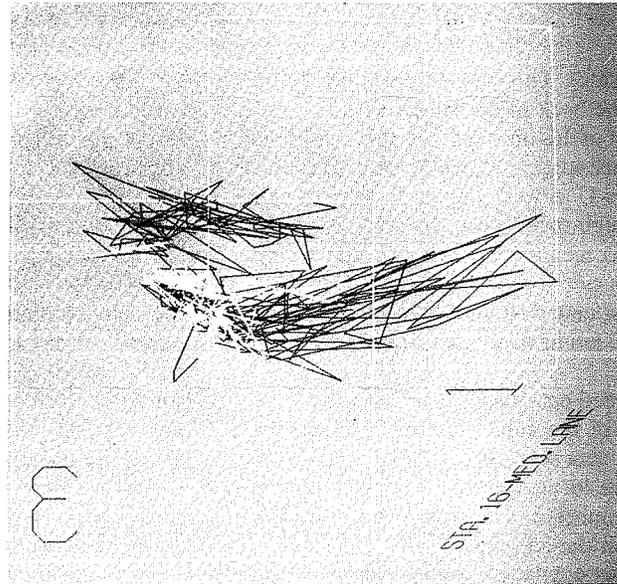
Figure 9 shows one possible schematic presentation of the shape of a surface for the Station 16 data set. The surface is projected onto the sides and bottom of a partial cube (see Figure 6) to show how the traditional views of the data are obtained. In Figure 9, the surface for uncongested data is almost vertical and extends from the "rear" of the cube to the front—like a board standing on edge. At the high-flow end, the surface falls downward and to the right. As speeds and flows decline, the edge of the surface nearest the reader rotates downward for higher flows compared with low flows over a narrow range of occupancy. The broken lines of the figure indicate a possible explanation for the speed-flow projection in which occupancies are outside the plotted range.

Comparison with Traditional Traffic Flow Theory

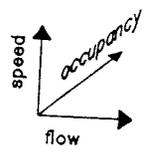
Traditional traffic flow theory has attempted to model these data using line functions and conventional statistical estimation techniques. This analysis implies that the data should not be represented by the standard line functions, with errors



8a:



8b:



8c:

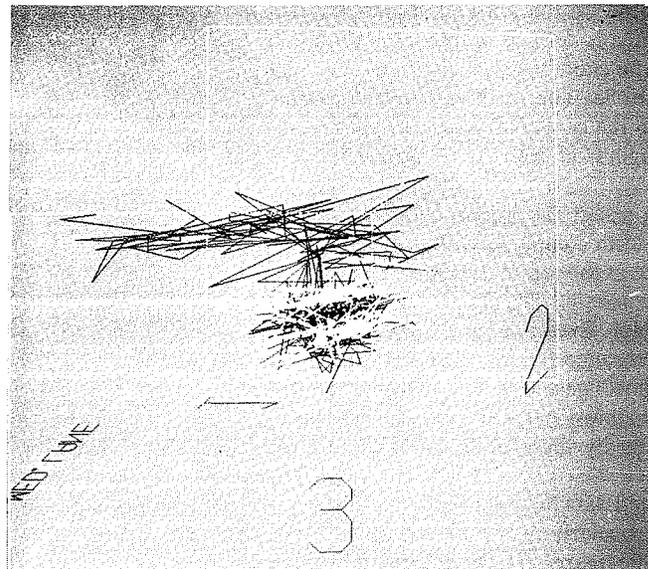


FIGURE 8 Oblique view of Station 16 speed-flow data.

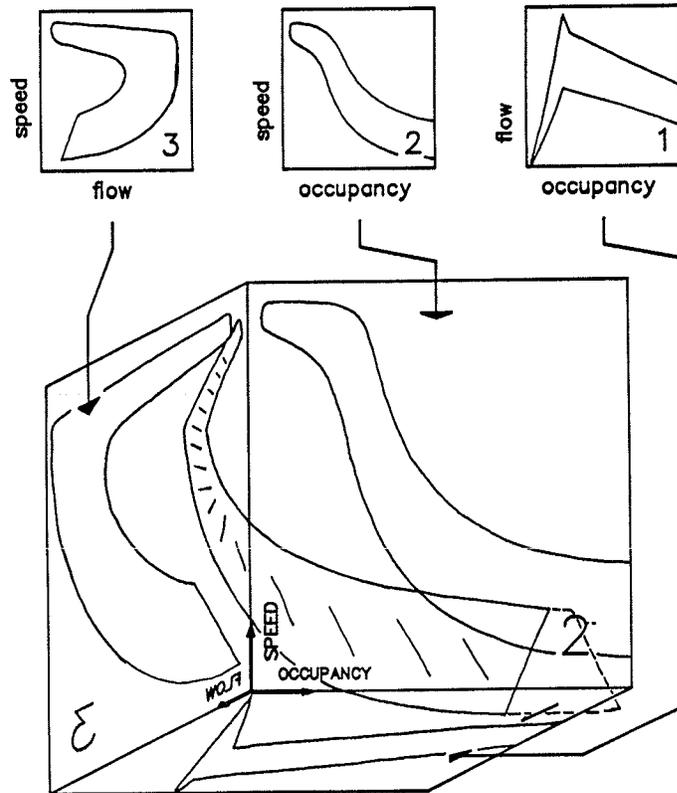


FIGURE 9 Schematic view of surface providing explanation for data scatter.

attributed to random variation in the data or errors in the estimation technique.

Scatter in the data is more correctly related to the three-dimensional model rather than to a single variable. It appears as though the scatter, particularly in the uncongested data, remains on a particular surface and is, therefore, not random with respect to a particular projection made at an angle to that surface. For this reason, statistical estimation using two-dimensional approaches is unlikely to yield consistent results in calibrating the three line functions. More complex three-dimensional estimation techniques may provide better results; however, such an approach would have to distinguish legitimate variation (i.e., on the surface) and error (i.e., variation above or below the surface). Therefore, it would seem nearly impossible to obtain a realistic representation of uninterrupted traffic flow theory using line functions.

Comparison with the Catastrophe Theory Model

This analysis provides more support for the catastrophe theory model of traffic flow than for the conventional model, but there are points of disagreement. There is strong support for three aspects of the model:

1. The clear indication of variation along what looks to be a plane for the uncongested data;
2. The fact that the congested or low-speed operations clearly do not occur on the same plane; and
3. The presence of apparent discontinuities in the form of two jumps from one side of the curve to the other.

However, there are two key points of disagreement as well:

1. The congested data do not clearly lie on a surface; and
2. There is some indication that congested operations, even down to 50 or 60 km/hr may still be occurring on the same plane as the uncongested operations.

These points of disagreement are not enough to reject the catastrophe theory model but do call for careful deliberation. As a result of this rethinking, one peculiarity of the data set used has been recognized. This particular station on the QEW lies between two onramps, both of which create bottlenecks. It is suspected that the congested data arise in two ways and that this affects their appearance. Some of the time, the queue from Highway 10 extends back through Station 16; this would give rise to the low-speed congested data. At other times, the queue from Highway 10 does not reach quite this far back but there is a queue that has formed upstream of Mississauga Road. At those times, Station 16 is recording vehicles that are accelerating from the upstream stop-and-go conditions, which would give rise to a pattern similar to that in the high-speed uncongested data. Persaud and Hurdle (12) describe this phenomenon clearly. Hence, the second point of disagreement with the catastrophe theory model has an explanation, and only the first point calls the model into question.

It seems that the best way to picture the three-dimensional representation is as a twisted U or V, in which both arms are more ribbons than lines. The ribbon for uncongested operations is at an oblique angle to all three conventional representations, but that angle is smallest for the flow-occupancy face of the cube. The ribbon for low-speed congested oper-

ations is clearly at a different angle than the first one, but more analysis needs to be done to determine if that arm is planar or a conical section of some kind and to decide exactly where the high-speed congested data fit.

There is an interesting overlap between these results and another current paper. Hall and Persaud (13) report that the so-called fundamental equation of identity (flow equals the product of speed and density) seems to hold true only for uncongested data. That equation also defines a surface that may coincide with the uncongested portion of the catastrophe theory surface. The fact that the congested data appear in this research to be situated on quite a different surface (or portion of one) is consistent with Hall and Persaud's results.

CONCLUSIONS

In traffic flow work as well as in other disciplines, new measurement techniques and instrumentation have frequently produced observations that could not easily be explained by conventional theory. The data available from current FTMSs would seem to be such a case. These systems measure all three key traffic variables (speed, flow, and concentration in the form of roadway occupancy), whereas earlier systems could often measure only two. Simple three-dimensional graphical treatment of the measured variables shows major weaknesses with conventional traffic flow theory, as depicted in standard diagrams of two-variable relationships.

Three specific problems have arisen. First, the assumptions about the distribution of error underlying normal statistical estimation techniques for two-variable equations clearly are inappropriate. Three variables are involved, and there is an interrelation of the error among them. The scatter shown in a standard two-variable portrayal of the data is perhaps more a function of the angle between that projection and the actual plane of the data than it is of the scatter in the data themselves.

Second, at least for the uncongested data, there does seem to be a plane along which all of the data fall. There is considerable scatter of data over the plane, but there seems to be little or no deviation from its surface. This observation is simply not addressed by conventional theory but may be of considerable benefit in analyzing what happens on freeways.

Third, it is clear from this analysis that the low-speed congested data do not lie on the same plane as the uncongested data. These data may or may not form a plane themselves, but a shift in the underlying relationship definitely occurs with the move into severe congestion.

As a consequence of these three problems, it would seem that the simple bivariate curves that have conventionally been used to represent traffic flow relationships are inadequate for depicting the underlying three-dimensional relationship. Rather than a thin U set at an angle in 3-space to the axes, a twisted, broad-armed U is needed, whose angle, with respect to the axes, changes with a move into congestion.

Although conventional theory cannot be supported with these results, some aspects of the recently proposed catastrophe theory model of traffic flow are confirmed. There remain some mismatches between these data and this theory, but it provides a closer fit than the conventional model.

These results have been obtained from the analysis of only one lane at one station on one roadway. While other data sets could lead to a different interpretation, that is doubtful. The data produced should not make this system unique. If

there is to be a further test of the same kind, however, it is imperative that all three variables be measured. Calculation of the third variable assumes the very thing that is to be tested here, and a paper by Hall and Persaud (13) suggests the calculations are suspect. If all three variables must be measured, then there is a strong reason for using occupancy, rather than density, as the third variable for traffic theory.

On the whole, these results suggest that it is time to rethink some of the fundamentals of traffic flow theory and make better use of the data currently available.

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